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Radio resource allocation to provide physical layer security in relay-assisted cognitive radio networks

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Abstract: In this study, the authors consider a cooperative communication framework based on one-way and two-way relays to provide secure communications for secondary users (SUs) within an orthogonal frequency-division multiple access-based underlay cognitive network. By proposing a radio resource allocation problem with the aim of maximising the secrecy sum-rate of SUs and solving it, they show that the deployment of relays is vital to achieve a non-zero secrecy sum-rate. Also, the impact of two-way relay in system performance improvement is clearly visible in comparison with one-way relay such that it can roughly double the resulting system secrecy sum-rate. The impact of different system parameters on the achievable secrecy sum-rate for both one-way and two-way relays is also investigated and compared through simulations.

1 Introduction

Dynamic spectrum access and sharing have been proposed to account for inefficient utilisation of spectrum caused by fixed spectrum assignment to licensed (primary) users [1, 2]. In cognitive radio networks, unlicensed users, called secondary users (SUs), are allowed to use the spectrum assigned to primary users (PUs) provided that a certain level of quality of service is guaranteed for PUs.

Among different spectrum access techniques by SUs, we consider an underlay method [3] where SUs can access the spectrum simultaneously with PUs provided that the resulting interference on PUs activity is below a desired threshold level. In other words, power and frequency allocation to SUs is subject to the so called interference threshold constraint. In an orthogonal frequency division multiple access (OFDMA) environment, such a constraint should be satisfied on any subcarrier.

Due to the broadcast nature of wireless channels, security has been as an important consideration in wireless networks as the transmitted signal will be vulnerable to eavesdropping and wiretapping. From physical layer point of view, the concept of secrecy rate was proposed in [4] over a channel model called the wiretap channel where the secrecy rate is defined as the rate between transmitter and receiver of the legitimate user minus the overheard rate by the eavesdroppers. Such a concept was extended to the case of cognitive radio networks in [5].

It should be noted that if the rate between the transmitter and the eavesdropper exceeds that of the main channel, the secrecy rate is zero which is usually the case. To overcome this issue, the help of relay cooperation has been proposed in some works for conventional (non-cognitive) networks. In [6], a novel role of relays in secure communication from source to destination is addressed where it is shown that the secrecy rate in the absence of relays tends to zero. The focus of many works in this regard has been on one-way relays [7–9]. In [7, 8], power allocation for secrecy rate maximisation is investigated using decode-and-forward (DF) relays. In [9], a DF relay-assisted OFDMA network is considered where in addition to power allocation, subcarrier allocation is also in place to maximise the average secrecy rate.

Recently, the two-way relay channel [10, 11], in which two nodes simultaneously exchange information through one assisting relay have received much research interest due to its high bandwidth efficiency and potential application to cellular networks. As for secure communications, El Gamal *et al.* [12] developed new achievable rate regions for the two-way wiretap channel. In [13, 14], two-way amplify-and-forward relay is used for providing secure communication in conventional networks.

In cognitive radio networks, providing a secure communication for SU's is as important as that of PU's. In general, due to the inclusion of additional constraints imposed by the primary service, the resulting problems will be more challenging. To the best of our knowledge, application of relays to provide secure communication in cognitive radio networks from SU's perspective has not received a deserving attention yet. In this regards, we can only mention the work of [15] where in a cognitive radio framework with untrusted SUs, the secrecy capacity *for PUs* is investigated using one-way relays.

To this end, our aim is to provide non-zero secrecy rate for the SUs within a cognitive radio network using one-way and two-way DF relaying. We propose a resource allocation problem with the aim of maximising the secrecy sum-rate of SUs. We show that due to the inclusion of the extra interference constraint, deployment of relays to extend the feasibility set of the problem is even more vital than the case of conventional networks to obtain a non-zero secrecy rate for SUs. Moreover, deploying two-way relays results in secrecy rates as high as twice of the rate obtained using one-way relays which suggests that the deployment of two-way relays is well worth the extra complexity.

The rest of this paper is organised as follows. In Section 2, the system model is described. The optimisation problem is formulated in Section 3. In Section 4, the proposed problem is solved using the dual decomposition approach. Simulation and numerical results are shown in Section 5 and finally conclusions are given in Section 6.

2 System model and notations

We consider an OFDMA-based cognitive radio network with N subcarriers and U SUs which are assisted by M decode and forward (DF) relays, see Fig. 1*a* for one-way mode and Fig. 1*b* for two-way mode. Moreover, each node operates in a half-duplex mode. For simplicity, in our model, we consider one eavesdropper trying to overhear the transmitted data and one PU. However, the proposed system model can be easily extended to multiple

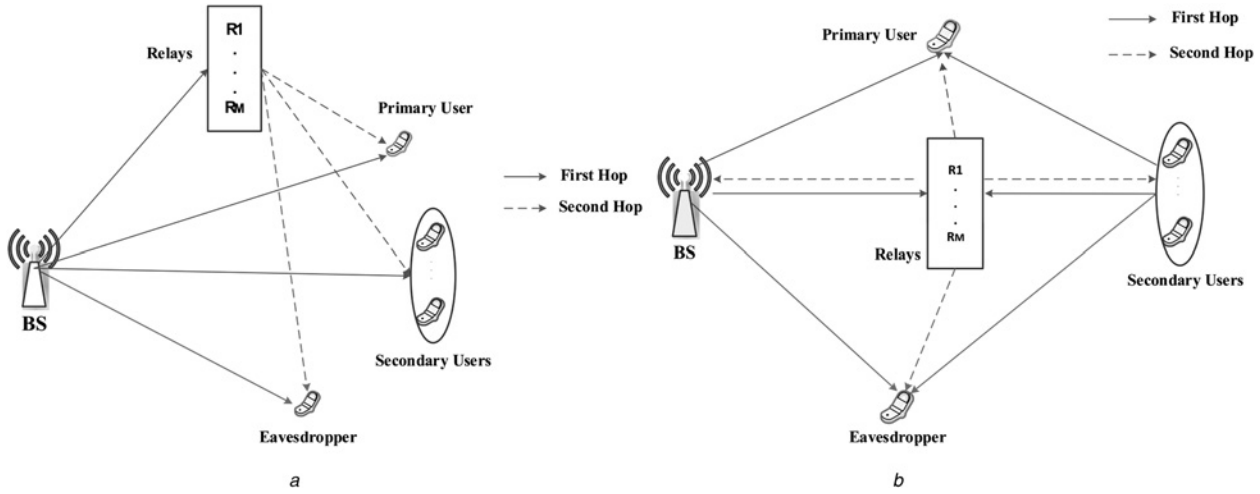

Fig. 1 *System model for different relay modes*

a One-way
b Two-way

eavesdroppers and PUs. The SUs can use the licensed spectrum provided that the interference to the PU does not exceed a threshold level Ith.

The channel state information (CSI) between arbitrary nodes a, and b on the ith subcarrier are characterised by G^i_{ab}, where it is assumed that they remain constant in each frame. We assume the overall effect of the thermal noise and interference resulting from PUs activity as an additive Gaussian random variables with zero mean [16]. To simplify the mathematical analysis, the noise variance is assumed to be constant over all subcarriers.

The allocated power that an arbitrary node a transmits over the ith subcarrier to b is characterised by P^i_{ab}. Consequently, $P^k_{\mathrm{sr}_m}$ denotes the power transmitted by the base station (BS) (source) to the mth relay over subcarrier k. Moreover, $P^k_{ut_m}$ denotes the power transmitted by the user u to the mth relay over subcarrier k. Finally, $P^l_{\mathrm{r}_m u}$ denotes the power transmitted by the mth relay to the uth user over subcarrier l. It is assumed that if a signal from source to a given relay m and from relay m to destination is transmitted over subcarriers k and l, respectively, to form a (k, l) pair, such a pair is assigned to only one relay m. Note that in general, $k \neq l$. The maximum allowable transmission powers that can be used at the BS, users and relays are denoted by P_{BS}, P_{U} and P_{R_m}, respectively.

2.1 One-way relay mode

For one-way relay mode, we consider the downlink connection and in-line with the literature of cooperative communicants, we refer to the secondary BS as source and the secondary receiver as the destination. Transmission is performed in two equal time slots. In the first time slot (hop), the source transmits signals on all subcarriers. Then, in the second time slot, the relays decode the received signals and forward them to the corresponding destinations. In both time slots, the SUs as well as the PU and the eavesdropper receive the transmitted signal.

2.2 Two-way relay mode

Among different two-way relay protocols, in this paper, we focus on the two-phase two-way relay protocols due to its better spectral efficiency [17]. In this scheme, it takes two time slots to exchange information between the two nodes. In the first time slot, which is termed as the multiple access channel (MAC) phase, the BS and users simultaneously transmit signals to the relays. Due to using a half-duplex transmitter–receiver, there is no direct link between BS and users. In the second time slot, which is known as the broadcast channel (BC) phase, the relays broadcast signals to BS and users. In both time slots, PU and eavesdropper also receive signals. Here, the channel reciprocity is assumed, i.e. $G^i_{ab} = G^i_{ba}$.

3 Problem formulation

3.1 One-way relay

To achieve the maximum transmission rate, we use the maximal ratio combining method at the destination [18]. Thus, the signal-to-noise ratio at the SU receiver and the eavesdropper node are, respectively, given by

$$\gamma_{u,c} = P^k_{\mathrm{sr}_m} G^k_{\mathrm{su}} + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m u}, \tag{1}$$

$$\gamma_{e,c} = P^k_{\mathrm{sr}_m} G^k_{\mathrm{se}} + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m e}. \tag{2}$$

In relay-based networks, the overall rate is the minimum of the rates of the two transmission hops [6]. Hence the secrecy rate for the uth user on the paired subcarrier (k, l) assigned to the mth relay is formulated as follows

$$\mathrm{Rate}^u_{m(k,l)} = \min\{R_1, R_2\}, \tag{3}$$

where R_1 and R_2 are the secrecy rates of first and second hops [4] and are obtained as follows [Note that if we do not consider the maximisation in (4), i.e. set $R_1 = 1/2[\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m}) - \log_2(1 + \gamma_{e,c})]$, if the channel gain between the source and the mth relay is too weak, i.e. $G^k_{\mathrm{sr}_m} \to 0$, then $R_1 \to 0$, and $\mathrm{Rate}^u_{m(k,l)} = \min\{R_1, R_2\} \to 0$. However, since the uth user can still get the achievable rate by receiving the signal directly from the source in the first transmission hop without being affected by $G^k_{\mathrm{sr}_m}$, the secrecy rate may still be greater than zero. Consequently to avoid this situation, we modify the expression corresponding to R_1 to (4).]

$$R_1 = \frac{1}{2}[\max\{\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m}), \log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{su}})\} - \log_2(1 + \gamma_{e,c})]^+, \tag{4}$$

$$R_2 = \frac{1}{2}[\log_2(1 + \gamma_{u,c}) - \log_2(1 + \gamma_{e,c})]^+, \tag{5}$$

where factor 1/2 is considered due to the fact that two time slots are used for transmission of one message. Without loss of generality, we

assume the link between source and relay is stronger than the one between source and destination. Therefore, R_1 is modified to

$$R_1 = \frac{1}{2}[\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m}) - \log_2(1 + \gamma_{e,c})]^+ \tag{6}$$

Our goal is to maximise the secrecy rate of the network under available transmission power budget provided that the imposed interference by the SUs on the PU is kept below a predefined threshold level. We define $t_{k,l,u}$ and $\pi_{k,l,m}$ such that if the subcarrier pair (k, l) is allocated to user u, then $t_{k,l,u} = 1$ and if the subcarrier pair (k, l) is allocated to the mth relay, then $\pi_{k,l,m} = 1$. Otherwise, $t_{k,l,u} = \pi_{k,l,m} = 0$. We also define the following sets: $\boldsymbol{P} = [P^k_{\mathrm{sr}_m}, P^l_{\mathrm{r}_m u}]$, $\boldsymbol{\Pi} = [\pi_{k,l,m}]$, $\boldsymbol{t} = [t_{k,l,u}]$.

Now the optimisation problem is formulated as (7) where constraints C1 and C2 indicate the limitations on the maximum allowable transmission power that can be utilised in source and relays, respectively, and constraints C3 and C4 are interference threshold constraints in the first and second time slots, respectively. Constraint C5 corresponds to subcarrier pairing which guaranties that each subcarrier pair is only assigned to one user. Similarly, constraint C6 ensures that each subcarrier pair is assigned to only one relay

$$\begin{aligned}
P^{\text{one-way}}: \quad & \max_{\boldsymbol{P},\boldsymbol{\Pi},\boldsymbol{t}} \quad \sum_{k,l,m,u} \pi_{k,l,m} t_{k,l,u} \, \mathrm{Rate}^u_{m(k,l)}, \\
\text{s.t.:} \quad & \text{C1:} \sum_{m=1}^{M} \sum_{k=1}^{N} P^k_{\mathrm{sr}_m} \le P_{\mathrm{BS}}, \\
& \text{C2:} \sum_{u=1}^{U} \sum_{l=1}^{N} P^l_{\mathrm{r}_m u} \le P_{R_m}, \quad \forall m, \\
& \text{C3:} \sum_{m=1}^{M} P^k_{\mathrm{sr}_m} G^k_{sp} \le I\text{th}, \quad \forall k, \\
& \text{C4:} \sum_{u=1}^{U} \sum_{m=1}^{M} P^l_{\mathrm{r}_m u} G^l_{r_m p} \le I\text{th}, \quad \forall l, \\
& \text{C5:} \sum_{u=1}^{U} \sum_{l=1}^{N} t_{k,l,u} = 1, \quad \forall k; \quad \sum_{u=1}^{U} \sum_{k=1}^{N} t_{k,l,u} = 1, \quad \forall l \\
& \text{C6:} \sum_{m=1}^{M} \pi_{k,l,m} = 1, \quad \forall k, l, \\
& t_{k,l,u} \in \{0, 1\}, \quad \pi_{k,l,m} \in \{0, 1\},
\end{aligned} \tag{7}$$

From (3), it can be seen that the maximum rate over the subcarrier pair (k, l) and the mth relay is achieved when $R_1 = R_2$ [16]. Then, we will have $P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m} = P^k_{\mathrm{sr}_m} G^k_{\mathrm{su}} + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m u}$. Therefore, the allocated power for the mth relay can be expressed as a function of the source power as follows

$$P^l_{\mathrm{r}_m u} = \left[\frac{G^k_{\mathrm{sr}_m} - G^k_{\mathrm{su}}}{G^l_{\mathrm{r}_m u}} P^k_{\mathrm{sr}_m} \right]^+. \tag{8}$$

Therefore, the optimisation problem can be re-formulated as

$$\begin{aligned}
P^{*\text{one-way}}: \quad & \max_{P^k_{\mathrm{sr}_m},\boldsymbol{\Pi},\boldsymbol{t}} \quad \sum_{k,l,m,u} \pi_{k,l,m} t_{k,l,u} \, \mathrm{Rate}^{*u}_{m(k,l)}, \\
\text{s.t.:} \quad & \text{C1} - \text{C3} - \text{C5} - \text{C6} \\
& \text{C2:} \sum_{u=1}^{U} \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{G^k_{\mathrm{sr}_m} - G^k_{\mathrm{su}}}{G^l_{\mathrm{r}_m u}} P^k_{\mathrm{sr}_m} \le P_{R_m}, \quad \forall m, \\
& \text{C4:} \sum_{u=1}^{U} \sum_{m=1}^{M} \sum_{k=1}^{N} \frac{G^k_{\mathrm{sr}_m} - G^k_{\mathrm{su}}}{G^l_{\mathrm{r}_m u}} P^k_{\mathrm{sr}_m} G^l_{\mathrm{r}_m p} \le I\text{th}, \ \forall l,
\end{aligned} \tag{9}$$

where

$$\mathrm{Rate}^{*u}_{m(k,l)} = \frac{1}{2}\left[\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m}) - \log_2\left(1 + P^k_{\mathrm{sr}_m} G^k_{se} + \frac{G^k_{\mathrm{sr}_m} - G^k_{\mathrm{su}}}{G^l_{\mathrm{r}_m u}} P^k_{\mathrm{sr}_m} G^l_{\mathrm{r}_m e}\right)\right] \tag{10}$$

3.2 Two-way relay

For the two-way relay scenario, each rate assignment is in fact a pair of rates denoted by $R = [r_{\mathrm{ul}}, r_{\mathrm{dl}}]$, where r_{ul} and r_{dl} show uplink and downlink transmission rates, respectively. It should be mentioned that by the word uplink we mean the transmission from the SUs to the relays in MAC phase and the transmission from the relays to the BS in BC phase. By downlink, we mean the transmission from the BS to the relays in MAC phase and the transmission from the relays to the SUs in BC phase.

The rate region for the two-way relay channel with DF operation is defined as the intersection of the rate regions corresponding to MAC and BC phases, i.e. [10]

$$\mathfrak{R}^{\mathrm{DF}} = \mathfrak{R}_{\mathrm{MAC}} \cap \mathfrak{R}_{\mathrm{BC}}, \tag{11}$$

where $\mathfrak{R}_{\mathrm{MAC}}$ and $\mathfrak{R}_{\mathrm{BC}}$ denote the secrecy rate regions for the MAC and BC phases, respectively, defined as follows [19, 20]

$$\begin{aligned}
\mathfrak{R}_{\mathrm{MAC}} = \{(r_{\mathrm{ul}}, r_{\mathrm{dl}}): \\
& r_{\mathrm{ul}} \le \frac{1}{2}[\log_2(1 + P^k_{ur_m} G^k_{ur_m}) - \log_2(1 + P^k_{ur_m} G^k_{ue})], \\
& r_{\mathrm{dl}} \le \frac{1}{2}[\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m}) - \log_2(1 + P^k_{\mathrm{sr}_m} G^k_{se})], \\
& r_{\mathrm{ul}} + r_{\mathrm{dl}} \le \frac{1}{2}[\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m} + P^k_{ur_m} G^k_{ur_m}) \\
& \quad - [\log_2(1 + P^k_{\mathrm{sr}_m} G^k_{se} + P^k_{ur_m} G^k_{ue})]\}
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
\mathfrak{R}_{\mathrm{BC}} = \{(r_{\mathrm{ul}}, r_{\mathrm{dl}}): \\
& r_{\mathrm{ul}} \le \frac{1}{2}[\log_2(1 + P^l_{\mathrm{r}_m u} G^l_{\mathrm{sr}_m}) - \log_2(1 + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m e})], \\
& r_{\mathrm{dl}} \le \frac{1}{2}[\log_2(1 + P^l_{\mathrm{r}_m u} G^l_{ur_m}) - \log_2(1 + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m e})]\}.
\end{aligned} \tag{13}$$

Therefore, the secrecy rate region can be expressed as follows:

$$\begin{aligned}
\mathfrak{R}^{\mathrm{DF}} = \{(r_{\mathrm{ul}}, r_{\mathrm{dl}}): \\
& r_{\mathrm{ul}} \le \frac{1}{2}\min\left\{\log_2\left(\frac{1 + P^k_{ur_m} G^k_{ur_m}}{1 + P^k_{ur_m} G^k_{ue}}\right), \log_2\left(\frac{1 + P^l_{\mathrm{r}_m u} G^l_{\mathrm{sr}_m}}{1 + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m e}}\right)\right\}, \\
& r_{\mathrm{dl}} \le \frac{1}{2}\min\left\{\log_2\left(\frac{1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m}}{1 + P^k_{\mathrm{sr}_m} G^k_{se}}\right), \log_2\left(\frac{1 + P^l_{\mathrm{r}_m u} G^l_{ur_m}}{1 + P^l_{\mathrm{r}_m u} G^l_{\mathrm{r}_m e}}\right)\right\}, \\
& r_{\mathrm{ul}} + r_{\mathrm{dl}} \le \frac{1}{2}\log_2\left(\frac{1 + P^k_{\mathrm{sr}_m} G^k_{\mathrm{sr}_m} + P^k_{ur_m} G^k_{ur_m}}{1 + P^k_{\mathrm{sr}_m} G^k_{se} + P^k_{ur_m} G^k_{ue}}\right)\right\},
\end{aligned} \tag{14}$$

Now the optimisation problem is formulated as

$$\begin{aligned}
P^{\text{two-way}}: \quad & \max_{\boldsymbol{R},\boldsymbol{P},\boldsymbol{\Pi},\boldsymbol{t}} \quad \sum_{k,l,m,u} \pi_{k,l,m} t_{k,l,u} \, (r_{\mathrm{ul}} + r_{\mathrm{dl}}) \\
\text{s.t.:} \quad & \text{C1} - \text{C6}
\end{aligned}$$

$$\text{C7:}\ \sum_{m=1}^{M}\sum_{k=1}^{N} P^{k}_{ut_m} \le P_{\text{U}}, \quad \forall u,$$

$$\text{C8:}\ \sum_{u=1}^{U}\sum_{m=1}^{M} P^{k}_{ut_m} G^{k}_{up} \le I\text{th}, \quad \forall k,$$

$$\text{C9:}\ (r_{\text{dl}}, r_{\text{ul}}) \in \Re^{\text{DF}}, \tag{15}$$

where $\boldsymbol{P} = [P^{k}_{\text{sr}_m}, P^{l}_{\text{r}_m u}, P^{k}_{ut_m}]$ and $R = [r_{\text{ul}}, r_{\text{dl}}]$. Moreover, constraints C7 and C8 indicate the limitations on the maximum allowable transmission power that can be utilised in of users and interference threshold constraint which is caused by users, respectively, and constraint C9 is the same as (14).

4 Joint power and subcarrier allocation and relay assignment

In the sequel, we solve the problems using the dual method [21]. To do this, for each problem, i.e. $P^{\text{one-way}}$ and $P^{\text{two-way}}$, the Lagrangian function, L, is separately obtained as follows

$$L^{\text{one-way}} = \sum_{k,l,m,u} \pi_{k,l,m} t_{k,l,u} \tilde{L}^{\text{one-way}}_{k,l,m,u}(\boldsymbol{P}, \beta, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}) + \beta P_{\text{BS}} + \sum_{m=1}^{M} \gamma_m P_{R_m} + \sum_{k=1}^{N} \lambda_k I\text{th} + \sum_{l=1}^{N} \mu_l I\text{th}, \tag{16}$$

and (see (17)). Where

$$\beta, \boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_M], \quad \boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_N],$$
$$\boldsymbol{\mu} = [\mu_1, \ldots, \mu_N], \quad \boldsymbol{\nu} = [\nu_1, \ldots, \nu_U],$$
$$\boldsymbol{\eta} = [\eta_1, \ldots, \eta_N], \quad \zeta = [\zeta_{klmu}, \zeta^{\text{dl1}}_{klmu}, \zeta^{\text{dl2}}_{klmu}, \zeta^{\text{ul1}}_{klmu}, \zeta^{\text{ul1}}_{klmu}]$$

are non-negative values of Lagrange multipliers for constraints C1–C4 and C7–C9 and (see (18) and (19)).

Then, the dual function g for each problem is defined as follows:

$$g^{\text{one-way}}(\beta, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{P^{k}_{\text{sr}_m} \succeq 0} \tilde{L}^{\text{one-way}}_{k,l,m,u}, \tag{20}$$

$$g^{\text{two-way}}(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = \max_{\boldsymbol{P}, \boldsymbol{R} \succeq 0} \tilde{L}^{\text{two-way}}_{k,l,m,u}. \tag{21}$$

Thus, the dual problem can be written as

$$\min_{\beta, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu} \succeq 0} g^{\text{one-way}}(\beta, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}), \tag{22}$$

$$\min_{\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\nu}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}, \boldsymbol{\zeta} \succeq 0} g^{\text{two-way}}(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\nu}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}, \boldsymbol{\zeta}). \tag{23}$$

In general, in (21) when $r_{\text{ul}} \to \infty$, $r_{\text{dl}} \to \infty$ the dual function would be unbounded. If we consider the part of $\tilde{L}^{\text{two-way}}_{k,l,m,u}$ with respect to r_{ul}, r_{dl}, we would have

$$r_{\text{ul}}(1 - \zeta_{klmu} - \zeta^{\text{ul1}}_{klmu} - \zeta^{\text{ul2}}_{klmu}) = \begin{cases} 0 & \text{if } 1 \le \zeta_{klmu} + \zeta^{\text{ul1}}_{klmu} + \zeta^{\text{ul2}}_{klmu} \\ \infty & \text{if } 1 > \zeta_{klmu} + \zeta^{\text{ul1}}_{klmu} + \zeta^{\text{ul2}}_{klmu}. \end{cases} \tag{24}$$

$$r_{\text{dl}}(1 - \zeta_{klmu} - \zeta^{\text{dl1}}_{klmu} - \zeta^{\text{dl2}}_{klmu}) = \begin{cases} 0 & \text{if } 1 \le \zeta_{klmu} + \zeta^{\text{dl1}}_{klmu} + \zeta^{\text{dl2}}_{klmu} \\ \infty & \text{if } 1 > \zeta_{klmu} + \zeta^{\text{dl1}}_{klmu} + \zeta^{\text{dl2}}_{klmu}. \end{cases} \tag{25}$$

Hence, to make sure the dual function is bounded [22], we set $\zeta^{\text{dl2}}_{klmu} = 1 - \zeta_{klmu} - \zeta^{\text{dl1}}_{klmu}$ and $\zeta^{\text{ul2}}_{klmu} = 1 - \zeta_{klmu} - \zeta^{\text{ul1}}_{klmu}$. With these conditions, r_{dl} and r_{ul} are removed from Lagrange function, i.e. (19).

In order to solve (22) and (23) for a given set of Lagrangian multipliers, firstly, (20) and (21) are solved where the resulting vectors of power and subcarrier allocation as well as relay assignment are denoted by $\boldsymbol{P}^*$, $\boldsymbol{\Pi}^*$ and $\boldsymbol{t}^*$, respectively. This can be accomplished in three steps:

(i) *Power allocation:* Assuming that subcarrier pair (k, l) is assigned to relay m, the power allocation can be obtained by solving the following problem for each set of (k, l, m, u)

$$\begin{aligned} \max_{\boldsymbol{P}} \quad & \tilde{L}_{k,l,m,u} \\ \text{s.t.:} \quad & \boldsymbol{P} \succeq 0\ . \end{aligned} \tag{26}$$

To solve (26), we use the Karush–Kuhn–Tucker (KKT) conditions [21]. It must be mentioned that since the problems $P^{\text{one-way}}$ and $P^{\text{two-way}}$ are non-convex optimisation problems with respect to $\boldsymbol{P}$, using KKT conditions leads to a sub-optimal solution.

To determine powers, KKT conditions are applied to (26) and the allocated powers, $\boldsymbol{P}^*$, can be obtained, see the Appendix.

(ii) *Relay assignment:* By substituting the allocated power vector $\boldsymbol{P}^*$ into $\tilde{L}_{k,l,m,u}$, the relay selection for each subcarrier pair (k, l) can be

$$L^{\text{two-way}} = \sum_{k,l,m,u} \pi_{k,l,m} t_{k,l,u} \tilde{L}^{\text{two-way}}_{k,l,m,u}(\boldsymbol{r}, \boldsymbol{P}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\eta}, \boldsymbol{\zeta}) + \beta P_{\text{BS}} + \sum_{m=1}^{M} \gamma_m P_{R_m} + \sum_{k=1}^{N} \lambda_k I\text{th} + \sum_{l=1}^{N} \mu_l I\text{th} + \sum_{u=1}^{U} \nu_u P_{\text{U}} + \sum_{k=1}^{N} \eta_k I\text{th}, \tag{17}$$

$$\tilde{L}^{\text{one-way}}_{k,l,m,u} = \frac{1}{2}\left[\log_2(1 + P^{k}_{\text{sr}_m} G^{k}_{\text{sr}_m}) - \log_2\left(1 + P^{k}_{\text{sr}_m} G^{k}_{se} + \frac{G^{k}_{\text{sr}_m} - G^{k}_{\text{su}}}{G^{l}_{\text{r}_m u}} P^{k}_{\text{sr}_m} G^{l}_{\text{r}_m e}\right)\right] - \beta P^{k}_{\text{sr}_m} - \gamma_m \frac{G^{k}_{\text{sr}_m} - G^{k}_{\text{su}}}{G^{l}_{\text{r}_m u}} P^{k}_{\text{sr}_m} - \lambda_k P^{k}_{\text{sr}_m} G^{k}_{sp} - \mu_l \frac{G^{k}_{\text{sr}_m} - G^{k}_{\text{su}}}{G^{l}_{\text{r}_m u}} P^{k}_{\text{sr}_m} G^{l}_{\text{r}_m p}. \tag{18}$$

$$\begin{aligned} \tilde{L}^{\text{two-way}}_{k,l,m,u} = {} & [r_{\text{ul}} + r_{\text{dl}}] + \zeta^{\text{dl1}}_{klmu}\left[\frac{1}{2}\log_2\left(\frac{1 + P^{k}_{\text{sr}_m} G^{k}_{\text{sr}_m}}{1 + P^{k}_{\text{sr}_m} G^{k}_{se}}\right) - r_{\text{dl}}\right] \\ & + \zeta^{\text{dl2}}_{klmu}\left[\frac{1}{2}\log_2\left(\frac{1 + P^{l}_{\text{r}_m u} G^{l}_{ut_m}}{1 + P^{l}_{\text{r}_m u} G^{l}_{\text{r}_m e}}\right) - r_{\text{dl}}\right] + \zeta^{\text{ul1}}_{klmu}\left[\frac{1}{2}\log_2\left(\frac{1 + P^{k}_{ut_m} G^{k}_{ut_m}}{1 + P^{k}_{ut_m} G^{k}_{ue}}\right) - r_{\text{ul}}\right] \\ & + \zeta^{\text{ul2}}_{klmu}\left[\frac{1}{2}\log_2\left(\frac{1 + P^{l}_{\text{r}_m u} G^{l}_{\text{sr}_m}}{1 + P^{l}_{\text{r}_m u} G^{l}_{\text{r}_m e}}\right) - r_{\text{ul}}\right] + \zeta_{klmu}\left[\frac{1}{2}\log_2\left(\frac{1 + P^{k}_{\text{sr}_m} G^{k}_{\text{sr}_m} + P^{k}_{ut_m} G^{k}_{ut_m}}{1 + P^{k}_{\text{sr}_m} G^{k}_{se} + P^{k}_{ut_m} G^{k}_{ue}}\right) - (r_{\text{ul}} + r_{\text{dl}})\right] \\ & - \beta P^{k}_{\text{sr}_m} - \gamma_m P^{l}_{\text{r}_m u} - \nu_u P^{k}_{ut_m} - \lambda_k P^{k}_{\text{sr}_m} G^{k}_{sp} - \mu_l P^{l}_{\text{r}_m u} G^{l}_{\text{r}_m \text{p}} - \eta_k P^{k}_{ut_m} G^{k}_{up}. \end{aligned} \tag{19}$$

determined by solving the following problem

$$\max_{\boldsymbol{\Pi}} \quad \sum_{k,l,m,u} \pi_{k,l,m} t_{k,l,u} \tilde{L}_{k,l,m,u} \qquad \text{s.t.:} \quad \text{C6.} \tag{27}$$

Now for each subcarrier paired (k, l), the relay that maximises the function $\tilde{L}$ will be selected, in other words

$$\boldsymbol{\Pi}^* = \begin{cases} 1 & m^* = \arg\max_m \tilde{L}_{k,l,m,u}, \\ 0 & \text{o.w.} \end{cases} \tag{28}$$

(iii) *Subcarrier pairing:* Once the power and relay allocation are determined for every subcarrier pair, the following dual function is obtained

$$\max_{\boldsymbol{t}} \quad \sum_{k,l,u} t_{k,l,u} \tilde{L}_{k,l,u} \qquad \text{s.t.:} \quad \text{C5.} \tag{29}$$

To solve (29), exactly one element in each row, column and height of the resulting Lagrangian matrix should be picked such that the summation is as large as possible. This is a standard linear assignment problem which can be efficiently solved by the Hungarian method [23].

After obtaining $\boldsymbol{P}^*, \boldsymbol{\Pi}^*, \boldsymbol{t}^*$ for the given set of Lagrange multipliers, the sub-gradient method can be used [24] to solve the minimisation of the dual function g in (22) and (23) to obtain a new set of Lagrange multipliers.

With the updated values of the Lagrange multipliers, the power allocation, relay assignment and subcarrier allocation are evaluated again and this process continues iteratively.

5 Simulation results

In this section, we carry out some simulations to evaluate the secrecy rate of the proposed framework for different system parameters. We assume that all the users and the eavesdropper are placed in a circle with 1 km diameter and randomly distributed inside it and the BS is located in the centre of this circle. The network platform in our numerical examples is shown in Fig. 2.

The channel power gain is characterised by $\chi d_{ab}^{-\iota}$ where d_{ab} is the distance between arbitrary nodes a and b, ι denotes the path loss exponent, and χ models the shadowing effect. The path loss exponent is set to 2 and the shadowing effect follows a log-normal distribution, i.e. $10\log_{10}(\chi) \sim N(0, 8\,\text{dB})$. We perform simulations for 1000 randomly generated CSI realisations, where the parameters in Table 1 are used in simulation.

The number of subcarriers is $N = 32$ and the different number of users and relays is assumed. The variance of the additive Gaussian noise plus interference is assumed to be 1.

In Fig. 3, we present the secrecy sum-rate against the number of relays for both cognitive radio and conventional networks in one-way and two-way relay frameworks. The resource allocation problem corresponding to the conventional network is the same as that of the cognitive case when the Ith tends to infinity. For the both cases, corresponding to cognitive radio network, we set the Ith = 1. For both cognitive radio and conventional cases, we have $U = 4$, $P_{\text{BS}} = 15$ dB, $P_{\text{U}} = 10$ dB and $P_{R_m} = 10$ dB. As can be seen, while in case of conventional network, the problem is feasible without any relays, a non-zero secrecy rate can only be achieved for the cognitive case when we use at least one relay. This can be intuitively justified as follows. By inclusion of the interference threshold constraint, the feasibility set of the problem becomes empty as opposed to the conventional case, resulting in a zero secrecy rate. By inclusion of relays, the feasibility set size is increased and this results in a non-zero secrecy rate. This shows the importance of using relays in the cognitive case. Nevertheless, for both cases, increasing the number of relays causes the secrecy sum-rate to increase.

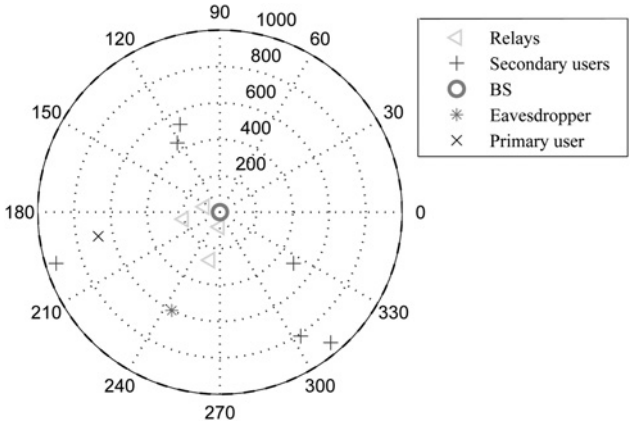

Fig. 2 *Network topology and user placement in simulation setup*

In other figures, we study the system performance for different parameters such as maximum transmit power of transmitters and number of users U, and relays M.

Figs. 4 and 5 show the secrecy sum-rate against the number of users for the different number of relays for one-way and two-way relays frameworks, respectively. Obviously, using two-way relays significantly improves the system performance in comparison with one-way relay. In most of the considered range, this rate improvement is between 50 and 100%. As can be seen, secrecy sum-rate grows by increasing the number of users. Moreover, by increasing the number of relays, the secrecy sum-rate increases.

Table 1 Simulation setup

Simulation parameters	Definition	Parameter value
G_{ab}	CSI between arbitrary nodes a and b	$\chi d_{ab}^{-\iota}$
χ	representing shadowing effect	$10\log_{10}(\chi) \sim N(0, 8\,\text{dB})$
d_{ab}	distances between arbitrary nodes a and b	all nodes randomly distributed inside a circle with 1 km diameter
ι	the path-loss exponent	2
thermal noise	the noise power is normalised to 1	Gaussian random variables with zero mean

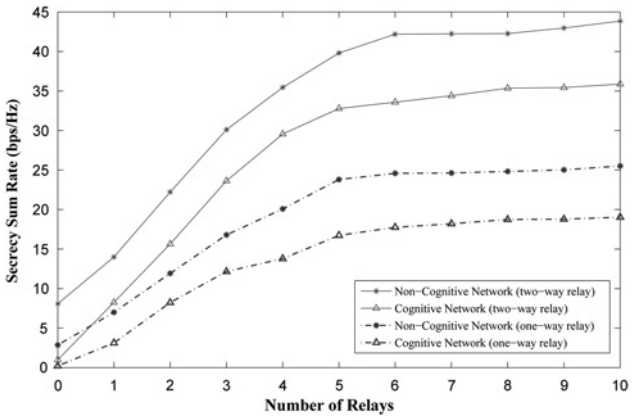

Fig. 3 *Secrecy sum-rate against number of relays, for conventional and cognitive radio networks as well as one-way and two-way relay frameworks. System parameters are $U = 4$, $P_{BS} = 15$ dB, $P_{R_m} = 10$ dB, $P_U = 10$ dB, $Ith = 1$*

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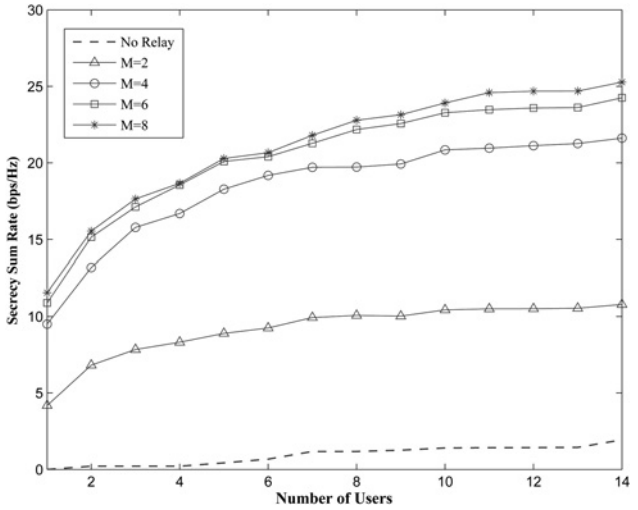

Fig. 4 *Secrecy sum-rate for one-way relay protocol against number of users, for different number of relays, M. System parameters are $P_{BS} = 15$ dB, $P_{R_m} = 10$ dB, $P_U = 10$ dB, Ith = 1*

However, such a growth becomes incremental when the number of relays gets larger. In the considered setup, doubling the number of relays from 4 to 8 results in only a negligible performance improvement. In fact the system performance is now limited by constraint C2.

In Fig. 6, we present the secrecy sum-rate versus the maximum secondary BS transmission power, P_{BS}, for different values of interference threshold. As clearly observed, increasing P_{BS} and Ith results in increasing secrecy sum-rate of the secondary service. Note that for smaller values of the interference threshold and for larger values of BS power, increasing P_{BS} does not necessarily result in increasing secrecy sum-rate of the secondary service. The reason is that the interference threshold constraint poses a restriction on the amount of the secondary BS transmit power. On the other hand, as the interference threshold becomes larger, the resulting sum-rate increase is incremental for a given P_{BS}. In this case, the system performance is limited by the constraint on allowable transmission power of the BS.

In Fig. 7, we present the secrecy sum-rate against the maximum relay transmission power, P_{R_m}, for different values of interference threshold. As can be seen, increasing P_{R_m} and Ith results in increasing secrecy sum-rate of the secondary service. Similar to Fig. 6, when the interference threshold becomes larger, the resulting sum-rate increase becomes incremental for a given P_{R_m}. In this case, the system performance is limited by the constraint on allowable transmission power of the relays.

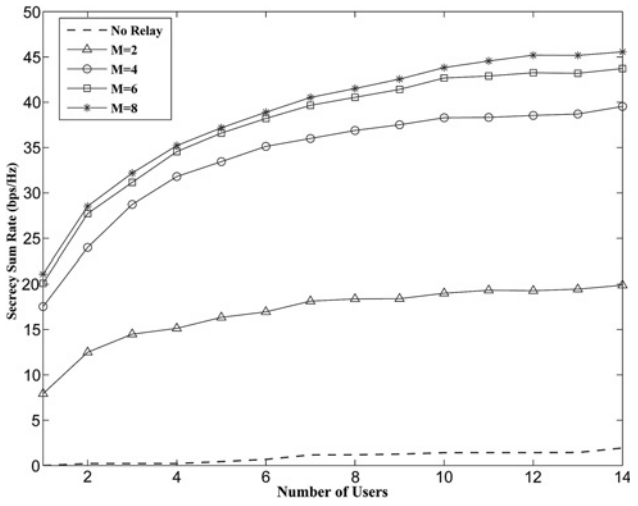

Fig. 5 *Secrecy sum-rate for two-way relay protocol against number of users, for different number of relays, M. System parameters are $P_{BS} = 15$ dB, $P_{R_m} = 10$ dB, $P_U = 10$ dB, Ith = 1*

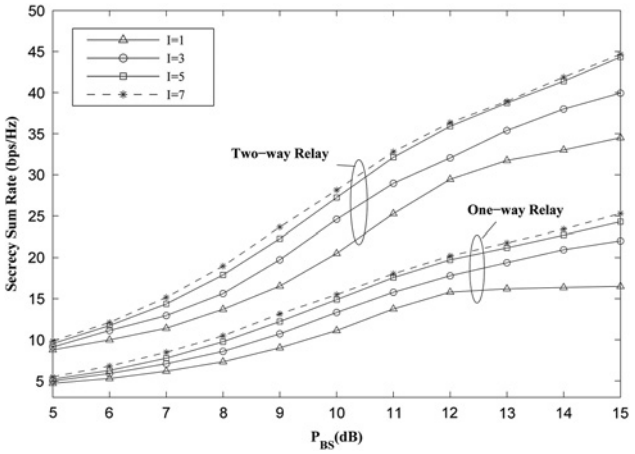

Fig. 6 *Secrecy sum-rate for both one-way and two-way relay protocols against available BS transmission power budget, P_{BS}, for different values of interference thresholds, I. System parameters are $U = 4$, $M = 4$, $P_{R_m} = 10$ dB, $P_U = 10$ dB*

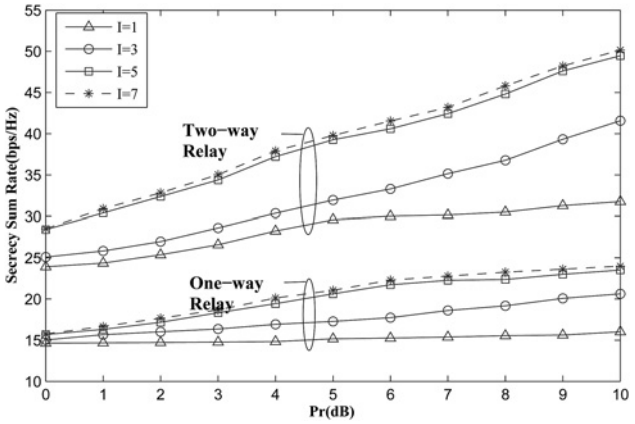

Fig. 7 *Secrecy sum-rate for both one-way and two-way relay protocols against available relays transmission power budget, P_r, for different values of interference thresholds, I. System parameters are $U = 4$, $M = 4$, $P_{BS} = 15$ dB, $P_U = 10$ dB*

6 Conclusions

In this paper, we investigated the impact of deploying one-way and two-way relays on the performance of an OFDMA-based underlay cognitive radio network in providing secure communications for SUs. By proposing a radio resource allocation problem with the aim of maximising the secrecy sum-rate of SUs and comparing it to the results obtained based on one-way relays, it was shown that deploying two-way relays can provide secrecy rates as high as twice of the rate obtained based on one-way relays. The impact of different system parameters such as maximum transmit power of relays and source, number of relays, and the interference threshold on the achievable secrecy sum-rate was also investigated through simulations.

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8 Appendix

8.1 Allocated power of $P^{\text{one-way}}$ and $P^{\text{two-way}}$

Taking the derivative of $\tilde{L}_{k,l,m,u}$ and setting it equal to zero we have

(i) One-way relay: The power allocation for both hops for user u which is assisted by relay m on subcarrier pair $(k,\ l)$ can be obtained as (see (30)) where

$$\alpha_1 = \frac{G^k_{\mathrm{sr}_m} - G^k_{\mathrm{su}}}{G^l_{\mathrm{r}_m u}}, \tag{30b}$$

$$\alpha_2 = 2\ln 2(\beta + \gamma_m \alpha_1 + \lambda_k G^k_{\mathrm{sp}} + \mu_l G^l_{\mathrm{r}_m p}\alpha_1), \tag{30c}$$

and

$$P^{l*}_{\mathrm{r}_m u} = \left[\frac{G^k_{\mathrm{sr}_m} - G^k_{\mathrm{su}}}{G^l_{\mathrm{r}_m u}} P^{k*}_{\mathrm{sr}_m}\right]^+. \tag{30d}$$

(ii) Two-way relay: $P^{k*}_{\mathrm{sr}_m}$, $P^{k^*}_{u\mathrm{r}_m}$ and $P^{l*}_{\mathrm{r}_m u}$ are obtained which are the roots of the following system of equations: (see (31a)–(31c)).

$$P^{k*}_{\mathrm{sr}_m} = \left[\frac{-(G^k_{\mathrm{sr}_m} + G^k_{\mathrm{se}} + G^l_{\mathrm{r}_m e}\cdot\alpha_1)}{2G^k_{\mathrm{sr}_m}(G^k_{\mathrm{se}} + G^l_{\mathrm{r}_m e}\cdot\alpha_1)} + \left(\sqrt{-G^k_{\mathrm{sr}_m} + G^k_{\mathrm{se}} + G^l_{\mathrm{r}_m e}\cdot\alpha_1} \times \frac{\sqrt{-G^k_{\mathrm{sr}_m} + G^k_{\mathrm{se}} + G^l_{\mathrm{r}_m e}\cdot\alpha_1 - \frac{4}{\alpha_2}G^k_{\mathrm{sr}_m}(G^k_{\mathrm{se}} + G^l_{\mathrm{r}_m e}\cdot\alpha_1)}}{2G^k_{\mathrm{sr}_m}(G^k_{\mathrm{se}} + G^l_{\mathrm{r}_m e}\cdot\alpha_1)}\right)\right]^+, \tag{30a}$$

$$\frac{\mathrm{d}\tilde{L}^{\text{two-way}}}{\mathrm{d}P^k_{\mathrm{sr}_m}} = \frac{\zeta^{\mathrm{dl1}}_{klmu}}{2\ln(2)}\left[\frac{G^k_{\mathrm{sr}_m}}{1 + P^k_{\mathrm{sr}_m}G^k_{\mathrm{sr}_m}} - \frac{G^k_{\mathrm{se}}}{1 + P^k_{\mathrm{sr}_m}G^k_{\mathrm{se}}}\right] + \frac{\zeta_{klmu}}{2\ln(2)}\left[\frac{G^k_{\mathrm{sr}_m}}{1 + P^k_{\mathrm{sr}_m}G^k_{\mathrm{sr}_m} + P^k_{u\mathrm{r}_m}G^k_{u\mathrm{r}_m}} - \frac{G^k_{\mathrm{se}}}{1 + P^k_{\mathrm{sr}_m}G^k_{\mathrm{se}} + P^k_{u\mathrm{r}_m}G^k_{ue}}\right] = 0, \tag{31a}$$

$$\frac{\mathrm{d}\tilde{L}^{\text{two-way}}}{\mathrm{d}P^k_{u\mathrm{r}_m}} = \frac{\zeta^{\mathrm{ul1}}_{klmu}}{2\ln(2)}\left[\frac{G^k_{u\mathrm{r}_m}}{1 + P^k_{u\mathrm{r}_m}G^k_{u\mathrm{r}_m}} - \frac{G^k_{ue}}{1 + P^k_{u\mathrm{r}_m}G^k_{ue}}\right] + \frac{\zeta_{klmu}}{2\ln(2)}\left[\frac{G^k_{u\mathrm{r}_m}}{1 + P^k_{\mathrm{sr}_m}G^k_{\mathrm{sr}_m} + P^k_{u\mathrm{r}_m}G^k_{u\mathrm{r}_m}} - \frac{G^k_{ue}}{1 + P^k_{\mathrm{sr}_m}G^k_{\mathrm{se}} + P^k_{u\mathrm{r}_m}G^k_{ue}}\right] - \eta_k G^k_{up} = 0, \tag{31b}$$

$$\frac{\mathrm{d}\tilde{L}^{\text{two-way}}}{\mathrm{d}P^l_{\mathrm{r}_m u}} = \frac{1 - \zeta^{\mathrm{dl1}}_{klmu} - \zeta_{klmu}}{2\ln(2)}\left[\frac{G^l_{u\mathrm{r}_m}}{1 + P^l_{\mathrm{r}_m u}G^l_{u\mathrm{r}_m}} - \frac{G^l_{\mathrm{r}_m e}}{1 + P^l_{\mathrm{r}_m u}G^l_{\mathrm{r}_m e}}\right] + \frac{1 - \zeta^{\mathrm{ul1}}_{klmu} - \zeta_{klmu}}{2\ln(2)}\left[\frac{G^l_{\mathrm{sr}_m}}{1 + P^l_{\mathrm{r}_m u}G^l_{\mathrm{sr}_m}} - \frac{G^l_{\mathrm{r}_m e}}{1 + P^l_{\mathrm{r}_m u}G^l_{\mathrm{r}_m e}}\right] - \gamma_m - \mu_l G^k_{up} = 0. \tag{31c}$$