Rendering Log Aesthetic Curves via Runge-Kutta Method

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Abstract. Log Aesthetic Curves (LAC) are visually pleasing curves which has been developed using monotonic curvature profile. Hence, it can be easily implemented in product design environment, e.g, Rhino 3D CAD systems. LAC is generally represented in an integral form of its turning angle. Traditionally, Gaussian-Kronrod method has been used to render this curve which consumes less than one second for a given interval. Recently, Incomplete Gamma Function was proposed to represent LAC analytically which decreases the computation time up to 13 times. However, only certain value of shape parameters (denoted as α) which dictates the types of curves generated for LAC, can be used to compute LAC. In this paper, the classical Runge-Kutta (RK4) method is proposed to evaluate LAC numerically to reduce the LAC computation time for arbitrary, α. The preliminary result looks promising where the evaluation time is decreased tremendously. This paper also demonstrates the accuracy control of LAC by reducing the stepsize of RK4. The computation time and the accuracy for various α, are also illustrated in the last section of this paper.

Keywords: Log-aesthetic curves, curve synthesis, industrial product design, computer graphic design.
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INTRODUCTION

Customers are attracted by the aesthetic appearance of the products before distinguishing its detailed functional capabilities. Thus, the aesthetic shapes of products dictates to the success of the industrial product [1,2]. In the field of Computer Aided Geometric Design (CAGD) and Computer Aided Design (CAD), the constructing of aesthetic curves has been actively investigated. This curve can be utilized in the design of highways, railway route and etc. [3]. The main characteristic of these curves is they have monotonic curvature profiles [4,5]. The study on curvature profile is discussed in detail in [6].

In 1999, Harada et al. [7] proposed a novel method to analyse the properties of planar curve called Logarithmic Distribution Diagram of Curvature (LDDC) where the relationship between the length frequencies of segmented curve with regards to its radius of curvature is plotted in a log-log coordinate system. Kanaya et al. [5] presented K-vector and later renamed it as Logarithmic Curvature Graph (LCG) [9,10]. If the gradient of LCG is constant, then the curve is categorized as aesthetic curves [5,7]. Therefore, LCG yields the analytic relationship between the interval of radius of curvature and its corresponding length frequency. Moreover, it is also suitable in calculating the aesthetic value of planar curve [9].

Miura [11] described the solution in detail for LCG and he presented the linear LCG as the general formula of Log Aesthetic Curve (LAC). There are four types of nature spiral that can be formed by LAC, i.e. logarithmic spiral, cornu spiral, circle involute and Nielson's spiral. The extension of LAC into Log Aesthetic Space Curve (LASC) had been discussed in [1,12]. Yoshida and Saito analysed the characteristics of the general formula and developed a method to control LAC alternatively. The research on combination of two linear LCG under certain condition had been done by Yoshida and Saito [13]. Furthermore, Yoshida et al. proposed LCG and Logarithmic Torsion Graph (LTG) for analysing the properties of arbitrary parametric curves [10]. In 2009, Levien and Sequin [14] proofed that LAC is the most promising curve for aesthetic design.

In the same year, Miura et al. [15] employed the variational principle into LAC and utilised it as digital filter. Moreover, Gobithaasan et al. [12] extend the analysis of LASC to Generalize LASC (GLASC) by adding two extra degree of freedoms into LASC. In 2009, Gobithaasan et al. [9] analysed Generalised Cornu Spiral (GCS), proposed by Ali et al. [16], and reported that the gradient of LCG can be presented as a linear function and GCS is indeed a generalized aesthetic curve. They further extended LAC to Generalized Log-Aesthetic Curve (GLAC) which has extra degree of freedom [2]. In 2012, Ziatdinov et al. [17] presented an analytic solution of LAC in terms of Incomplete Gamma Function which reduced the computation time up to 13 times. Recent progresses include shape

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analysis on GLAC [18], its drawable region which is wider than LAC [19] and the implementation of G2 LAC as plug-in for Rhino CAD system [20].

This paper proposes a numerical method which is called classical Runge-Kutta method to compute LAC. The final section reports measurement of computation time and its truncation error using classical Runge-Kutta method are compared with the analytic method, proposed by Ziatdinov et al. [17] for a better way to improve the performance of LACs' computation.

LOG AESTHETIC CURVE

In this section, Logarithmic Curvature Graph (LCG) which is the straight line (linear LCG), is used to describe about LAC equation. The equation for LCG which has a slope, α , is the fundamental equation of LAC [11]:

$$\log(\rho \frac{ds}{d\rho}) = \alpha \log \rho + C \tag{1}$$

where s is the arc length of a curve,

ρ is the radius of curvature, and

C is the constant.

The following is obtained by differentiating and substituting $\Lambda = e^{-C}$ (Λ is the shape parameter of LAC) into the equation where Λ is in the range $[0, \infty]$,

$$\frac{ds}{d\rho} = \frac{\rho^{\alpha-1}}{\Lambda}. \tag{2}$$

Integrating equation (2) and rewriting ρ in terms of arc length, s :

$$\rho = \begin{cases} e^{\Lambda s} & \alpha = 0 \\ (1+\Lambda\alpha s)^{\frac{1}{\alpha}} & Otherwise \end{cases} \tag{3}$$

ρ can be expressed in terms of turning angle, $\theta(s)$ by using equation (2) into $\frac{d\theta(s)}{ds} = \frac{1}{\rho}$ and then integrating it we obtain:

$$\rho = \begin{cases} e^{\Lambda\theta(s)} & \alpha = 0 \\ (1+(\alpha-1)\Lambda\theta(s))^{\frac{1}{\alpha-1}} & Otherwise \end{cases} \tag{4}$$

ρ varies from 0 to ∞ , s and $\theta(s)$ has various upper boundary and lower boundary which depending on the α value. The upper boundary and lower boundary of s and $\theta(s)$ with respect to α can be obtained from [2,21].

$$P(\theta) = \begin{cases} \{\int_0^\theta e^{\psi\Lambda} \cos(\psi) d\psi, \int_0^\theta e^{\psi\Lambda} \sin(\psi) d\psi\} & \alpha = 1 \\ \{\int_0^\theta (1+(\alpha-1)\Lambda\psi)^{\frac{1}{\alpha-1}} \cos(\psi) d\psi, \int_0^\theta (1+(\alpha-1)\Lambda\psi)^{\frac{1}{\alpha-1}} \sin(\psi) d\psi\} & Otherwise \end{cases} \tag{5}$$

At the $\theta(s) = 0$, the point defined by equation (5) goes through the origin and its tangent vector is $[1,0]^T$. Referring to equation (3), the curvature radius equation in terms of arc length, s is written as follows:

$$\rho = \begin{cases} e^{\Lambda s} & \alpha = 0 \\ 1+\Lambda s & \alpha = 1 \\ (1+\Lambda\alpha s)^{\frac{1}{\alpha}} & Otherwise \end{cases} \tag{6}$$

Substitute equation (6) into $d\theta = \int \frac{1}{\rho} ds$. Integrate the equation with s where $\theta(s) = 0$ when $s = 0$.

The turning angle of LAC, $\theta(s)$ is:

$$\theta(s)=\begin{cases}\dfrac{1-e^{-\Lambda s}}{\Lambda} & \alpha=0\\ \dfrac{\log(1+\Lambda s)}{\Lambda} & \alpha=1\\ \dfrac{-1+(1+\Lambda\alpha s)^{1-\frac{1}{\alpha}}}{(\alpha-1)\Lambda} & Otherwise\end{cases} \tag{7}$$

The point on LAC $C(s)$ whose arc length, s is defined on the complex plane as

$$C(s)=\begin{cases}\{\int_0^s \cos(\dfrac{1-e^{-\Lambda\mu}}{\Lambda})d\mu, \int_0^s \sin(\dfrac{1-e^{-\Lambda\mu}}{\Lambda})d\mu\} & \alpha=0\\ \{\int_0^s \cos(\dfrac{\log(1+\Lambda\mu)}{\Lambda})d\mu, \int_0^s \sin(\dfrac{\log(1+\Lambda\mu)}{\Lambda})d\mu\} & \alpha=1\\ \{\int_0^s \cos(\dfrac{-1+(1+\Lambda\alpha\mu)^{1-\frac{1}{\alpha}}}{(\alpha-1)\Lambda})d\mu, \int_0^s \sin(\dfrac{-1+(1+\Lambda\alpha\mu)^{1-\frac{1}{\alpha}}}{(\alpha-1)\Lambda})d\mu\} & Otherwise\end{cases} \tag{8}$$

Both equations (5) and (8) can used to render the points on LAC. The differences of these two equations are equation (8) is stable when the curve consist of inflection point ($\rho \to \infty$) and equation (5) is stable when the parameter Λ approaches 0. Hence, equation (8) will be utilized when $\alpha \le 0.5$ and $\Lambda > 1\times10^{-2}$ .Otherwise equation (5) will be used. In the next section, classical Runge Kutta (RK4) will be used to solve the numerical integration.

CLASSICAL RUNGE-KUTTA METHOD

Runge-Kutta methods can be represented in the form of Butcher tableau. Let an initial value problem be specified as follows:

$$y' = f(t,y), y(t_0)=y_0 \tag{9}$$

$$y_{n+1} = y_n + \sum_{i=1}^{s} c_i k_i \tag{10}$$

where

$k_1 = hf(t_n, y_n),$

$k_2 = hf(t_n + a_1 h, y_n + b_{1\ 1} k_1),$

$k_3 = hf(t_n + a_2 h, y_n + b_{2\ 1} k_1 + b_{2\ 2} k_2),$

$\vdots$

$k_s = hf(t_n + a_{s-1} h, y_n + b_{s\ 1} k_1 + b_{s\ 2} k_2 + \cdots + b_{s\ s} k_{s-1}),$

with h as integration step size from [22]. Table 1 shows the value of coefficients for classical Runge-Kutta method in butcher tableau form.

TABLE 1. Butcher Tableau of Classical Runge-Kutta method

k	a_k	b_k			c_k
		1	2	3	
1	0	0			1/6
2	1/2	1/2			1/3
3	1/2	0	1/2		1/3
4	1	0	0	1	1/3

LAC IN DIFFERENTIATION FORM

Equation (5) and (8) are both the general formulas for LAC. Since both equations are integrated, LAC equations in the form of differentiation are required if both equations are applied into classical Runge-Kutta method. Hence LAC formulas in the form of first order ODE can be written:

$$\frac{dP(\theta(s))}{d\psi}=\begin{cases}\{e^{\psi\Lambda}\cos(\psi),e^{\psi\Lambda}\sin(\psi)\} & \alpha=1\\ \{(1+(\alpha-1)\Lambda\psi)^{\frac{1}{\alpha-1}}\cos(\psi),(1+(\alpha-1)\Lambda\psi)^{\frac{1}{\alpha-1}}\sin(\psi)\} & Otherwise\end{cases} \tag{11}$$

and

$$\frac{dC(s)}{d\mu}=\begin{cases}\{\cos(\frac{1-e^{-\Lambda\mu}}{\Lambda}),\sin(\frac{1-e^{-\Lambda\mu}}{\Lambda})\} & \alpha=0\\ \{\cos(\frac{\log(1+\Lambda\mu)}{\Lambda}),\sin(\frac{\log(1+\Lambda\mu)}{\Lambda})\} & \alpha=1\\ \{\cos(\frac{-1+(1+\Lambda\alpha\mu)^{1-\frac{1}{\alpha}}}{(\alpha-1)\Lambda}),\sin(\frac{-1+(1+\Lambda\alpha\mu)^{1-\frac{1}{\alpha}}}{(\alpha-1)\Lambda})\} & Otherwise\end{cases} \tag{12}$$

with the initial value $x_0=0$ and $y_0=0$. In this paper, we used equation (12) to compute LAC segment.

PERFORMANCE METRIC

The performance metric of each method is compared in terms of computation time ($O(n)$) and truncation error ($O(h)$). Mathematica Version 8 has been used to generate the result. A simple computer built with Intel Pentium@ D CPU 2.80GHz with 4GB RAM was used to carry out this research. The classical Runge-Kutta method was programmed using Mathematica Version 8 along with Gaussian-Kronrod method and Incomplete Gamma Function. These methods are used to compare against classical RK method to elucidate its performance in computation LAC. The step size h for classical method is fixed to $\{0.1,0.01,0.001\}$ and $\theta(s)\in[0,1]$.

Computation Time

We have generated LAC segment of each configurations for 5 times and the average value of computation time are reported in Appendices A and B. Appendix A shows the computation time of each method in average are 0.0215222 seconds for classical Runge-Kutta method, 0.198989 seconds for Incomplete Gamma Function and 2.14036 seconds for Gaussian-Kronrod. It clearly indicated that classical Runge-Kutta method shorten the computation time for rendering LAC segment up to 9.3 times and 99.4 times correspond to Incomplete Gamma Function and Gaussian-Kronrod respectively. Since we utilize the fixed step size, $h=0.001$, we may reduce the computation time by increasing the value of h to 0.01 or 0.1 but compromising the accuracy.

Truncation Error

Incomplete Gamma Function is an analytic function, which is interpreted by simple and exact series representation [17]. Now that Incomplete Gamma Function is the exact solution for some α, we have distinguished the truncation error for classical RK method and Gaussian-Kronrod by comparing with Incomplete Gamma Function. Appendix B displays the truncation error for classical RK method and Gaussian-Kronrod for several values of α and Λ. The truncation error for classical RK method is approaching closer to the exact value when the fixed step size reduces.

CONCLUSION AND FUTURE WORK

This paper proposes the usage of classical Runge-Kutta method to render LAC instead of Gaussian-Kronrod or Incomplete Gamma Function. We have compared the performance metric of classical RK method with Incomplete Gamma Function which was introduced in [17] to show its efficiency and effectiveness. Furthermore, we have divided the classical RK method into three different step sizes ($h = 0.1, 0.01, 0.001$) to show the accuracy when the step size reduces. The classical RK method renders LAC segment up to 9.3 times faster than Incomplete Gamma Function and 99.4 times faster than Gaussian-Kronrod. Moreover, the truncation error of classical RK method is substantially small. A noticeable speed can be experienced when one renders a piecewise LAC similar to techniques stated in [8,20], using LAC as a fairing method [15] and extending LACs to form aesthetic surfaces. Our next work-in-progress includes automated control of stepsize with defined accuracy using adaptive Runge-Kutta methods.

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APPENDIX A: TIMESTAMP IN SECOND

Parameter		Classical Runge-Kutta method (RK4)	Other methods	
α	Λ		IGF	Gaussian-Kronrod
-100	0.00825	0.0218	0.1344	2.1244
-10	0.0758	0.0184	0.1282	2.1218
-1	0.417	0.0186	0.1154	2.131
-0.1	0.758	0.0222	0.109	2.134
-0.01	0.825	0.0154	0.1092	2.1434
0	0.833	0.0158	0.0934	2.1094
0.01	1	0.0252	0.0996	2.3432
0.1	1	0.0186	0.1842	2.1528
0.9	1	0.0252	0.9298	2.131
0.99	1	0.0186	0.3838	2.128
1.1	1	0.0192	0.1714	2.1216
$\frac{6}{5}$	1	0.022	0.1498	2.128
$\frac{5}{4}$	1	0.019	0.1528	2.0998
$\frac{4}{3}$	1	0.0214	0.1468	2.1188
$\frac{3}{2}$	1	0.0312	0.1248	2.1214
2	1	0.022	0.1186	2.1026
10	1	0.025	0.209	2.1468
100	1	0.0278	0.2216	2.1684
Average		0.0215222	0.198989	2.14036

APPENDIX B: ERROR ESTIMATE VS. STEPSIZE

Parameter		Classical Runge-Kutta method (RK4)			Gaussian-Kronrod
		Step size			
α	Λ	0.1	0.01	0.001	
2	1	1.87158×10^{-8}	4.95603×10^{-12}	5.20952×10^{-16}	5.2521×10^{-16}
	10	1.7742×10^{-7}	4.72517×10^{-11}	5.27501×10^{-15}	1.67228×10^{-15}
	100	1.76919×10^{-6}	4.71476×10^{-10}	5.24345×10^{-14}	1.87276×10^{-14}
$\frac{4}{3}$	1	2.3415×10^{-8}	5.95807×10^{-12}	9.65177×10^{-16}	7.4534×10^{-16}
	10	4.76795×10^{-6}	1.22994×10^{-9}	1.56416×10^{-13}	3.0141×10^{-14}
	100	4.02401×10^{-3}	1.06458×10^{-6}	1.05203×10^{-10}	3.62686×10^{-11}
$\frac{10}{9}$	1	2.5857×10^{-8}	6.32757×10^{-12}	2.24969×10^{-15}	2.19362×10^{-15}
	10	2.48698×10^{-4}	4.32208×10^{-8}	3.87514×10^{-10}	3.07773×10^{-10}
	100	6617.31	1.1435	0.409439	0.51485

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