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Spectrum combinatorial double auction for cognitive radio network with ubiquitous network resource providers

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Abstract: Spectrum auction is an emerging economic scheme to stimulate both primary spectrum operators (POs) and secondary users (SUs) to be involved in spectrum sharing. Previous spectrum auction works mostly assume each PO can only have one type spectrum or each SU can only buy homogeneous spectrum bands from the same PO. However, in a ubiquitous network scenario, each PO possesses heterogeneous spectrum resources such as WiFi, 3G and each SU may request different types of spectrum bands from the same PO. Existing auction schemes cannot be used to effectively solve the problem. Therefore, the authors come out with a lightweight combinatorial double auction to tackle this challenge. Since spectrum combinatorial double auction problem is NP-hard, the authors develop a general greedy algorithm G-Greedy to solve the problem. Inspired by the recent group-buying discounts, they also invent an enhanced scheme E-Greedy to further optimise total utility. They theoretically prove the economy properties of the proposed schemes such as individual rationality, budget balance and truthfulness. Simulation results show that both of the two algorithms can yield higher utilities and are effective.

1 Introduction

With tremendously increasing number of ubiquitous wireless communication devices, spectrum resource has become scarcity. On one hand, due to the static allocation of wireless bands, some spectrums are not fully utilised. On the other hand, spectrum starvation is becoming ever more severe. Cognitive radio (CR) [1] is a promising paradigm to mitigate the contradiction between spectrum under-utilisation and starvation problems. With the capability to sense, detect and access the frequency bands that are not currently being occupied, the CR technology permits secondary users (SUs) to exploit unused bands owned by primary users (PUs) to enhance spectrum utilisation efficiency.

To optimally allocate and utilise spectrums in CR networks (CRNs), there have been extensive literatures dealing with this issue. Traditional methodologies such as game model and Markovian decision process on opportunistic spectrum access can be found in a recent survey [2]. Different from those in [2], auction-based spectrum allocation in CRNs is an essential requirement mainly for three reasons: first, it is efficient [3] to handle spectrum allocation, which stimulates methodological innovation from an economical point of view [4]. Second, with SUs' changing transmission requirements and PUs' on–off behaviours, auction can handle market changes with perceived fairness and allocation efficiency [5]. What is more, compared with most game models in which statistical information of primary channels is a priori request [2], auction is more suitable for dynamic spectrum supply and demand. Third, although economic tools such as price [6], game theory [7] and auction have been widely applied on spectrum marketing in recent years [8], auction is the preeminent due to its fairness and efficiency.

In this paper, we mainly study spectrum auction problem with heterogeneous spectrum providers in a ubiquitous network setting. This can be characterised as multiple primary operators (POs) with multiple SUs and each PO has multiple types of spectrum bands, which is different from one PO that has different bands but not different types. Although in the latter network (one PO has different bands but not different types) scenario, each homogeneous spectrum owned by one PO may be divided into multiple sub-bands; however, each PO possesses the same kind of spectrum. When PUs utilising that primary spectrum come back, SUs should vacate the spectrum immediately. This may cause additional network delay and congestion. The motivation of this work (one PO has multiple types of spectrum bands) is multi-fold. First, with the explosively growing number of wireless communication devices such as smartphones, laptops and tablets, bandwidth demand is becoming larger than ever before. Mobile data offloading for cognitive M2M communication [9] is able to deal with burst bandwidth requirement by delivering part of application data traffic which is originally flowed to the cellular network over other networks such as TV broadcast networks. In this way, congestion is alleviated and spectrum utilisation efficiency is achieved by CR technology [10]. Second, when each PO has multiple types of spectrums and each SU may bid a combination of different types of spectrums from the same PO (Just as same as [11], we assume each SU can only get channels from the same PO.), SUs' transmission delay can be mitigated due to spectrum redundancy in case of some primary bands are interrupted by the corresponding PUs. Third, due to the fast mobility of SUs, for example, in a cognitive vehicular network, vehicles on the road that act as SUs may traverse through multiple POs. Therefore, SUs may bid from a set of POs, which is much more practical.

To solve spectrum combinatorial double auction problem with the scenario proposed in this paper, existing methods cannot be directly applied. The work in [11] was the first work to study sellers' heterogeneity, however, it only considered buying the same spectrum from the same PO, which was not practical. As in real situations, one SU may require different types of spectrums such as data offloading [12, 13]. For combinatorial double auction, PUs send their redundant spectrums to auction agents which are named as POs for profits and SUs act as buyers buy the resource from POs. Existing methods like [14] proposed a subgradient algorithm to find near optimal solutions to the combinatorial double auction problem. The problem in [14] was decomposed into multiple subproblems and the subgradient algorithm was used to iteratively derive the optimal assignment. However, auction was not used and each bidder could be attached to multiple sellers. In this paper, we focus on the scenario where each SU bidder can only be attached to one PO. In [11], a spectrum combinatorial auction scheme was proposed. However, each PO in [11] had homogeneous bands and each SU bids for a bundle spectrums of different types from a combination of POs. Although [15] formulated the grid resource combinatorial double auction problem as a traditional winner determination problem, the algorithm was suitable for the scenario proposed in [11], and not for this paper. As just as same as [14], each SU in [15] may attach to multiple POs.

Multi-round executions of methods like McAfee scheme [16] for the homogeneous scenario are not effective for this paper, because the execution result of the corresponding algorithm may be one SU that attaches to multiple POs. However, for the heterogeneous scenario in this paper, the spectrum bundle is acquired from one PO. How to efficiently allocate the spectrum bundles to SUs when there are large numbers of POs and SUs in the network system is the main challenge. In real situations, for example, in urban cities mobile devices such as smartphones or on-board devices [17] in a vehicular network have limited computational ability and they are moving very fast. Efficient algorithms should be invented to cope with spectrum allocation problem before those devices moving out of the service scope of POs. Second, in a typical auction mechanism, auction properties like truthfulness, budget balance and so on should be guaranteed. Therefore, how to design an efficient combinatorial double auction to meet these economic properties is another challenge. Third, during the auction scheme, SUs pay for the bidding channels to meet their requirements and POs sell the PUs' channels for profits. How to stimulate and attract spectrum buyers to participate in spectrum auction and enhance total utility in the auction should be carefully considered. To solve the above challenges, we propose two lightweight combinatorial double auction schemes for spectrum allocation.

The major contributions of this paper can be summarised as follows:

- We first analyse the situation where there are multiple different types spectrum bands owned by one PO in the ubiquitous network, then we formulate the spectrum combinatorial double auction problem into a binary integer programming problem. To reduce the computational complexity, we design a polynomial time algorithm, the G(General)-Greedy algorithm to solve the total utility maximisation problem.
- By relaxing the original problem formulation equations, through experiment, we examine the upper bound of total utility of the spectrum combinatorial double auction mechanism.
- Motivated by the group-buying discount in electronic commerce [18], where sellers offer products and services at significantly reduced price on condition that the quantity of buyers reaches to a certain number, we propose a group discount spectrum allocation scheme, the E(Enhanced)-Greedy algorithm to further optimise total utility. This scheme can attract SUs to participate in spectrum auction, which is more practical.
- The proposed two greedy algorithms are efficient because of their polynomial time complexity running time. We prove that the proposed auction algorithms are truthful, budget balanced and has individual rationality.
- We design various experiments to examine the relationships between total utility and the number of PUs and SUs. Both allocation satisfaction ratio and allocation efficiency have been carefully studied. Simulation results show, both G-Greedy and E-Greedy algorithms can yield higher utilities and are effective. This work can be seen as a methodological innovation in spectrum auction.

The rest of the article is structured as follows. Section 2 introduces related works on auction schemes. Section 3 describes the system model and we formally define the auction problem. Algorithms for the auction scheme are illustrated in Section 4. We evaluate the proposed algorithms in Section 5 and Section 6 concludes this paper.

2 Related works

Spectrum auction as an efficient spectrum allocation method has been studied in various literatures. Generally, there are four main categories of spectrum auctions: single, double, combinatorial and combinatorial double auctions. Compared with single auction [19–23], double auction is more suitable due to its fairness and efficiency [24]. In a double auction, both buyers and sellers submit their bids to a central auctioneer who will determine winners, charge for them and pay for sellers. For combinatorial auction [25, 11], SUs can buy a combination of spectrums from POs. The work in [25] was the first to examine heterogeneous demands of time–frequency resources from the perspective of buyers but neglects the heterogeneous spectrum offerings from PUs.

Lin *et al.* [26] introduced a flexible spectrum auction scheme to maximise social utility. In [27], time slots were divided and traded to meet both long-term and short-term SUs' demands. However, both of them were confined to the situation where there is only one spectrum seller (or PO) in the system. In the real situation, *there are always many providers and it is hard for SUs to determine which provider to bid.* For double auctions, most papers failed to regard spectrum bands as non-identical resources. The work in [28] was the first to consider spectrum reusability, while [22] also took SUs' spectrum reusability into consideration and each SU could bid for multiple bands. Li *et al.* [29] proposed a double auction scheme where SUs could bid for different number of channels and diverse time slots from the same PU. In [30], a multi-unit double auction scheme was developed to meet SUs' partial demands. The authors assumed that each PU possessed multi-unit spectrum sub-bands provided that all the sub-bands were identical. In reality, *each PO may have multiple types of spectrums and this makes it hard to design efficient spectrum allocation mechanisms based on auction theory.* In [25], the spectrum was reused in a time–frequency division manner and the problem was modelled as a combinatorial auction. However, the proposed method cannot tackle heterogeneous spectrums. Although [31] was the first to study multiple spectrum owners and [32] examined the tradeoff between auction efficiency and robustness with multiple SUs and multiple PUs, none of them aimed at multiple types spectrum bands. The work in [7] was the first to tackle combinatorial double auction using game theory. The computational complexity on solving differential equations makes it difficult to be really deployed. Different from [7], we focus on designing lightweight heuristic algorithms for spectrum combinatorial double auction with each PO possesses multiple types of spectrums. Meanwhile, we try to design an efficient mechanism to attract SUs buying the vacant spectrums from POs.

Another research topic related to this work is group-buying scheme, which is an emerging electronic commerce model [33]. However, there are few works applying group-buying discount to specific research fields such as spectrum auction. In 2013, Lin *et al.* [34] first introduced group buying into the spectrum market. He [34] proposed a three-stage auction framework for spectrum group buying to conquer the difficulty that one single SU could not afford a whole channel whereas a group of SUs could buy. This cannot be directly used in the setting of this paper where each one of SUs should obtain the demanded resource independently. Therefore, different from existing work, we utilise the group-buying discount scheme to offer spectrum resources at a cheaper price to encourage SUs to participate in the auction.

3 System model and problem formulation

3.1 System model

We consider a CRN system in a ubiquitous network where there are multiple spectrum sellers with heterogeneous spectrums and multiple CR users with differentiating spectrum demands shown in Fig. 1. We assume that each PU is operated by a PO and each PO possesses multiple types of spectrums from different wireless networks in a ubiquitous network setting. Without loss of

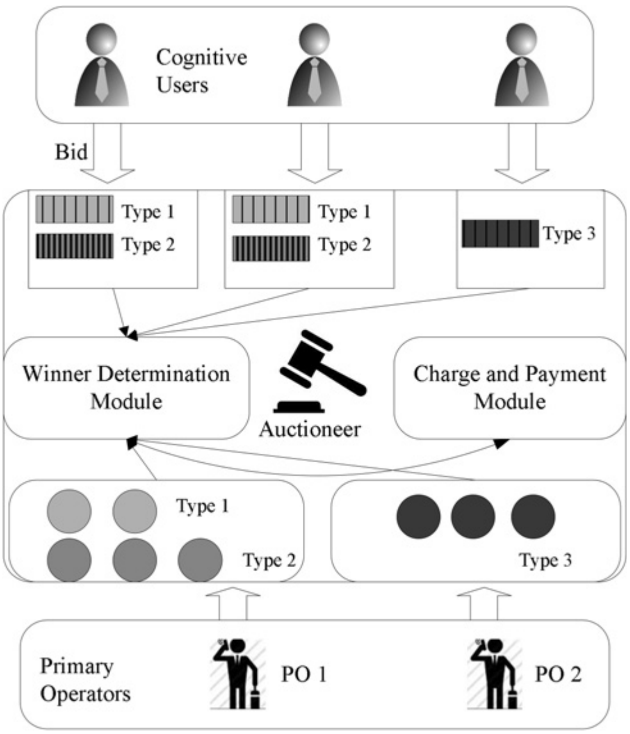

Fig. 1 *System model for combinatorial spectrum double auction*

generality, we assume there are K types of spectrum bands owned at each PO which are distinguished by their spectrum bandwidth. For example, 6 MHz for TV band and 200 KHz for GSM band [11]. To buy and sell the spectrums, POs and SUs should submit their bids to a central auctioneer. The auctioneer is in charge of collecting bids, computing and allocating spectrums, charging for SUs and paying for POs.

In the system, there are m POs (sellers) and n SUs (buyers). The set of buyers' bids is denoted by B and $B = \{B_1, B_2, \ldots, B_n\}$. The ith element B_i in the set B denotes the bid set of the ith SU, where $i \in \{1, 2, \ldots, n\}$. For each B_i, $i \in \{1, 2, \ldots, n\}$, is denoted by a three-tuple $B_i = \{\boldsymbol{d_i}, t_i, v_i\}$ where $\boldsymbol{d_i} = (d_i^1, d_i^2, \ldots, d_i^K)$. The element d_i^k in $\boldsymbol{d_i}$ means the number of the kth type spectrum bands demanded by the ith SU for all $k \in \{1, 2, \ldots, K\}$. t_i denotes the time span that the ith SU wants to reserve. v_i is the truthful value evaluated by the ith SU. Just as [35, 25], we assume that all the SUs in the system are single minded which means each SU can only submit one channel bundle.

Let S denote the POs' bids and $S = \{S_1, S_2, \ldots, S_m\}$. The m elements in set S are from m different POs. The bid of the jth PU is denoted by S_j for $j \in \{1, 2, \ldots, m\}$. $S_j = (\boldsymbol{o_j}, \boldsymbol{q_j}, \boldsymbol{\theta_j})$ where $\boldsymbol{o_j} = (o_j^1, o_j^2, \cdots, o_j^K)$ and $\boldsymbol{q_j} = (q_j^1, q_j^2, \cdots, q_j^K)$. o_j^k is the kth type spectrum owned by the jth PO. q_j^k is the kth type spectrum's unit price per time slot at the jth PO. In this way, the price of the heterogeneous spectrum bundle can be calculated. For example, there are three kinds of spectrum bands, they are 3G, WiFi and 2G with unit price vector {\$0.3, \$0.4, \$0.5}. Each element in the set from left to right denotes the unit price of 3G spectrum, WiFi spectrum and 2G spectrum accordingly and forms a spectrum bundle. Therefore, the total price of one bundle containing one 3G channel, two WiFi channels and three 2G channels is \$2.6 and is calculated as $1 \times \$\,0.3 + 2 \times \$\,0.4 + 3 \times \$\,0.5$.

3.2 Problem formulation

Let M denote the set of POs in the network system and $|M| = m$. Let N denote the set of SUs where $|N| = n$. After executing the auction algorithm at the auctioneer, an allocation matrix can be formed. The allocation matrix is an $N \times M$ matrix and is denoted by $\boldsymbol{X}_{n \times m}$. Each element in matrix $\boldsymbol{X}$ is denoted as x_{ij} and is defined as

$$x_{ij} = \begin{cases} 1, & \text{if user } i \text{ is associated to PO } j \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

We assume SUs are all single minded and it is hard for a SU to get spectrum resources from different POs [36]. Therefore, each SU can acquire at most one bid, which means one single SU can only get the demanded spectrum bands from the same PO. Hence

$$\sum_{j=1}^{m} x_{ij} \le 1, \quad \forall 1 \le i \le n \tag{2}$$

Since spectrum bands are limited at each PO, the sum of the requested spectrum bands should not exceed the spectrum bandwidth limit of the PO. Thus

$$\sum_{i=1}^{n} x_{ij} d_i^k \le o_j^k, \quad \forall 1 \le j \le m, \quad 1 \le k \le K \tag{3}$$

Let c_i denote the auctioneer's final charging price for the ith SU. Then, the utility of buyer SU i is

$$u_i = \sum_{j=1}^{M} x_{ij}(v_i - c_i) \tag{4}$$

where $v_i = \sum_k d_i^k (1 + \gamma_i) t_i$. In [11], the valuation of SU j is defined as $v_j = d_j \log\,(1 + \gamma_j)$, where d_j is the bandwidth demand, γ_j is the signal-to-noise ratio at SU j. This definition is based on the channel capacity expression which is common in spectrum auction literatures. Like [33], the valuation of the virtual machine contains a time length factor, in this paper we consider each SU may occupy the channel for some time, thus the time length should be considered as part of the valuation function. The longer the SU occupies the spectrum bands, the higher valuation it is. In this paper, for SU i, the valuation of the kth spectrum is denoted as $d_i^k(1 + \gamma_i)t_i$, then the total valuation for SU i is $\sum_k d_i^k(1 + \gamma_i)t_i$. Let p_j denote the final payment to the jth PO by the auctioneer. Similarly, the utility of seller PO j is

$$\begin{aligned} u_j &= p_j - v_j \\ &= p_j - \sum_{i=1}^{N} x_{ij} \sum_{k=1}^{K} (d_i^k q_j^k t_i) \end{aligned} \tag{5}$$

Therefore, given the bids vector of SUs $\boldsymbol{B}$ and the bids vector of POs $\boldsymbol{S}$, the target is to design efficient auction algorithms to determine the spectrum allocation matrix $\boldsymbol{X}$. In this way, the charge vector $\boldsymbol{C}$ and payment vector $\boldsymbol{P}$ can also be determined. It should be noted that there are three main optimisation objects of combinatorial double auction. The first is to maximise the utility of sellers, the second is to maximise the utility of the auctioneer and the third is to maximise the total utility of both buyers and sellers. Thus, in this paper we concentrate on the third optimisation target. Hence, the optimisation target of this scheme can be formally described as

$\underline{\mathcal{P}1}$:

$$\max\,(\sum_{i=1}^{N} u_i + \sum_{j=1}^{M} u_j) \tag{6}$$

$$\text{s.t.} \quad (\text{C1})\text{:} \sum_{j=1}^{m} x_{ij} \le 1, \quad \forall 1 \le i \le n \tag{7}$$

$$\text{(C2)}: \sum_{i=1}^{n} x_{ij} d_i^k \le o_j^k, \quad \forall 1 \le j \le m, \quad 1 \le k \le K \tag{8}$$

$$\text{(C3)}: x_{ij} \in \{0, 1\}, \quad \forall 1 \le i \le n, \quad 1 \le j \le m \tag{9}$$

where (6) is the solution target. Constraint C1 denotes that each SU can only be served by one PO at one time. C2 is the total capacity constraint at each PO. C3 is the binary constraint. According to [37, 25], the combinatorial auction is NP-hard, thus the problem defined by (6)–(9) is also NP-hard.

3.3 Economic properties

The goal of this paper is to design an efficient combinatorial double auction scheme to best achieve the following economic properties: individual rationality, budget balance and truthfulness.

(i) *Individual rationality*: An auction is individually rational if no winner's utility is negative.
(ii) *Budget balance*: For a particular auction, budget balance requires that the auctioneer's revenue is not negative. This property is utilised to motivate the auctioneer to participate in the auction scheme.
(iii) *Truthfulness*: An auction is truthful if given other players' strategy profile and the fixed auction scheme, a player cannot improve its utility by submitting any bid that is not the same with its true bid.

On the basis of (6)–(9), in the next section, we will derive the upper bound of $\mathcal{P}1$ and propose a polynomial-time approximation algorithm. Then, we design two greedy combinatorial double auction schemes and compare the performance of the above algorithms.

4 Spectrum allocation mechanism

4.1 Upper bound

In this subsection, we derive the upper bound of $\mathcal{P}1$ based on the method proposed in [11]. It should be noted that the complexity of $\mathcal{P}1$ comes from the fact that for a demanded spectrum resource, the number of different types of spectrums requested from the PO is different. Therefore, to ease of solution, we try to relax constraint C2 by allowing SUs to bid fractional number of channels as

$$\sum_{i=1}^{n} \sum_{k=1}^{K} d_i^k x_{ij} \le \sum_{k=1}^{K} o_j^k, \quad \forall 1 \le j \le m \tag{10}$$

Define $(\pi_1, \pi_2, \ldots, \pi_m)$ as positive multipliers. Then, the relaxed problem of $\mathcal{P}_1$ can be written as

$\underline{\mathcal{P}_2}$:

$$\max \quad \sigma = \left(\sum_{i=1}^{N} u_i + \sum_{j=1}^{M} u_j\right) \tag{11}$$

$$\text{s.t.} \quad \text{(C1}'\text{)}: \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \sum_{k=1}^{K} d_i^k x_{ij} \le \sum_{j=1}^{m} \pi_j \sum_{k=1}^{K} o_j^k, \quad \forall 1 \le j \le m \tag{12}$$

$$\text{(C2}'\text{)}: \sum_{i=1}^{n} x_{ij} d_i^k \le o_j^k, \quad \forall 1 \le j \le m, \quad 1 \le k \le K \tag{13}$$

$$\text{(C3}'\text{)}: x_{ij} \in \{0, 1\}, \quad \forall 1 \le i \le n, \quad 1 \le j \le m \tag{14}$$

Therefore, the optimal value of π_j would maximise the target in $\mathcal{P}2$. According to [11], the optimal choice of π_j for all $j \in m$ is $\pi_j = \omega$, where ω can be any positive constant. To further simplify the solution target, we assume the bidding price of the buyer equals its true valuation. Let $x_i' = \sum_{j=1}^{m} x_{ij}$, which denotes whether the ith buyer wins the auction or not. Therefore, the problem defined in $\mathcal{P}2$ can be rewritten as

$\underline{\mathcal{P}3}$:

$$\max \quad \sigma = \sum_{i=1}^{n} x_i' \sum_{k=1}^{K} d_i^k q_j^k t_i \tag{15}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{k=1}^{K} d_i^k x_i' \le \sum_{k=1}^{K} o_j^k, \quad \forall 1 \le j \le m \tag{16}$$

$$x_i' \in \{0, 1\}, \quad \forall 1 \le i \le n \tag{17}$$

According to the relaxed problem $\mathcal{P}3$, by adopting integer programming algorithm or branch and bound algorithm, the solution to $\mathcal{P}3$ can be derived. However, due to the computational complexity during the problem solving procedure, in the following subsections, we first present an approximation algorithm to derive the upper bound, then we present two greedy spectrum combinational double auction mechanisms used to solve the original problem.

4.2 Approximation algorithm to derive the upper bound

In this subsection, we present an approximation algorithm to derive the upper bound of the solution target. After receiving all the bids from SUs, the auctioneer will sort the bids of buyers in a decreasing order while sorting the total number of spectrums of each PO in an increasing order. Different from [38, 39], in this paper we define the 'size' of the bid bundle B_i as $s_i = \sum_{k=1}^{K} w_k d_i^k t_i$. w_k is the weight [39] of the kth type spectrum. Then, the bid density for SU i is defined as $bd_i = v_i / \sqrt{s_i}$. Therefore, we have

$$\frac{v_1}{\sqrt{s_1}} \ge \cdots \ge \frac{v_i}{\sqrt{s_i}} \ge \cdots \frac{v_n}{\sqrt{s_n}} \tag{18}$$

$$\sum_{k=1}^{K} o_1^k \le \sum_{k=1}^{K} o_2^k \le \cdots \le \sum_{k=1}^{K} o_j^k \le \cdots \le \sum_{k=1}^{K} o_m^k \tag{19}$$

Next, we will derive a feasible solution using the algorithm proposed below. Denote σ as the total bidding price of all the winning bidders, let r_j be the number of channels remained at PO j, and ε_i be the flag variable on the assignment status between SU i and PO j and ε_i is defined as

$$\varepsilon_i = \begin{cases} 0, & \text{SU } i \text{ is not attached to any PO} \\ j, & \text{SU } i \text{ is attached to PO } j \end{cases} \tag{20}$$

Then, we design an approximation algorithm, that is, the approximation algorithm to derive the auction upper bound (ADUB) algorithm. ADUB first initialise the allocation flag variable and then traverses each of the POs and execute the ALlocation and payment calculation (ALC) or SUs. The outer 'for' circulation of ADUB has a complexity of $O(m)$ and the inter 'for' circulation's complexity is $O(n)$. Therefore, the total complexity of the ADUB algorithm is $O(mn)$.

4.3 G-Greedy allocation algorithm

G-Greedy allocation algorithm is based on the bid density [39, 38]. In the first phase, bids of POs are sorted in ascending order according to the bid density while bids of SUs are sorted in descending order. This is used to give priority of SUs' bids from the highest to the

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Algorithm 3

Input: M,N,V,W // The number of POs M, the number of SUs N, valuation matrix V and weight vector W
Output: X,C,P

Phase 1: Calculate and sort the bids of POs and SUs
$E \leftarrow \emptyset$
for $i = 1$ to N **do**
  $bd_i \leftarrow v_i/\sqrt{s_i}$ // calculate bid density of SU i
  $E = E \cup \{bd_i\}$
end for
Sort bid densities in E in descending order
$F \leftarrow \emptyset$
for $j = 1$ to M **do**
  $bd_j \leftarrow v_j/\sqrt{s_j}$ // calculate bid density of PO j
  $v_j = \sum_{k=1}^{K} q_j^k o_j^k$
  $F = F \cup \{bd_j\}$
end for
Sort bid densities in F in ascending order
Phase 2: Allocate the spectrum bands
$X_{N\times M} \leftarrow 0$ // initialize allocation matrix X
for $e = 1$ to $|E|$ **do**
  Assume the eth bid density is bd_i
  for $f = 1$ to $|F|$ **do**
    Assume the fth bid density is bd_j
    $flag = 1$
    for $k = 1$ to K **do**
      if $(d_i^k > o_j^k)$ **then**
        $flag = 0$ // demand>supply
      end if
      $p_j = d_i^k q_j^k t_i$
    end for
    if $(flag = 0 || p_j > v_i)$ **then**
      continue
    end if
    $X_{ij} \leftarrow 1$ // SU i is assigned to PO j
    $o_j^k \leftarrow o_j^k - d_i^k$// update spectrum resources at PO j
  end for
end for
Phase 3: Charge for SUs and calculate payments to POs
$C \leftarrow \emptyset$
for each winner SU i **do**
  $c_i \leftarrow \sum_{k=1}^{K} d_i^k q_j^k t_i$
  $C \leftarrow C \cup c_i$
end for
$P \leftarrow \emptyset$
for each PO j **do**
  $p_j \leftarrow \sum_{i=1}^{N} c_i x_{ij}$
  $P \leftarrow P \cup p_j$
end for

Fig. 2 *G-Greedy allocation algorithm*

lowest and to prioritise bids of POs from the cheapest to the most expensive level.

Then, the bid density for SU i is defined as $bd_i = v_i/\sqrt{s_i}$. For SU i, the larger the bid density bd_i is, the higher chances are there to win a bid.

Similarly, the 'size' of the jth PO is defined as $s_j = \sum_{k=1}^{K} w_k o_j^k$ and the price of all the spectrum bundles at PO j is $p_j = \sum_{k=1}^{K} q_j^k o_j^k$. Therefore, the bid density bd_j for PO j is $bd_j = p_j/\sqrt{s_j}$.

The next procedure is to allocate the heterogeneous spectrum resources owned at each PO. Then, for each SU i in the priority sequence which is sorted according to the bid density, if PO j can fulfil the demands of SU i, then the allocation will start. If the price of all the spectrum bands for SU i at PO j is higher than the ith SU's valuation, then the searching and matching procedure will continue. After each successful allocation, the amount of spectrum bands at the PO will decrease. The allocation algorithm is described in Algorithm 3 (see Fig. 2).

4.4 E-Greedy algorithm

In this subsection, we propose the E-Greedy algorithm. Group discount or group bargaining has been employed in electronic commerce [18] and used by Lin *et al.* [34] for compositional buying, not for group discount. To the best of the authors' knowledge, our recent paper [33] was the first to address group discount in cloud computing. Different from [33], in this section, we extend [33] to address the spectrum sharing and allocation problem in a ubiquitous wireless network.

Algorithm 4

Input: M,N,V,W // The number of POs M, the number of SUs N, valuation matrix V and weight vector W
Output: X

Phase 1: Calculate and sort the bids of POs and SUs
$E \leftarrow \emptyset$
for $i = 1$ to N **do**
  $bd_i \leftarrow v_i/\sqrt{s_i}$ // calculate bid density of SU i
  $E = E \cup \{bd_i\}$
end for
Sort bid densities in E in descending order
$F \leftarrow \emptyset$
for $j = 1$ to M **do**
  for each discount level D_j^l in D_j **do**
    $bd_j^l \leftarrow (1 - D_j^l)v_j/\sqrt{s_j}$ // calculate the leveraged bid density of PO j on level l
    $F = F \cup \{bd_j^l\}$
  end for
end for
Sort bid densities in F in ascending order
Phase 2: Allocate the spectrum bands
for $j = 1$ to M **do**
  $G_j \leftarrow \emptyset$ //G_j denotes the set of SUs that attach to PO j
end for
for $f = 1$ to $|F|$ **do**
  Assume the fth level bid density is bd_j^l
  $G' \leftarrow G_j$ // store current G_j in G'
  for $e = 1$ to E **do**
    Assume the eth bid density is bd_i
    if SU i with B_i is satisfied with PO j towards spectrum bands on the lth level discount and its own true valuation **then**
      $G_j \leftarrow G_j \cup \{B_i\}$
    end if
  end for
  if $|G_j| < \xi_j^l$ **then**
    $G_j \leftarrow G'$ // allocation failed in this round
  end if
end for
Transform the allocation results into the allocation matrix X

Fig. 3 *E-Greedy allocation algorithm*

First of all, we define the group discount function as $D{:}\mathbb{N} \rightarrow [0, 1)$. The argument in the function represents the PO's group size G, which belongs to $\{0, 1, 2, \ldots\}$. The dependent variable D is discount rate that is in the range [0,1). Without loss of generality, we assume there are $|G_j|$ SUs that attach to PO j and D_j denotes the discount rate at PO j when there are $|G_j|$ SUs. For example, when unit price of one type spectrum bands is \$3.0, the group discount rate is 0.3 when there are 30 SUs that are assigned to the PO. Therefore, the new unit price is \$ $3.0 \times (1 - 0.3) =$ \$ 2.1. Suppose there are $|G_j|$ SUs that attach to PO j, then the group discount rate is calculated as $D_j(|G_j|) = D_j(\sum_{i=1}^{N} x_{ij})$. By applying group discount scheme, (5) should be modified as [33]

$$u_j = p_j - v_j = p_j - \sum_{i=1}^{N} x_{ij} \sum_{k=1}^{K} (d_i^k q_j^k t_i) D_j\left(\sum_{i=1}^{N} x_{ij}\right) \quad (21)$$

Suppose the lth level discount at PO j is denoted as D_j^l. We redefine the baseline bid density for PO j as bd_j^0 and $bd_j^0 = p_j/\sqrt{s_j}$. Then, the lth bid density of the PO j is $bd_j^l = (1 - D_j^l)bd_j^0$. We also define the jth PO's discount level threshold as ξ_j^l. That is, when there are ξ_j^l SUs attach to PO j, then the level discount is D_j^l. For example, suppose $l \in \{0, 1, 2\}$, $D_j^l \in \{0, 0.1, 0.2\}$. Let $bd_j^0 = 0.02$, $\xi_j^0 = 0$, $\xi_j^1 = 10$ and $\xi_j^2 = 20$. Then, $bd_j = \{0.02, 0.018, 0.016\}$.

The first phase is just as similar as the corresponding phase in the G-Greedy algorithm. In the second phase, E-Greedy traverses the lists of bid densities and assigns the spectrum demands to POs iteratively. Without loss of generality, we assume that the first leveraged bid density in the sorted leveraged bid densities is bd_j^l which means the lth level bid density for PO j. Then, the algorithm will traverse the sorted list of SUs, each of whose demanded spectrum resource could be fulfilled at the jth PO and the true valuation of the SU is no less than the charged price of PO j with discount D_j^l. If the accumulated number of satisfied SUs reached to the discount level threshold ξ_j^l at PO j, then all SUs will be assigned to the PO and the remained leveraged bid densities at PO j will be abolished. Otherwise, no SUs will be matched to the PO in the current round. Then, E-Greedy searches for the next level bid density and continues execution of the procedure. The detail algorithm is shown in Algorithm 4 (see Fig. 3).

After allocating spectrum bands for SUs, the next step is to design charge and payment scheme. A frequently used payment scheme is the Vickery [40] style, which charges the winner the highest price of all the non-winning bids. Therefore, based on Vickery scheme, the payment is calculated by multiplying the 'size' value of SU i with the highest bid density of the non-winning bids [39]. To guarantee individual rationality, the final charge chooses the maximum value between the Vickery style price and the charge price of the PO that is attached by SU i. Then, the payment for a PO is the total charges of SUs that are attached to it. The algorithm is presented in Algorithm 5 (see Fig. 4).

4.5 Algorithm analysis and auction properties

4.5.1 Time complexity analysis

Theorem 1: The time complexity for G-Greedy is $O(N\log N)$. The time complexity for E-Greedy is $O(N^2)$.

Proof: For G-Greedy algorithm, the complexity for sorting the bids is $O(N\log N) + O(M\log M)$. The second phase of G-Greedy is $O(NMK)$. The overall complexity is $(N\log N)$ when $N \gg M$ and

Algorithm 5

Input: Winner set W
Output: C,P

Phase 1: Calculate charges for SUs
$C \leftarrow \emptyset$
for all user i in winner set W **do**
  $W_i \leftarrow \{l : i \notin W \Rightarrow l \in W\}$ // calculate bid density of SU i
  Choose a user i with the highest bid density in W_i
  if $W_i \neq \emptyset$ **then**
    $c_i \leftarrow max\{\frac{\sqrt{s_i}v_l}{\sqrt{s_l}}, \sum_{k=1}^{K} d_i q_j^k D_j(|G_j|)\}$
  else
    $c_i \leftarrow \sum_{k=1}^{K} d_i q_j^k D_j(|G_j|)$
  end if
$C \leftarrow C \cup \{c_i\}$
end for
Phase 2: Calculate payments to POs
$P \leftarrow \emptyset$
for each PO j **do**
  $p_j \leftarrow \sum_{i=1}^{N} c_i x_{ij}$
  $P \leftarrow P \cup p_j$
end for

Fig. 4 *Charge and payment for E-Greedy*

$N \gg K$. Since there would be at most N discount levels for each PO, there are NM bid densities in the worst case. The loop for spectrum allocation will traverse the N SUs for each leveraged bid density, thus it takes $O(MN^2)$ time. The overall time complexity of E-Greedy is $O(N^2)$ when $N \gg M$. □

4.5.2 Economic properties analysis

Theorem 2: The auction participants in the proposed schemes are individually rational.

Proof: In G-Greedy, for the true valuation of SU i, we assume $v_i = \sum_k d_i^k (1 + \gamma_i) t_i > c_i$, where γ_i is the signal noise ratio at SU i, then SUs in G-Greedy are individually rational. We will illustrate through simulation that this additional constraint does not prohibit the effectiveness of the proposed scheme. That is, in a special condition, G-Greedy may not be individually rational for the auction, but by carefully choosing the parameters, G-Greedy may guarantee individual rationality. This will not affect the resource allocation procedure which is the main target, but may result in a lower total utility [38]. For E-Greedy, individual rationality is strongly guaranteed. The charge scheme for winner SUs is the maximum value of the Vickery price and the ask price of the assigned PO. To prove the individual rationality for a SU, we have to prove that both of the Vickery price and the ask price are no more than the true valuation of the user. For the Vickery price of SU i, it is $v_l/\sqrt{s_l}$ and $(v_l/\sqrt{s_l}) \leq (v_i/\sqrt{s_i})$. Otherwise, SU l will be allocated before SU i. Therefore, $\sqrt{s_i}v_l/\sqrt{s_l} < \sqrt{s_i}v_i/\sqrt{s_i} = v_i$. The payment of a PO is the accumulation of the charge for SUs, which is no less than the ask price of the PO. Therefore, the POs' individual rationality is guaranteed. □

Theorem 3: The auctioneer in the auction is budget balanced.

Proof: The payment to each PO is the accumulation of charges for SUs that are attached to it. Hence, the total payment of POs will be equal to total charges for all winning SUs. Therefore, budget balance is also guaranteed. □

Theorem 4: The greedy spectrum allocation mechanisms are truthful.

Proof: If a SU who attempted to achieve a higher utility through misrepresenting the true valuation [38], then there could be two cases.

(i) *Case 1*: bid<true valuation. Then, it might be rejected by the auction scheme since the bid was too low to get a positive utility. Suppose there could be one of the winners due to the changes in the bid density sequence, because the SUs' bid densities were arranged in descending order while the POs' bid densities were arranged in ascending order. Then, the SU might win the higher bid PO to obtain a lower utility.
(ii) *Case 2*: bid>true valuation. Then, it might need more spectrum resources than it was actually demanded. Hence, some of spectrum bands were wasted. The SU could only acquire a higher utility by trading with an extremely low bid PO. However, no SUs would like to sacrifice its demands to do so. □

5 Performance evaluation

The presented evaluation metrics are: (i) total utility, (ii) satisfaction ratio, which is the ratio between the number of winning SUs to the number of all SUs and (iii) allocation efficiency, which is the ratio between the number of allocated channels to the total number of channels provided at POs. For comparison purpose, we also implement the random allocation method, which assigns each type of spectrum bands randomly to SUs.

5.1 Methodology

We evaluate the proposed G-Greedy and E-Greedy algorithms with random bids and time behaviours.

Although we cannot compare the proposed greedy algorithms with existing works because there are no prior literatures that have achieved the same economic properties in an combinatorial spectrum double auction setting with each PO possesses heterogeneous spectrums, we evaluate the proposed algorithms with the random allocation method. Meanwhile, we examine the auction upper bound based on Algorithms 1 and 2 (see Figs. 5 and 6) in the next simulation subsection. To better compare the proposed algorithms, we implement a multi-round scheme and within each round, the spectrum double auction algorithm for homogeneous scenario (where each PO possesses homogeneous

Algorithm 1

$\sigma = 0$ // initialize total payment
Rearranging the sequence of bidders according to eq.(18)
Rearranging the sequence of sellers according to eq.(19)
for $i = 1$ to n **do**
  $\epsilon_i = 0$ // initialize the allocation flag variable
end for
for $j = 1$ to m **do**
  $r_j = \sum_{k=1}^{K} o_j^k$
  Execute ALC()
end for

Fig. 5 *Approximation algorithm to derive the auction upper bound*

Algorithm 2

Input: m,n,K,d_i^K,r_j
Output: σ, ϵ_i

for $i = 1$ to n **do**
  if $\epsilon_i = 0$ and $\sum_{k=1}^{K} d_i^k \leq r_j$ **then**
    $\epsilon_i = j$ // PO j is assigned to SU i
    $r_j = r_j - \sum_{k=1}^{K} d_i^k$ // calculate the remained resource
    $\sigma = \sigma + \sum_{k=1}^{K} d_i^k q_j^k t_i$ // total payment
  end if
end for

Fig. 6 *Allocation and payment calculation algorithm ALC* spectrum bands) is executed. We adopt the typical algorithm, the McAfee double auction [16] for each round. At the end of each round, one type spectrum bands will be allocated from POs to SUs. Since there are K types spectrum bands, there are K rounds for the multi-round McAfee double auction. Then, we compare the bidder satisfaction ratio with the proposed methods in this paper. That is because the algorithms proposed in this paper are not time slot based. Therefore, although the modified homogeneous algorithm is a multi-round scheme, we choose the bidder satisfaction ratio as the performance evaluation metric.

We implement G-Greedy and E-Greedy using Matlab 7.1 to test their performances. The default parameters used during the simulation are listed in Table 1. At most 900 bidders arrive during the simulation. Their demands for each type spectrum bands are taken from the range [0, 5] within an average of ten time slots. We run our program 250 times and take the average value under each condition. The total utility, satisfaction ratio as well as allocation efficiency are studied and compared under various situations.

5.2 Simulation setting

By default we assume there are three different types of spectrums, for example, WiFi, GSM and 3G. We set their weights to a uniformly distributed variable $w_k = \text{rand}() \times 10$. To generate SU's bid, we assume that the demanded number of each type spectrum is not >6 and the required time slots are <100. For ease of illustration, we set the maximum value per bandwidth at \$1.1 and the upper bound of the valuation is calculated by multiplying \$1.1 with the number of requested spectrum bands as well as the number of time slots. To generate PO's bid, we assume that each PO possesses no more than 1000 spectrum bands for each type spectrum among all the K types spectrum bands. The unit price is randomly generated in $[0.1w_k \pm 0.05]$, in which $0.1w_k$ denotes the basic unit price of the kth type spectrum. For example, a PO's unit price of the GSM band is generated within the range 0.2 ± 0.05. For the group discount rule, the group discount level is generated within [0, 2] that means there are at most two discount levels in the group discount system and 0 means there is no group discount. For simplicity, the first discount level is set when there are a quarter SUs that attach to a single PO and the discount rate is 20%. The second discount level is set when there are one third SUs in the group and the discount rate is 40%. To reflect the randomness character of the discount levels at different POs, we introduce a random variable which is denoted as δ and $\delta = \pm 0.05$. Therefore, the final discount of each PO is the basic discount plus the random variable δ.

5.3 Simulation and results

First, we examine the relationship between the number of POs and the total utility achieved with different algorithms shown in Fig. 7. In this simulation, we set the ratio between the number of POs m and the number of spectrum types K as a constant, that is, $m/K = 5$. Here, the total utility is divided by 1000 to show clearly in the figure. It shows that the upper bound is achieved by relaxation and for all the algorithms, the total utility grows with the growing number of POs. That is because when there are more POs in the system, more requirements can be satisfied. Compared with total utility, both G-Greedy and E-Greedy outperform the random method. The G-Greedy algorithm is about 52.46% times larger than the random method in average while the E-Greedy algorithm has a higher increase compared with the random method on auction utility, which averaged to 63.05%. What is more, E-Greedy is averagely 21.73% times higher than G-Greedy on total utility.

Table 1 Default parameters

SU's bid	
demanded spectrum of each type	$d_i^k \in [0, 5]$
requested time slot	$t_i \in [1, 10]$
true valuation	$v_i \in [0, \sum_k (1+\gamma_i) d_i^k t_i], \quad \gamma_i = 0.1$
PO's bid	
number of spectrum offerings	$o_j^k \in [0, 1000]$
unit price	$q_j^k = 0.1w_k \pm 0.05$
PO's group discount rule	
levels of discount	$l_j \in [0, 2]$
group size	$\{N/4, N/3\}$
discount	$D_j^l = 20\% l + \delta, \quad l \in \{1, 2\}$

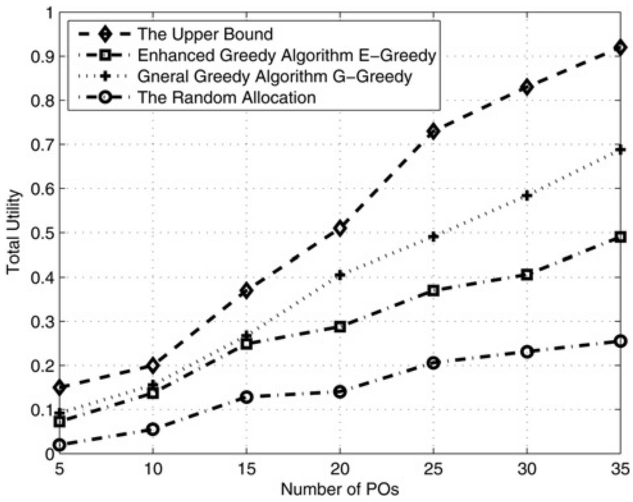

Fig. 7 *Performance of the proposed algorithms on total utility with the growing number of POs*

To examine the satisfaction ratio with the growing number of SUs, we fixed the number of POs as $m = 5$ and the type of spectrums K is fixed at $K = 3$. The results show that bidder satisfaction ratio decreases as the number of SUs increases. That is because the number of auctioned channels is limited. The results are shown in Fig. 8. What else can be seen from Fig. 8 is that both G-Greedy and E-Greedy have a higher bidder satisfaction ratio than the random method. For example, when there are 100 SUs, the satisfaction ratio of G-Greedy and E-Greedy is 0.577 and 0.653 accordingly while for the random method, the ratio is about 0.491. On average, G-Greedy has a 13.6% times higher bidder satisfaction ratio than the random method while E-Greedy has a 24.9% times better bidder satisfaction ratio. What is more, the McAfee-based algorithm is close to the random method. That is because in different rounds, SUs may be assigned to different POs.

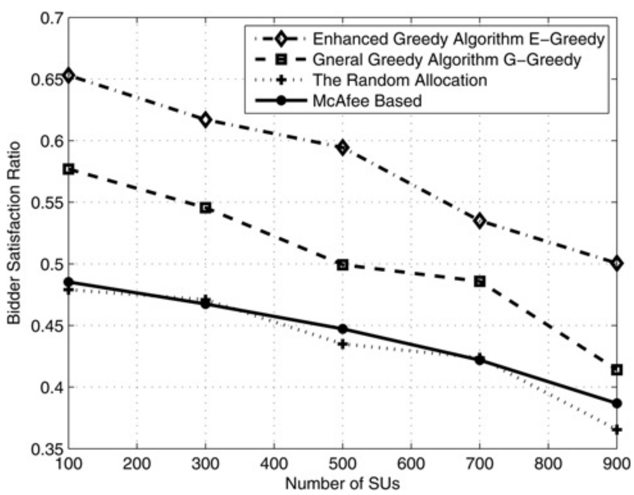

Fig. 8 *Bidder satisfaction ratio vs. number of SUs*

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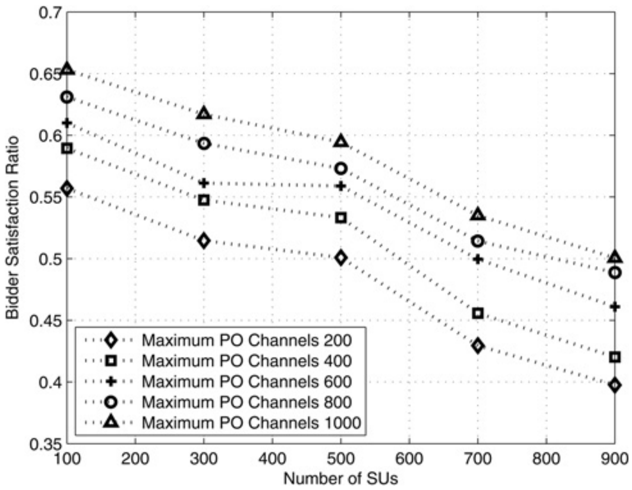

Fig. 9 *Bidder satisfaction ratio vs. number of SUs with changing channels*

The feasible solution is the intersection of each round's allocation result.

In the third experiment, we still observe the relationship between the bidder satisfaction ratio and the increasing number of SUs, but with changing number of maximum bidding channels from 200 channels to 1000 channels. We show the influence on the number of maximum bidding channels per PO in Fig. 9 taking the E-Greedy algorithm. When the number of traded channels per PO gets larger, the satisfaction ratio of SUs becomes larger accordingly. That means, more SUs can fulfil their tasks.

The fourth experiment observes the relationship between the total utility and the increasing number of SUs. We compare the differences between the proposed algorithms and the random allocation method with changing values m and K. The results are shown in Fig. 10. The dotted lines are the results when $m = 5$ and $K = 3$ while the dashed lines represent $m = 9$ and $K = 4$. Under the two scenarios, as the number of SUs increases from 100 to 900, total utility grows for all algorithms. However, the random allocation method has a poor utility comparing with the proposed algorithms. For example, when $m = 5$, $K = 3$ and the number of SUs $n = 500$, the total utility for G-Greedy is 1.34 and for E-Greedy is 2.63, whereas the utility is about 1.04 for the random method. Therefore, total utility of G-Greedy is about 22.3% higher than the random method and E-Greedy has about 60.45% utility gain than the random method. What else can be seen is that the total utility of random method prone to converge to a constant with the growing number of SUs. When it comes to the proposed methods, our algorithms keep a growing trend under the above scenarios. With the growing of PO number m from $m = 5$ to $m = 9$ and the maximum spectrum type number K from $K = 3$ to $K = 4$, the total utility grows accordingly. As there are more channels offered by the POs, more SUs' requirement can be guaranteed. On average, when $m = 5$, $K = 3$, the performance gain of G-Greedy to the random allocation method is 25.44% with a performance gain of E-Greedy to the random method 61.03%. When $m = 9$, $K = 4$, the performance gain of G-Greedy to the random allocation method is 30.99% with a performance gain of E-Greedy to the random method 58.23%.

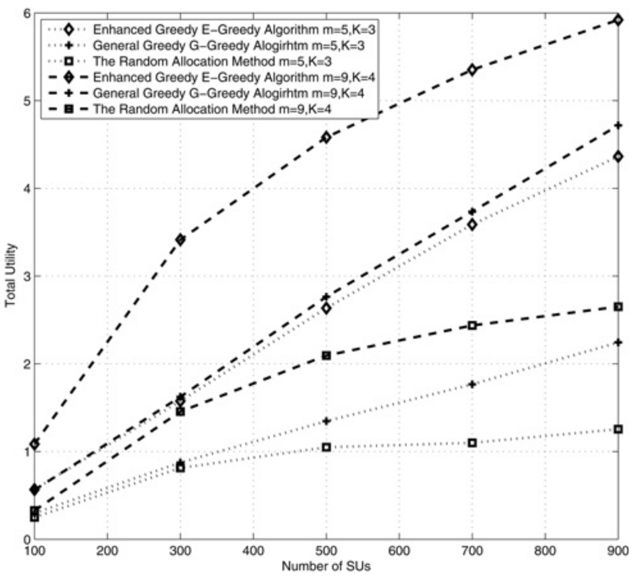

Fig. 10 *Total utility vs. number of SUs with changing m and K*

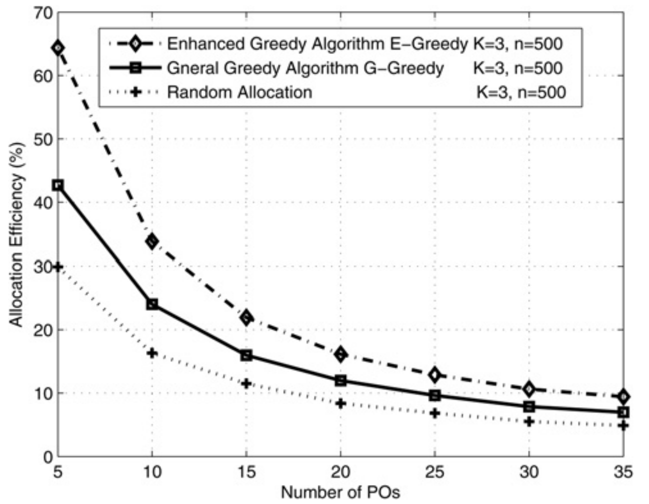

Fig. 11 *Allocation efficiency vs. number of POs*

The fifth experiment observes the allocation efficiency with the number of POs. In this experiment, we set the number of SUs at $n = 500$ and let $K = 3$. The results that correspond to this study are depicted in Fig. 11. With the growing number of POs, the allocation efficiency drops quickly when the number of POs is <15 and almost reaches to a constant when the number of POs reaches 35. That is because when K is fixed, then the spectrum demands of SUs will stay. For each algorithm, the increase of PO numbers will result in lower allocation efficiency due to the stable allocation result and the higher number of offered spectrums. It is obvious that, both G-Greedy and E-Greedy have higher allocation

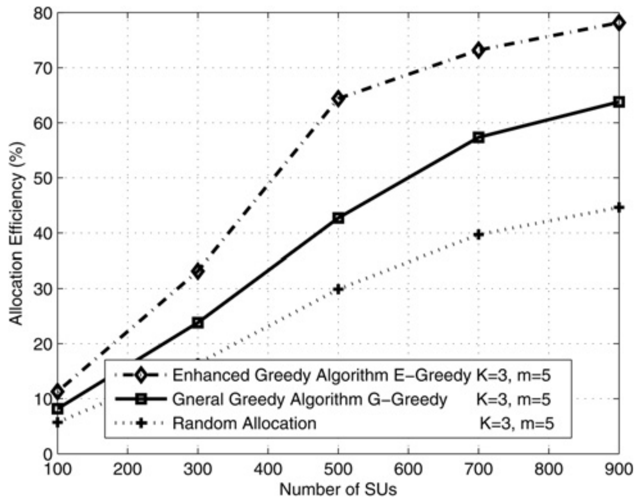

Fig. 12 *Allocation efficiency vs. number of SUs*

efficiency than the random allocation method. For example, when there are 20 POs, under the given setting, the allocation efficiency for E-Greedy and G-Greedy is 16.13 and 11.9%, respectively. Both of them are higher than the 8.4% allocation efficiency of the random allocation method.

Finally, we examine the allocation efficiency with the number of SUs. The results are shown in Fig. 12. This figure illustrates that the allocation efficiency grows with the increasing number of SUs and among the three different methods, both G-Greedy algorithm and E-Greedy algorithm outperform the random allocation method.

From the aforementioned discussions, we can now draw the conclusion that G-Greedy and E-Greedy are efficient to allocate spectrum resources under the setting discussed in this paper and E-Greedy is better than G-Greedy.

6 Conclusion

In this work, a new lightweight spectrum combinatorial double auction scheme is proposed for CR networks with multiple heterogeneous spectrum bands owned by POs and demanded by multiple SUs. Different from existing works, our scheme can fit for a ubiquitous setting where there are different kinds of spectrum bands owned at the same PO. We construct a combinatorial double auction mechanism to allocate the spectrums and design a greedy algorithm named G-Greedy to solve the allocation problem. Then, we further enhance the performance of G-Greedy by employing group discount scheme E-Greedy. By adopting a Vickrey-like mechanism, we analyse the economic properties of the proposed schemes and show that the schemes are economically robust. Numerical results indicate that the proposed algorithms are better than the random method and the group discount scheme outperforms the random allocation scheme and the McAfee-based scheme.

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