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Double threshold-based cooperative spectrum sensing for a cognitive radio network with improved energy detectors

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Abstract: Performance of cooperative spectrum sensing in a cognitive radio (CR) network is investigated where each CR node uses an improved energy detector (IED) to sense the primary user (PU), and makes a local decision regarding the presence of PU using double thresholds (DTHs). The local hard decisions are combined at fusion centre (FC) to obtain the global decision. The advantage of a DTH-based system over a single threshold based one is, a CR node can opt for no decision when a decision variable lies in the fuzzy zone between two thresholds. Such censoring reduces the transmission overhead between CR and FC without significantly affecting the receiver operating characteristics. In this study, the performance of the abovementioned CR network has been assessed in terms of the average number of normalised transmitted sensing bits (k_{nor}), the total error probability ($P_{e,n}$) and optimal number of CR users that ensures minimum $P_{e,n}$. It was observed that k_{nor} increases as the signal power raise factor of IED increases or failed sensing probability decreases. Further, the agility of the network improves as the PU death rate increases. Impact of reporting channel on the sensing performance has also been indicated.

1 Introduction

1.1 Motivation

Recently, cognitive radio (CR) emerged as a smart and agile technology to meet the demand for wireless services. It omits the confliction regarding under-utilised licensed band and scarcity in unlicensed band [1–4]. In CR networks, a secondary user uses the licensed band if the unlicensed band is not vacant. It is imperative to check the presence of primary user (PU) in that particular licensed band before accessing it, to avoid possible interference with the PU. The corresponding techniques are collectively known as *spectrum sensing* (SS). Time-varying distortions over the sensing wireless link, namely fading and shadowing, may result in sensing failures (false alarm/missed detection). The availability of multiple CR nodes opens up the possibility to exploit the idea of cooperation among CR users, which may circumvent the problem to a great extent [5, 6]. In cooperative SS (CSS), a set of CR sensors perform SS and independently send their local decision to the fusion centre (FC) for further processing. Finally, FC will take a decision regarding the presence or absence of PU. If the number of CR users is too high and all the cognitive sensors send their report to the FC, the required bandwidth is high even if one bit quantisation is used. In such situations, double threshold (DTH)-based detection technique may be useful in order to increase the spectrum efficiency and bandwidth utilisation [7].

1.2 Related literature

Although the idea of cooperation in detecting problems were addressed long back [8], the idea of cooperation among sensing nodes, as mentioned in the previous subsection, were first described by Cabric *et al.* [5] and Ghasemi and Sousa [6]. In particular, the authors showed that detection probability (P_{d}) can be substantially improved with CSS when compared with the traditional non-CSS. The performance of a CR network can further be enhanced if each CR uses an improved energy detector (IED) instead of a conventional energy detector (CED) [9, 10]. In an IED, the signal power raise factor (SPRF) parameter ($p; p \geq 0$) is not necessarily restricted to $p = 2$. Quite evidently, for $p = 2$, the IED structure reduces to simple signal squarer, i.e. it behaves like a CED. The order of improvement is measured by the reduction in overall errors due to misjudgment, which is the sum of false alarm probability (P_{f}) and missed detection probability (P_{m}). The optimal number of CR users, to minimise P_{f} and P_{m} in a cooperative network, has been investigated in [11].

Next, we would like to cite some other important articles which point out the common assumptions pertaining to the CR literature and presents performance evaluations when these assumptions become invalid. For example, the reporting channels between CR and FC, which are used for reporting the CRs local binary decisions to FC, are often considered to be noiseless for the sake of simplicity. However, in practice, these reporting channels are not free from noise or fading mechanisms. In [12, 13], the authors investigated the performance of SS over noisy and faded reporting channels. Another simplistic assumption encountered in the text [14, 15] is that the reporting channels are considered to be dedicated. In [16], the authors proposed a simple CR system without any dedicated reporting channels. The CRs send their local decisions over orthogonal sub-channels and thus avoid requirement for extra spectrum band.

Steering our attention to the articles dealing with DTH-based SS, we would like to start with [17], where the authors showed that the DTH-based detection reduces the average number of transmitted local decision bits which in turn reduces the sensing time and improves the agility of overall network. In [18], the authors proposed a weighting judgment method in DTH detection to improve the performance of SS. The judgment is made using reliability factor depending on signal-to-noise ratio (SNR) values. In a recent article by Lin *et al.* [19], a hierarchical decision process was proposed where the overall decision is made on the basis of both local decisions and centre fusion decision. The local decisions are made by CRs which are not lying in the 'no decision' region and the centre fusion decision is made from soft combining of the energy values which fall between the two thresholds at FC.

1.3 Contributions of this paper

In this paper, our goal is to characterise a CSS system where each CR is utilising an IED and the CRs take hard decisions regarding the presence of the PU (presence, absence, or no decision) by comparing the received signal against two thresholds. The activity of PU is modelled as a two-state discrete-time Markov process with traffic birth rate and death rate. The local SS decision of each CR is transmitted to FC through dedicated reporting channels. The decisions from CRs are fused at FC using OR logic, which means if even a single CR has observed that the PU is present, the FC will take decision in favour of the presence of PU which in turn results in lesser (compared with AND or majority logic) interference on PU. The total error probability ($P_f + P_m$) is minimised when the optimal number of CRs cooperate in the decision process. The analytical expressions leading to the numerical evaluation of optimal number of CRs have been presented in this paper, for both the classical single threshold (STH)-based CRs as well as for the more sophisticated DTH-based CR elements. The performance of DTH-based system is compared on the basis of the metric average number of normalised transmitted sensing bits (k_{nor}). Further, we proposed an algorithm to find the thresholds of DTH detection. The agility of the network is investigated at different 'no decision' probabilities, at different traffic birth–death rates, and for different values of SPRF parameter (p).

The specific contributions of this paper are as follows:

- An analytical expression is derived to estimate the detection probability over Rayleigh faded channel. The PU signal is assumed to follow Gaussian statistics [11], in contrast to the majority of the literature where a deterministic PU signal was assumed. To the best of the authors' knowledge this is a new result.
- An analytical expression is derived to estimate the optimal number of CRs required for cooperation which minimises total probability of error considering a two-state Markovian activity model of the PU. Performance with the optimal number of CRs has been assessed under several network conditions.
- The impact of the imperfect reporting channel on the sensing performance has been assessed.
- Two algorithms have been proposed, Algorithm 1 (see Fig. 1) is used to obtain the thresholds in DTH-based local sensing under IED, for a given constraint on false alarm probability and Algorithm 2 (see Fig. 2) is used to find out the detection probability.
- An analytical framework has been developed for assessing agility gain of the network.

1.4 Organisation of this paper

The remaining of this paper is organised as follows. Section 2 begins with a formal description of the CSS model under study. We introduce the equations governing PU activity model for CRs equipped with IEDs thereafter. This is followed by the analytical

Algorithm 1

Require: $Q_f \Leftarrow 0.01, N \Leftarrow 10, 0 \le p \le 10, 0.001 \le b_0 \le 0.1$
1: $l_1 \Leftarrow$ length (b_0)
2: $l_2 \Leftarrow$ length (p)
3: **for** $i = 1$ **to** l_1 **do**
4: $\quad \Delta_0(i) \Leftarrow b_0(i)^{1/N}$
5: $\quad P_x(i) \Leftarrow 1 - [Q_f/(1 - b_0(i))]$
6: $\quad$ **for** $j = 1$ **to** l_2 **do**
7: $\quad\quad F_{W_i|H_0}(\lambda_2(i,j), p) \Leftarrow [P_x(i) + b_0(i)]^{1/N}$
8: $\quad\quad P_{df} \Leftarrow 1 - F_{W_i|H_0}(\lambda_2(i,j), p)$
9: $\quad\quad p \Leftarrow j$
10: $\quad\quad$ Evaluate $\lambda_2(i,j) = g_2(F^{-1}_{W_i|H_0}(1 - P_{df}), p)$
11: $\quad\quad F_{W_i|H_0}(\lambda_1(i,j), p) \Leftarrow F_{W_i|H_0}(\lambda_2(i,j), p) - \Delta_0(i)$
12: $\quad\quad P^*_{df} \Leftarrow 1 - F_{W_i|H_0}(\lambda_1(i,j), p)$
13: $\quad\quad$ Evaluate $\lambda_1(i,j) = g_1(F^{-1}_{W_i|H_0}(1 - P^*_{df}), p)$
14: $\quad$ **end for**
15: **end for**

Fig. 1 *Determination of threshold (λ_1 and λ_2) values*

Algorithm 2

Require: Initialise parameters like$\sigma_s^2, \sigma_n^2, N, u, b_0, p, P(H_0)$ and $P(H_1)$
1: Initialise the range ofQ_f
2: Find λ_1 and λ_2 using Algorithm 1
3: $count = 0$
4: Signal,$s \Leftarrow N_R(0, \sigma_s^2) + jN_I(0, \sigma_s^2)$
5: Signal,$n \Leftarrow N_R(0, \sigma_n^2) + jN_I(0, \sigma_n^2)$
6: Sensing channel, $h \Leftarrow \sqrt{h_R^2 + h_I^2}$
7: Received signal at CR, $y \Leftarrow hs + n$
8: Signal energy, $E = |y|^2$
9: **if** $E > \lambda_2$ **then**
10: $\quad d(i) = 1$
11: **else**
12: $\quad d(i) = 0$
13: **end if**
14: Repeat the steps 4 to 13 for N times.
15: $FC_{deci-stat} = sum(d)$
16: **if** $FC_{deci-stat} > 1$ **then**
17: $\quad count = count + 1$
18: **end if**
19: Repeat the steps 2 to 18 for different values of p for simulation times.
20: Evaluate detection probability, $Q_d = count/numberofsimulation$

Fig. 2 *Determination of detection probability* models for deriving different sensing probabilities over fading environments in Sections 3 and 4. In Section 5, the analytical expression for the optimal number of CRs is discussed. Improvement in the agility of the network is discussed in Section 6. The results and discussion are presented in Section 7, and finally Section 8 concludes the paper.

2 System model

Let us consider a CSS system consisting of one PU, N number of CRs, and one secondary base station (SBS) which consists of a FC, as shown in Fig. 3. We assume that both the sensing channels and reporting channels are affected by noise as well as fading effects. The CR system is time slotted, and at the beginning of every time slot, it has to take a decision about the presence of PU in the respective channel. The energy detector of each CR comes up with a local decision on the basis of the received signal at its sensing channel input. The decisions from CRs, sent via the reporting channels, are fused at FC. A CR is allowed to transmit data in the respective time slot when the fusion result indicates that PU is absent.

The equivalent received signal at the energy detector of the ith CR is

$$y_i(t) = \begin{cases} n_i(t), & H_0 : \text{PU absent;} \\ h_i s_i(t) + n_i(t), & H_1 : \text{PU present;} \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, N$, $s(t)$ is the transmitted signal by PU, $n_i(t)$ is additive white noise, h_i denotes the complex channel fading amplitude, and the two hypotheses, null hypothesis (H_0) and alternative hypothesis (H_1), represents the absence and presence of PU, respectively.

In a detection cycle (T_f), each CR user first senses the spectrum over time τ and transmits data over the remaining time ($T_\text{f} - \tau$) of the frame. Let the received signal at the sensing channel input of each CR is sampled at a rate f_s and K be the number of samples, i.e. $K = \tau f_\text{s}$. The test statistics of the energy detector are given as [9]

$$W_{i,K} = \frac{1}{K}\sum_{i=1}^{K} |y_i|^p; \quad p > 0 \tag{2}$$

where p is the SPRF parameter. The test statistic for a CED ($p = 2$) can be approximated by a Gaussian distribution for a large number of samples under H_0 and H_1 as shown in [9]. Most of the existing literature [10, 11] considered only a single sample ($K = 1$) of the received signal from PU in the IED to obtain the statistics of the observable, while sum of a number of received samples, if considered may represent a more accurate analysis of IED as presented in [9], but with increased mathematical complexities. As our prime focus is not an exact analysis of IED, the approximate

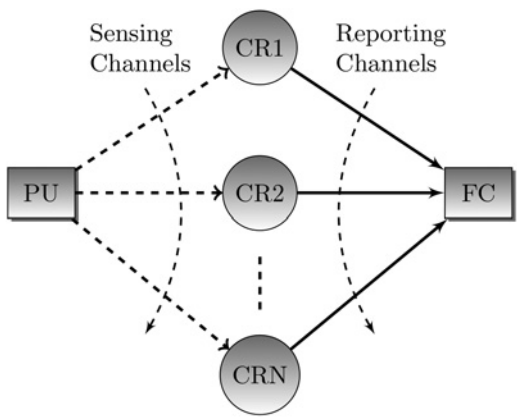

Fig. 3 *System model for cooperative SS*

popular analysis of IED as existing in the literature is captured in our present work to make the desired analysis, focusing mainly on joint interactions of DTH and IED, mathematically tractable. The existing literatures studied IED or DTH-based sensing separately, but none of them has studied their joint impact to the best of our knowledge. Moreover, while most of the existing literatures on CSS considered reporting channel as ideal or binary symmetric channel, our present work models it as a faded one which is a more realistic scenario in practice. Thus we claim that our work is novel in its own sense. Following the previous argument, the received energy statics are calculated from a single sample

$$W_i = |y_i|^p; \quad p > 0 \tag{3}$$

in this paper, which serves as a satisfactory lower bound.

Let $f^{\{\text{R,C}\}}_{y_i|H_0}$ and $f^{\{\text{R,C}\}}_{y_i|H_1}$ be the conditional probability density functions (PDFs) of received signals at the CR under H_0 and H_1. The superscript (R or C) is used to differentiate between the real valued and complex valued signals.

In case of real valued PU signal and real valued noise, we assume that the PU transmitted signal, $s(t) \sim \mathcal{N}(0, \sigma_\text{s}^2)$, is Gaussian with zero mean and variance σ_s^2 [11], and the noise is Gaussian, $n_i \sim \mathcal{N}(0, \sigma_n^2)$, with zero mean and variance σ_n^2. Thus the PDF under no signal condition, $f^\text{R}_{y_i|H_0}$, is free from fading mechanism experienced in sensing channel

$$f^\text{R}_{y_i|H_0}(y) = \left(1/\sqrt{2\pi\sigma_n^2}\right)\exp\left[-y^2/(2\sigma_n^2)\right] \tag{4}$$

whereas $f^\text{R}_{y_i|H_1}$ depends on the sensing channel fading statistics. In absence of fading, i.e. for an additive white Gaussian noise (AWGN) channel, the distribution, $f^\text{R}_{y_i|H_1}$, can be expressed as

$$f^\text{R}_{y_i|H_1}(y) = \left[1/\sqrt{2\pi(\sigma_n^2 + \sigma_\text{s}^2)}\right]\exp\left[-y^2/\left\{2\left(\sigma_n^2 + \sigma_\text{s}^2\right)\right\}\right] \tag{5}$$

On the other hand, if the received PU signal is complex Gaussian and noise is circularly symmetric complex Gaussian (CSCG) the distributions, $f^\text{C}_{y_i|H_0}$ and $f^\text{C}_{y_i|H_1}$, can be written as

$$f^\text{C}_{y_i|H_0}(y) = (y/\sigma_n^2)\exp\left[-y^2/(2\sigma_n^2)\right] \tag{6}$$

$$f^\text{C}_{y_i|H_1}(y) = \left[y/(\sigma_n^2 + \sigma_\text{s}^2)\right]\exp\left[-y^2/\left\{2\left(\sigma_n^2 + \sigma_\text{s}^2\right)\right\}\right] \tag{7}$$

Under steady-state conditions the PU activity may be modelled with a two-state birth–death (B–D) process. If between two successive sampling instants the PU starts transmitting, the system state changes from H_0 to H_1, while the opposite happens when a PU becomes inactive. The state remains unchanged if the PU remains active (or inactive) in two consecutive sampling instants [20]. If α (birth rate) and β (death rate) are the probability fluxes between the above mentioned states, we may derive the state probabilities from the following steady-state equations

$$P(H_0)\alpha = P(H_1)\beta \tag{8}$$

and

$$P(H_0) + P(H_1) = 1 \tag{9}$$

as

$$P(H_0) = \beta/(\alpha + \beta) \tag{10}$$

and

$$P(H_1) = \alpha/(\alpha + \beta) \tag{11}$$

where $P(\cdot)$ denotes the probability.

Later, using the basic model described above, we evaluate the receiver operating characteristic (ROC) at CR (local ROC) and at FC (overall ROC), which are presented in Sections 3 and 4, respectively.

3 Local ROC analysis

3.1 Sensing probabilities using STH

3.1.1 Case 1: real-valued Gaussian PU signal and real-valued noise: We assume that each CR contains an IED which computes the decision statistics [19], $W_i = |y_i|^p$; $p > 0$ where p is the SPRF parameter. The corresponding cumulative distribution function (CDF) for AWGN channel is

$$F_{W_i}(y, p) = P\left(|y_i| \le y^{1/p}\right) = P\left(y_i \le y^{1/p} | y_i \ge 0\right) - P\left(y_i \le -y^{1/p} | y_i \le 0\right) \quad (12)$$

which, when differentiated with respect to y, results in the PDF of the output at IED

$$f_{W_i}(y, p) = (1/p) y^{(1-p)/p} f_{y_i}\left(y^{1/p}\right) + (1/p) y^{(1-p)/p} f_{y_i}\left(-y^{1/p}\right) \quad (13)$$

For $p = 2$, statistics of W_i reduces to that of the CED. The PDF of W_i under H_0 and H_1 can be obtained from (4), (5), and (12) as follows

$$f^{\rm R}_{W_i|H_0}(y, p) = \left(\sqrt{2} y^{(1-p)/p} / (p\sqrt{\pi\sigma_n^2})\right) \exp\left[-y^{2/p}/(2\sigma_n^2)\right] \quad (14)$$

$$f^{\rm R}_{W_i|H_1}(y, p) = \left(\sqrt{2} y^{(1-p)/p} / \left(p\sqrt{\pi(\sigma_n^2 + \sigma_{\rm s}^2)}\right)\right) \times \exp\left[-y^{2/p}/\left\{2\left(\sigma_n^2 + \sigma_{\rm s}^2\right)\right\}\right] \quad (15)$$

Now we invoke two SS metrics [21], namely the *false alarm probability*

$$P_{\rm f} = P(W_i > \lambda | H_0) = \int_\lambda^\infty f_{W_i|H_0}(y)\,{\rm d}y$$

and *missed detection probability*

$$P_{\rm m} = P(W < \lambda | H_1) = \int_0^\lambda f_{W_i|H_1}(y)\,{\rm d}y$$

where λ is the detection threshold. Substituting the expression of $f^{\rm R}_{W_i|H_0}(.,.)$ from (14) in the generalised expression for $P_{\rm f}$, and after some algebra, one obtains an integral of the form $\int_\lambda^\infty x^{a-1}\exp(-x)\,{\rm d}x$ which can be further simplified using the definition of complementary incomplete gamma function, $\Gamma(\cdot,\cdot)$, [22] to obtain

$$P^{\rm R}_{\rm f} = \left(1/\sqrt{\pi}\right)\Gamma\left[1/2, \lambda^{2/p}/\left(2\sigma_n^2\right)\right] \quad (16)$$

Similarly, the value of $P^{\rm R}_{\rm m}$ at each CR can be obtained by putting the value of $f^{\rm R}_{W_i|H_1}(.,.)$ from (15) in the generalised expression for $P_{\rm m}$ and utilising the function definition of incomplete gamma function, $\gamma(\cdot,\cdot)$ [22]

$$P^{\rm R}_{\rm m} = \left(1/\sqrt{\pi}\right)\gamma\left[1/2, \lambda^{2/p}/\left\{2\left(\sigma_n^2 + \sigma_{\rm s}^2\right)\right\}\right] \quad (17)$$

Probability of the complementary event, i.e. the detection probability, $P^{\rm R}_{\rm d} = 1 - P^{\rm R}_{\rm m} = P(W_i > \lambda | H_1)$, can be found easily from (17)

$$P^{\rm R}_{\rm d} = \left(1/\sqrt{\pi}\right)\Gamma\left[1/2, \lambda^{2/p}/\left\{2\left(\sigma_n^2 + \sigma_{\rm s}^2\right)\right\}\right] \quad (18)$$

3.1.2 Case 2: complex-valued Gaussian PU signal and CSCG noise: The PDF of W_i, under H_0 and H_1 for complex-valued Gaussian PU signal and CSCG noise, can be obtained using (6) and (7) as

$$f^{\rm C}_{W_i|H_0}(y, p) = \left(y^{(2-p)/p}/(p\sigma_n^2)\right)\exp\left[-y^{2/p}/\left(2\sigma_n^2\right)\right] \quad (19)$$

$$f^{\rm C}_{W_i|H_1}(y, p) = \left(y^{(2-p)/p}/p(\sigma_n^2 + \sigma_{\rm s}^2)\right) \times \exp\left[-y^{2/p}/\left\{2\left(\sigma_n^2 + \sigma_{\rm s}^2\right)\right\}\right] \quad (20)$$

Thus the false alarm probability ($P^{\rm C}_{\rm f}$), detection probability ($P^{\rm C}_{\rm d}$) and missed detection probability ($P^{\rm C}_{\rm m}$) can be derived in the following manner

$$P^{\rm C}_{\rm f} = \int_\lambda^\infty f^{\rm C}_{W_i|H_0}(y, p)\,{\rm d}y = \exp\left[-\lambda^{(2/p)}/\left(2\sigma_n^2\right)\right] \quad (21)$$

$$P^{\rm C}_{\rm m} = \int_0^\lambda f^{\rm C}_{W_i|H_1}(y, p)\,{\rm d}y = 1 - \exp\left[-\lambda^{(2/p)}/\left\{2\left(\sigma_n^2 + \sigma_{\rm s}^2\right)\right\}\right] \quad (22)$$

$$P^{\rm C}_{\rm d} = 1 - P^{\rm C}_{\rm m} = \exp\left[-\lambda^{(2/p)}/\left\{2\left(\sigma_n^2 + \sigma_{\rm s}^2\right)\right\}\right] \quad (23)$$

3.1.3 Detection probabilities in faded environment: Let us now consider the case when the sensing channel is Rayleigh faded. The CDF of decision statistics W_i under H_1 hypothesis is dependent on fading mechanism in sensing channel. As a result, the detection probabilities in fading environment, $P_{\rm d}(\gamma)$, becomes a function of SNR, $\gamma = \sigma_{\rm s}^2/\sigma_n^2$, and its average value may be found as [6]

$$\bar{P}^{\{\rm R,C\}}_{{\rm d}Fad} = \int_0^\infty P^{\{\rm R,C\}}_{\rm d}(\gamma) f_\gamma(\gamma)\,{\rm d}\gamma \quad (24)$$

where $f_\gamma(\gamma)$ is the PDF of SNR, and $P^{\rm R}_{\rm d}$ and $P^{\rm C}_{\rm d}$ are defined in (16) and (21), respectively. On the other hand, the CDF of W_i under H_0 hypothesis is independent of fading. Hence, there is no need to calculate the false alarm probabilities for fading channels separately.

Lemma 1: For real-valued Gaussian PU signal and real-valued noise, the average detection probability over Rayleigh faded sensing environment is

$$\bar{P}^{\rm R}_{\rm dFad} = {\rm erfc}(\sqrt{q}) + \exp(L)\left[\exp\left(-2\sqrt{Lq}\right) - \sqrt{q/\pi}\int_0^1 t^{-3/2}\exp(-q/t)\exp(-Lt)\,{\rm d}t\right]$$

where $q = \lambda^{2/p}/(2\sigma_n^2)$ and $L = 1/\bar{\gamma}$.

Proof: Using the relation between $\Gamma(1/2,\cdot)$ and ${\rm erfc}(\cdot)$ from [23, (6.1)], and the definition of SNR (γ), the detection probability in AWGN channel, $P^{\rm R}_{\rm d} = (1/\sqrt{\pi})\Gamma\left[1/2, \lambda^{2/p}/\left\{2(\sigma_n^2 + \sigma_{\rm s}^2)\right\}\right]$ can be written as $P^{\rm R}_{\rm d} = {\rm erfc}\left[\sqrt{\lambda^{2/p}/\left\{2\sigma_n^2(\gamma+1)\right\}}\right]$. If the fading amplitude follows a Rayleigh distribution, the SNR follows an exponential PDF, $f_\gamma(\gamma) = (1/\bar{\gamma})\exp(-\gamma/\bar{\gamma})$; $\gamma \ge 0$ [6], and the average detection probability may be calculated as

$$\bar{P}^{\rm R}_{{\rm d}Fad} = (1/\bar{\gamma})\int_0^\infty {\rm erfc}\left[\sqrt{\lambda^{2/p}/\left\{2\sigma_n^2(\gamma+1)\right\}}\right]\exp(-\gamma/\bar{\gamma})\,{\rm d}\gamma$$

Using integration by parts and then using the relation $\mathrm{d}/\mathrm{d}z\{\mathrm{erfc}(z)\} = -2/\pi \exp(-z^2)$, the expression is simplified to

$$\bar{P}_{\mathrm{d}Fad} = \mathrm{erfc}(\sqrt{q}) + \sqrt{q/\pi}\exp(L) \int_1^{\infty} t^{-3/2}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t$$

where $t = \gamma + 1$, $q = \lambda^{2/p}/(2\sigma_n^2)$, and $L = 1/\bar{\gamma}$. The integral part, $\mathcal{I}_1 = \int_1^{\infty} t^{-3/2}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t$, can be expressed as a difference of two integrals

$$\mathcal{I}_1 = \int_0^{\infty} t^{-3/2}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t - \int_0^{1} t^{-3/2}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t$$

The first integral, bounded by 0 to ∞ can be replaced by $\sqrt{\pi/q}\exp(-2\sqrt{Lq})$ using [24, (2.3.16.3)] and doing some algebra thereafter. The second integral has a finite range and can be computed numerically. □

Lemma 2: For complex-valued Gaussian PU signal and CSCG noise, the average detection probability over Rayleigh faded sensing environment is

$$\bar{P}^{\mathrm{C}}_{\mathrm{dFad}} = L\exp(L)\left[\sqrt{4q/L}K_1\left(\sqrt{4qL}\right) - \int_0^1 t^{-3/2}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t\right]$$

where $K_1(.)$ is the modified Bessel function of first order and second kind.

Proof: For the complex signal/noise case, the average detection probability may be calculated as

$$\bar{P}^{\mathrm{C}}_{\mathrm{dFad}} = (1/\bar{\gamma})\int_0^{\infty} \exp(-\lambda^{(2/p)}/2\sigma_n^2(\gamma+1))\exp(-\gamma/\bar{\gamma})\,\mathrm{d}\gamma$$

Continuing the notational consistency for t, L, and q from Lemma 1, the average detection probability can be rewritten as $\bar{P}_{\mathrm{dFad}} = L\exp(L)\int_1^{\infty}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t$. Just like the previous case, the integral part, $\mathcal{I}_2 = \int_1^{\infty}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t$, can be expressed as a difference of two integrals

$$\mathcal{I}_2 = \int_0^{\infty}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t - \int_0^{1}\exp(-q/t)\exp(-Lt)\,\mathrm{d}t$$

The first integral can be evaluated through modified Bessel function $\sqrt{4q/L}K_1(\sqrt{4qL})$ using [24, (2.3.16.1)]. □

3.2 Sensing probabilities using DTH

In STH-based detection technique, all the CRs send their decisions (0 or 1) to FC, while in case of a DTH-based system, the decisions lying in the 'no decision' region need not be reported to FC. As a result, transmission overhead can be reduced and precious channel bandwidth may be saved.

A CR operating with DTHs, say λ_1 and λ_2 (where $\lambda_2 \ge \lambda_1$), sends a single decision bit D_i to FC according to following rule [25]

$$D_i = \begin{cases} 0 & W_i < \lambda_1 \\ \text{No decision} & \lambda_1 < W_i < \lambda_2 \\ 1 & W_i > \lambda_2 \end{cases} \tag{25}$$

i.e. the decision goes in favour of H_0 (PU absent) when the decision variable (W_i) is smaller than lower threshold (λ_1), and the CR decides in favour of H_1 (PU present) when the decision variable (W_i) exceeds the upper threshold (λ_2). The CR remains silent (no decision) when the decision statistics lies in the range between the two thresholds.

Following the development in the previous subsections, it is quite straightforward to write the expressions for detection probability, $P_{\mathrm{dd}} = P(W_i \ge \lambda_2|H_1)$, and the missed detection probability, $P_{\mathrm{dm}} = P(W_i \le \lambda_2|H_1)$, at each CR under DTH-based detection when the sensing channel is Rayleigh faded by simply substituting λ with λ_2 in the results presented in Lemmas 1 and 2. The performance metrics for the real and the complex cases are given below.

3.2.1 Case 3: real-valued Gaussian PU signal and real-valued noise: If the PU signal is real Gaussian and noise is real valued, the detection probability and missed detection probability under DTH are given by

$$P^{\mathrm{R}}_{\mathrm{dd}} = \mathrm{erfc}(\sqrt{q_2}) + \exp(L)\left[\exp(-2\sqrt{Lq_2}) - \sqrt{q_2/\pi}\int_0^1 t^{-3/2}\exp(-q_2/t)\exp(-Lt)\,\mathrm{d}t\right] \tag{26}$$

$$P^{\mathrm{R}}_{\mathrm{dm}} = 1 - P^{\mathrm{R}}_{\mathrm{dd}} \tag{27}$$

where $q_2 = {\lambda_2}^{2/p}/(2\sigma_n^2)$ and L is defined in Lemma 1. Note that the extra letter d in the suffix is added to avoid confusion with similar quantities for STH.

The false alarm probability, $P_{\mathrm{df}} = P(W_i \ge \lambda_2|H_0)$, the expression remains unchanged (for both AWGN and fading) except the fact that λ is now replaced by λ_2. However, to avoid confusion, we denote it as P_{df}, i.e.

$$P^{\mathrm{R}}_{\mathrm{df}} = (1/\sqrt{\pi})\Gamma\left[1/2, \lambda_2^{2/p}/(2\sigma_n^2)\right] \tag{28}$$

3.2.2 Case 4: complex-valued Gaussian PU signal and CSCG noise: If the PU signal is complex Gaussian and noise is CSCG, the above mentioned detection probability, missed detection probability and false alarm probability may be expressed as

$$P^{\mathrm{C}}_{\mathrm{dd}} = L\exp(L)\left[\sqrt{4q_2/L}K_1(\sqrt{4q_2L}) - \int_0^1 \exp(-q_2/t)\exp(-Lt)\,\mathrm{d}t\right] \tag{29}$$

$$P^{\mathrm{C}}_{\mathrm{dm}} = 1 - P_{\mathrm{dd}} \tag{30}$$

$$P^{\mathrm{C}}_{\mathrm{df}} = \exp(-q_2) \tag{31}$$

where L and q_2 are as defined in Section 3.2.1.

3.3 Determination of thresholds (λ_1 and λ_2)

We now define two new probability metrics for characterising the new 'no decision' stage, which arise due to the introduction of DTHs, $\Delta_1 = P(\lambda_1 < W_i < \lambda_2|H_1)$ and $\Delta_0 = P(\lambda_1 < W_i < \lambda_2|H_0)$. They can be evaluated by integrating the conditional PDFs, $f_{W_i|H_1}$ and $f_{W_i|H_0}$, over the range λ_1 to λ_2. The relationship of these quantities with the conditional PDFs given by (14) and (15) and (13), i.e. for the real valued case, are presented graphically in Fig. 4.

The readers may note here that the 'no decision' probability, $\Delta_{0,1}$, can be represented in terms of the conditional CDFs (CCDFs), $F_{W_i|H_{0,1}}(\lambda, p)$, of the decision statistics as $\Delta_{0,1} = F_{W_i|H_{0,1}}(\lambda_2, p) - F_{W_i|H_{0,1}}(\lambda_1, p)$ where $F_{W_i|H_{0,1}}(\lambda, p) = \int_0^{\lambda} f_{W_i|H_{0,1}}(y, p)\,\mathrm{d}y$. We assume that at FC, on the average K out of N local decisions are reported. Let, k_{nor} denotes the normalised average number of

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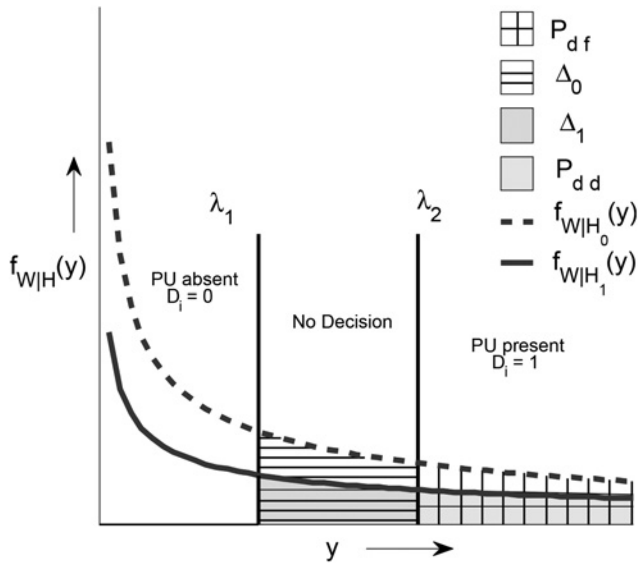


Fig. 4 *Distribution of PDF with DTH*

sensing transmitted bits. This implies [7]

$$k_{\text{nor}} = K/N = 1 - P(H_0)\Delta_0 - P(H_1)\Delta_1 \tag{32}$$

where $K \le N$ results in $k_{\text{nor}} \le 1$. If no CR sensor responds to the FC ($K = k_{\text{nor}} = 0$), the situation is referred to as *failed sensing*. In such situation, receiver requests all the CR users to perform SS again. Let, $b_{0,1}$ denotes the failed sensing probabilities under $H_{0,1}$. As there are N cooperating CRs, we have

$$b_{0,1} = \Delta_{0,1}^N = [F_{W_i|H_{0,1}}(\lambda_2, p) - F_{W_i|H_{0,1}}(\lambda_1, p)]^N$$

It may be noted that $b_{0,1}$ is a variable quantity. Further, we can characterise P_{dd}, P_{df} and P_{dm} in terms of the CCDFs as $P_{\text{dd}} = 1 - F_{W_i|H_1}(\lambda_2, p)$, $P_{\text{dm}} = F_{W_i|H_1}(\lambda_2, p)$, and $P_{\text{df}} = 1 - F_{W_i|H_0}(\lambda_2, p)$, respectively. The thresholds (λ_1 and λ_2) are found using Algorithm 1 (Fig. 1). It is evident from the above discussion that the λ_2 is a function of $F^{-1}_{W_i|H_0}(1 - P_{\text{df}})$ and p, which can be denoted as $\lambda_2 = g_2(F^{-1}_{W_i|H_0}(1 - P_{\text{df}}), p)$. On the other hand, λ_1 is a function of $F^{-1}_{W_i|H_0}(1 - P^*_{\text{df}})$ and p, given as $\lambda_1 = g_1(F^{-1}_{W_i|H_0}(1 - P^*_{\text{df}}), p)$, where $P^*_{\text{df}} = 1 - F_{W_i|H_0}(\lambda_1, p)$. In Algorithm 1 (Fig. 1), two probabilities, namely, $P_x = P(W < \lambda_1|H_0)$ and $P_y = P(W < \lambda_1|H_1)$, are considered for minimising notational complexity. Finally, for a CR network with N nodes that can operate with a desired Q_f value, an algorithm for finding the value of the two thresholds (λ_1 and λ_2) is as mentioned below. The algorithm basically finds a set of possible values of two thresholds depending on the two parameters b_0 and the design parameter p, respectively, for a fixed value of Q_f. For ideal and noise less reporting channels, $Q_f = Q_{\text{f,id}}$, and the corresponding analytical expressions are available in (33), whereas for the imperfect reporting channel $Q_f = Q_{\text{f,imp}}$, given by (39).

Algorithm 1 (Fig. 1) generates a matrix of sensing thresholds where row is for the different values of b_0 and column is for the different values of p.

4 Overall ROC analysis

4.1 Overall ROC analysis for ideal noiseless reporting channels

Let us consider the case when $K \ge 1$. The probabilities of correct sensing are $(1 - b_{0,1})$, since the probabilities of the complementary events, failed sensing, are $b_{0,1}$. The overall false alarm probability ($Q_{\text{f,id}}$) and detection probability ($Q_{\text{d,id}}$) can be represented as $Q_{\text{f,id}} = (1 - b_0)P(W_c > \lambda_2|H_0)$ and $Q_{\text{d,id}} = (1 - b_1)P(W_c > \lambda_2|H_1)$, where W_c denotes the decision statistics at the FC output. As a result of censoring the individual CR decisions lying in the fuzzy region $\lambda_1 < W_i < \lambda_2$, we always have $P(W_c < \lambda_1|H_{0,1}) + P(W_c > \lambda_2|H_{0,1}) = 1$. This particular result enables us to express $Q_{\text{f,id}}$ and $Q_{\text{d,id}}$ in the form

$$Q_{\text{f,id}} = (1 - b_0)\left[1 - P(W_c < \lambda_1|H_0)\right] \tag{33}$$

$$Q_{\text{d,id}} = (1 - b_1)\left[1 - P(W_c < \lambda_1|H_1)\right] \tag{34}$$

Later, it is easy to verify that when the FC implements OR logic, we have

$$P(W_c < \lambda_1|H_0) = \sum_{K=1}^{N} \binom{N}{K} [F_{W_i|H_0}(\lambda_1, p)]^K [F_{W_i|H_0}(\lambda_2, p) - F_{W_i|H_0}(\lambda_1, p)]^{N-K} \tag{35}$$

which reduces to $\left[F_{W_i|H_0}(\lambda_2, p)\right]^N - b_0$. Similarly, it can be shown that, $P(W_c < \lambda_1|H_1) = \left[F_{W_i|H_1}(\lambda_2, p)\right]^N - b_1$. Further, noting that $F_{W_i|H_0}(\lambda_2, p) = 1 - P_{\text{df}}$, and $F_{W_i|H_1}(\lambda_2, p) = 1 - P_{\text{dd}}$, we may rewrite (33) and (34) as

$$Q_{\text{f,id}} = (1 - b_0)\left[1 - (1 - P_{\text{df}})^N + b_0\right] \tag{36}$$

$$Q_{\text{d,id}} = (1 - b_1)\left[1 - (1 - P_{\text{dd}})^N + b_1\right] \tag{37}$$

which can be evaluated after substituting the expressions of P_{dd} and P_{df} as available in Section 3. Another quantity of interest, the overall missed detection probability ($Q_{\text{m,id}}$), can be found using the relation, $Q_{\text{m,id}} = 1 - Q_{\text{d,id}}$.

4.2 Overall ROC analysis for imperfect (noisy and faded) reporting channels

In practice, the reporting channels are not free from fading and noise. Let us assume that binary phase-shift keying (BPSK) modulated local decisions are made at the CRs during DTH-based SS, and the selected CRs send their local binary decisions to FC over the corresponding reporting channels. Further, the fading coefficient of a reporting channel is assumed to be fixed over decision symbol period. In this case, the signal at the FC received from the ith selected CR is

$$y_i = h_i m_i + n_i; \quad i = 1, 2, \ldots, K \tag{38}$$

where h_i is the reporting channel co-efficient, $m_i \in (+\sqrt{E_b}, -\sqrt{E_b})$, and n_i denotes AWGN. For BPSK modulation, the threshold at FC is set to zero. This implies that the false alarm probability, detection probability and missed detection probability at FC for each CR can be written as $P_{\text{f,imp}} = P(y_i > 0|H_0)$, $P_{\text{d,imp}} = P(y_i > 0|H_1)$, and $P_{\text{m,imp}} = (1 - P_{\text{d,imp}})$, respectively.

The overall false alarm probability ($Q_{\text{f,imp}}$), detection probability ($Q_{\text{d,imp}}$) and missed detection probability ($Q_{\text{m,imp}}$) for K number of

CRs for imperfect reporting channel can be derived as

$$
\begin{aligned}
Q_{\mathrm{f,imp}} &= P(K \geq 1|H_0)P(D_{\mathrm{overall}} = 1, |H_0) \\
&= (1 - b_0)\left[1 - (1 - P_{\mathrm{f,imp}})^K + b_0\right]
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
Q_{\mathrm{d,imp}} &= P(K \geq 1|H_1)P(D_{\mathrm{overall}} = 1, |H_1) \\
&= (1 - b_1)\left[1 - (1 - P_{\mathrm{d,imp}})^K + b_1\right]
\end{aligned} \tag{40}
$$

$$
Q_{\mathrm{m,imp}} = 1 - Q_{\mathrm{d,imp}} \tag{41}
$$

where $D_{\mathrm{overall}} = f(D_1, D_2, \ldots, D_K)$ is the overall decision at FC, f is a fusion function and the ultimate decision based on D_{overall} is given by

$$
D_{\mathrm{overall}} = \begin{cases} 1 \Rightarrow & \text{PU present} \\ 0 \Rightarrow & \text{PU absent} \end{cases} \tag{42}
$$

5 Optimal number of CRs in CSS

In a CCR network, the optimisation of number of CRs that are cooperating is required to minimise the total error rate. The total error rate is the sum of false alarm probability and missed detection probability. Further, the optimal number of CRs (n_{opt}) is often lesser than the available CRs (N), and if $n_{\mathrm{opt}} < N$, the delay of the CR network in taking decision regarding the vacant spectrum is also reduced [11].

The total error $P_{\mathrm{e},n}$ for SS for ideal and noise less reporting channel with n cooperating nodes can be written as

$$
\begin{aligned}
P_{\mathrm{e},n} &= P(H_0)Q_{\mathrm{f,id}}(n) + P(H_1)Q_{\mathrm{m,id}}(n) \\
&= P(H_0)Q_{\mathrm{f,id}}(n) + P(H_1)(1 - Q_{\mathrm{d,id}}(n))
\end{aligned} \tag{43}
$$

Similarly the total error $P_{\mathrm{e},n}$ for imperfect reporting channel with n cooperating nodes can be written as

$$
\begin{aligned}
P_{\mathrm{e},n} &= P(H_0)Q_{\mathrm{f,imp}}(n) + P(H_1)Q_{\mathrm{m,imp}}(n) \\
&= P(H_0)Q_{\mathrm{f,imp}}(n) + P(H_1)(1 - Q_{\mathrm{d,imp}}(n))
\end{aligned} \tag{44}
$$

Lemma 3: The optimal number of CRs in the cooperative network is $n_{\mathrm{opt}} = \min(N, \lceil n \rceil)$ where $n = \ln[((1 - b_0)\beta/(1 - b_1)\alpha)(P_{\mathrm{df}}/(1 - P_{\mathrm{dm}}))]/\ln[P_{\mathrm{dm}}/(1 - P_{\mathrm{df}})]$ for ideal noise less reporting channel and $n = \ln[((1 - b_0)\beta/(1 - b_1)\alpha)(P_{\mathrm{f,imp}}/(1 - P_{\mathrm{m,imp}}))]/\ln[P_{\mathrm{m,imp}}/(1 - P_{\mathrm{f,imp}})]$ for imperfect reporting channel.

Proof: Let N be the number of cooperating CRs perform local SS using DTH. The selected CRs report their decisions at the FC through ideal and noise less reporting channel. The local decisions are combined using OR logic at FC to take the overall decision about the presence of PU.
To maximise the performance of SS, $P_{\mathrm{e},n}$ should be minimised. Differentiating (44) with respect to n results in

$$
\begin{aligned}
\frac{\mathrm{d}P_{\mathrm{e},n}}{\mathrm{d}n} &\simeq P_{\mathrm{e},(n+1)} - P_{\mathrm{e},n} \\
&= P(H_0)[Q_{\mathrm{f}}(n + 1) - Q_{\mathrm{f}}(n)] + P(H_1)[Q_{\mathrm{d}}(n) - Q_{\mathrm{d}}(n + 1)]
\end{aligned} \tag{45}
$$

Inserting the expressions for false alarm, (36), and missed detection, (37) probabilities at the FC in (45), we have

$$
\frac{\mathrm{d}P_{\mathrm{e},n}}{\mathrm{d}n} = P(H_0)(1 - b_0)(1 - P_{\mathrm{df}})^n P_{\mathrm{df}} - P(H_1)(1 - b_1)P_{\mathrm{dm}}^n(1 - P_{\mathrm{dm}}) \tag{46}
$$

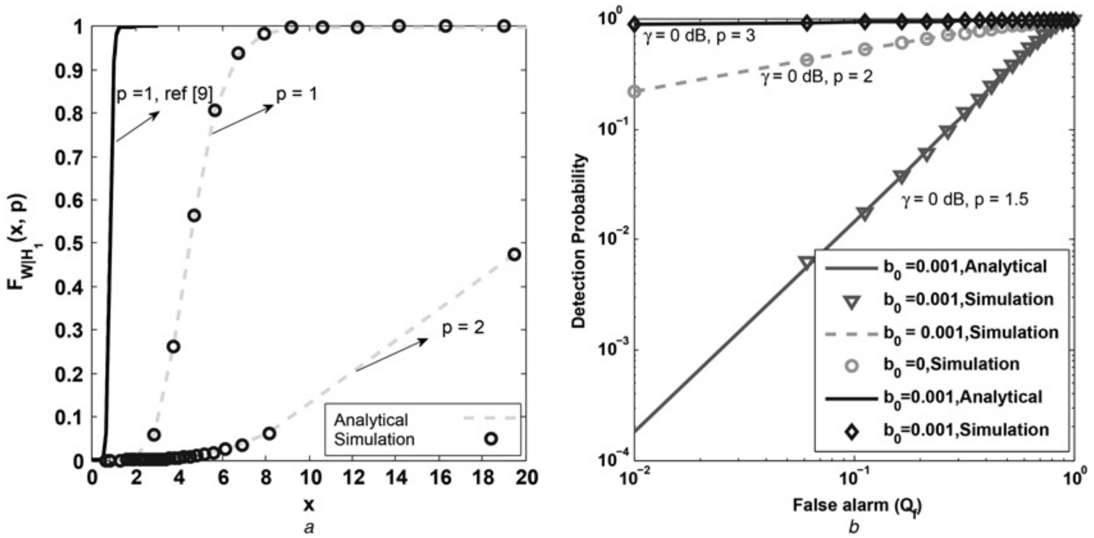
The optimum number of CRs can be obtained when $\mathrm{d}P_{\mathrm{e},n}/\mathrm{d}n = 0$. Equating the right-hand side of (46) to zero, and after applying some algebra we obtain

$$
(P_{\mathrm{dm}}/(1 - P_{\mathrm{df}}))^n = [P(H_0)/P(H_1)][((1 - b_0)/(1 - b_1)][P_{\mathrm{df}}/(1 - P_{\mathrm{dm}})] \tag{47}
$$

Further, replacing $P(H_0)$ and $P(H_1)$ from (10) and (11) in the above mentioned equation and thereafter taking logarithm on both sides we obtain

$$
n = \ln[\{(1 - b_0)\beta/(1 - b_1)\alpha\}\{P_{\mathrm{df}}/(1 - P_{\mathrm{dm}})\}]/\ln[P_{\mathrm{dm}}/(1 - P_{\mathrm{df}})] \tag{48}
$$

Similarly, the value of n for imperfect reporting channel can be



a

b

Fig. 5 *Variation of CDF and detection probability for several values of SPRF parameter (p)*
a Variation of CDF
b Effect of false alarm probability on the detection probability

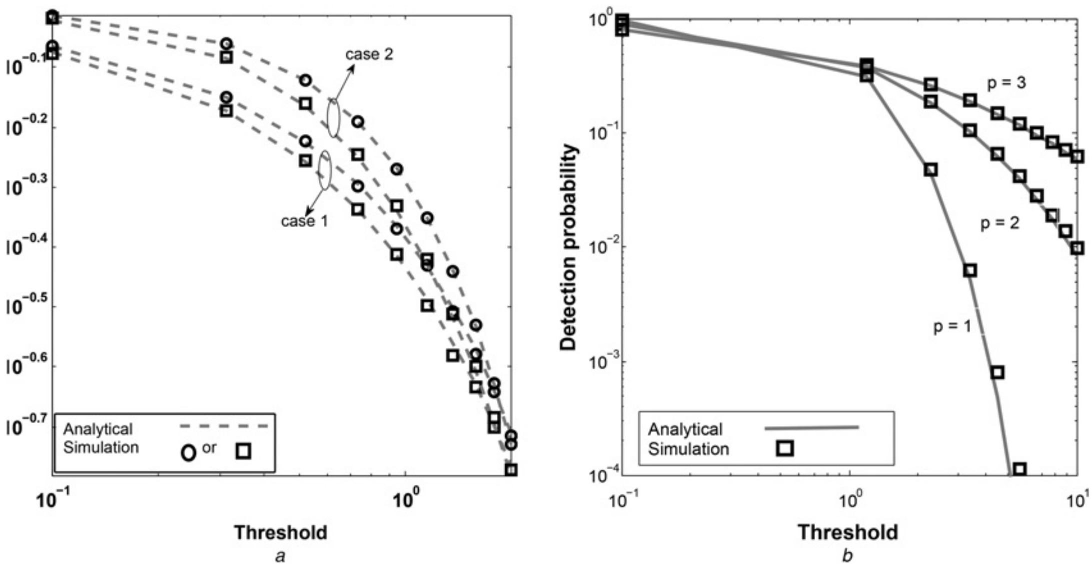


Fig. 6 *Variation of detection probability of with detection threshold*

a Validation of simulation testbed

b Effect of SPRF parameter on the detection probability

obtained as

$$n = \ln[\{(1 - b_0)\beta/(1 - b_1)\alpha\}\{P_{\mathrm{f,imp}}/(1 - P_{\mathrm{m,imp}})\}]/\ln[P_{\mathrm{m,imp}}/(1 - P_{\mathrm{f,imp}})] \tag{49}$$

6 Agility improvement

The communication overhead can be reduced if DTH-based energy detection is used because the CRs with decision statistics in no decision region remain silent instead of responding to FC. Hence, the number of transmitted sensing bits in the reporting channel and detection time can be reduced and thus agility of the overall network will be improved. The total sensing time (T) which consists of local sensing time (T_{LS}) and time required for polling n CRs, ready with sensing decision, is given as

$$T = T_{\mathrm{LS}} + n\,T_{\mathrm{PC}} \tag{50}$$

where T_{PC} is time for polling each CR. For STH-based detection, the total sensing time is

$$T_{\mathrm{ST}} = T_{\mathrm{LS}} + N\,T_{\mathrm{PC}} \tag{51}$$

However, for DTH-based detection, the total sensing time is given by

$$T_{\mathrm{DT}} = T_{\mathrm{LS}} + K\,T_{\mathrm{PC}} \tag{52}$$

The agility gain can be defined as $\mu = T_{\mathrm{ST}}/T_{\mathrm{DT}}$. From (10), (11), and (32), we may write K in terms of α and β as $K = N[1 - \{\beta/(\alpha + \beta)\}\Delta_0$

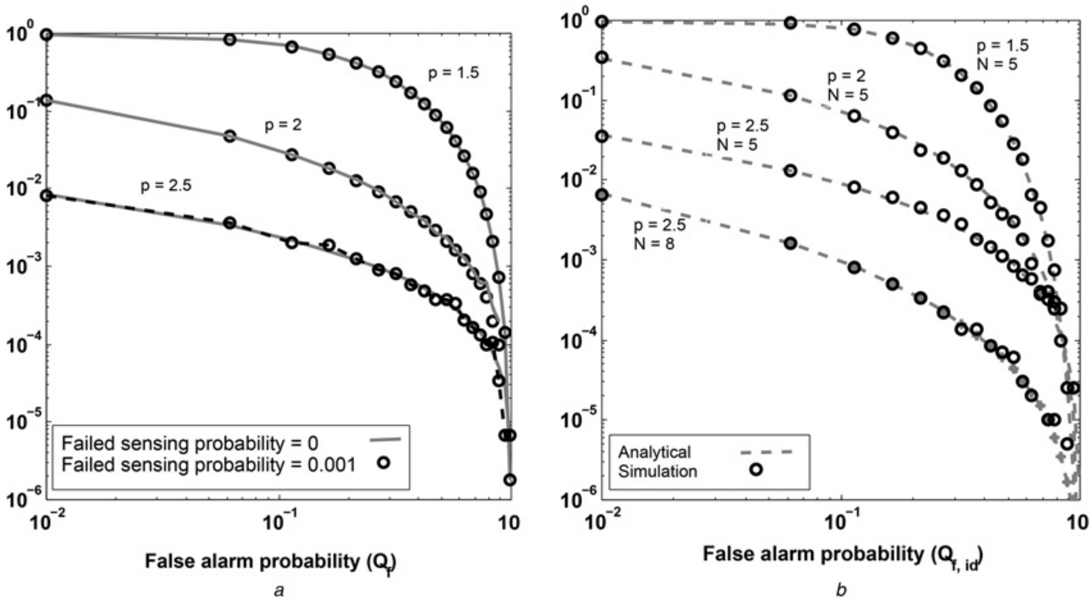


Fig. 7 *Effect of SPRF parameter, fail sensing probability and number cooperating CRs on ROC*

a Effect of SPRF parameter and fail sensing probability on ROC

b ROC of complex valued Gaussian signal and CSCG noise under DTH

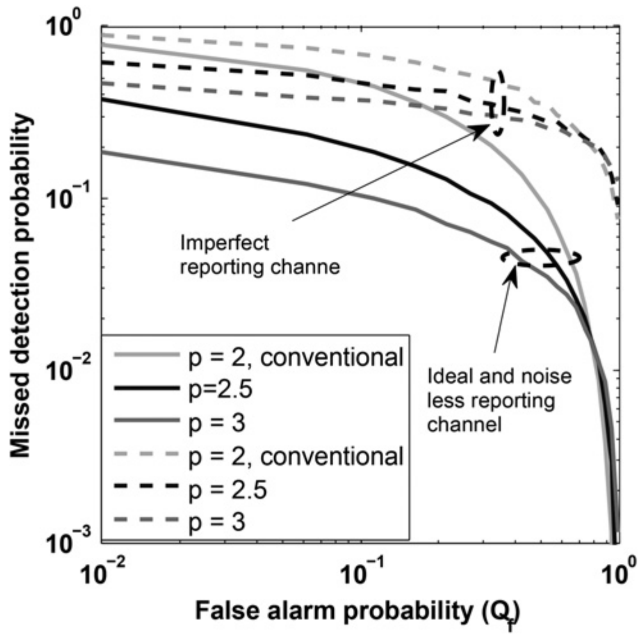


Fig. 8 *ROC for ideal noise less reporting channel and imperfect reporting channel*

$-\{\alpha/(\alpha+\beta)\}\Delta_1]$. Next utilising the expression of K, we obtain the expression for agility gain as

$$\mu = 1 + \frac{NT_{PC}\left[\{\beta/(\alpha+\beta)\}\Delta_0 + \{\alpha/(\alpha+\beta)\}\Delta_1\right]}{T_{LS} + NT_{PC}\left[1 - \{\beta/(\alpha+\beta)\}\Delta_0 - \{\alpha/(\alpha+\beta)\}\Delta_1\right]} \tag{53}$$

7 Results and discussion

We have developed a simulation test bed in MATLAB on the basis of the analysis presented in the earlier sections, and the analytical and simulation results for Rayleigh faded sensing channel are graphically presented here. The default values of parameters used for all the subsequent plots are, $p=2$, $b_0=0.01$, $P(H_0)=P(H_1)=0.5$, $\sigma_s^2=1$, $\sigma_n^2=5$, and $\alpha=\beta=0.5$ unless mentioned otherwise.

In Fig. 5*a*, CDF under H_1 condition has been investigated. It is observed that the analytical and simulation results are well matched and the CDF decreases as the value of IED parameter 'p' increases. In Fig. 5*b*, the detection probability is investigated with respect to the false alarm probability for several values of b_0 and p. It is also observed that the detection probability increases as false alarm probability increases for fixed values of γ, b_0, and p. It is observed that use of DTH has no significant impact on the performance of SS as compared with the case of STH while the performance can be improved if the value of IED parameter increases.

To verify our simulation test bed, simulation results for $N=1$, $\sigma_s^2=1$, $\sigma_n^2=0.5$, $p=1.5$ are superimposed on the corresponding analytical plot in Fig. 6. It was found that there is an exact match. The performance has been investigated for case 1 (real signal/noise) and case 2 (complex signal/noise) for both Gaussian sensing channel (black circles) and Rayleigh faded sensing channel (black squares) to show the impact of fading on the sensing performance. It is seen in Fig. 6*a*, in general, the detection probability (P_d) decreases as the predefined threshold at the energy detector of CR increases and the channel fading degrades the sensing performance. In Fig. 6*b*, the performance has been investigated for the complex signals and CSCG noise (case 2). The variation of P_d against λ has been investigated for different values of SPRF parameter, $p=1$, 2, 3, considering STH ($b_0=0$) at CR. It was found that for a particular value of threshold (λ), detection probability can be improved if the value of p increases.

In Fig. 7*a*, the ROC for case 1 has been investigated under STH ($b_0=0$) as well as under DTH ($b_0=0.001$), for $N=8$ and for different values of $p(=1, 2, 3)$ where b_0 is the failed sensing probability. It is found that the analytical results are well matched with the simulation results for different values of p while $b_0=0$ and $b_0=0.001$. As expected, Q_m decreases as Q_f increases, and for a particular value of Q_f, Q_m reduces when p is increased. It is observed that the performance for $b_0=0.001$ is almost overlapped with the performance for $b_0=0$, for all possible values of p under consideration. The results demonstrate that the use of DTH-based SS reduces the transmission overhead without degrading the performance in comparison to STH-based detection. In Fig. 7*b*, the ROC has been investigated for case 2 under ideal noise less reporting channel. It is found that the simulated results are well matched to the analytical results. The performance is investigated

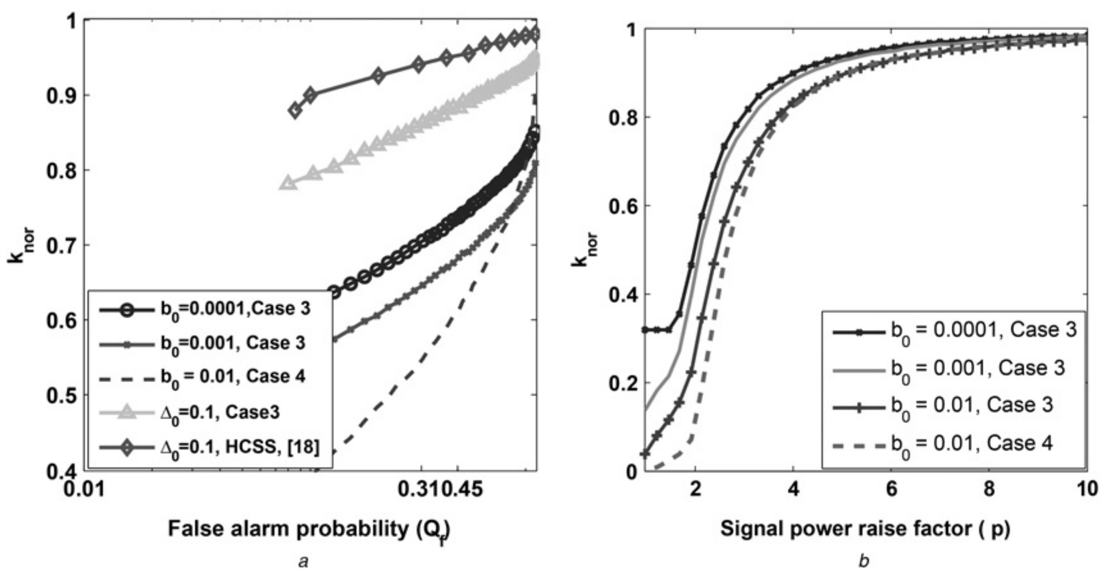


Fig. 9 *Normalised average number of transmitted bits*

a Variation with false alarm probability
b Variation with SPRF parameter

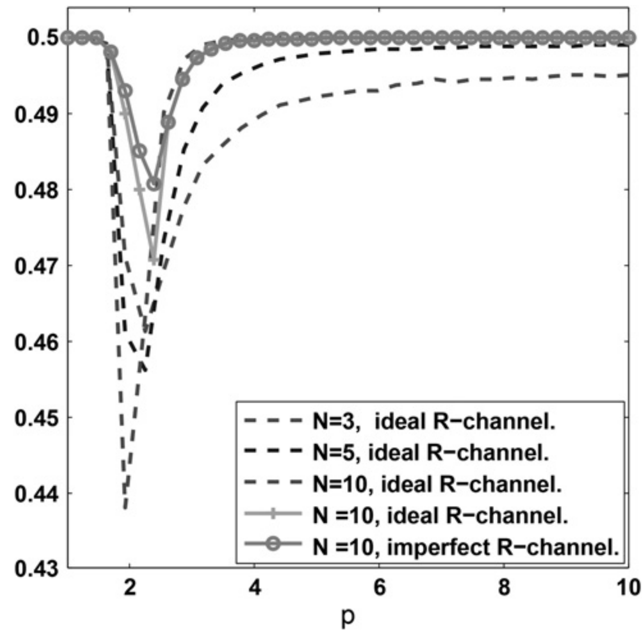


Fig. 10 *Total error as a function of SPRF parameter for different number of CR users*

for $N = 5$, 8 and $b_0 = 0.001$. It is observed that the missed detection probability decreases as the value of SPRF parameter increases or as the number of cooperating CRs increases.

In Fig. 8, the ROC has been investigated for case 2. The performance is compared between two scenarios: ideal noise less reporting channel and imperfect reporting channel, for $N = 4$, $\sigma_s^2 = 2$, $\sigma_n^2 = 1$, and $b_0 = 0.001$. It is observed that the proposed scheme outperforms the conventional scheme while $p > 2$ and the fading in reporting channel degrades the sensing performance significantly.

In Fig. 9, the performance is investigated in terms of the normalised average number of transmitted bits for case 3 (marked with bold lines) and case 4 (dashed lines). Case 3 and case 4, as defined in Section 3.2, refers to the real and complex signal models for the CR network using DTH. In Fig. 9*a*, the normalised average number of transmitted bits (k_{nor}) is shown as a function of Q_f for $b_0 = 0.0001$, 0.001, 0.01, and $N = 8$. It is observed that k_{nor} increases as Q_f increases for a fixed value of b_0. On the other hand, k_{nor} decreases for a particular value of Q_f as b_0 increases. When b_0 increases, the 'no decision' region increase, which in turn reduces the k_{nor} as reflected in the results. It is also found that the number of normalised average number of transmitted bits for case 4 is less compare to case 3. The performance of hierarchical CSS (HCSS) of [19] is also compared with the scheme of case 3 and it is found that the performance of case 3 outperforms the HCSS scheme. Fig. 9*b* shows the variation of (k_{nor}) with SPRF parameter for the same 8 user case. It is found that k_{nor} decreases as the b_0 increases for fixed value of p or while case 4 is used instead of case 3 for fixed value of p and b_0. On the other hand, k_{nor} increases as the value of p increases for a fixed value of b_0. For example, at $b_0 = 0.01$, k_{nor} increases from 0.23 to 0.699 when p increases from 2 to 3.

The variation of the total error against the SPRF parameter for different number of cooperating CR users and different reporting channel (R-channel) environment have been investigated in Fig. 10 for case 3 (dashed lines) and case 4 (bold lines). It is found that the total error initially reduces with p increasing and reaches its minimum value, and thereafter the total error starts increasing with the p. Hence, it is evident that there exists a minimum total error corresponding to an optimal p value for a given number of cooperating CRs. As an indicative example, with $N = 10$ CRs, the total error will be minimum when p is equal to 2. The results also reveal that if the number of cooperating CRs reduces, the minimum total error increases and the minimum occurs at higher values of p. It is also observed that the total error increases while the reporting channel is imperfect and noisy.

In Fig. 11, the optimal number of CR users (n_{opt}) is shown as a function of SPRF parameter. It was found that for a particular number of cooperating CR users, the required n_{opt} for taking decision about the spectrum status decreases as p increases. In Fig. 11*a*, the simulation results are well matched to the analytical results for $b_0 = 0.0001$. The required optimal number of CR users decreases from 8 to 5 when p increases from 4 to 5 when $N = 10$, $\sigma_s^2 = 2$, $\sigma_n^2 = 0.5$, and $b_0 = 0.001$. On the other hand, for a fixed value of p, the required optimal number of CRs increases while the fail sensing probability increases. For example, when $N = 10$ and $p = 5$, the n_{opt} increases from 4 to 5 as b_0 increases from 0.0001 to 0.001. In Fig. 11*b*, we have investigated the n_{opt} with respect to the death rate (β) and observed that n_{opt} decreases as the death rate increases.

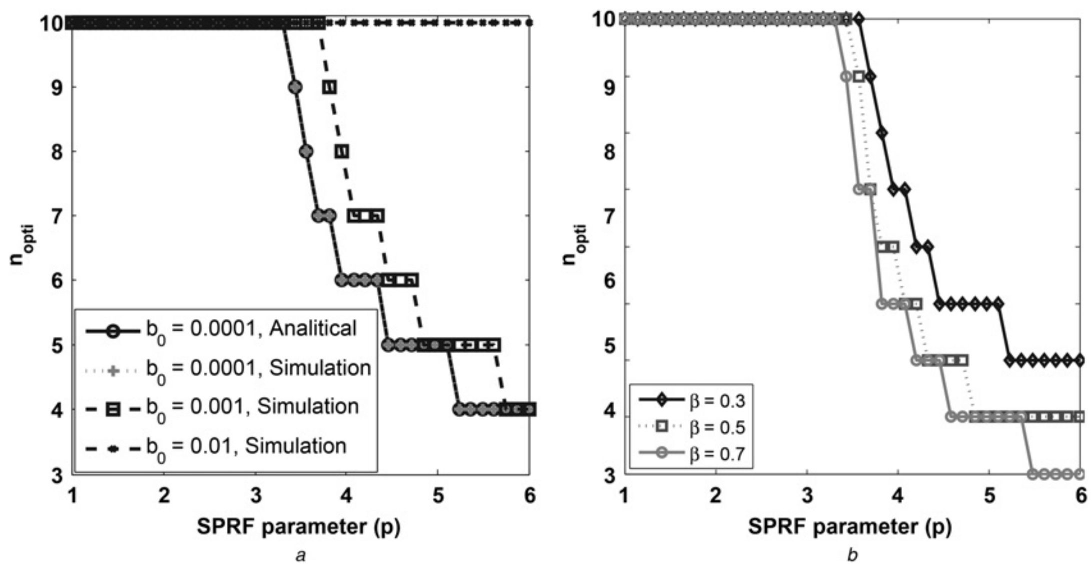


Fig. 11 *Effect of SPRF parameter on optimal number (n_{opt}) of CR users*

a Variation of failed sensing probability
b Variation of death rate

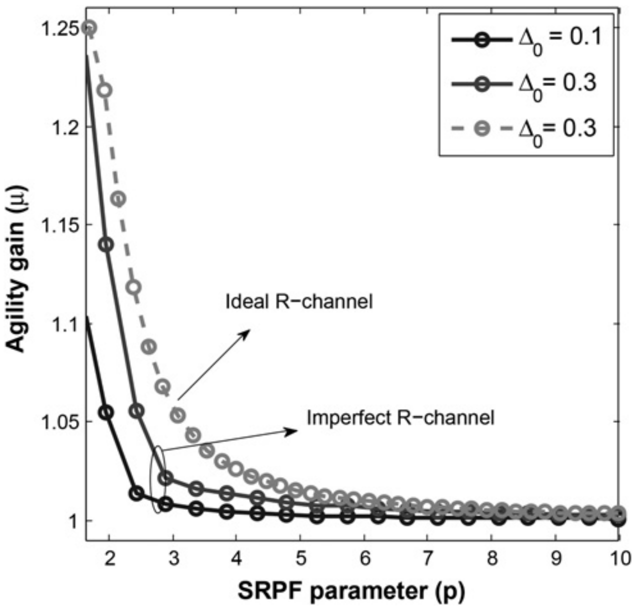


Fig. 12 *Agility gain as a function of SPRF parameter*

In Fig. 12, the agility gain is shown as a function of SPRF parameter for case 4. The performance has been investigated for $N = 8$, $Q_f = 0.01$, and $\Delta_0 = 0.1$, 0.3. From the results, we can say that the agility gain of the network can be improved if the probability of 'no decision' (Δ_0) increases. The average number of normalised sensing bits to be transmitted is reduced when Δ_0 increases, which in turn increases the agility of the network. The agility gain has also been investigated for ideal reporting channel and imperfect reporting channel. It is observed that the agility gain degrades if the reporting channel is imperfect.

8 Conclusion

The joint impact of DTH-based local censoring and use of IED at each of the cooperating CR nodes on the performance of SS have been studied, and an algorithm has been proposed for finding the thresholds for a given set of system parameters. The study led to a novel analytical expression of detection probability for Gaussian PU signal and Rayleigh faded sensing channel. Also, an analytical framework has been developed for the calculation of the optimal number of cooperating CR users which minimises the total error probability. This study is useful in designing a bandwidth constrained CR network as we have found that the impact of SPRF and PU activity parameters on the average number of normalised sensing bits is significant. The number of sensing bits increases with increase in p and decrease in the death rate (β) of PU activity. It is interesting to note that the optimal number of CR users, on the other hand, decreases with the increase in p and β. Further, the agility gain (due to the introduction of DTH) improves with the increase in Δ_0 while the same decreases as p increases. The effect of imperfect reporting channel on the sensing performance is noticeable. It increases the missed detection probability and reduces the agility gain.

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