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Joint admission control and beamforming design for the interference cognitive radio network with partial channel state information case

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Abstract: In order to effectively utilise the spectrum of wireless network, secondary users (SUs) are often authorised to share spectrum with primary users (PUs) in cognitive networks. To guarantee effective transmission of PUs and SUs, the interference from the SUs to the PUs is limited and only the SUs that have the information rates (rates) than the corresponding thresholds are allowed to transmit their data. In order to select proper SUs to transmit and to maximise their achievable weighted sum rate (WSR) under rate requirement and interference constraint, the authors propose a joint admission control and beamforming design algorithm for partial channel state information (CSI) case. The WSR maximisation problem is first constructed by using approximation and convex relaxation, according to the statistical property of the CSI. Then a penalty factor is introduced to check the feasibility of such problem and instruct the admission of the SUs. Beyond the feasible SUs set, the WSR is maximised by updating the achievable rate of each user with rate profile technique and polyblock outer approximation method. Simulation results show that the proposed beamforming can achieve higher WSR than the beamforming of the existing baseline method.

1 Introduction

Cognitive radio (CR), which allows unlicensed secondary users (SUs) to transmit with primary users (PUs) either opportunistically or concurrently, is a promising approach to alleviate spectrum scarcity [1].

For opportunistic transmission, SUs may transmit when they detect a spectrum hole [2]. Such schemes usually work when the spectrum is severely underutilised, otherwise SUs might not have sufficient opportunities to gain channel access. On the other hand, SUs may access the primary spectrum concurrently with PU under the permissions of the PUs and under the condition that the SUs do not cause harmful interference to the PUs [3, 4]. In order to access more users and get more efficient utilisation of the spectrum, the scenario that the PUs share spectrum with SUs has become more and more appealing and the spectrum-share transmission has been extensively studied [5–7].

To guarantee the transmission of all PUs and SUs, the interference from the SUs is limited such that the PUs can receive their data effectively. On the other hand, the transmission of the SUs should satisfy their quality of service (QoS). For example, the SUs should meet their minimum required information rate (rate). If a SU achieves a low rate and induces high interference, then the transmission of the SU should be prohibited. Thus, rate and interference thresholds are given for the SUs to guarantee the effective transmission of the network. To maximise the weighted sum rate (WSR) of the SUs over such rate requirement and interference constraint (RI-WSR), beamformings of SUs are designed to suppress such interference. On the other hand, if the system cannot meet the rate requirement and the interference constraint, call admission control should be implemented to select proper SUs.

Most of the existing beamforming design methods for CR only consider the achievable sum rate or the mean-square error (MSE), they neglect the minimum rate requirement of the SUs [3, 5–9]. Furthermore, if there are several SUs transmitting concurrently with PUs, the QoS of SUs or the interference constraints may never be all satisfied because of their various rate demands and different channel state informations (CSIs). Thus, existing work that considers the CR network may be infeasible. To maximise the WSR, the authors in [5] propose a beamforming approach to maximise the achievable sum rate for the SUs based on game theory. The game-theory-based beamforming method finds the Nash equilibrium by considering the inter-user interference (IUI), but does not usually achieve the Pareto optimality. In [6], the dual decomposition method is introduced to solve the WSR problem in cognitive network. Approximate method is used to construct the optimisation problem. The IUI is replaced by a constant, such that the optimisation problem can be easily solved and an approximate-optimal solution is obtained. However, when there are several SUs in the network, we even do not know which user is active, it is difficult to estimate the approximate interference of users, especially when the CSI is imperfect. Existing beamforming design techniques in [10, 11] are designed for downlink users, only local optimal solutions are obtained when maximising the WSR of the cell and the interference from the transmitters to the receivers cannot be controlled. Thus, new designs should be made for effective communications for CR networks.

In this paper, we consider admission control and beamforming design for spectrum sharing cognitive wireless network. A beamforming algorithm that to maximise the WSR of SUs under rate requirement and interference constraint is proposed. To make this problem feasible, an admission control method is proposed to select proper SUs to transmit in the network. The admission control method is based on a penalty factor that introduced in the rate requirement constraint. The penalty factor, which belongs to a QoS margin variable, is used to indicate the admission of the SUs. If the rate requirement constraint can be satisfied, the QoS margin variable will be zero, otherwise, the QoS margin variable will greater than zero. Thus, if the problem is feasible, the users can transmit their data, otherwise, the user, with the maximum QoS margin variable will be prohibited to transmit in the network. Then a polyblock outer approximation method is employed to design the beamforming algorithm to maximise the WSR of the SUs. The polyblock outer approximation method has been proved of guaranteed convergence and global optimality [12]. The main contributions of this paper are concluded as follows:

- Owing to complex wireless propagation environments, such as multipath fading, shadowing and path losses of wireless channels, the practical CSI is often imperfect. The probability constraints of the rate and overall interference are given according to the practical application, and the corresponding closed forms are deduced by using the statistical property of the channel states.
- In order to control the admission under the rate requirement and the interference constraint, a feasible check problem is constructed to make the WSR problem feasible. To effectively perform the admission control process, a penalty factor is introduced to judge the active and inactive users. Furthermore, to conveniently process the problem, successive convex approximation (SCA) method and semi-definite program relaxation (SDR) method are adopted for the problem formulations and to deduce the admission control of the SUs.
- The polyblock outer approximation method is explored to update the achievable SINR under the rate requirement and the interference constraint. Difference to the former work that adopts the polyblock outer approximation method [12–14], the polyblock outer approximation method is jointly used with the SCA method and the SDR method in this paper. Owing to the non-convexity of the WSR problem under partial CSI, there is some rate loss caused by the SCA method and the rank reduction procedure. We prove that the polyblock outer approximation method is still convergence and the simulation results demonstrate its validity compared with the existing baseline method.

By jointly using the polyblock outer approximation method and rate profile technique, the RI-WSR problem are transposed into solvable convex programs for partial CSI case. Simulation results show that the joint admission control and beamforming design algorithm can achieve higher WSR compared with the existing baseline method.

The rest of the paper is organised as follows. Section 2 describes the system model. The proposed algorithm for admission control and WSR maximisation with partial CSI is introduced in Section 3. The performance of the proposed algorithm is compared with existing method by simulation in Sections 4 and 5 gives the conclusion of this paper.

Notations: Vector is denoted by bold-face lower-case letter and bold-face upper-case letter is for matrix. $|\cdot|_2$ and $\mathrm{tr}(\cdot)$ represent the norm and the trace operations, respectively. $(\cdot)^{\mathrm{H}}$ and $\mathcal{D}(\cdot)$ denote the Hermitian transpose and diagonal matrix, respectively.

2 System model

There are K pairs SUs and one pair PU transmitting in the network as shown in Fig. 1. In order to manager the transmission of the SUs and reduce the feedback of the network, a secondary monitor centre (SMC) is introduced in the CR network. The kth transmitter is equipped with M_k antennas, each receiver is equipped with one antenna and receives its data from the corresponding transmitter. The signal received by the kth secondary receiver can be expressed as

$$y_k = \boldsymbol{h}_{kk}^{\mathrm{H}}\boldsymbol{v}_k x_k + \sum_{l=1,l\neq k}^{K} \boldsymbol{h}_{lk}^{\mathrm{H}}\boldsymbol{v}_l x_l + \boldsymbol{h}_{pk}^{\mathrm{H}}\boldsymbol{v}_p x_p + n_k \tag{1}$$

where $\boldsymbol{h}_{lk} \in \mathbb{C}^{M_l\times 1}$ is the channel vector from the lth transmitter to the kth receiver; $\boldsymbol{v}_l \in \mathbb{C}^{M_l\times 1}$ is the beamforming vector of the lth transmitter; x_l is the signal transmitted by the user l with distribution $\mathcal{N}(0, 1)$; n_k is a zero-mean additive white Gaussian noise (AWGN) vector of the kth pair SUs with distribution $\mathcal{N}(0, \delta_k^2)$. The achievable rate of the kth user can be expressed as

$$r_k = \log\left(1 + \frac{|\boldsymbol{h}_{kk}^{\mathrm{H}}\boldsymbol{v}_k|^2}{\sum_{l=1,l\neq k}^{K} |\boldsymbol{h}_{kl}^{\mathrm{H}}\boldsymbol{v}_l|^2 + \sigma_k^2}\right)$$

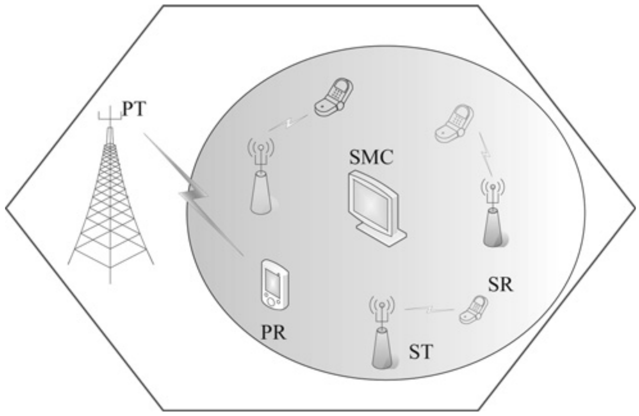

Fig. 1 *Illustration of the concurrently transmission of PU and SUs*

Receiver of each transmission link in the network topology is interfered by other transmitters

Plotted lines represent the desired transmission links

SMC is introduced in the CR network to reduce the feedback of the network and calculate the beamforming vectors of the SUs

where $\sigma_k^2 = |\boldsymbol{h}_{kp}^{\mathrm{H}}\boldsymbol{v}_p|^2 + \delta_k^2$ denotes the equivalent noise of the kth SU. $\mathrm{SINR}_k = |\boldsymbol{h}_{kk}^{\mathrm{H}}\boldsymbol{v}_k|^2/\left(\sum_{l=1,l\neq k}^{K} |\boldsymbol{h}_{kl}^{\mathrm{H}}\boldsymbol{v}_l|^2 + \sigma_k^2\right)$ represents the signal-to-interference-and-noise ratio (SINR) of the user k.

For partial CSI, we consider the case that only the statistical information of the channels is available at the receivers. It is assumed that $\boldsymbol{h}_{lk} \sim \mathcal{N}(0, \boldsymbol{C}_{lk})$, where $\boldsymbol{C}_{lk} \succeq 0$ denotes the channel covariance matrix and is known to the secondary transmitters. The random variables $\boldsymbol{h}_{kk}^{\mathrm{H}}\boldsymbol{v}_k$ and $\boldsymbol{h}_{lk}^{\mathrm{H}}\boldsymbol{v}_l$ are independent exponentially distributed with parameters $1/\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k$ and $1/\boldsymbol{v}_l^{\mathrm{H}}\boldsymbol{C}_{lk}\boldsymbol{v}_l$.

The interference from the SUs to the PU should not too much such that the PU can effectively receive its own data. Thus, there will be a transmission outage when the interference from the SU to the PU is higher than a given threshold. As the CSI uncertainty, there is no solution to constrain the interference of all users universally. To mitigate the interference of the SUs, we maintain the outage probability that because of the interference below a threshold. Assume the allowed interference from the kth SU to PU is b_k, then, the constraint can be written as

$$\Pr(|\boldsymbol{h}_{kp}^{\mathrm{H}}\boldsymbol{v}_k|^2 \geq b_k) \leq \varepsilon, \quad \forall k \tag{2}$$

where ε is the interference allowed outage probability of the kth SU; $\varepsilon = 0$ indicates that the interference power greater than b_k is not allowed and $\varepsilon = 1$ indicates that the constraint completely allows the power of interference greater than b_k. The inequality (2) is in fact a cumulative distribution function (CDF) of an exponential random variable with parameter $1/\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kp}\boldsymbol{v}_k$. Inequality (2) can be expressed as

$$\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kp}\boldsymbol{v}_k \leq b_k/\ln(1/\varepsilon) \tag{3}$$

Inequality (3) shows that the interference covariance is proportional to the interference threshold b_k. For a given ε, the larger the b_k, the more interference power is allowed to PU. As there are more than one users in the network, in order to constrain the total interference, according to (3), we relax b_k as a number smaller than a given threshold ρ and constrain the total interference $\sum_{k=1}^{K} b_k \leq \rho$. Thus, the interference constraints for all SUs can be written as

$$\Pr(|\boldsymbol{h}_{kp}^{\mathrm{H}}\boldsymbol{v}_k|^2 \geq b_k) \leq \varepsilon, \quad \forall k \tag{4}$$

$$\sum_{k=1}^{K} b_k \leq \rho \tag{5}$$

where ρ is the equivalent interference threshold for all SUs. From the PU point of view, as long as the whole interference is not larger than the given threshold, the PU can effectively decode its own data and transmit normally. The PU only cares the whole interference that it receives, the individual interference of the SU is not very important. As a result, for practical application, only the whole interference constraint is considered for the PU in this problem.

On the other hand, the beamforming should be designed to guarantee the QoS of the SUs. Thus, a rate threshold is set to guarantee the rate of SUs. The constraint can be expressed as

$$\Pr\left(r_k \le r_k^{\min}\right) \le \epsilon_k$$

where $r_k^{\min}$ is the rate threshold of the kth SU and ε_k is the QoS outage probability.

To maximise the WSR of the SUs under the rate requirement and the interference constraint, the optimisation problem is constructed as

$$\text{P1} \quad \max_{\boldsymbol{v}_k} \sum_{k=1}^{K} w_k r_k \tag{6a}$$

$$\text{s.t.} \quad \Pr\left(r_k \le r_k^{\min}\right) \le \epsilon_k, \quad \forall k \tag{6b}$$

$$\Pr(|\boldsymbol{h}_{kp}^{\mathrm{H}}\boldsymbol{v}_k|^2 \ge b_k) \le \varepsilon, \quad \forall k \tag{6c}$$

$$\sum_{k=1}^{K} b_k \le \rho \tag{6d}$$

$$\|\boldsymbol{v}_k\|_2^2 \le p_k, \quad \forall k \tag{6e}$$

where w_k is the rate weight of the kth user and p_k is the maximum power of the transmitter k.

3 Proposed algorithm for admission control and weighted sum-rate maximisation

In the following subsections, the feasibility of the WSR problem with partial CSI is first checked, then the proposed beamforming design algorithm is given to maximise the WSR of the network.

3.1 Feasibility and admission control

We first find the closed form of the probability function (6b). Then we construct the admission control problem and introduce QoS margin variables to check if it is feasible.

According to [15, 16], (6b) can be equivalently written as the following closed form

$$1 - \exp\left(\frac{-\gamma_k\sigma_k^2}{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k}\right)\prod_{l\ne k}\left[\frac{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k}{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k + \gamma_k\boldsymbol{v}_l^{\mathrm{H}}\boldsymbol{C}_{lk}\boldsymbol{v}_l}\right] \le \epsilon_k \tag{7}$$

where $\gamma_k = 2^{r_k^{\min}} - 1$. A QoS margin variable λ_k is introduced to control the admission of the kth SUs, then (6b) becomes

$$1 - \exp\left(\frac{-\gamma_k\sigma_k^2}{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k}\right)\prod_{l\ne k}\left[\frac{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k}{\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{C}_{kk}\boldsymbol{v}_k + \gamma_k\boldsymbol{v}_l^{\mathrm{H}}\boldsymbol{C}_{lk}\boldsymbol{v}_l}\right] \le \epsilon_k + \lambda_k \tag{8}$$

Let $\boldsymbol{V}_k = \boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}$, by using SDR, (8) can be written as

$$(1 - \epsilon_k - \lambda_k)\exp\left(\frac{\gamma_k\sigma_k^2}{\mathrm{tr}(\boldsymbol{C}_{kk}\boldsymbol{V}_k)}\right)\prod_{l\ne k}\left[1 + \gamma_k\frac{\mathrm{tr}(\boldsymbol{C}_{lk}\boldsymbol{V}_l)}{\mathrm{tr}(\boldsymbol{C}_{kk}\boldsymbol{V}_k)}\right] \le 1 \tag{9}$$

To make (9) tractable, let $e^{x_{lk}} \triangleq \mathrm{tr}(\boldsymbol{C}_{lk}\boldsymbol{V}_k)$ and $e^{\mu_k} \triangleq 1 - \epsilon_k - \lambda_k$. Then, the feasibility problem of (6) can be formulated as

$$\min \sum_{k=1}^{K} b_k + T\sum_{k=1}^{K}\lambda_k^2 \tag{10a}$$

$$\text{s.t.} \quad e^{\mu_k + \gamma_k\sigma_n^2 e^{-x_{kk}}}\prod_{l\ne k}\left[1 + \gamma_k e^{x_{lk} - x_{kk}}\right] \le 1, \quad \forall l, k \tag{10b}$$

$$1 - \epsilon_k - \lambda_k \le e^{\mu_k}, \quad \forall k \tag{10c}$$

$$\mathrm{tr}(\boldsymbol{C}_{lk}\boldsymbol{V}_l) \le e^{x_{lk}}, \quad \forall l, k \tag{10d}$$

$$\mathrm{tr}(\boldsymbol{C}_{kk}\boldsymbol{V}_k) \ge e^{x_{kk}}, \quad \forall k \tag{10e}$$

$$\mathrm{tr}(\boldsymbol{C}_{kp}\boldsymbol{V}_k) \le b_k/\ln(1/\varepsilon), \quad \forall k \tag{10f}$$

$$\mathrm{tr}(\boldsymbol{V}_k) \le p_k, \quad \forall k \tag{10g}$$

where T is a very large number, such as 10^7. By introducing the penalty factor λ_k, problem (10) is feasible always. Thus, we can judge the feasibility of problem (6) by the object of (10) and the value of the penalty factor λ_k. If problem (6) is feasible, λ_k will be zero, and the object of (10) will not greater than ρ. If $\lambda_k > 0$ or the object of (10) is great than ρ, problem (6) is infeasible, then admission control technique is adopted according to λ_k. Problem (10) is non-trivial because of its non-convexity. To solve this problem by convex optimisation methods, the first-order Taylor series approximation is used to construct the problem.

Constraints (10c) and (10d) are non-convex. The first-order lower bounds of e^{μ_k} at $\mu_k^{(\phi)}$ is

$$e^{\mu_k^{(\phi)}}(\mu_k - \mu_k^{(\phi)} + 1) \tag{11}$$

Similarly, the first-order lower bounds of $e^{x_{lk}}$ at $x_{lk}^{(\phi)}$ is

$$e^{x_{lk}^{(\phi)}}(x_{lk} - x_{lk}^{(\phi)} + 1) \tag{12}$$

By applying the approximate expressions (11) and (12), we obtain the following convex problem

$$\begin{aligned}
&\min \sum_{k=1}^{K} b_k + T\sum_{k=1}^{K}\lambda_k^2 \\
&\text{s.t.} \quad e^{\mu_k + \gamma_k\sigma_n^2 e^{-x_{kk}}}\prod_{l\ne k}\left[1 + \gamma_k e^{x_{lk} - x_{kk}}\right] \le 1, \quad \forall l, k \\
&1 - \varepsilon_k - \lambda_k \le e^{\mu_k^{(\phi)}}(\mu_k - \mu_k^{(\phi)} + 1), \quad \forall k \\
&\mathrm{tr}(\boldsymbol{C}_{lk}\boldsymbol{V}_l) \le e^{x_{lk}^{(\phi)}}(x_{lk} - x_{lk}^{(\phi)} + 1), \quad \forall l, k \\
&\mathrm{tr}(\boldsymbol{C}_{kk}\boldsymbol{V}_k) \ge e^{x_{kk}}, \quad \forall k \\
&\mathrm{tr}(\boldsymbol{C}_{kp}\boldsymbol{V}_k) \le b_k/\ln(1/\varepsilon), \quad \forall k \\
&\mathrm{tr}(\boldsymbol{V}_k) \le p_k, \quad \forall k
\end{aligned} \tag{13}$$

This convex problem can be easily solved by convex optimisation tools, such as CVX [17] or Sedumi [18]. SCA method is used to obtain the solution $\boldsymbol{V}_k$ and λ_k of this problem. After having the optimal solutions x_{lk}^* and μ_k^* of the ϕth step iteration by solving (13), take them as the initial approximate points of the $(\phi+1)$th step, that is, let $x_{lk}^{(\phi+1)} = x_{lk}^*$ and $\mu_k^{(\phi+1)} = \mu_k^*$, $\forall l, k$. Continue this process until convergence. Then the approximate object of (13), which is denoted as I, can be obtained by the iterative SCA method. According to [19, 20], I is an upper bound of (10). Thus, if the obtained $\lambda_k = 0$ and $I \le \rho$, the problem is feasible; otherwise, the admission control approach is used according to the solution obtained by the SCA method. The proposed admission control algorithm is based on the obtained λ_k, we conclude the admission control approach as Algorithm 1 (see Fig. 2).

Algorithm 1

Initialise the secondary user set $\mathcal{K}'$.

Loop

  Initialise $\mathbf{V}_k^{(0)}$ and $\lambda_k^{(0)}, \forall k \in \mathcal{K}'$. Let $\phi := 0$.

  Loop

    Let $\phi := \phi + 1$;

    Solving problem (13) to obtain $\mathbf{V}_k^*$, λ_k^*, x_{lk}^* and $\mu_k^*, \forall l, k \in \mathcal{K}'$;

    Let $x_{lk}^{(\phi+1)} := x_{lk}^*$ and $\mu_k^{(\phi+1)} := \mu_k^*, \forall l, k \in \mathcal{K}'$;

  until convergence.

  If $\sum_{k=1}^{K} \lambda_k > 0$ or $I > \rho$, let $k := \text{argmax}(\lambda_k)$. Update the feasible set $\mathcal{K}' := \mathcal{K}' \backslash \{k\}$.

until $\sum_{k=1}^{K} \lambda_k = 0$ and $I \leq \rho$.

Output: Active secondary user set $\mathcal{K} := \mathcal{K}'$.

Fig. 2 *Penalty-based admission control algorithm*

3.2 Updating the achievable rate r_k

By using the admission control approach, we obtain the active user set $\mathcal{K}$ to make this problem feasible. The minimum WSR of the network is $\sum_{k=1}^{K} w_k r_k^{\min}$. To maximise the WSR of the network, we update the rate of each user to increase the WSR.

Let $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K]$ (where $\alpha_k \geq 0$ and α_k, $\forall k$ satisfies $\sum_{k=1}^{K} \alpha_k = 1$) denotes the target ratio between the users' achievable rate and the users' sum-rate [21]. For a given $\boldsymbol{\alpha}$, the maximum rate R_{sum}^* can be obtained by solving the following problem

$$\max_{R_{\text{sum}}, \boldsymbol{v}_k} R_{\text{sum}} \tag{14a}$$

$$\text{s.t.} \quad \Pr\big(r_k \leq \boldsymbol{s}_k(R_{\text{sum}})\big) \leq \epsilon_k, \quad \forall k \tag{14b}$$

$$\Pr(|\boldsymbol{h}_{kp}^{\text{H}} \boldsymbol{v}_k|^2 \geq b_k) \leq \varepsilon, \quad \forall k \tag{14c}$$

$$\sum_{k=1}^{K} b_k \leq \rho \tag{14d}$$

$$\|\boldsymbol{v}_k\|_2^2 \leq p_k, \quad \forall k \tag{14e}$$

where $\boldsymbol{s}(R_{\text{sum}}) = \boldsymbol{r}^{\min} + \boldsymbol{\alpha}(R_{\text{sum}} - \sum_{k=1}^{K} r_k^{\min})$ represents the achievable rate vector of SUs and $\boldsymbol{r}^{\min} = [r_1^{\min}, \ldots, r_K^{\min}]$. For a given set of rate $\boldsymbol{s}_k(R'_{\text{sum}}) > r_k^{\min}$, $\forall k$, we need to check if the problem is feasible. If the problem is feasible, the rate is then updated to increase the WSR. For the given $\boldsymbol{\alpha}$, the maximum achievable WSR R_{sum} can be obtained by the following bisection method which is outlined as Algorithm 2 (see Fig. 3).

Algorithm 2

Input: $\tilde{R}_{\text{sum}}^{\min}$, $\tilde{R}_{\text{sum}}^{\max}$, α and a small value $\iota > 0$. $\tilde{R}_{\text{sum}}^{\max}$ is an upper bound. $\tilde{R}_{\text{sum}}^{\min} = \sum_{k=1}^{K} w_k r_k^{\min}, k \in \mathcal{K}$, where $\mathcal{K}$ is a feasible user set. Let $\phi = 0$.

Loop

  $R_{\text{sum}} := \frac{1}{2}(\tilde{R}_{\text{sum}}^{\min} + \tilde{R}_{\text{sum}}^{\max})$;

  Loop

    Let $\phi := \phi + 1$;

    Solve problem(13) to update I_p^*, $\mathbf{V}_k$, x_{lk}^* and $\mu_k^*, \forall l, k \in \mathcal{K}$;

    Let $x_{lk}^{(\phi+1)} := x_{lk}^*$ and $\mu_k^{(\phi+1)} := \mu_k^*, \forall l, k \in \mathcal{K}$;

  until convergence or $I_p^* \leq \rho$ and $\sum_{k=1}^{K} \lambda_k = 0$;

  If $I_p^* \leq \rho$, setting $\tilde{R}_{\text{sum}}^{\min} := R_{\text{sum}}$; else, setting $\tilde{R}_{\text{sum}}^{\max} := R_{\text{sum}}$;

until $\tilde{R}_{\text{sum}}^{\max} - \tilde{R}_{\text{sum}}^{\min} \leq \iota$;

Output: The approximate rate R_{sum} and the corresponding $\mathbf{V}_k$.

Fig. 3 *Bisection method for finding the maximum achievable WSR R_{sum} with a given rate-profile vector α*

3.3 Updating rate profile by polyblock outer approximation approach

In Section 3.2, the approximate WSR $R_{\rm sum}$ with the given rate-profile vector α is obtained. In this subsection, we employ the polyblock outer approximation method to update the rate-profile vector $\boldsymbol{\alpha}$ to further increase the WSR. We give a brief introduction of the polyblock outer approximation method [12–14, 22].

Let $\boldsymbol{v} = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_K]$, $f_k(\boldsymbol{v}) = |\boldsymbol{h}_{kk}^{\rm H}\boldsymbol{v}_k|^2$, $g_k(\boldsymbol{v}) = \sum_{l=1,l\neq k}^{K} |\boldsymbol{h}_{kl}^{\rm H}\boldsymbol{v}_l|^2 + \sigma_k^2$ and $\text{SINR}_k(\boldsymbol{v}) = f_k(\boldsymbol{v})/g_k(\boldsymbol{v})$. Introduce a new variable $\boldsymbol{z} = [z_1, \ldots, z_K]$. Let $\mathcal{G} = \{\boldsymbol{z} | 0 \le z_k \le 1 + \text{SINR}_k(\boldsymbol{v}), \quad \boldsymbol{v} \in \mathcal{X}\}$, where

$$\mathcal{X} = \left\{ \begin{array}{l} \boldsymbol{v} | \exp\left(\dfrac{-\gamma_k\sigma_k^2}{\boldsymbol{v}_k^{\rm H}\boldsymbol{C}_{kk}\boldsymbol{v}_k}\right) \\ \prod\limits_{l\neq k}\left[\dfrac{\boldsymbol{v}_k^{\rm H}\boldsymbol{C}_{kk}\boldsymbol{v}_k}{\boldsymbol{v}_k^{\rm H}\boldsymbol{C}_{kk}\boldsymbol{v}_k + \gamma_k\boldsymbol{v}_l^{\rm H}\boldsymbol{C}_{lk}\boldsymbol{v}_l}\right] \ge \epsilon_k + 1 \\ \boldsymbol{v}_k^{\rm H}\boldsymbol{C}_{kp}\boldsymbol{v}_k \le b_k/\ln(1/\varepsilon), \\ \sum\limits_{k=1}^{K} b_k \le \rho,\ \|\boldsymbol{v}_k\|_2^2 \le p_k, \quad \forall k \end{array} \right\}$$

$\mathcal{G}$ can be represented as a compact normal set (A set $\mathcal{S}$ is called normal if $\boldsymbol{z}' \le \boldsymbol{z}$ and $\boldsymbol{z} \in \mathcal{S}$ implies $\boldsymbol{z}' \in \mathcal{S}$) with non-empty interior. Let $\Phi(\boldsymbol{z}) = \sum_{k=1}^{K} w_k \log(z_k)$. Then problem P1 can be reformulated as

$$\max \Phi(\boldsymbol{z}),\ \text{s.t.}\ \boldsymbol{z} \in \mathcal{G} \tag{15}$$

Define $\mathcal{H} \in [\boldsymbol{z} | z_k \ge 2^{r_k^{\min}}, \forall k]$, $\mathcal{H}$ is a reverse normal set (A set $\mathcal{S}$ is called reverse normal if $\boldsymbol{z}' \ge \boldsymbol{z}$ and $\boldsymbol{z} \in \mathcal{S}$ implies $\boldsymbol{z}' \in \mathcal{S}$). Then, (15) can be written as the following canonical monotonic optimisation problem (MP)

$$\max_{\Phi}(\boldsymbol{z}),\ \text{s.t.}\ \boldsymbol{z} \in \mathcal{G} \cap \mathcal{H} \tag{16}$$

The polyblock outer approximation method is used to obtain the global solution. The global solution is obtained by constructing a sequence of deflating polyblocks that approximate the maximum WSR of the feasible region.

For polyblock $\mathcal{S}^n$, denoting $\mathcal{V}^n$ as its proper vertex set. Consequently, the polyblock $\mathcal{S}^n$ is fully determined by its proper vertices $\mathcal{V}^n$. Find $\boldsymbol{z}^n \in \mathcal{V}^n$ that maximises the objective function

$$\boldsymbol{z}^n = \operatorname*{argmax}_{\boldsymbol{z}\in\mathcal{V}^n} \Phi(\boldsymbol{z}) \tag{17}$$

If $\boldsymbol{z}^n \in \mathcal{G} \cap \mathcal{H}$, then it solves the MP (16), else, building a branched polyblock $\mathcal{S}^{n+1} \subset \mathcal{S}^n$ to exclude $\boldsymbol{z}^n$. Repeat this procedure will lead to a sequence of polyblocks approximating $\mathcal{G} \cap \mathcal{H}$:$\mathcal{S}^1 \supset \mathcal{S}^2 \supset \cdots \supset \mathcal{G} \cap \mathcal{H}$. Thus, the polyblocks $\mathcal{S}^n$ shrinks step by step, leading to the corresponding vertex closely approximate to the optimal WSR of the feasible region. Replacing $\boldsymbol{z}^n$ in $\mathcal{V}^n$ with K new vertices

$$\boldsymbol{z}^n(k) = \boldsymbol{z}^n - (z_k^n - \pi_k^{\mathcal{G}}(\boldsymbol{z}^n))\kappa_k \tag{18}$$

where $\pi^{\mathcal{G}}(\boldsymbol{z}^n)$ is the projection of $\boldsymbol{z}^n$ on $\mathcal{G}$, that is, the intersection point of the SINR boundary and the line $\xi\boldsymbol{z}^n$ which connects the two points $\gamma = [2^{r_1^{\min}}, \ldots, 2^{r_K^{\min}}]$ and $\boldsymbol{z}^n$; κ_k is a unit vector with the only non-zero in the kth entry. Construct the new vertices as $\mathcal{V}^{n+1} = (\mathcal{V}^n \backslash \boldsymbol{z}^n) \cup \boldsymbol{z}^n(k) \cap \mathcal{H}$, and then remove improper vertices. Continue this process until $\boldsymbol{z}^n \in \mathcal{G} \cap \mathcal{H}$. For practical implementation, when $\max\{(z_k^n - \pi_k^{\mathcal{G}}(\boldsymbol{z}^n))/z_k^n\} \le \varsigma$, the algorithm can be terminated, where ς is the error tolerance level.

After getting $\boldsymbol{z}^n$, the rate profile is updated as

$$\boldsymbol{\alpha} = \frac{\log(\boldsymbol{z}^n) - \boldsymbol{r}^{\min}}{\sum_{k=1}^{K}\log(z_k^n) - \sum_{k=1}^{K} r_k^{\min}} \tag{19}$$

Consequently, the projection of $\boldsymbol{z}^n$ on $\mathcal{G}$ can be expressed as $\pi^{\mathcal{G}}(\boldsymbol{z}^n) = 2^{s(R_{\rm sum})}$, where $R_{\rm sum}$ can be obtained by Fig. 3 for the given $\boldsymbol{\alpha}$. The initial polyblock $\mathcal{S}^0$ is constructed that contains the feasible set of Problem P1. Let $\bar{\boldsymbol{H}}_k = \boldsymbol{h}_k\boldsymbol{h}_k^{\rm H}/\sigma_k^2$, solving the following objective function, we can obtain $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_K]$

$$\begin{aligned} \max\ & \beta_k = 1 + \text{tr}(\boldsymbol{v}_k^{\rm H}\bar{\boldsymbol{H}}_k\boldsymbol{v}_k) \\ \text{s.t.}\ & |\boldsymbol{h}_{kp}^{\rm H}\boldsymbol{v}_k|^2 \le b_k/\ln(1/\varepsilon) \\ & \|\boldsymbol{v}_k\|_2^2 \le p_k, \forall k \end{aligned} \tag{20}$$

Then $\mathcal{G} \in [0, \boldsymbol{\beta}]$ is a compact normal set that contains the feasible region of Problem P1.

The proposed algorithm for solving the RI-WSR in cognitive networks is concluded as Algorithm 3 (see Fig. 4). We show the proposed joint admission control and beamforming design algorithm in Fig. 5.

When we have the desired $\boldsymbol{V}_k$, there is no guarantee that the obtained $\boldsymbol{V}_k$ is rank one. As a result, if the obtained $\boldsymbol{V}_k$ is not rank one, certain techniques, such as randomised procedure [7] or eigenvalue decomposition (EVD) approximation method [23], are required to find the approximate optimal beamforming. To reduce the rate loss caused by the approximation, the obtained beamforming vector $\boldsymbol{v}_k$ is calculated according to the principal

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Algorithm 3

Input: Initialise an accuracy level $\varsigma > 0$. Initialise $\mathcal{V}^0 = \{\boldsymbol{\beta}\}$, Initialise $n = 0$;

1: **Loop**

2: Find $\boldsymbol{z}^n$ that maximises the objective function $\boldsymbol{z}^n := \text{argmax}_{\boldsymbol{z}^n\in\mathcal{V}^n}\Phi(\boldsymbol{z})$;

3: For $\boldsymbol{z}^n$, obtain $\boldsymbol{\alpha}$ according to (19). Calculate the maximum achievable WSR $R_{\rm sum}$ and the corresponding $\mathbf{V}_k$ by using Algorithm 2;

4: Update $\pi^{\mathcal{G}}(\boldsymbol{z}^n) := 2^{s(R_{\rm sum})}$;

5: Construct a smaller polyblock $\mathcal{S}^{n+1}$ with vertex set $\mathcal{V}^n$, $\boldsymbol{z}^n(k) := \boldsymbol{z}^n - (z_k^n - \pi_k^{\mathcal{G}}(\boldsymbol{z}^n))\boldsymbol{\kappa}_k$. Update $\mathcal{V}^{n+1} := (\mathcal{V}^n\backslash\boldsymbol{z}^n) \cup \boldsymbol{z}^n(k) \cap \mathcal{H}$, remove improper vertices and let $n := n + 1$;

6: **until** $\max\{(z_k^n - \pi_k^{\mathcal{G}}(\boldsymbol{z}^n))/z_k^n\} \le \varsigma$;

Output: $\mathbf{v}_k, k \in \mathcal{K}$ obtained by rank reduction of $\mathbf{V}_k$.

Fig. 4 *Proposed algorithm for maximising the WSR for SUs*

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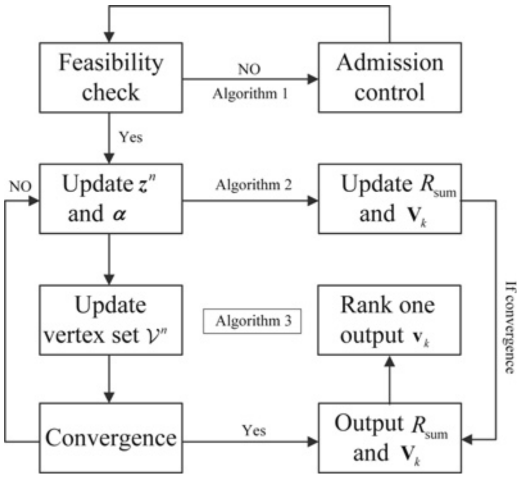

Fig. 5 *Illustration of the process of the proposed algorithm*

eigenvector of V_k. Take the EVD of V_k, $V_k = U_{kv}\mathcal{D}(d_{kv1}, \dots, d_{kvN})U_{kv}^{\mathrm{H}}$, where $U_{kv} = [u_{kv1}, \dots, u_{kvM}]$ and $d_{kv1} \geq \dots \geq d_{kvM}$. Then the beamforming vector is calculated as $v_k = \sqrt{d_{kv1}}u_{kv1}$.

3.4 *Convergence and application analysis of the proposed algorithm*

Proposition 1: The proposed algorithm is convergent and the limit point of the iteration is a stationary point of (6).

Proof: As the sequence of polyblocks satisfying: $\mathcal{S}^1 \supset \mathcal{S}^2 \supset \dots \supset \mathcal{G} \cap \mathcal{H}$ and the minimum WSR of the network is $\sum_{k=1}^{K} w_k r_k^{\min}$, we can obtain $\lim_{n\to\infty}\|z^n - z^n(k)\| \to 0$. From (18), we can have $\lim_{n\to\infty}\|z^n - z^n(k)\| = \lim_{n\to\infty} z_k^n - \pi_k^{\mathcal{G}}(z^n) \to 0$, which indicates $z^n \to \pi^{\mathcal{G}}(z^n)$ when $n\to\infty$. As $\pi^{\mathcal{G}}(z^n) = 2^{s(R_{\text{sum}})}$, R_{sum} is obtained by SCA method from Fig. 3. The SCA method usually obtains a lower bound of (14) [19]. Thus, sub-optimality of SCA method results in suboptimal solution of Fig. 4 and the obtained solution of Fig. 4 is a lower bound of problem (6) if the obtained V_k is rank one.
Furthermore, there may be some loss from the rank reduction process if the obtained V_k is not rank one. The rank reduction process only takes place after we have the solution of Fig. 4, it is a separate problem and may output suboptimal solution, but it takes little effort on the convergence of Fig. 4 as analysed above. Simulation results show that the second largest eigenvalue of V_k is usually 10^7 times smaller than the principal eigenvalue, thus the obtained V_k can be approximated as rank one. Thus, the rate loss of the rank reduction process usually can be ignored. Overall, the proposed algorithm is convergent and the obtained solution of the proposed algorithm is suboptimal. □

The proposed algorithm can be implemented as follows. First, each receiver estimates the CSI and transmits it to the SMC. After receiving the CSI of each user, the SMC implements the proposed algorithms to control the admission and calculate the corresponding beamforming vector for each active SU. After obtaining the beamforming vectors, the SMC transmits the corresponding beamforming vector to each active SU and simultaneously transmits the indicator message to the SUs to make them silent if they are forbidden to transmit.

The SMC is used to transmit the CSI and calculate the beamforming vectors. With the help of the SMC, the proposed algorithm can be implemented more effectively. Compared with the global CSI feedback from one user to another, the centre-based feedback of the SMC can save lots of overhead [24]. On the other hand, with the help of the SMC, the beamforming vectors can be computed by the SMC when the CSI is available, this can reduce the calculating times and the complexity of the STs.

4 Numerical results and analysis

In this section, we evaluate the performance of the proposed algorithm with simulation results. For all the PU and the SUs, each transmitter is equipped with three antennas. It is assumed that every SU has identical transmit power constraint p_k. The beamforming vector v_p at PU is randomly generated uniformly on the unit-norm sphere $\|v_p\|^2 = 1$. The beamforming vectors of the SUs are obtained by using Fig. 4, where $\delta_k = 0.1$ and w_k is set to a random value between 0 and 1.

4.1 *Admission control*

Fig. 6 shows the number of active SUs of the proposed algorithm. There are ten pairs of SUs in the network and $h_{lk} \in \mathcal{N}(0, I)$. The power of all the SUs are set to $p_k = 5$ W and the interference constraints are relaxed. In order to conveniently show the relationship of the rate thresholds and the active number of SUs, the rate thresholds of all SUs are set to a value of the same. Then the simulation result can be shown directly in one figure as the rate thresholds of all SUs share the same coordinate axis. If the QoS is satisfied, all the SUs of the feasible set are allowed to transmit, otherwise, the user with the maximum λ_k is prohibited to transmit. When the rate threshold becomes higher, the number of SUs that the network can support becomes fewer. This is because when the rate threshold gets higher, the feasible region becomes smaller, and the original problem may be infeasible. From the first constraint of (13), we can see the left side of the inequality is proportional to γ. This means that if $r_k^{\min}$ becomes higher, the left side of the inequality of the first constraint of (13) becomes bigger, this may cause the value of the left side of the inequality greater than 1. Thus, to make this problem feasible, the number of the components in the production of this constraint should be reduced to a smaller value, and this would result in the blocked of some SUs.

Fig. 6 also reflects the relationship of the supported number of SUs and the outage probability. The higher the outage probability, the higher proportionality of the rate that allowed to be smaller than the given threshold the system obtains, and the more SUs the network can support. When the outage probability gets higher, the feasible region becomes larger, so the number of the blocked SUs becomes smaller.

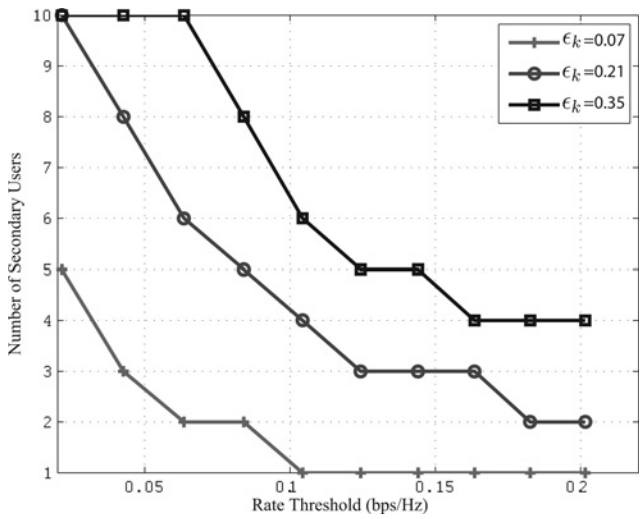

Fig. 6 *Number of active SUs against rate threshold*

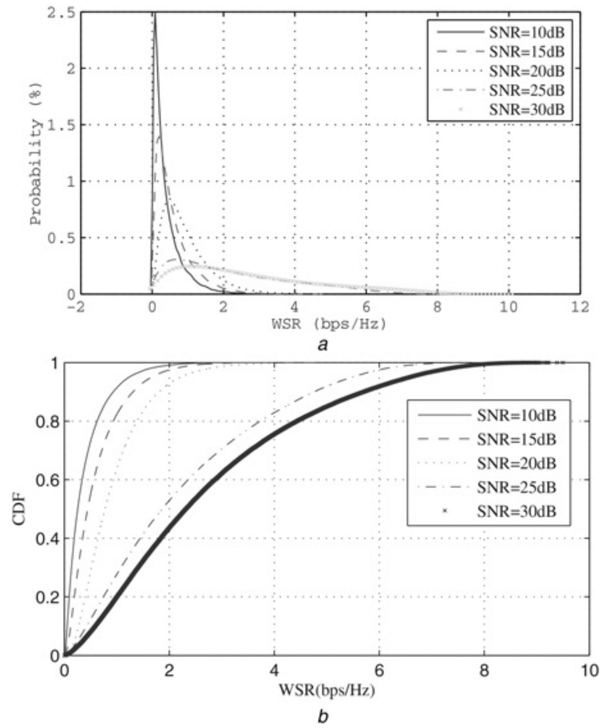

Fig. 7 *Probability distributions a and CDF b of achievable WSRs*
a $\rho = 5$ and $\varepsilon = 0.35$
b $\rho = 5$ and $\varepsilon = 0.35$

4.2 Performance of the achievable WSR

Fig. 7 shows the probability distributions and CDF of the achievable WSRs with different SNRs. The simulation results are obtained by 10^5 channel realisations, where $\boldsymbol{h}_{lk} \in \mathcal{N}(0, \boldsymbol{I})$. There are three SUs and one PU transmitting in the network. After having the desired $\boldsymbol{V}_k$, if $\text{rank}(\boldsymbol{V}_k) = 1$, then $\boldsymbol{v}_k$ is obtained by using the EVD $\boldsymbol{V}_k = \boldsymbol{v}_k \boldsymbol{v}_k^{\text{H}}$. If $\text{rank}(\boldsymbol{V}_k) > 1$, we get the $\boldsymbol{v}_k$ according to the maximum eigenvalue and eigenvector. Simulation results show that the second largest eigenvalue is more than 10^7 times smaller than the largest eigenvalue, thus, the obtained $\boldsymbol{V}_k$ can be seen as rank one. In order to reflect the effectiveness of rate performance of the proposed algorithm and get the same conditions to compare with the baseline method, the rate constraint (6b) is dropped in the simulations. This is equivalent to the particular condition that 0 rate threshold is set in problem (6).

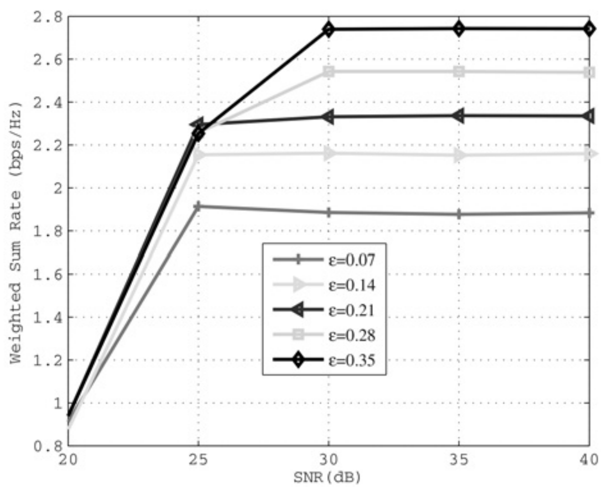

Fig. 8 *WSR of different allowed interference thresholds*

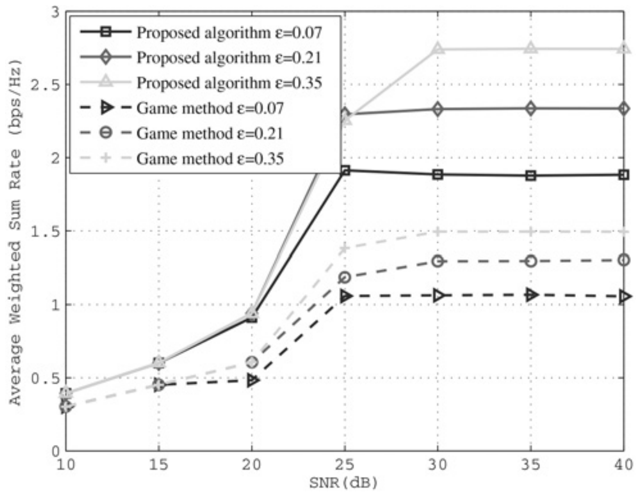

Fig. 9 *WSR of different schemes against SNR*

Fig. 7 shows that when SNR vary from 10 to 30 dB, the big components of the obtained WSRs become more, this has thus resulted in the increase of the average WSR of the SUs. The peak point of the curve in the probability figure represents the number of users that achieve the corresponding rate is the maximum. When the SNR becomes higher, the peak point shifts right, which means that the overall rates that users achieve become higher. From the CDF figure, we can see when SNR = 30 dB, the users get much better rates than those of other SNRs. For example, when SNR = 30 dB, the probability that the achieved rate less than 1 bps/Hz is about 0.2, while the probability is about 0.3 when SNR = 25 dB and about 0.8 when SNR = 15 dB.

We illustrate the average WSR of different outrage probabilities with partial CSI in Fig. 8. Three SUs and one PU are transmitting in the network. $\boldsymbol{v}_p$, δ_k, w_k, ρ and ε are set the same as Fig. 7. The simulation results are obtained by making a count of 10^5 channel realisations. Fig. 8 shows that the larger the allowed outage probability, the higher average WSR the SUs can achieve.

Fig. 9 shows the average WSR of different methods varies with SNR. The game method [5] is compared with the proposed algorithm. The simulation parameters $\boldsymbol{v}_p$, δ_k, w_k, ρ and ε are set the same as Fig. 7. The game method gets a Nash equilibrium other than a global solution, and the rate solution of the user k depends on other K–1 users that are transmitting simultaneously in the network. The proposed algorithm adopts polyblock outer approximation algorithm to get a near optimal WSR when there is no rate requirement. Simulation results show that the proposed algorithm performs better than the game method whenever the SNR is.

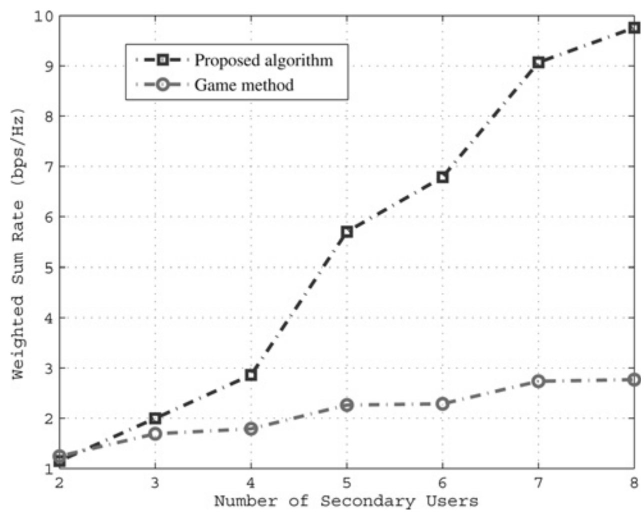

Fig. 10 *WSR of different schemes against SU number*

Fig. 10 depicts the WSR against the number of SUs with partial CSI. The SNR is set to 5 dB. $\boldsymbol{h}_{lk} \in \mathcal{N}(0, \boldsymbol{I})$. $\rho = 4K/3$, where K represents the number of the SUs. $\delta_k = 0.1$ and $\varepsilon = 1/e^2 \simeq 0.14$. The game method [5] is compared with the proposed algorithm. The proposed algorithm can get higher WSR than the game method. Simulation results show the enhanced performance of the proposed algorithm, especially when the user number is larger. The more number of the SUs, the higher interference between the SUs. The proposed algorithm adopts the polyblock outer approximation method and rate profile method. The polyblock outer approximation method can obtain optimal solution if the optimal projection of z^n on $\mathcal{G}$ can be found. When the rate requirement is set to 0, there is no loss of the rate from the SCA method, thus, only rate loss produced by rank reduction method. Our simulation results show that the loss of the rank reduction is little and can be omitted, thus, near optimal WSR is obtained by the proposed algorithm. Simulation results show that the proposed algorithm performs well in high interference networks.

4.3 Complexity analysis

The simulation results in Section 4.2 show that the proposed algorithm performs better at achieving WSR than the game method with partial CSI. However, the proposed algorithm usually has higher complexity.

The complexity of solving a convex problem depends on the dimensions of the variables and the number of the constraints. Obtaining a beamforming covariance matrix from an SDR problem by using CVX needs about $O(M^{3.5}\log(1/\nu))$ iterations [23], where ν controls the convergence accuracy. This process is needed by both the game method and the proposed algorithm. Thus, the complexity of the whole calculation depends on the convergence rate. However, the polyblock outer approximation method usually converges slowly and the bisection method in Fig. 3 is time-consuming. Thus, the proposed algorithm usually has higher complexity than the game method.

Fig. 11 shows the convergence of the polyblock outer approximation method. Case I is set for a particular case. The channel covariance is set to the unit matrix with dimension $M \times M$, that is, $\boldsymbol{C} = \boldsymbol{I}$. Case II is set for a general case, the channel covariance is set to a random positive symmetric matrix $\boldsymbol{C} \succcurlyeq 0$. Simulation results show that when $\boldsymbol{C}$ is a diagonal matrix, the polyblock outer approximation converges fast and only several iterations are needed. At this time, the needed iterations are comparable with that of the game method. However, when the matrix $\boldsymbol{C}$ is a general positive semi-definite matrix, the polyblock outer approximation converges very slowly. This will result in higher complexity of the proposed algorithm than the game method.

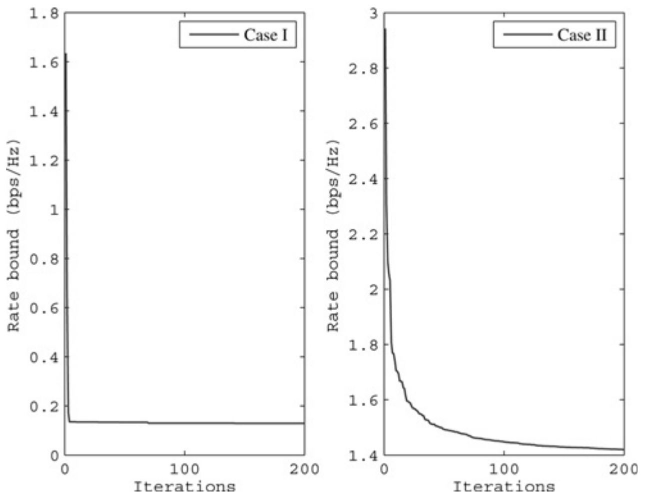

Fig. 11 *Convergence of polyblock outer approximation algorithm, $\rho = 5$, $\delta_k = 0.1$ and $\varepsilon = 0.07$*

5 Conclusion

This paper proposes a joint admission control and beamforming design algorithm to maximise the WSR of SUs under the rate requirement and the interference constraint in the spectrum-sharing-based CR networks. As the QoS requirement and the interference constraint, the feasibility is first checked before the problem is being solved. By introducing a penalty factor, the admission can be performed by the instructions of the obtained penalty factor values. As non-convexity of such problem, the polyblock outer approximation method is used to update the achievable WSR of the SUs. By using the polyblock outer approximation method and the rate profile technique, the non-convex RI-WSR problem are decomposed into convex subproblems, then efficient optimisation tools can be used to obtain the solutions of the original problem. Simulation results show that the proposed algorithm can get higher WSR compared with the existing baseline method in partial CSI case.

6 Acknowledgments

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