IET Communications

*Research Article*

# Improved power loading scheme for orthogonal frequency division multiplexing based cognitive radio

ISSN 1751-8628
Received on 31st July 2014
Revised on 26th May 2015
Accepted on 17th July 2015
doi: 10.1049/iet-com.2014.1208
www.ietdl.org

*Dipen Bepari* ✉, *Debjani Mitra*

*Department of Electronics Engineering, Indian School of Mines, Dhanbad – 826004, Jharkhand, India*
✉ *E-mail: dipen.jgec04@gmail.com*

**Abstract:** A subcarrier power allocation scheme for orthogonal frequency division multiplexing (OFDM) based cognitive users has been proposed for spectrum sharing with primary users (PU) allowing secondary users (SU) to coexist in same as well as adjacent frequency bands by underlay and interweave approaches respectively. The SU sum capacity of reliable communication has been maximised under the constraints of total power, each sub-channel power and PU aggregate interference limits. Based on the quality of channel gains and spectral distance from PU bands, the powers of subcarriers adjacent to PU bands are suitably controlled till the interference constraint of PUs are satisfied. Controlled subcarrier deactivation achieved by augmenting this 'n-adjacent' power redistribution step to traditional OFDM water-filling gives a powerful tool for efficiently maintaining high SU sum capacity without exceeding PU interference tolerance limit for a wide range of total power budget. Different water levels get assigned for the adjacent and non-adjacent subcarrier groups. In terms of SU capacity and PU interference, the proposed algorithm outperforms some of the existing schemes having almost same complexity as the proposed one. The improvement is significant when the adjacent subcarriers channel gains are relatively 'good' or 'bad' compared with the non-adjacent ones.

## 1 Introduction

Dynamic spectrum access through cognitive radio (CR) technology is widely used to address the problem of inefficient spectral usage in both spatial and temporal domain replacing the existing policy of assigning fixed bands only to licensed users [1–3]. The spectral usage activity information of licensed primary users (PU) made available to unlicensed secondary users (SU) by spectrum sensing enables the latter to adopt flexible, dynamic opportunistic spectrum access schemes. The awareness about spectrum usage and existence of PU in a geographical area is known to SU by spectrum sensing and adaptive power control can help in the intelligent coexistence of both PUs and SUs [2, 4]. Literature projects orthogonal frequency division multiplexing (OFDM) as a popular transmission technology for cognitive radio network (CRN) [5, 6]. The use of orthogonal subcarriers not only mitigates frequency selective fading, but also helps in efficient spectrum utilisation. The use of OFDM modulation by the SUs, gives them two important benefits without any additional overhead. Firstly, fast Fourier transform (FFT) operation in OFDM receiver enables spectrum sensing and secondly in the scenario of rapidly changing channel and spectrum, the provision of adaptive control of the flexible subcarrier structure enables spectrum pooling, spectrum shaping, and efficient resource allocation. The traditional water-filling and uniform-power-variable-rate algorithms maximising the SU sum capacity for optimal subcarrier power allocation in OFDM subcarriers is a widely discussed problem [7, 8]. It becomes challenging in the domain of CRNs accommodating PU-SU coexistence under interference protection to PUs. Meeting the interference constraint is relatively easier when PU-SU transmit in the same band. However, when the OFDM-CR users and PUs are spectrally adjacent (exist in side-by-side bands), non-orthogonality of the OFDM subcarriers causes mutual interference, and meeting of PU interference constraint by optimal water-filling becomes difficult. Moreover, in OFDM based CRNs, frequency band is divided into several sub-channels, each corresponding to some licensed PU user band. By sub-channel is meant an unoccupied band spanned by a group of subcarriers that would be used for CR transmission. The interference limit of each such PU has to be met and thus there is a need to consider sub-channel transmit power constraints in the power loading algorithms [9]. From spectrum sensing data, the unoccupied PU sub-channels and the spectral distance between the CR-subcarriers and the PU bands can be made known. Several new subcarrier power allocation schemes based on modified versions of conventional water-filling have been proposed in works of [8–17], wherein the system model considers the PUs existing in the same as well as adjacent frequency bands as that of a single user OFDM-SU. The iterative partitioned water-filling (IPWF) algorithm of Wang *et al.* [9] based on convex optimisation is capable of maximising the capacity under both sum and sub-channel power constraints. To include the effects of sub-carrier side-lobes, a reformulated algorithm finds the optimal power allocation recursively by decoupling the sub-channel power constraints phase-by-phase. The power-increment and power-decrement water-filling process of [18] for OFDM-based CR system is faster and of lower complexity as compared with IPWF. Reducing of PU interference is also approached by the use of raised cosine pulses and other windowing techniques as in [6]. Peng *et al.* [10] have addressed power allocation including the interference from subcarrier side-lobes. PU activity dependent subcarrier availability factor has been incorporated into the capacity function in [11], to develop a risk-return model. The Lagrange based optimal power allocation of [12] maximises downlink transmission capacity keeping the aggregate interference acceptable to PUs. The suboptimal algorithm proposed in [13] is for a probabilistic model of interference threshold eliminating channel quality feedback from the PU to SU. In the works of [11, 13], the suggested scaling and step-ladder methods are capable of reducing the complexity of power allocation to quite an extent. The relatively simple method of nulling the adjacent subcarriers power is also effective, but at the cost of reduced spectral efficiency [14].

Motivated from the nulling technique, the power distribution strategy proposed here is capable of minimum sacrifice of spectral efficiency. It adopts controlled deactivation of the subcarriers adjacent to the PU band. In our algorithm, after the sum and sub-channel power constraints have been met, in the last step, instead of allocating zero power to $n$ adjacent subcarriers, the power in these subcarriers are reallocated until interference power constraint is strictly satisfied. Hence, a different water level is achieved for the '*n-adjacent*' subcarriers. Starting from $n = 1$, the algorithm increments the value of $n$, redistributing the power till the desired interference constraint is met. This ensures minimum loss of spectral efficiency and maximum opportunity for 'good' quality adjacent subcarriers to take part in transmission. The main contribution of the paper is the new heuristics based '*n-adjacent*' power redistribution algorithm capable of meeting low PU interference tolerances with high SU sum capacity for a wide range of total power budget. The step-ladder and nulling method does not guarantee the fulfilment of interference constraint beyond a certain level of power budget. Moreover, for the suboptimal method of [13], it may so happen that none of the constraints are met strictly unless scaling is performed as in [11], to compensate for the loss of transmission capacity. Further, the impact of quality of channel gains (of the adjacent subcarriers) on the power control strategies has not been considered in any of the related works. The nulling method tends to lose out in capacity when channel gains for the adjacent subcarriers are excellent. The proposed strategy here coined as '*n-adjacent*' is capable of addressing these issues. By selecting a minimum number of adjacent subcarriers and determining a different water level for the adjacent group, the interference constraint is strictly met, at the same time a high level of capacity is achieved.

The remainder of this paper is outlined as follows: in Section 2 system model and problem formulation has been discussed. The proposed power loading scheme for SU sum capacity maximisation described in Section 3. Comparative performance evaluation of proposed power allocation schemes and existing power loading schemes such as scaling, step-ladder, nulling and other strategies are analysed in Section 4. Finally Section 5 concludes the paper.

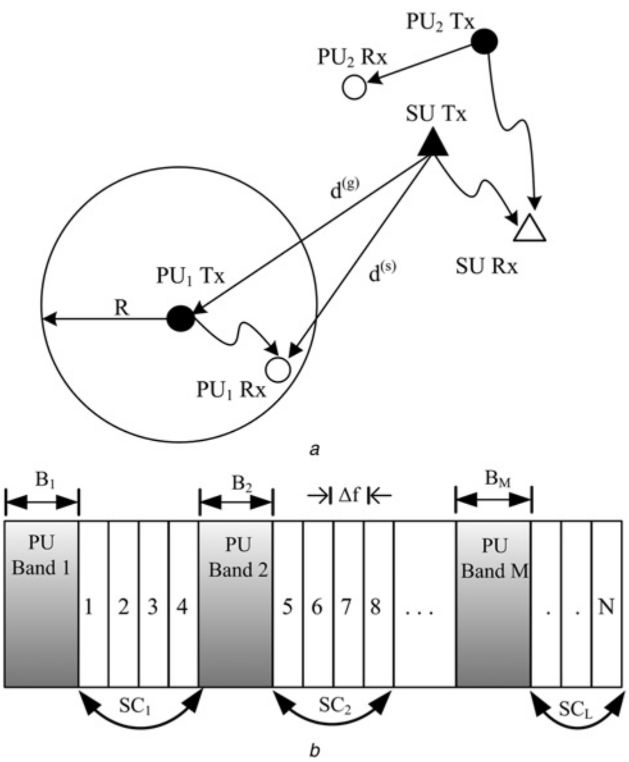


**Fig. 1** *Cognitive radio system model*

*a* Spatial domain
*b* Frequency domain

## 2 System model and problem formulation

### 2.1 System model

A commonly used OFDM based CR system model in both space and frequency domain is depicted in Fig. 1 as in [9, 11]. Depending on geographical location of PU, two types of spectrum access schemes have been considered: *underlay* and *interweave*. For $PU_1$ assumed to be geographically located far away from SU, accurate information of signal of $PU_1$ is not available and underlay access for secondary communication can be set up through an interference constraint maintaining a specific signal-to-interference-noise ratio quality of service at PU receivers. The interference limit policy and geo-location spectrum occupancy database of all such licensed bands is usually available to SUs. In general, if a SU transmitter is able to detect primary receivers within a distance say $d^{(s)}$, then its transmission power must be confined below or equal to $P^{tx}(d^{(s)})^{\beta}$, where $P^{tx}$ is the prescribed maximum interference power level allowed by the primary system, and $\beta$ is the path loss exponent. In absence of cooperation from PU, the direct detection of primary receivers is difficult and is instead approached by detecting primary transmitters. Let $d^{(g)}$ denotes the distance between SU transmitter and PU transmitters as shown in Fig 1. If $R$ is the transmission range of the primary transmitters, i.e., the primary receivers are located within a distance of $R$ from it, then $d^{(s)}$, the minimum distance between SU transmitter and PU receiver will be $d^{(s)} = d^{(g)} - R$. For every such PU, $R_j$ and $P_j^{tx}$, respectively denotes the radius of the protection area and the corresponding power limit to be maintained at the boundary of this area. Therefore, for underlay scheme, $P_j^{sc}$, the total power of $j$th sub-channel constraint should be

$$P_j^{sc} \triangleq P_j^{tx}(d_j^{(g)} - R_j)^{\beta_j} \tag{1}$$

For all unheard PU transmitters and in absence of their location information, $d^{(g)}$ is set as the radius of the SU reliable sensing region as also used in [9]. The sensing radius depends upon the detection sensitivity of the sensing algorithm used by the spectrum detector and the extent of cooperative sensing. Thus SU underlay $PU_1$ with both transmitting concurrently in the same band.

For $PU_2$ assumed to be geographically co-located with SU, following an interweave approach, coexistence is assumed in frequency bands adjacent to each other. PU activities are well-detected and information of PU signal as well as spectral holes is available to the SU. In this scheme, before transmission, if a PU signal is detected in a sub-channel then SU allocates 'zero' power to that sub-channel, i.e. $P_j^{sc}$, the total power of $j$th sub-channel constraint as $P_j^{sc} = 0$. The spectral distance between the SU subcarriers and the PU band is significant and the SU transmission needs to be highly flexible in term of the spectral shape of transmit signal for efficient and opportunistic use of spectrum [19]. The typical spectrum arrangement is as shown in Fig. 1*b* where $B_1$, $B_2$, $B_3$, …, $B_M$ denote the $M$ numbers of PU occupied bands. $N$ numbers of subcarriers are grouped into $L$ sub-channels corresponding to the unoccupied bands of PUs. The interval of every subcarrier in the SU band is $\Delta f$. Averaging the frequency bins in the FFT stages of the OFDM based CR receiver can easily implement energy detection based sensing. Knowledge of PU spectral occupancy in the sub-channels can thus be sensed without any additional hardware [5]. The PU's activity is assumed to be uncorrelated.

If $P_i$ is the power allocated to $i$th subcarrier then the achievable rate $r_i$ for ideal modulation and coding scheme is given by [19]

$$r_i = \Delta f \log_2\left(1 + \frac{|G_i^{ss}|^2 P_i}{\sigma^2 + \sum_{j=1}^{M} J_i^j}\right) \tag{2}$$

where, $G_i^{ss}$ is the estimated channel gain between SU transmitter to its corresponding receiver of $i$th subcarrier, $\sigma^2$ is additive white Gaussian noise (AWGN) and $J_i^j$ is the interference introduced by signals from the $j$th PU's band to the $i$th subcarrier in the SU user band. In general, it is given as [10, 12]


*IET Commun.*, 2015, Vol. 9, Iss. 16, pp. 2033–2040

$$J_i^j = |G_m^{\mathrm{ps}}|^2 \int_{d_{im}-\Delta f/2}^{d_{im}+\Delta f/2} [E\{I^{\mathrm{PU}}(\omega)\}]\mathrm{d}\omega \quad (3)$$

where, $G_m^{\mathrm{ps}}$ is the channel gain between $m$th PU transmitter and SU receiver, $d_{im}$ is spectral distance between $i$th OFDM subcarrier and $m$th PU band. The PU signal is usually denoted as some specific random process, whose power density spectrum after FFT processing in the OFDM receiver, $E\{I^{\mathrm{PU}}(\omega)\}$ has to be known to get the interference factor accurately. $\omega$ is the frequency normalised to the sampling frequency. For the purpose of algorithm development, $J_i^j$ has been approximated as AWGN, the assumption being valid under the existence of a large number of PUs [12].

### 2.2 Problem formulation

*2.2.1 Power constraints:* The two power constraints considered in the optimisation are (i) total power constraint of the OFDM block to be used by the SU and (ii) individual sub-channel power constraint for each of the $L$ sub-channels. $P_{\mathrm{total}}$, the total SU transmitter power constraint is expressed as

$$\sum_{i=1}^{N} P_i \le P_{\mathrm{total}} \quad (4)$$

Due to the existence of the first type of PUs as expressed through (1) and for giving protection to the undetected PUs, SU subcarriers in a given sub-channel must use suitable transmission powers such that the total power in a sub-channel is below the threshold power level $P_j^{\mathrm{sc}}$. If $S_j$ is the sum power allotted to sub-channel $j$, then to avoid interference to $\mathrm{PU}_1$ we must maintain

$$S_j \le P_j^{\mathrm{sc}} \quad \forall j \in \{1, 2, \ldots, L\} \quad (5)$$

where $S_j = \sum_{i\ \mathrm{s.t.}\ \phi(i)=j} P_i$. As there are $L$ numbers of sub-channels spanning the $N$ subcarriers, it follows that $\sum_{j=1}^{L} S_j = \sum_{i=1}^{N} P_i \le P_{\mathrm{total}}$. The term $\phi(i)=j$ implies the $i$th subcarrier belongs to sub-channel $j$.

*2.2.2 Interference constraints:* There are two major mutual interference factors in any OFDM based CR when the PU and SU are coexisting on adjacent sub-bands: (i) the interference introduced by the PU band on the SU subcarriers as considered by (3) and (ii) the sum interference introduced by SU subcarriers to the PU. In general, the second type of interference can be addressed by different signal-processing techniques minimising power leakage in the side lobes. For mathematical optimisation therefore, we need to consider the term $I_i^m$, denoting interference introduced by the $i$th subcarrier of the SU on the $m$th PU sub-channel as considered in all related works [9, 12].

$$I_i^m = |G_m^{\mathrm{sp}}|^2 P_i T_{\mathrm{s}} \int_{d_{im}-B_m/2}^{d_{im}+B_m/2} [sinc(f\,T_{\mathrm{s}})]^2 \mathrm{d}f \quad (6)$$

where $G_m^{\mathrm{sp}}$ is the channel gain between SU transmitter and $m$th PU receiver, $T_{\mathrm{s}}$ is the symbol duration and $B_m$ denotes occupied bandwidth of $m$th PU band. Defining $k_i^m = T_{\mathrm{s}} \int_{d_{im}-B_m/2}^{d_{im}+B_m/2} [sinc(f\,T_{\mathrm{s}})]^2 \mathrm{d}f$ the factor can be written in compact form as $I_i^m = k_i^m P_i |G_m^{\mathrm{sp}}|^2$. The $k_i^m$ factors physically represent the association of each individual sub-carrier in terms of their spectral distance from the PU bands. The subcarriers which are immediately adjacent to PU band will have higher $k_i^m$ values and causes higher amount of interference. The aggregate interference on the primary system consisting of $M$ PU bands, subjected by the $N$ subcarriers will thus have to be restricted to $I_{\mathrm{th}}$, an interference threshold so that $\sum_{i=1}^{N}\sum_{m=1}^{M} I_i^m \le I_{\mathrm{th}}$.

As adopted by [13], the primary system interference presented from all the SU subcarriers to all the $M$ PU sub-channels are specified by the following probabilistic interference constraint [13]

$$\Pr\left(\sum_{i=1}^{N}\sum_{m=1}^{M} I_i^m \le I_{\mathrm{th}}\right) \ge P_{\mathrm{a}} \quad (7)$$

This can be alternately expressed as

$$\Pr\left(\sum_{i=1}^{N}\sum_{m=1}^{M} |G_m^{\mathrm{sp}}|^2 k_i^m P_i \le I_{\mathrm{th}}\right) \ge P_{\mathrm{a}} \quad (8)$$

Here, Pr denotes probability and $P_{\mathrm{a}}$ is the minimum probability of the aggregate interference being maintained below $I_{\mathrm{th}}$. For Rayleigh distributed amplitude gain, the distribution of $|G_m^{\mathrm{sp}}|^2$ corresponds to an exponential distribution of mean $\lambda_m$. Considering $\lambda_1 = \lambda_2 = \cdots = \lambda_M = \lambda$, (8) is approximated as

$$1 - \exp\left(\frac{-I_{\mathrm{th}}}{2\lambda^2 \sum_{i=1}^{N}\sum_{m=1}^{M} k_i^m P_i}\right) \ge P_{\mathrm{a}} \quad (9)$$

If

$$I_{\mathrm{eff}} = \frac{I_{\mathrm{th}}}{2\lambda^2(-\ln(1-P_{\mathrm{a}}))} \quad (10)$$

Then, (9) can be expressed as

$$\sum_{i=1}^{N} P_i K_i \le I_{\mathrm{eff}} \quad (11)$$

where, $K_i = \sum_{m=1}^{M} k_i^m$. Therefore, (11) incorporates the PU interference constraint to be used in the optimisation function, which may now be expressed in totality as

$$R = \max_{\boldsymbol{P}} \sum_{i=1}^{N} \Delta f \log_2\left(1 + \frac{|G_i^{ss}|^2 P_i}{\sigma^2 + \sum_{j=1}^{M} J_i^j}\right) \quad (12)$$

subject to:

$$P_i \ge 0 \quad \forall i \in \{1, 2, \ldots, N\} \quad (13)$$

$$S_j \le P_j^{\mathrm{sc}} \quad \forall j \in \{1, 2, \ldots, L\} \quad (14)$$

$$\sum_{j=1}^{L} S_j = \sum_{i=1}^{N} P_i \le P_{\mathrm{total}} \quad (15)$$

$$\sum_{i=1}^{N} P_i K_i \le I_{\mathrm{eff}} \quad (16)$$

The power allocation problem considered here assumes the prior knowledge of: $P_{\mathrm{total}}$, $P_j^{\mathrm{sc}}$, $I_{\mathrm{eff}}$ and $K_i$ factor of each subcarrier. Throughout this paper we have mainly focused on the situation that $P_{\mathrm{total}} \le \sum_{j=1}^{L} P_j^{\mathrm{sc}}$. For $P_{\mathrm{total}} > \sum_{j=1}^{L} P_j^{\mathrm{sc}}$, the power allocation problem is modified to a problem with two constraints: each sub-channel power constraint (14) and interference power constraint (16). The total power constraint ceases to be significant in the optimisation procedure.

## 3 Proposed power loading scheme

We propose here a suboptimal algorithm to maximise capacity (12) under the constraints (13)–(16), outlined in four steps, out of which the first two steps are same as approached in most of the works [9, 11]. Step III and step IV represent the implementation of the new '*n-adjacent*' strategy for meeting the interference constraint. The performance of the proposed strategy is compared with some similar heuristic suboptimal schemes discussed in the above mentioned works such as step-ladder, scaling and nulling approaches which are capable of reducing the complexity of the allocation problem. In step ladder approach gradually increasing powers are assigned to subcarriers farther from the PU bands. In the scaling method, the subcarrier powers are scaled down by a factor to compensate the overshoot of PU interference. The nulling strategy of assigning zero power to adjacent subcarriers is excellent for maintaining interference limit, however it imposes capacity loss to the SU system. The basic objective in all these schemes is to allot more power to the subcarriers which are spectrally distant from the PU bands.

### 3.1 Step I: Fulfilment of total power constraint

Power is allocated to each subcarrier using traditional water-filling under total power constraint. The power of each subcarrier is determined as:

$$P_i^{(\mathrm{I})} = \max\left\{0,\ \frac{1}{\alpha} - h_i^{-1}\right\} \quad \forall i \in \{1, 2, \ldots, N\} \tag{17}$$

where $\frac{|G_i^{ss}|^2}{\sigma^2 + \sum_{j=1}^{M} J_i^j}$ denoted as $h_i$ is the effective carrier to noise ratio (CNR), α is the Lagrange constant and $(1/\alpha)$ is the water level calculated from

$$\sum_{i=1}^{N} \max\left\{0,\ \frac{1}{\alpha} - h_i^{-1}\right\} = P_{\mathrm{total}} \tag{18}$$

$P_i^{(\mathrm{I})}, \forall i \in \{1, 2, \ldots, N\}$ is passed to the next step.

### 3.2 Step II: Fulfilment of each sub-channel power constraint

Now, based on $P_i^{(\mathrm{I})}, \forall i \in \{1, 2, \ldots, N\}$, the $L$ number of sub-channels are divided in two sets $X$ and $Y$ such that if $S_j \le P_j^{\mathrm{sc}}$, then $j \in X$ and if $S_j > P_j^{\mathrm{sc}}$, then $j \in Y$. For empty $Y$, $P_i^{(\mathrm{II})} = P_i^{(\mathrm{I})}$ and is passed to step III. Otherwise, water-filling algorithm is executed for each sub-channel of set $Y$ with power budget equal to per sub-channel power constraint $P_j^{\mathrm{sc}}$ to get $|Y|$ number of different water levels for set $Y$. The powers of subcarrier belongs to set $X$ is also redistributed using (17) with a new power budget of $P_{\mathrm{total}} - \sum_{j \in Y} P_j^{\mathrm{sc}}$. The modified power vector of each subcarrier$P_i^{(\mathrm{II})}, \forall i \in \{1, 2, \ldots, N\}$ is passed to the next step.

### 3.3 Step III: Selection of n-adjacent group of subcarriers for power redistribution

Based on $P_i^{(\mathrm{II})}, \forall i \in \{1, 2, \ldots, N\}$, if $\sum_{i=1}^{N} P_i^{(\mathrm{II})} K_i \le I_{\mathrm{eff}}$ then $P_i^{(\mathrm{II})}, \forall i \in \{1, 2, \ldots, N\}$ is set as the final power allocation vector. Otherwise, depending upon the interference tolerance level $I_{\mathrm{eff}}$, the next strategy is to judiciously redistribute power only for those subcarriers which are adjacent to the PU bands. The subcarriers which are far away from the PU band usually do not contribute significant interference to PU. We introduce a term here, '*n-adjacent*' to denote the $n$ number of subcarriers adjacent to a PU band. Thus '*1-adjacent*' will mean the single subcarrier existing on both side of each PU spectrum. Similarly, if the powers of two consecutive subcarriers on either side of each PU spectrum need to be modified to meet the interference constraint, we refer the case as '*2-adjacent*', and so on. The following two sub-steps can be used to select the minimum value of $n$ and we start with $n = 1$.

(i) $I_{\mathrm{far}}$, the aggregate interference from the subcarriers that are not '*n-adjacent*' to the PU bands is computed

$$I_{\mathrm{far}} = \sum_{i=1}^{N} P_i^{(\mathrm{II})} K_i \quad i \ne n\text{-adjacent} \tag{19}$$

(ii) An interference decision variable $I_{\mathrm{dc}} = I_{\mathrm{eff}} - I_{\mathrm{far}}$, is introduced to check if $I_{\mathrm{dc}} > 0$: If $I_{\mathrm{dc}} < 0$, $n$ is incremented by one, and the above two sub-steps are repeated till $I_{\mathrm{dc}} > 0$.

In this paper the simulation results reported have been restricted to *3-adjacent* cases having considered a small subset of subcarriers. Although for relatively lower values of $I_{\mathrm{eff}}$, there is a need to consider higher values of '$n$' in practical OFDM based CR systems. The powers of the non-*n adjacent* subcarriers are kept unchanged and '*n adjacent*' subcarriers power are modified in next step.

### 3.4 Step IV: Power reallocation algorithm for the 'n-adjacent' group of subcarriers

The idea here is to treat the adjacent group of non-contiguous subcarriers as a separate sub-channel for which the effective interference threshold is $I_{\mathrm{dc}}$ and the reallocation strives to make

$$\sum_{j=1}^{N_a} K_j P_j = I_{\mathrm{dc}} \tag{20}$$

where $N_a = n(2M - 1)$ is the number of subcarriers in the '*n-adjacent*' group arranged in an ascending order with respect to their inverse CNR, i.e., $h_j^{-1} \le h_{j+1}^{-1}$ for any $j \in 1, 2, 3, \ldots, N_a - 1$ and $K_j$ and $P_j$ are respectively their interference factors and subcarrier powers. Our approach aims towards selective nulling or assigning of low powers to poor quality subcarriers. We would like to determine an optimal water-level $W_{\mathrm{o}}$ for the adjacent group of subcarriers such that the constraint (20) is strictly satisfied. The subcarrier with highest CNR, that is, $SC_1$ (Fig. 2) is initially allocated power $P_1$ corresponding to a water level of $h_2^{-1}$. This implies that $P_j = 0$, for $j = 2, 3, 4, \ldots, N_a$, i.e., zero power is assigned to the rest of the subcarriers of the *n- adjacent* group. There are now two possibilities for the value of $K_1 P_1$.

#### 3.4.1 Case I: $K_1 P_1 \ge I_{dc}$:

This case implies that water-level has to be decreased below $h_2^{-1}$ and power allocated to $SC_1$ is modified as $P_1 = (I_{\mathrm{dc}}/K_1)$.

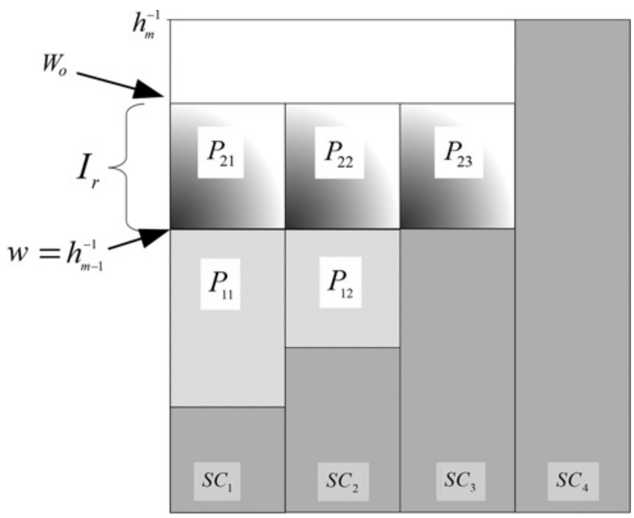


**Fig. 2** *Power distribution for the 'n-adjacent' group*


IET Commun., 2015, Vol. 9, Iss. 16, pp. 2033–2040

*3.4.2 Case II: $K_1P_1 < I_{dc}$:* This case implies that more number of subcarrier can now be allowed to take part in transmission. Hence water level has to be increased to $h_3^{-1}$ and power $P_1$ and $P_2$ are allocated respectively to $SC_1$ and $SC_2$ corresponding to this new water level.

If $K_1P_1 + K_2P_2$ is still less than $I_{dc}$, water-level is increased to $h_4^{-1}$ and so on. This procedure is repeated till the sum interference term of (20) just exceeds $I_{dc}$ or all the $N_a$ subcarriers have been considered. If a water-level corresponding to the inverse CNR of say, the $m$th subcarrier causes the sum interference term of (20) to just exceed $I_{dc}$, then finite amounts of power can be allocated to $N_x = (m-1)$ number of subcarriers for a water level that lies between $h_m^{-1}$ and $h_{m-1}^{-1}$. If the sum interference term is still less than $I_{dc}$ after having considered a water-level of $h_{N_a}^{-1}$, then the water level can be selected above $h_{N_a}^{-1}$ and corresponding powers can be allocated to $N_x = N_a$ numbers of subcarriers.

Let $P_{1m}$ denote the power allocated corresponding to the water level $w = h_{N_x}^{-1}$ for $m = 1, 2, \ldots, N_x$ and $\sum_{m=1}^{N_x} K_m P_{1m}$ be the corresponding interference. Then from Fig. 2, additional equal powers $P_{2m}$ can be distributed among the $N_x$ number of subcarriers so as to bridge the residual interference $I_r = I_{dc} - \sum_{m=1}^{N_x} K_m P_{1m}$, if any. Thus

$$P_{2m} = \frac{I_r}{\sum_{m=1}^{N_x} K_m}, \quad m = 1, 2, 3, \ldots, N_x \tag{21}$$

$P_j^{adj}$, the total allocated power for the $N_x$ number of subcarriers in the '*n-adjacent*' group will thus finally be

$$P_j^{adj} = P_{1m} + P_{2m}, \quad m = 1, 2, 3, \ldots, N_x, j = 1, 2, 3, \ldots, N_x \tag{22}$$

The optimum water level, $W_o$ of the '*n-adjacent*' group is obtained by adding $P_j^{adj}$ to its corresponding $h_j^{-1}$, $(j = 1, 2, \ldots, N_x)$. Once $W_0$ is known, we can easily find the power of adjacent subcarrier $P_i^{(III)}$ where '$i$' is '*n-adjacent*' subcarriers according to its original position.

For maintaining the overall sub-channel power constraint (14), the final power $P_i^a$ for the '*n-adjacent*' group will be

$$P_i^a = \min\left\{P_i^{(III)}, P_i^{(II)}\right\} \quad i = n\text{-adjacent subcarriers} \tag{23}$$

After step-II, $(|Y| + 1)$ numbers of water-levels are capable of meeting the $L$ sub-channel power constraints and hence finally there will be maximum $(|Y| + 2)$ number of water levels including the optimum water level $W_0$ selected as above. For instance, in Fig. 3, subcarriers 1, 6 and 7 are '*1-adjacent*' to the PU bands and the two sub-channels have different water levels, say $W_1$ and $W_2$ after step II. Water level of '*1-adjacent*' subcarrier is reduced to $W_0$ for fulfilling interference constraint. Hence, three water levels are used here in the final power allocation to satisfy all the constraints and achieving maximum spectral efficiency.

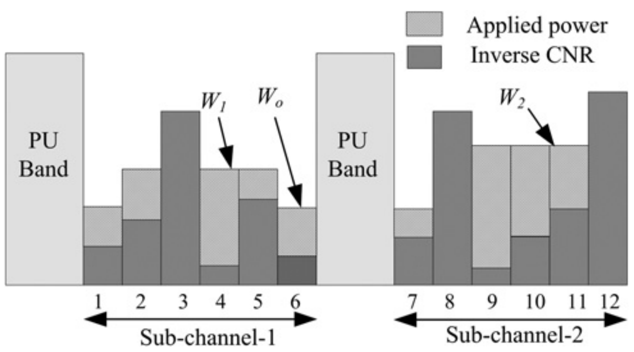


**Fig. 3** *Different water-levels for the adjacent and non-adjacent subcarriers*

**Table 1** Complexity comparison of different algorithms

| Different algorithms | Complexity |
|---|---|
| '*n-adjacent*' method | $O(NL) + O(n) + O(N_x)$ |
| suboptimal method | $O(LN) + O(N\log(N))$ |
| scaling method | $O(NL)$ |
| step-ladder method | $O(NL)$ |
| one-nulling method | $O(1)$ |

It is to be noted that for $W_o \le W_{min}^{(II)}$, interference constraint and for $W_o > W_{min}^{(II)}$, at least one sub-channel power constraint is satisfied strictly where $W_{min}^{(II)}$ be the minimum water level after step II among $(|Y| + 1)$ different water-level. Both the total power and sub-channel power constraints are met strictly if the algorithm ends at step-II.

In our system model, $N$ numbers of available subcarriers are grouped into $L$ sub-channels. In the worst case scenario, step-II can be solved in $O(N)$ and there would be maximum of $L$ iterations. Hence the complexity of step II is order of $O(NL)$. The complexity of step III wherein the minimum value of $n$ selected is $O(n)$ while step IV allocating power to $N_x$ number of subcarriers in the selected '*n-adjacent*' group is solvable in $O(N_x)$. As $n \ll N_x \ll N$ and step-II has the highest complexity among all the steps, worst case complexity of the algorithm is $O(NL)$. It is almost same as some of the reported methods as shown in Table 1. The one nulling method although has the lowest complexity suffers from very poor spectral efficiency.

## 4 Performance evaluation

The performance of the proposed algorithm has been validated and compared with some of the identified methods in literature such as the scaling, step-ladder and nulling schemes.

### 4.1 Scaling, step-ladder and nulling schemes

The scaling method as discussed in [11] scales down the subcarrier powers until interference constraint is met strictly as per the following

$$\boldsymbol{P}^s(t) = \frac{I_{eff}\boldsymbol{P}^s(t-1)}{\sum_{i=1}^{N} K_i P_i^{(II)}} \tag{24}$$

where, $\boldsymbol{P}^s(t)$ and $\boldsymbol{P}^s(t-1)$ are the subcarriers power of $t$th and $(t-1)$th iteration respectively.

The step ladder scheme considered in [11, 12] satisfies the interference constraint by assigning higher power to the subcarriers which are far from the PU bands, that is, in inverse proportion to their respective $K_i$ factors. The redistributed step-ladder power, $P_i^{sl}$ is

$$P_i^{sl} = \frac{P_{sl}}{K_i}, \quad \forall i \le N, i\forall j \tag{25}$$

where, $P_{sl}$ is the allocated power obtained after step II of meeting each sub-channel power constraint, i.e.,

$$\sum_{i \text{ s.t.} \phi(i)=j} P_i^{sl} = \sum_{i \text{ s.t.} \phi(i)=j} P_i^{(II)}$$

Another suboptimal algorithm discussed in [13] is also based on the $K_i$ factors dependent mainly on the spectral distances between SU subcarriers and PU band. This scheme distributes the power $P_i^s$ as

$$P_i^s = \frac{I_{th}}{2K_i N\lambda^2(-\ln(1-P_a))} \quad (i = 1, 2, \ldots, N) \tag{26}$$

The final power for $i$th subcarrier $P_i^{so}$ is allocated as

$$P_i^{so} = \min\{P_i^{s}, P_i^{(II)}\} \quad \forall i \tag{27}$$

Unlike '*n-adjacent*' method, this suboptimal method needs scaling of power until one constraint satisfies strictly. The nulling mechanism of allocating zero power to the '*n-adjacent*' subcarriers is perhaps the simplest method of restricting interference to the PU bands. However, it is at the cost of loss of sum rate capacity for the secondary user and sacrificing transmission opportunity for the adjacent subcarriers which may have excellent channel gain at their frequencies. In the current work, the performance of the proposed algorithm has been compared with the case of one-nulling method that assigns zero power to a single adjacent carrier on either side of the PU band.

### 4.2 Simulation parameters

We consider a simple OFDM based CR system for the SU accessing two opportunistic sub-channels each with six available subcarriers. The distribution of spectrum between the collocated PU and SU is as shown in Fig. 3. Sub-channel 1 comprises of the group of subcarriers from 1 to 6 while 7 to 12 comprise sub-channel 2. The channel is assumed to be Rayleigh faded with average channel power gain of 1. Numerical simulations have been performed with parameters $N = 12$, $P_a = 0.95$, $I_{th} = 1 \times 10^{-6}$ watt, $\lambda = 0.15$, $P_{total} = 5 \times 10^{-5}$ watt, $P_1^{sc} = 1 \times 10^{-5}$ watt, $P_2^{sc} = 4 \times 10^{-5}$ watt, $\sigma^2 = 1.5 \times 10^{-6}$, $\Delta f = 0.125$ MHz, $K_1 = 0.45$, $K_2 = 0.35$, $K_3 = 0.3$, $K_4 = 0.45$, $K_5 = 0.6$, $K_6 = 0.7$, $K_7 = 0.85$, $K_8 = 0.75$, $K_9 = 0.65$, $K_{10} = 0.4$, $K_{11} = 0.2$ $K_{12} = 0.1$, $J_i^1 = 6 \times 10^{-6}$ watts and $J_i^2 = 6.5 \times 10^{-6}$ watts. The results of capacity and PU interferences are obtained after averaging over 100, 000 independent simulation runs.

### 4.3 Simulation results and analysis

Fig. 4 shows the average achievable system capacity with respect to total power budget that is obtained for the proposed algorithm in comparison with the suboptimal, step-ladder, scaling and the one nulling methods for a fixed value of $I_{th}$. The suboptimal method referred here is the one proposed in [13] and it has been used without scaling. For lower power budget (approximately below $1 \times 10^{-5}$ watt) all the methods (except suboptimal and nulling mechanism) are almost equally efficient in terms of SU sum capacity, as interference constraint is easily satisfied. Beyond a certain level of power budget, the maximum SU sum capacity tends to saturate as interference constraint dominantly restricts the allocation of higher powers to all the individual subcarriers. Notably, the proposed algorithm is better than the suboptimal method and scaling techniques. The higher capacity outputs

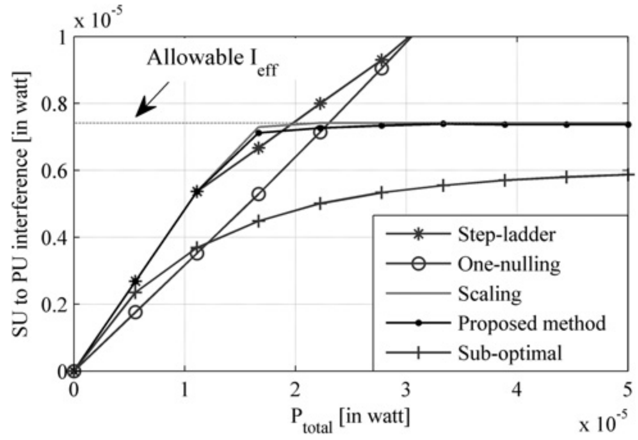


**Fig. 5** *Introduced interference by proposed algorithm and different power allocation algorithm with respect to total power budget ($I_{th} = 1 \times 10^{-6}$ watt)*

obtained for the step-ladder and nulling method in the higher power budget regions are actually not useful in the CR system as the corresponding interference presented to the PU by the SU subcarriers actually exceed the maximum allowable limits.

This is clear from Fig. 5 comparing all the methods in terms of the average interference introduced to PU by SU for the same range of power budget and $I_{th}$. For instance, in Fig. 4, capacity output for the step-ladder method starts increasing beyond a power budget of around $2 \times 10^{-5}$ watt but in Fig. 5, it starts violating the interference constraint near about that value of total power. The strength of the proposed algorithm in maintaining the interference constraint even for high total power budget is clearly established. The scaling method is also equally effective in meeting the interference constraint as the proposed one, but the average sum capacity of the latter is slightly more.

The SU to PU interference however, must be dependent on $I_{th}$, the PU tolerance limit of the SU sub-channels and Fig. 6 gives a good measure of the strong impact of this single factor on the SU sum interference. It is observed that our proposed '*n-adjacent*' algorithm, when executed for a fixed value of $I_{th}$ ($1 \times 10^{-6}$ watt), is capable of determining a maximum value of total power budget that can maintain the PU interference constraint. Beyond this value as shown by $P_{max}$ in Fig. 6, the power allocation is modified by step 3 to increase the value of $n$. For example *2-adjacent* is unable to meet $I_{th} = 1 \times 10^{-6}$ watt beyond $P_{max} = 2.75 \times 10^{-5}$ watt, but adopting *3-adjacent* based allocation solves the problem. This is an additional benefit of our proposed algorithm that without compromising on capacity, interference constraints can be met strictly for even high levels of power budget.

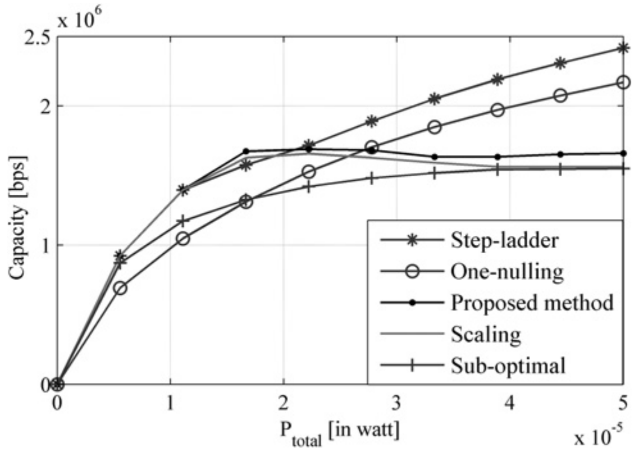


**Fig. 4** *Relation between system capacity verses power budget for proposed algorithm and different power allocation algorithm ($I_{th} = 1 \times 10^{-6}$ watt)*

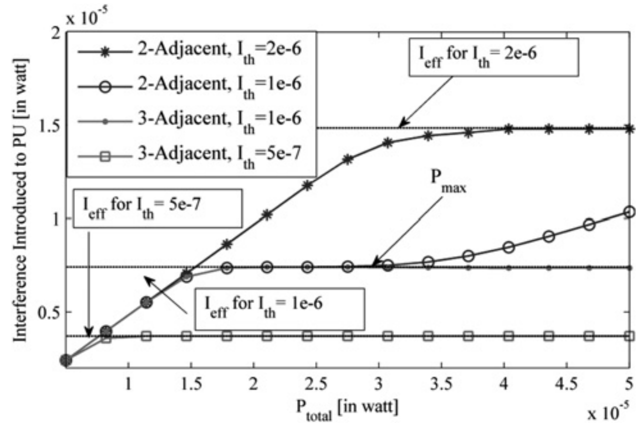


**Fig. 6** *Relation between total power and interference introduced to PU for different interference threshold*

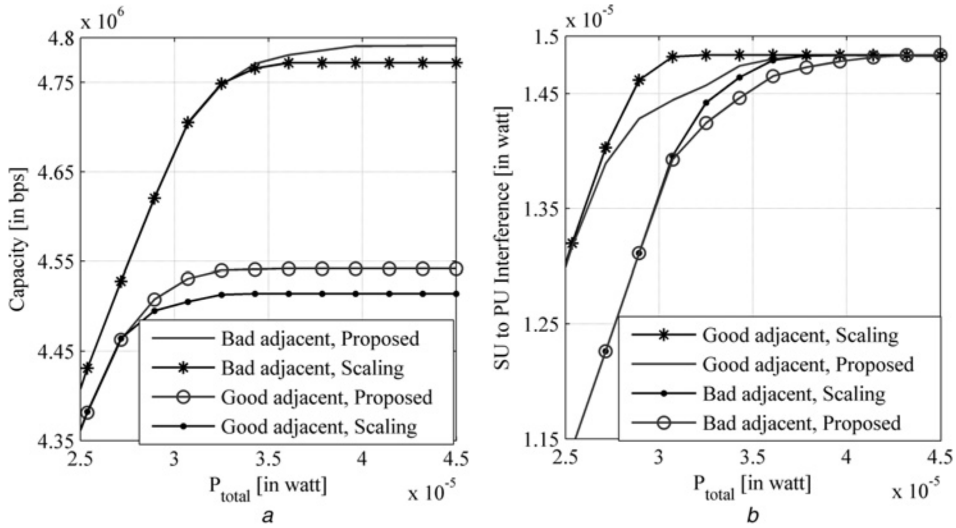


**Fig. 7** *Performance comparison of proposed and scaling method for both the 'good' and 'bad' adjacent in terms of*

*a* Capacity and
*b* Interference

The SU sum capacity and interference performances of our proposed algorithm are almost same as the scaling method for generalised Monte Carlo simulations run for Rayleigh faded channels of unit average power. However, when the 'channel quality' of the SU subcarriers adjacent to the PU band are taken into consideration, the benefit of considering the proposed '*n-adjacent*' method is clearly established as depicted through Figs. 7 and 8. Two cases have been considered: (i) Good adjacent: when the subcarriers of the '*n-adjacent*' group are having higher channel gains as compared with the non-adjacent ones. (ii) Bad adjacent: when the subcarriers of the '*n-adjacent*' group are having relatively poorer channel gains as compared with the non-adjacent ones. Fig. 7 shows how the two methods (proposed and scaling) fare in terms of capacity and interference for a particular value of $I_{th} = 2 \times 10^{-6}$ watt, under these two cases. The following observations are noted in reference to Figs. 7*a* and *b*:

(i) The overall capacity is more influenced by the quality of non-adjacent subcarriers. For both the methods, the achievable SU capacity is thus seen to be much less for the 'good adjacent' case as compared with the 'bad adjacent' one. This is because, in the 'bad adjacent' case, the non-adjacent subcarriers having higher gains and low values of $K_i$ are all allocated high values of power. The adjacent subcarriers, on the other hand, with poor gains and high $K_i$ factors have zero or low values of allotted power.
(ii) The SU sum interference is more influenced by the quality of adjacent subcarriers. For both the scaling and '*n-adjacent*' methods, the interference is much less when the adjacent subcarriers are 'bad', as this usually leads to assignment of relatively lower values of power and vice versa.
(iii) The proposed method shows significant improvement of capacity as well as reduction of total PU interference in comparison with the scaling method for both the 'good adjacent and 'bad adjacent' cases.

The magnitude of capacity improvement and PU interference reduction as compared with the 'scaling' method has been computed by taking their respective differences and the corresponding measures shown through Figs. 8*a* and *b*. Two noteworthy observations here are:

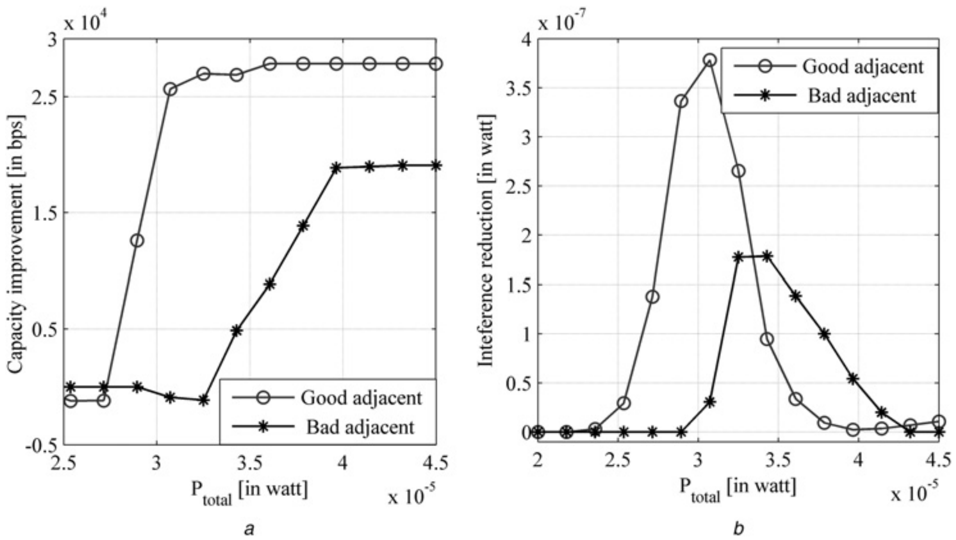


**Fig. 8** *The magnitude of performance enhancement of proposed method over scaling for both the 'good' and 'bad' adjacent subcarriers in terms of*

*a* Capacity improvement and
*b* Interference reduction

(i) The capacity improvement and PU interference reduction of the proposed method over the scaling method is significantly higher for the good adjacent case in comparison with the 'bad adjacent' case for a certain range of total power budget. For example, in Fig. 8*a*, between a total power budget of $3.5 \times 10^{-5}$watt to $4.5 \times 10^{-5}$ watt, the use of the '*n-adjacent*' algorithm improve the capacity approximately 1.5 times if the channel gains are relatively higher for adjacent subcarriers bands. On the interference front too, a similar trend is observed.
(ii) The total power budget values for which maximum improvement in capacity and reduction in interference takes place are not same. This implies that the analysis for the entire power range will be extremely useful in deciding the most optimal value of total power budget that should be used. Dynamic channel state information (CSI) is usually available through channel estimation techniques applied over successive OFDM frames and depending on the 'quality' of the adjacent subcarriers, power allocation may also be changed suitably.

## 5 Conclusion

In this paper we introduce a new heuristic sub optimal power allocation algorithm for interference free coexistence of OFDM based CR users sharing spectrum is a joint interweave and underlay fashion. Termed as '*n-adjacent*', the strategy determines a minimum number of adjacent subcarriers near the PU band whose power is judiciously assigned with a water-level that is mostly different from the non-adjacent subcarriers of the same sub-channel. Numerical simulation results show that the proposed '*n-adjacent*' power reallocation based water-filling in OFDM can efficiently maintain high SU sum capacity without exceeding a specified PU interference tolerance limit for a wide range of total power budget. The capacity and interference metrics of the proposed algorithm are shown to be better when compared with the suboptimal method of [13], one nulling, and step-ladder method. The complexity is almost same as these methods. When the variation of channel gain over all the subcarriers is not very high, then the performance of the proposed method in meeting the interference constraint is almost same as that of the scaling approach of [11], although capacity wise, our method is slightly better. However, when the channel qualities of subcarriers adjacent to PU bands are either 'good' or 'bad' in comparison with the non-adjacent ones, the additional benefits of the '*n-adjacent*' algorithm is clearly established over the scaling method in terms of increased SU capacity and reduced PU interference. By the use of higher values of '*n*', the '*n-adjacent*' algorithm also enables CR users to share spectrum with PU at low interference tolerance levels, without having to sacrifice their own capacity. The simple power loading algorithm proposed here is quite efficient as a powerful application for OFDM based CR systems. The impact of actual $K_i$ interference factors tested for standard OFDM system model and analysis related to the imperfect CSI is currently a part of ongoing work.

## 6 References


1 Federal Communications Commission: 'Spectrum policy task force'. Report ET Docket no. 02-135, November 2002
2 Ian, F.A., Won-Yeol, L., Mehmet, C.V., *et al.*: 'NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey', *Comput. Netw.*, 2006, **50**, (13), pp. 2127–2159
3 Beibei, W., K., J.R.L.: 'Advances in cognitive networks: a survey', *IEEE J. Sel. Topics Signal Process.*, 2011, **5**, (1), pp. 5–23
4 Simon, H.: 'Cognitive radio: brain-empowered wireless communications', *IEEE Trans. Sel. Areas Commun.*, 2005, **23**, (2), pp. 201–220
5 Hisham, A.M., Tevfik, Y., Hüseyin, A.: 'OFDM for cognitive radio: Merits and challenges', *IEEE Wirel. Commun.*, 2009, **16**, (2), pp. 6–15
6 Tuan, L.V., Hieu, D.C., Kinh, N.V., *et al.*: 'Full-filling subcarrier power allocation in OFDMA-based cognitive radio systems', *Wirel. Eng. Tech.*, 2014, **5**, pp. 11–18
7 Mohammed, A., Pei, X., Muhammad, A.I., *et al.*: 'Low complexity subcarrier and power allocation algorithm for uplink OFDMA systems', *EURASIP J. Wirel. Commun. Netw.*, 2013, **98**, pp. 1–6
8 Kibeom, S., Mohseni, M., Cioffi, J.M.: 'Optimal resource allocation for OFDMA downlink systems'. Proc. IEEE Int. Conf. on Information Theory, Seattle, May 2006, pp. 1394–1398
9 Peng, W., Ming, Z., Limin, X., *et al.*: 'Power allocation in OFDM-based cognitive radio systems'. Proc. IEEE Int. Conf. GLOBECOM, Washington, November 2007, pp. 4061–4065
10 Peng, W., Xiaofeng, Z., Limin, X., *et al.*: 'A general power allocation algorithm for OFDM-based cognitive radio systems'. Proc. IEEE Int. Conf. on Communication, Dresden, June 2009, pp. 1–5
11 Ziaul, H., Gaurav, B., Ekram, H., *et al.*: 'Energy-efficient power allocation in OFDM-based cognitive radio systems: a risk-return model', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (12), pp. 6078–6088
12 Gaurav, B., Md. Jahangir, H., Vijay, K.B.: 'Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (11), pp. 4710–4718
13 Gaurav, B., Md. Jahangir, H., Vijay, K.B.: 'Adaptive power loading for OFDM-based cognitive radio systems with statistical interference constraint', *IEEE Trans. Lett.*, 2011, **10**, (9), pp. 2786–2791
14 Timo, W., Loerg, H., Albert, K., *et al.*: 'Mutual interference in OFDM-based spectrum pooling systems'. Proc. IEEE Int. Conf. on Vehicular Techology, May 2004, pp. 1873–1877
15 Tariq, A., Michael, S., Lutz, L.: 'Single and multiple carrier designs for cognitive radio systems'. Proc. IEEE Int. Conf. on Communication, Cape Town, May 2010, pp. 1–6
16 Jianwu, Z., Jingxiu, C., Xiaorong, X.: 'Worst subcarrier avoiding water-filling subcarrier allocation scheme for OFDM-based CRN', *J. Electron. (China)*, 2012, **29**, (3–4), pp. 205–210
17 Ivan, C., Sinja, B., Michael, S.: 'Subcarrier weighting: a method for sidelobe suppression in OFDM systems', *IEEE Commun. Lett.*, 2006, **10**, pp. 444–446
18 Qilin, Q., Andrew, M., Yaoqing, L.Y.: 'An efficient water-filling algorithm for power allocation in OFDM-based cognitive radio'. Proc. Int. Conf. on Systems and Informatics, Yantai, May 2012, pp. 2069–2073
19 Thomas, M., Cover, J.T.: 'Elements of information theory' (John Wiley and Sons, 2006, 2nd edn.)



*IET Commun.*, 2015, Vol. 9, Iss. 16, pp. 2033–2040