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Cost-Based Stackelberg Potential Game for Cognitive Radio Ad Hoc Network*

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In cognitive radio ad hoc networks, the opportunistic access of vacant wireless channel opens a new frontier for efficient spectrum utilization as in many situations, a wide range of spectrum is not even partially utilized by the license owners (primary users, PUs). While the idea seems to be lucrative for spectrum hungry users without licenses, a natural competition between potential stake holders arises, which needs to be regulated in order to efficiently utilize available resources and avoid chaos. With the introduction of unlicensed users in licensed bands, the operations and interests of PUs need to be protected, hence the spectrum owners are given an advantage and control over the multiple access policy (a leader-follower scenario). In this work, we address the problems in spectrum access and channel selection equilibrium in a leader (PU)-follower (secondary user, SU) setup. In contrast to previous game formulations that lack efficient power and pricing schemes, we present a cooperative Stackelberg potential game for cognitive players. A dynamic cost function is articulated to induce awareness in players to mitigate the effects of selfish choices in spectrum access while at the same time steer the distributed network towards achieving Nash equilibrium. The proposed scheme is mutually beneficial for all players and focuses on improving the network performance and power efficiency. We design the network potential function such that the nodes have performance based incentives to cooperate and achieve a Nash equilibrium solution for efficient channel acquisition and capacity. Simulation results show fast convergence in channel selection strategies and increase in capacity for the entire network.

Keywords: Cognitive radio; potential game; Stackelberg game; ad hoc networks; cost based games.

1. Introduction

The cognitive radio system involves advanced software and hardware techniques to opportunistically utilize unused licensed spectrum by carefully following the

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etiquettes and considering the requirements of the primary users (PUs) or the peer secondary users (SUs). Sophisticated spectrum sensing mechanism is required to avoid transmission overlap between PUs and SUs.[1] While the signal processing and physical layer requirements prepare a network for connectivity, the nodes are independent in making decisions in choosing a spectrum that is most appropriate for best performance.[2] In addition to accommodating SUs, it is pivotal to protect the rights of PUs while at the same time increase spectrum efficiency. The main concern arising in implementing cognition in ad hoc networks occurs due to the restrictions imposed by the PU. Moreover, as all SUs attempt to access channels, their actions create conflicts and interference, deteriorating performance. This leads to a depletion of resources including bandwidth and power.

The independent decision making results in a natural competition among users, therefore it is important to device a mechanism to inhibit the channel acquisition policy in order to avoid chaos.[2–10] A game theoretic solution is an attractive choice in regulating a competition where multiple users actively participate and try to optimize their utility by acting selfishly. In the presence of multiple users that are competing for a shared resource, a global mechanism assists in monitoring the affects of the choice of players not only on themselves but the entire network. In addition, it is important for the nodes to attain equilibrium so that there is no qualm to select a different channel.

There are many methods to achieve convergence in different game models. In Ref. 3, the authors present the convergence of a cooperative game for temporal spectrum sharing, but users can enter or leave the network only at the beginning of the game, and are *controlled* to wait for the existing players to establish their strategies if they try to enter at a later time. Although the convergence is achieved in this model, efficient power and channel allocation and the involvement of PU is not completely addressed. In another work,[4] the convergence of two different game models is discussed based on Cournot and Bertrand games[11] for two user CR spectrum sharing. Li and Xie[5] discuss a repeated game for improving transmission rate of CRs using clustering. The convergence is observed for two SUs, which along with the transmit power and pricing affects the transmission rate. The cooperation in cognitive game for an incomplete channel information is discussed in Ref. 6. Their work presents a Baeysian game where SUs are allowed channel access at the expense of forwarding packets for the licensed users. This creates an additional delay and higher power consumption by the SUs for relaying as channel cost. These works do not incorporate a competitive environment of a large number of CRs striving for a single channel and lack a dynamic pricing model.

Another effective formulation to address CR issues is through potential games. Potential game is the class of games in which individual incentives for changing strategy can be expressed as a composite potential function, providing a complete picture of the game for homogeneous players (i.e., players having similar objectives). These games are more adaptive and require minimum information exchange for the management of entities and convergence of strategies (Nash equilibrium).[7] The concepts of potential games to encourage cooperation among CRs is exploited in Ref. 8. This work, however, excludes the role of PU and lacks efficient allocation of power to individual users based on network performance.

The remainder of this paper is organized as follows: Section 2 presents a background of Stackelberg games and its application in CRs. Section 3 describes the system model. Section 4 discusses the problem formulation. Section 5 explains the proposed utility and potential function. Section 6 provides the transmission power for efficient performance, while Secs. 7 and 8 present the numerical simulations and also results, respectively.

2. Stackelberg Game

A game is defined by a set of players, a set of strategies or actions, and a set of payoffs called utilities earned as a result of these actions. A natural and rational attitude of cognitive competition is the adoption of selfish behavior by all players, with every user preserving its individual interests. However, it may not prove beneficial due to limited availability of resources and cause collisions, hence a favorable solution is to behave in a cooperative manner. Players (PUs and SUs) are provided incentives to instigate cooperation in the form of price, relaying or rewards.

In economics, when two firms selling homogeneous goods compete sequentially for a common market, the firm that moves first is sometimes able to capture the market. The second firm can devise its strategy for the quantity it should sell based on the first firm's move. The total quantity of the product these two collectively sell should equal the demand for that product. The firm or player, which moves first is called the leader, and the firm or player that moves afterwards is called follower. This leader-follower game is called a Stackelberg game. Hence, the Stackelberg model comprise a strategic game where players compete as leader and follower. The leader preemptively chooses its action knowing that the followers are observing its actions. This allows the leader to govern the system in its best possible interest. The followers, being rational players, observe the leader's action and act accordingly to meet the market demand. It should be noted that although follower can only act after the leader, its actions may effect the leader. Hence, leader must be careful about its chosen action so as to guide the follower along a Nash equilibrium path.

In ad hoc CR networks, this leader-follower game model can be effectively employed. This game setup ensures the supremacy of PU which is the most important aspect of cognition. The work presented in Ref. 9, discuss a potential game played as Stackelberg model where PU is the leading player making the first move and SUs are the followers. Their approach addresses the channel and power allocation issue, where players opt from a joint set of channels and power levels. The work in Ref. 9, however, considers only the overlay system with four discrete power levels and channels, for a fixed channel cost determined by the leader. The pricing mechanism is not dynamic and fair, which restricts the system and does not effectively optimize transmit power and network performance.

As stated above, the previous works incorporating cooperation lack the efficient distribution of resources, such as transmission power and bandwidth, compromising the network performance. This work differs from previous formulations by incorporating performance based and efficient pricing and power allocations. We present the competition among cognitive users for both overlay and underlay systems as a cooperative Stackelberg potential game. The potential function incorporates spectrum efficiency along-with fair pricing to improve network performance and power consumption. In this work, we explore the convergence or Nash equilibrium of proposed Stackelberg potential game formulated to incorporate cooperation among all players (PUs and SUs) so that a stable solution for improved network performance is achieved.

The methodology adopted in this work employs Stackelberg game formulation for achieving an efficient solution. PU is considered the leader of Stackelberg game, making the first move and causing the follower SUs to obey its rules. The PU promotes cooperation among the followers by instituting a price for each SU according to the level of interference an entering CR creates and the interference already existing on the channel. The payoff of each player is determined by the spectral efficiency achieved and the paid price. The proposed potential function includes performance affecting parameters and maximizing this function ensures improved performance with proficient resource consumption.

3. System Model

We consider a cognitive radio ad hoc network with N available channels, each owned and led by a PU which sets the performance criteria for its respective channel. The total number of cognitive transmitter–receiver pairs (followers) competing for these channels is M. The path loss model is assumed to be proportional to the inverse squared distance between two nodes. The link gain from ith SU transmitter to the jth receiver is given by G_{ij} and vice-versa, while the link gain between the transmitter and receiver of ith CR pair is given by G_{ii}. Similarly, the channel gain between a primary transmitter and cognitive receiver i is given by G_{0i} and vice-versa. Transmission power of PU and SU are given by p_0 and p_i, respectively.

We consider the case when $M > N$, hence CRs must compete for the channel. This competition initiates a game where all players strive to access an available channel. Here, M is the set of players and N is the number of strategies from the strategy set $s = \{s_1, s_2, \ldots, s_M\}$, where $s_i \in [1, N]$. This set contains all possible strategies a player can opt, forming a strategy space.

The Stackelberg game is applied in this work to illustrate the dynamics of competition. Since PUs are the mandatory players, being committed to the spectrum monopoly, they are the natural leader of this game. The followers, in this case, are the new entrants or SUs striving for a possible abode in this game for transmission bandwidth. The action set of leader PU is the decision to transmit over a channel or leave it vacant for SUs. The utility obtained by user i as a result of these actions is given by U_i. The leader, in this case, being the owner of spectrum resources sets a price for channel usage. An intelligent design of a cost function can significantly alleviate the effects of inappropriate channel selection by the sensing nodes. The followers (SUs) are motivated to opt for low interference channels that are also economical.

In this paper, we consider two different types of systems. The first model involves co-existing PUs and SUs (spectrum underlay), while the second approach deals with accessing vacant channels (spectrum overlay), where CRs can only compete for a channel with no licensed transmissions. As the number of SUs is greater than the number of channels, more than one SU transmit over a channel. This causes CRs to create interference for each other. For these two models, we define four interference terms based on the fact that only the players, which opt for the same channel are a source of interference for each other.

$$\begin{aligned}
I_{iv} &= \sum_{j=1, j\neq i, s_i=s_j}^{M} p_j G_{ji}\,, \quad \text{observed by SU } i \text{ in overlay}\,, \\
I_{iu} &= \sum_{j=0, j\neq i, s_i=s_j}^{M} p_j G_{ji}\,, \quad \text{observed by SU } i \text{ in underlay}\,, \\
I'_{iv} &= \sum_{j=1, j\neq i, s_i=s_j}^{M} p_i G_{ij}\,, \quad \text{created by SU } i \text{ in overlay}\,, \\
I'_{iu} &= \sum_{j=0, j\neq i, s_i=s_j}^{M} p_i G_{ij}\,, \quad \text{created by SU } i \text{ in underlay}\,.
\end{aligned}$$

The last two primed terms serve to encourage cooperation, making the users considerate about their behavior towards other network users. Based on the interference terms defined above, we can write the spectral efficiency of a cognitive radio system from Ref. 10 as:

$$\mu_i = \log_2(1 + \beta\gamma_i)\,, \tag{1}$$

where γ_i is the signal-to-interference-plus-noise-ratio (SINR) modified accordingly for underlay or overlay. $\beta = \frac{1.5}{\ln(0.2/B_o)}$, and B_o is the target bit-error-rate (BER) required for successful transmission. This spectral efficiency is a measure of spectrum utilization. Higher spectral efficiency is a desirable feature for licensed and unlicensed users. Hence, the payoff of a player is measured by the spectral efficiency offered by a particular strategy. The spectrum efficiency for overlay case is thus defined as:

$$\mu_{iv} = \log_2\left(1 + \frac{\beta p_i G_{ii}}{I_{iv} + N_o}\right). \qquad (2)$$

Similarly, the spectral efficiency function in underlay case is simplified to:

$$\mu_{iu} = \log_2\left(1 + \frac{\beta p_i G_{ii}}{I_{iu} + N_o}\right). \qquad (3)$$

4. Problem Formulation

We model the overlay and underlay spectrum access problems as cooperative Stackelberg games. If the leader PU is transmitting over a channel or reclaims it in the middle of the game, the corresponding action set of CRs in overlay case is to wait for transmission to end or to opt for an alternate channel among other vacant options. In case of underlay, all CRs may not vacate the channel if they can satisfy the condition imposed by PU.

In order to introduce cooperation in the game, we propose a dynamic cost model. The cost function is responsible for providing spectrum opportunities to SUs by generating revenue for PU. Instead of charging all users with the same price that may not be fair for some players, we model a pricing scheme where the cost of every channel varies for each user according to the performance. The users creating low interference levels are encouraged by offering a discounted price. In addition, the cost function depends on the number of competing cognitive users, more users create more competition and higher interference, increasing the channel cost. The channel cost increases with the transmit power over that channel. The users that transmit with higher power must pay a higher price due to higher level of interference induced. The PU's goal is to sell the channel at a cost that is profitable for it but at the same time should enable it to attract more SUs. The objective of cost function is not just to earn revenue for the PU, but to enable more SUs to be accommodated, so that the spectrum efficiency is improved.

5. Utilities and Potential Function

As stated earlier, a game is completely defined by three quantities: a set of players (which, in this case are the PUs and SUs), a set of actions (which involve the choice of channels and transmission power of users), and the payoffs or utilities $U_i \in U$ for each player i. The first two quantities are defined in the previous section. In this section, we explain the most important quantity affecting the decisions of players and the outcome of the game, called payoff or utility. A player i opts for an action $a_i \in [k, p_i], k = 1, \ldots, N$, which provides the highest payoff under the circumstances created by all the other players involved in the game.

The utility function of players depend on the efficient utilization of bandwidth and the required price to achieve this efficiency. The payoff of the leader PU increases with increase in spectral efficiency and the revenue generated from follower SUs. The payoffs of SUs increase with higher spectral efficiency but decrease with the price paid for channel access. The utility functions for the overlay and underlay cases can be respectively summarized as:

$$U_{iv} = \mu_{iv} - C_{iv}\,,$$
$$U_{iu} = \mu_{iu} - C_{iu}\,,$$

where C_{iv} is the price user i pays to gain channel access in overlay systems, and C_{iu} is the price paid for underlay cognition scenario.

In overlay systems, the cost increases with the threshold SINR level ζ required by the SU for a channel. If the number of available channels is large, the competition to among them is greater, as SUs have more options to choose from and correspondingly the cost is kept low. Based on this discussion, the cost function for overlay model of a channel required from a cognitive user trying to access it is given by:

$$C_{iv} = \frac{p_i M \zeta}{N}\left(\lambda I'_{iv} - (1-\lambda) I_{iv}\right), \tag{4}$$

where λ is the weight assigned to the interference a user creates over a channel, which incorporates the case of imperfect channel estimation for the Bayesian game.[6] The first summation term represents the interference created by a user which acts as a cooperating factor. This cost function depicts that the cost for a CR increases with the interference it creates and decreases with the interference it observes.

In underlay scheme, the SUs are required to keep the interference level within a certain limit so as not to hinder PU's transmissions. PU charges the SUs for a channel based on the interference level created by them. The higher interference level tolerated by a PU on a channel, I_{th}, reduces the cost by encouraging SUs to choose that particular channel. In this case, the CRs are not required to monitor the presence of PU and interference by PU is always accommodated. This encourages high performance CRs and the CRs that create higher interference for the PU are discouraged. However, setting the threshold I_{th} too stringent increases the cost and discourages the users to opt for it. Thus, the cost function for underlay case becomes:

$$C_{iu} = \frac{p_i M \zeta}{N I_{\text{th}}}\left(\lambda I'_{iu} - (1-\lambda) I_{iu}\right). \tag{5}$$

The PU's utility increases with the revenue generated by SU, i.e., price paid for channel access and decrease with the interference created by SUs. The PU on channel j in overlay has the utility, which can be obtained from the revenue obtained from SUs, i.e.,

$$U_{pj} = \sum_{k=1, s_k = s_j}^{M} \left(\mu_{kv} + C_{kv}\right). \tag{6}$$

The leader PU j's utility in underlay cases can be obtained as:

$$U_{pj} = \sum_{k=1, s_k = s_j}^{M} (\mu_{ku} + C_{ku})\,. \tag{7}$$

Potential games have an added advantage of providing the complete behavior of all players in a single well defined global function. This function is useful for catering individual player's needs as well as the overall performance of the network. The proposed potential function incorporates the spectral efficiency measure and cost in its design. Higher spectral efficiency improves the potential function, whereas higher cost deteriorates performance by lowering the value of potential function. Mathematically, if $\mathbf{V}$ is the potential function and U_i is the utility of player i, the potential game can be defined under the following condition[9]:

$$U_i(a_i', a_{-i}) - U_i(a_i'', a_{-i}) > 0 \Longleftrightarrow \mathbf{V}(a_i', a_{-i}) - \mathbf{V}(a_i'', a_{-i}) > 0$$

$\forall\, a_{-i} \in A$, $\forall\, a_i'$, $a_i'' \in A$, where, a_i', $a_i'' \in A$ are the actions taken by player i, and $a_{-i} \in A$ are the actions of opponents of i.

In spectrum underlay case, the overall network performance decreases when the interference level set by PU is low. In spectrum overlay approach, the potential function is simplified as there is no interference limit set by the PU due to its absence. The designed potential function for the overlay case is given as:

$$V_v = \sum_{i=1}^{M} (\mu_{iv} - C_{iv})\,. \tag{8}$$

Similarly, for underlay case, the SUs are required to keep their interference level below the level I_{th}, yet achieve a sufficient SINR level to establish successful transmissions. The potential function formulated for spectrum underlay scheme is given by:

$$V_u = \sum_{i=1}^{M} (\mu_{iu} - C_{iu})\,. \tag{9}$$

6. Power Allocation

The proposed potential function is a convex function which considers power and spectrum efficiency, besides cost and interference. In order to implement power control at the transmitter nodes, we optimize the potential function for transmission power of nodes in an effort to increase overall network performance. This allows efficient power allocation to users, creating a balance between successful transmission and acceptable interference. The potential function given in (8) and (9) can be optimized with respect to power. In order to achieve this, we compute the first derivative of potential function with respect to power, which for overlay system, can be written as:

$$\frac{\partial V_v}{\partial p_i} = \frac{\beta G_{ii}}{\ln(2)(N_o + I_{iv} + \beta p_i G_{ii})} - 2\frac{M\zeta}{N}\lambda p_i G_{iv} + \frac{M\zeta}{N}(1-\lambda)I_{iv}\,.$$

Similarly, for the underlay potential game, we have:

$$\frac{\partial V_u}{\partial p_i} = \frac{\beta G_{ii}}{\ln(2)(N_o + I_{iu} + \beta p_i G_{ii})} - 2\frac{M\zeta}{NI_{\rm th}}\lambda p_i G_{iu} + \frac{M\zeta}{NI_{\rm th}}(1-\lambda)I_{iu}\,,$$

where $G_{iu} = \sum_{j=0}^{M} G_{ij}$ for underlay systems, and $G_{iv} = \sum_{j=1}^{M} G_{ij}$ for overlay systems. Equating the above expression to zero yields the transmission power solution for underlay case as:

$$\begin{aligned} p_i^* &= \frac{1}{4B_i\lambda G_{iu}}\Big[\lambda' B_i I_{iu} - 2\lambda(I_{iu}+N_o)G_{iu} \\ &\quad + \sqrt{4\lambda G_{iu}\{(I_{iu}+N_o)\{\lambda G_{iu}(I_{iu}+N_o) + B_i\lambda' I_{iu}\} + 2B_i^2 NI_{\rm th}/(M\zeta\ln(2))\} + B_i^2\lambda'^2 I_{iu}^2}\Big]\,. \end{aligned}$$

Similarly, for spectrum overlay, the power is obtained as:

$$\begin{aligned} p_i^* &= \frac{1}{4B_i\lambda G_{iv}}\Big[\lambda' B_i I_{iv} - 2\lambda(I_{iv}+N_o)G_{iv} \\ &\quad + \sqrt{4\lambda G_{iv}\{(I_{iv}+N_o)\{\lambda G_{iv}(I_{iv}+N_o) + B_i\lambda' I_{iv}\} + 2B_i^2 N/(M\zeta\ln(2))\} + B_i^2\lambda'^2 I_{iv}^2}\Big]\,, \end{aligned}$$

where $B_i = \beta G_{ii}$, and $\lambda' = 1 - \lambda$. The above expression provides the transmit power level required by the cognitive users to ensure better network performance.

7. Numerical Results and Simulations

The simulation setup shown here considers three identical channels ($N = 3$) and five cognitive users ($M = 5$), although the game is completely implementable for higher number of SUs and channels. These CR pairs are uniformly distributed in an area of $200\,\mathrm{m}^2$. The noise variance N_o is assumed to be 10^{-5} and threshold SINR ζ is taken as 20 dB and the parameter $I_{\rm th}$ is assumed as 10 dB for the purpose of these simulations. The probabilistic parameter λ is considered to be 0.5. Initially, the PU is assumed absent and all three channels are vacant. The game is played among SUs only, which initially choose a channel randomly at the beginning of the game and then decide their actions according to the proposed potential game. These strategies are chosen in favor of the channels offering maximum utility to the players. Since the game is sequential in nature, these choices are made by players moving one at a time, while the other players maintain their previous strategies. The game is played iteratively and players have a chance to review their actions. By repeatedly playing in the proposed fashion, all players reach a point where they no longer desire to change their strategies, as no further gain in payoff or network performance can be achieved and an equilibrium is attained. As shown in Fig. 1, when the game reaches iteration 30,

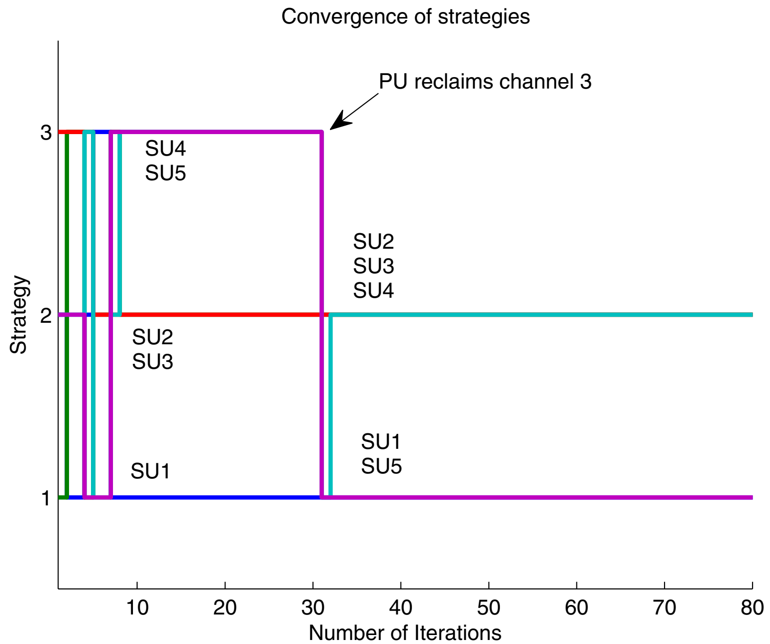

Fig. 1. Plot of convergence of strategies for spectrum overlay system (note the absence of SU on channel 3 due to the presence of PU) (color online).

the PU appears at channel 3. If the game is played in overlay mode, the SUs transmitting at channel 3 must vacate it and switch to other available channels. This creates additional interference to the users already transmitting over channels 1 and 2, and some of them might switch their options too. The Nash equilibrium for this case is depicted in Fig. 1. This figure shows that whenever a PU appears (in this case, after 30 iterations over channel 3), the cognitive users must re-adjust their strategies and the choice of reclaimed channel 3 becomes nonexistent. The game is now played for the remaining 2 channels instead of 3.

In case of underlay mode, however, all players do not completely switch to other channels, some may continue using that channel while others can make a switch so as to retain the tolerable interference level for PU. This scenario is shown in Fig. 2, where the channel reclaimed by the PU is not ignored by the SUs. Instead, in this case, SUs modify their strategies according to the level of interference suffered by SUs and tolerance level of the PU. The convergence in underlay cases may take longer time to establish as compared to overlay. However, the convergence time for both the cases is much better than the overlay case discussed in Ref. 9. The interference and power levels achieved at stability are also improved. The underlay scheme also provides more spectral opportunities as the PU's channel can still be used and is not completely excluded from the possible set of actions. Another interesting observation

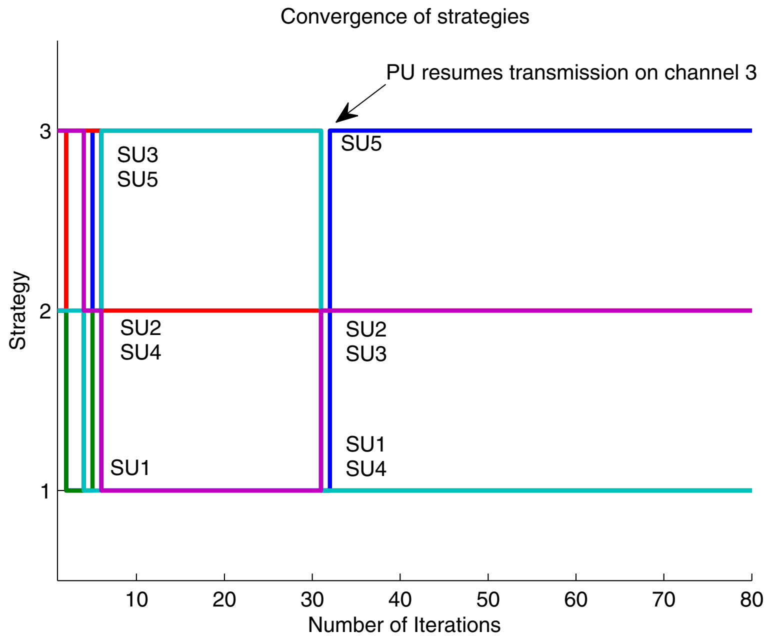

Fig. 2. Plot of convergence of strategies for spectrum underlay system (color online).

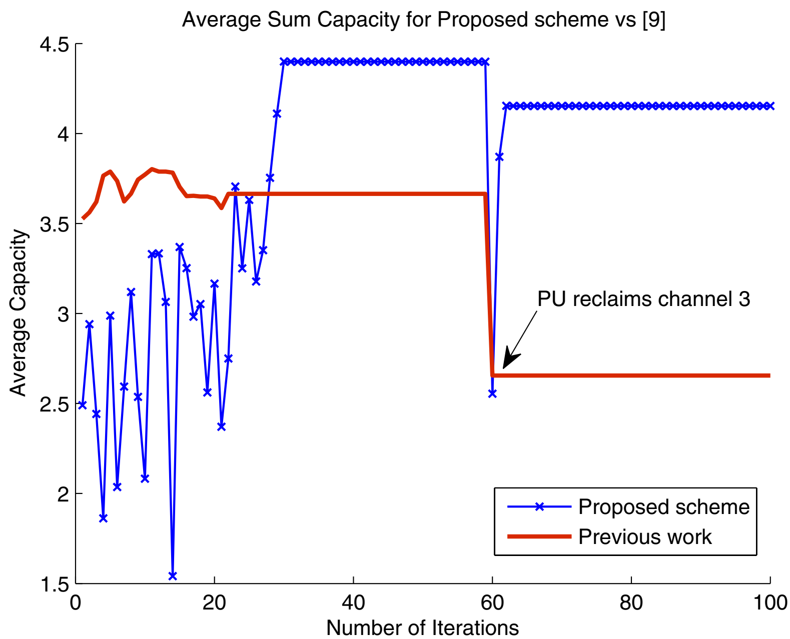

Fig. 3. Comparison of average sum capacity of proposed work with Ref. 9 for 25 CR pairs and 3 channels (color online).

is that in Ref. 9, the users keep changing their power levels even after the convergence of channel selection, which means that power levels converge much later than the channel choices. No reason has been provided for this kind of behavior. In the proposed model, power levels converge as soon as channel acquisition achieves Nash equilibrium. The results are valid for any number of users and channels. A comparison of average sum capacity of the proposed work with Ref. 9 is shown in Fig. 3. This comparison reveals that the proposed scheme performs better. The performance is improved for a more congested network in case of proposed method compared to the previous methods.

8. Conclusion

In this paper, we present the convergence of transmission power and channel selection strategies in cognitive radio ad hoc networks. We formulate the problem as a potential game based on power and cost. The problem is approached as a Stackelberg game, where the PU is the leading authority to set transmission parameters for the follower cognitive players. The CR nodes choose their strategies for the maximum value of proposed potential function. These strategies are opted for the improved potential function to provide better choices for the CR players, the PU, as well as the entire network. The convergence and average capacity for proposed scheme is improved compared to existing methods.[9] The underlay and overlay scenarios are separately discussed with underlay providing more spectrum opportunities. The action set comprising of transmission power and channel opted in these games provide better network performance with efficient resource consumption.

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