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Variable active antenna spatial modulation

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Abstract: In this study, variable active antenna spatial modulation (VASM) technique, which allows the selection of any number of antennas (but not a constant number of active antennas) to transmit a symbol at the same signalling interval, is presented. In the first step of the development of VASM, antenna selection method is determined to use the minimum number of transmit antennas. Complexity analysis of maximum likelihood decoder is given for the new scheme. In the second step, to reduce the correlation effect, a novel antenna selection method called VASM improved (VASMi) that minimises the pairwise error probability is introduced for exponentially correlated channels. For 6, 8, and 10 bit/s/Hz spectral efficiencies, bit error rates are obtained and compared with spatial modulation (SM) and generalised SM (GSM) for different antenna and signal constellation configurations. VASMi outperforms SM and GSM in both uncorrelated and exponentially correlated multiple-input multiple-output channels for similar antenna configurations.

1 Introduction

One practical solution to improve the spectral efficiency and reliability of radio communication for higher data rates is multiple-input multiple-output (MIMO) technology, which attains multiple data streams from multiple antennas [1]. Owing to their implementation simplicity and low decoding complexity, space-time bloc codes (STBCs) have superior potential for MIMO systems [2, 3]. Orthogonal STBCs (OSTBCs), as a special class of STBCs, is very attractive because of their linearly increasing decoding complexity with respect to the number of symbols in signal constellation which restricted to at most 3/4 symbols per channel use for more than two transmit antennas [4]. On the other hand, Bell Labs layered space-time architecture (BLAST) [5] is one of the spatial multiplexing systems to obtain high data rates and, consequently, high spectral efficiencies. The most basic form is known as V-BLAST (vertical-BLAST) [6], which causes high level of inter-channel interference (ICI). A novel MIMO transmission scheme known as spatial modulation (SM) is introduced by Mesleh *et al.* in [7, 8] as an alternative to V-BLAST transmission, which removes the ICI completely between the transmit antennas and decreases the complexity of maximum likelihood (ML) decoder. In SM, the indices of antennas are used to convey information in addition to the conventional two-dimensional signal constellations such as M-ary phase shift keying and M-ary quadrature amplitude modulation (M-QAM). Only one antenna is selected at each instance and therefore, the number of antennas increases exponentially according to the number of bits that represents the indices of antennas. An optimal ML decoder is presented in [9] for SM scheme, which improves error rate approximately in the amount of 4 dB. This optimal decoder makes an exhaustive search over the aforementioned three-dimensional space and provides better error performance than V-BLAST and maximal ratio combining. On the other hand, space shift keying (SSK) modulation for MIMO channels, proposed by Jeganathan *et al.* in [10], which eliminates amplitude/phase modulation and transmits only the antenna indices to decrease the decoding complexity, results in no performance improvement in error rate. Generalised form of SSK (GSSK) is introduced in [11], where various combinations of transmit antenna indices are used unlike SM that activates only one antenna during each transmission interval to eliminate ICI totally. In [12], GSSK is applied to SM and generalised SM (GSM) is introduced by Younis *et al.* SM and STBC are combined in [13, 14] as STBC-SM, which transmits STBC over a MIMO system using different combinations of antennas. It is observed that STBC-SM provides signal-to-noise ratio (SNR) gains of 3.8 dB for 3 bit/s/Hz and 3.4 dB for 6 bit/s/Hz over SM. Trellis coded modulation (TCM) and SM are jointly designed to take advantage of both coding gain and diversity in [15, 16]. This new scheme SM-TCM provides ~6 dB SNR gain compared with SM in uncorrelated Rayleigh channels while using 8 and 16 state encoders, and in correlated channels this gain increases with correlation. In [17], an adaptive SM transmission scheme, which is based on a developed modulation order selection criterion, is proposed, so as to minimise the conditioned pairwise error probability (PEP) for each channel realisation.

In this paper, a novel MIMO transmission technique, called variable active antenna spatial modulation (VASM), which allows variable number of antennas at each instance, is introduced. In the proposed antenna selecting method, number of active antennas is not constant unlike GSM and SM. In this paper, the total number of antennas needed is decreased for smaller M compared with the conventional SM by a proposed selection method. Therefore, high bit rates with practically realisable number of antennas become possible. Additionally, for exponentially correlated channels, a novel method for antenna selection is derived and indicated as VASMi (VASM improved). Bit error probability and unconditional pairwise probability (UPEP) are derived. Bit error performances are obtained for exponentially correlated Rayleigh fading channels and compared with SM and GSM for similar antenna configurations.

The rest of the paper is organised as follows: In Section 2, the system model is given and VASM scheme is introduced. Bit error rate (BER) performances are given in the case of uncorrelated Rayleigh fading channel in Section 3. In Section 4, a novel method for antenna selection in exponentially correlated channels and bit error probability are derived, and then in Section 5, BERs are compared with SM and GSM. Section 6 includes the main conclusions of the paper.

Notation: Bold lowercase is used for column vectors. $(\cdot)^*$ denotes complex conjugation. $(\cdot)^{\mathrm{H}}$ denotes Hermiston of a matrix. For a complex variable x, $\mathrm{Re}\{x\}$ denotes the real part of x. The probability of an event is denoted by $\mathrm{P}[\cdot]$. $X \sim CN(0, \sigma_x^2)$ represents the distribution of a circularly symmetric complex Gaussian random variable (r.v.) X. $\boldsymbol{F}_X(x;\upsilon_1, \upsilon_2)$ represents the F-distribution of r.v. X with degrees of υ_1 and υ_2. $\boldsymbol{G}(x;v)$ represents the chi-square distribution with degree of v. $\|\cdot\|$ stands for the Frobenius norm. The cardinality of the vector $\boldsymbol{u}$ is denoted by $n(\boldsymbol{u})$.

2 System model

In the proposed system model given in Fig. 1, independent and identically distributed (i.i.d.) binary information stream $\boldsymbol{u}$ is split into two parts. The first part is used for selecting antennas and the second part determines the modulated signal from the signal constellation. Input bit stream $\boldsymbol{u} = (\boldsymbol{u}^1, \boldsymbol{u}^2)$, where $\boldsymbol{u}^1$ indicates the first part and $\boldsymbol{u}^2$ indicates the second part. Here, k is the spectral efficiency, and n_T is the number of transmit antennas. More than one antenna can be selected in this antenna selection method. The important point is to find a method that represents the first part of the input with minimum number of antenna. The minimum number of antenna can be attained by choosing antennas if the value of the corresponding bits is one in $\boldsymbol{u}^1$. For example, if $\boldsymbol{u}^1 = (1,0,1)$, the first and the third antennas will be activated and the output vector of the VASM mapper becomes $\boldsymbol{s} = (s, 0, s)$, where $\boldsymbol{s}$ is the signal determined in the modulator using $\boldsymbol{u}^2$.

Here, the problem is that if $\boldsymbol{u}^1$ is all zero, there will be no activated antennas and thus no transmission occurs. Therefore, the need of one more antenna arises for all zero vector. In Table 1, the output vector of the VASM mapper for 3 bit input as $\boldsymbol{u}^1$ is given. The last antenna is used for all zero input vectors. If there are n_T number of transmit antennas, input vector to the VASM mapper consists of $n_\mathrm{T} - 1$ elements. In other words, number of antennas must be one element more than the number of elements of input vector. The input vectors to the VASM mapper and the modulator are in the form of $\boldsymbol{u}^1 = (u_1, u_2, \ldots, u_{n_\mathrm{T}} - 1)$ and $\boldsymbol{u}^2 = (u_{n_\mathrm{T}}, u_{n_\mathrm{T}} + 1, \ldots, u_k)$. The MIMO channel is characterised by an $n_\mathrm{T} \times n_\mathrm{R}$ matrix $\boldsymbol{H}$, whose entries are i.i.d. r.v.s having the $CN(0,1)$ distribution, where n_R denotes the number of receive antennas. It is assumed that $\boldsymbol{H}$ remains constant during the transmission of a frame and takes independent values from one frame to another. Furthermore, $\boldsymbol{H}$ is assumed to be perfectly known at the receiver, but is not known at

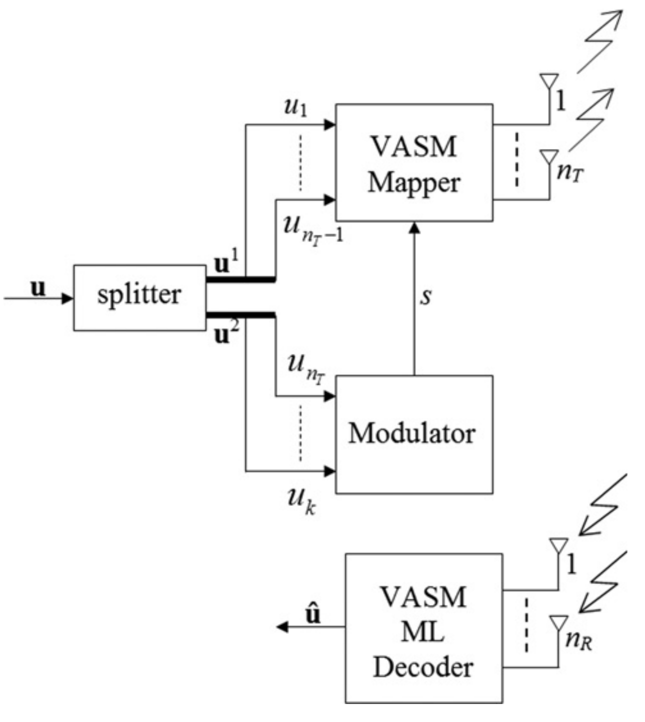

Fig. 1 *Block diagram of the VASM system*

Table 1 Output vector of the VASM mapper for 3 bits input

$\boldsymbol{u}^1$	s
(0,0,0)	(0,0,0,s)
(1,0,0)	(s,0,0,0)
(0,1,0)	(0,s,0,0)
(0,0,1)	(0,0,s,0)
(1,1,0)	(s,s,0,0)
(1,0,1)	(s,0,s,0)
(0,1,1)	(0,s,s,0)
(1,1,1)	(s,s,s,0)

the transmitter. The transmitted signal is corrupted by an n_R-dimensional additive complex Gaussian noise vector with i.i.d. entries distributed as $CN(0,N_0)$. At the receiver, a ML decoder is employed to provide an estimate $\hat{\boldsymbol{u}}$ of the input bit sequence.

To bring out the difference between SM and VASM in the system design, examples can be given for $k = 6$ and 10 bit/s/Hz. For $k = 6$ bit/s/Hz and 4-QAM, in SM, $\boldsymbol{u}^2$ consists of 2 bits and $\boldsymbol{u}^1$ consists of 4 bits. Thus, $n_\mathrm{T} = 2^{n(\boldsymbol{u}^1)} = 2^4 = 16$ antennas are needed. If the number of antennas wanted to be decreased, for example to 8, $\boldsymbol{u}^1$ and $\boldsymbol{u}^2$ both consist of 3 bits and 8-QAM is used in modulator. In VASM, for 4-QAM, $\boldsymbol{u}^2$ consists of 2 bits and $\boldsymbol{u}^1$ consists of 4 bits. $n_\mathrm{T} = n(\boldsymbol{u}^1) + 1 = 5$ antennas are needed. For $k = 10$ bit/s/Hz and 4-QAM, in SM, $\boldsymbol{u}^2$ consists of 2 bits and $\boldsymbol{u}^1$ consists of 8 bits. Thus, $n_\mathrm{T} = 2^{n(\boldsymbol{u}^1)} = 2^8 = 256$ antennas are needed which is not realisable. If the number of antennas wanted to be decreased, for example to 8, $\boldsymbol{u}^1$ consists of 3 and $\boldsymbol{u}^2$ consists of 7 bits and 128-QAM is used in modulator which decrease the performance of BER. In VASM, for 4-QAM, $\boldsymbol{u}^2$ consists 2 bits and $\boldsymbol{u}^1$ consists of 8 bits. $n_\mathrm{T} = n(\boldsymbol{u}^1) + 1 = 9$ antennas are needed which is realisable. The spectral efficiency of the VASM scheme, which can be formulated as

$$m = \log_2(M) + n_\mathrm{T} - 1 \text{ [bit/s/Hz]}, \tag{1}$$

is greater than that of SM schemes, according to the same number of transmit antennas while the spectral efficiency of SM is $m = \log_2(M \times n_\mathrm{T})$ bit/s/Hz. For the same spectral efficiency m and the same number of transmit antennas n_T, the ratio of modulation indexes can be determined as

$$\frac{M_\mathrm{SM}}{M_\mathrm{VASM}} = \frac{2^{n_\mathrm{T}-1}}{n_\mathrm{T}}. \tag{2}$$

M_SM and M_VASM represent the modulation indexes of SM and VASM, respectively. For the spectral efficiency $m = 10$ and $n_\mathrm{T} = 8$, the ratio of modulation index $M_\mathrm{SM}/M_\mathrm{VASM} = 16$ which results in 128-QAM for SM and 8-QAM for VASM. If there is one more antenna such as nine instead of eight, modulation index of VASM can be decreased one level and the ratio becomes $M_\mathrm{SM}/M_\mathrm{VASM} = 2^{n_\mathrm{T}}/n_\mathrm{T}$, thus 4-QAM can be used instead of 8-QAM which improves the performance of BER.

2.1 ML decoder for the VASM system

In this subsection, ML decoder is formulated for the VASM scheme. Quasi-static Rayleigh flat fading MIMO channel with n_T transmit and n_R receive antennas is considered. The received signal vector $\boldsymbol{y}$ can be expressed as

$$\boldsymbol{y} = \sqrt{\frac{\rho}{\mu}}\boldsymbol{s}\boldsymbol{H} + \boldsymbol{\eta} \tag{3}$$

where $\boldsymbol{H}$ is $n_\mathrm{T} \times n_\mathrm{R}$ channel matrix, μ is a normalisation factor to ensure that ρ is the average SNR at each receive antenna. μ depends on number of active antennas during a signalling interval for a constant ρ. $\boldsymbol{s}$ and $\boldsymbol{\eta}$ denote $1 \times n_\mathrm{T}$ transmission and $1 \times n_\mathrm{R}$ noise vectors. $\boldsymbol{H}$ and $\boldsymbol{\eta}$ are assumed to be i.i.d. complex Gaussian random variables with zero mean and unit variance. Channel matrix $\boldsymbol{H}$ is assumed to be known at the receiver but not at the transmitter. The constructed ML decoder makes an exhaustive search over $M \times 2^{n(\boldsymbol{u}^1)}$ combination of transmission to decide the output vector of the mapper that minimise the metric as shown below

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{y} - \sqrt{\frac{\rho}{\mu}} \boldsymbol{sH} \right\|^2. \tag{4}$$

$\boldsymbol{x}$ is the spatially modulated transmitted signal and $\hat{\boldsymbol{x}}$ is the output of the ML decoder, where $\boldsymbol{x} = (A, s)$ and A is a set of active antennas. To indicate the effect of the number of active antennas, the metric can be given as

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{y} - \sqrt{\frac{\rho}{\mu}} \left(\sum_{i=1}^{n_\mathrm{T}} s_i h_{i1} \quad \sum_{i=1}^{n_\mathrm{T}} s_i h_{i2} \quad \cdots \quad \sum_{i=1}^{n_\mathrm{T}} s_i h_{in_\mathrm{R}} \right) \right\|^2 \tag{5}$$

where h_{ij} indicates the channel parameter between ith transmit and jth receive antennas and, s_i is the ith element of the output vector $\boldsymbol{s}$ and $s_i \in \{0, s\}$. The number of active antennas is denoted by n_C, and it gives us the number of $\boldsymbol{s}'$ in the output vector. Therefore, each summation in (5) consists of n_C elements, hence, the metric becomes

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{y} - \sqrt{\frac{\rho}{\mu}} n_\mathrm{C} s \left(\sum_{i \in A} h_{i1} \quad \sum_{i \in A} h_{i2} \quad \cdots \quad \sum_{i \in A} h_{in_\mathrm{R}} \right) \right\|^2. \tag{6}$$

$\boldsymbol{s}$ and active antennas are detected by minimising the metric in (6).

2.2 Complexity analysis of the ML decoder

The comparison of the complexity between methods is generally examined according to the number of multiplications. In contrast to ML decoder of conventional SM, there are $n_\mathrm{C} \times n_\mathrm{R}$ additions in (6), but this can be neglected, also as a multiplication operation, there is only one multiplication because of n_C. However, for some $\boldsymbol{s}$ vectors, the value of n_C is 1, thus multiplication is not needed. For a constant $\boldsymbol{s}$, there are $2^{n(\boldsymbol{u}^1)} = 2^{n_\mathrm{T}} - 1$ possible channel combinations and n_C takes value of 1 for n_T times. Therefore, $2^{n_\mathrm{T}} - 1 - n_\mathrm{T}$ multiplication is employed additionally for a constant $\boldsymbol{s}$. In ML decoder of SM, there are $(n_\mathrm{R} + 1) \times n_\mathrm{T}$ multiplications for all combinations of channels, but in VASM, this will be approximately $(n_\mathrm{R} + 2) \times 2^{n_\mathrm{T}} - 1$ for all combinations of channels because of n_C, in the case of ignoring the values of 1 for n_C. Finally, having M different $\boldsymbol{s}'$, ML decoder employs $M_\mathrm{SM} \times (n_\mathrm{R} + 1) \times n_\mathrm{T}$ multiplications for SM and $M_\mathrm{VASM} \times (n_\mathrm{R} + 2) \times 2^{n_\mathrm{T}} - 1$ for VASM.

To compute the Frobenius norm, n_R multiplications are needed. Therefore, $\hat{\boldsymbol{x}}$ can be decided after $M_\mathrm{SM} \times (n_\mathrm{R} + 1) \times n_\mathrm{T} \times n_\mathrm{R}$ multiplications for SM and $M_\mathrm{VASM} \times (n_\mathrm{R} + 2) \times 2^{n_\mathrm{T}} - 1 \times n_\mathrm{R}$ multiplications for VASM. The ratio of complexities

$$\frac{C_\mathrm{SM}}{C_\mathrm{VASM}} = \frac{M_\mathrm{SM} \times (n_\mathrm{R} + 1) \times n_\mathrm{T} \times n_\mathrm{R}}{M_\mathrm{VASM} \times (n_\mathrm{R} + 2) \times 2^{n_\mathrm{T} - 1} \times n_\mathrm{R}} = \frac{n_\mathrm{R} + 1}{n_\mathrm{R} + 2}. \tag{7}$$

In the case of $k = 10$ bit/s/Hz, $n_\mathrm{T} = 8$, $n_\mathrm{R} = 4$, the complexity of ML decoders can be compared using the number of multiplications. In SM, $n(\boldsymbol{u}^1) = 3$ and $M = 128$, the total number of multiplications $C_\mathrm{SM} = 20480$. In VASM, $n(\boldsymbol{u}^1) = 7$ and $M = 8$, the total number of multiplications $C_\mathrm{VASM} = 24576$, which gives the complexity ratio of 5/6 and is a comparable value against SM.

3 BER performance in uncorrelated Rayleigh fading channels

In this section, simulation results for the VASM scheme in different configurations are introduced and comparisons with SM and GSM are made. For $k = 6$, 8, and 10 bit/s/Hz spectral efficiencies, BER performances are obtained using computer simulation for four receive antennas, $n_\mathrm{R} = 4$. Quasi-static Rayleigh fading channels with $CN(0,1)$ distribution are considered.

First, $k = 6$ bit/s/Hz spectral efficiency is considered. The first configuration for SM is $\boldsymbol{u}^1 = 3$ bits and $\boldsymbol{u}^2 = 3$ bits which results in eight transmit antennas and 8-QAM. The second configuration for SM is $\boldsymbol{u}^1 = 2$ bits and $\boldsymbol{u}^2 = 4$ bits which needs four transmit antennas and 16-QAM. The increase in BER performance with decreasing modulation index which is predicted and stated in the beginning of this paper can be seen from Fig. 2. The configuration for VASM is $\boldsymbol{u}^1 = 4$ bits and $\boldsymbol{u}^2 = 2$ bits which needs five transmit antennas for 4-QAM. The BER performance of this configuration

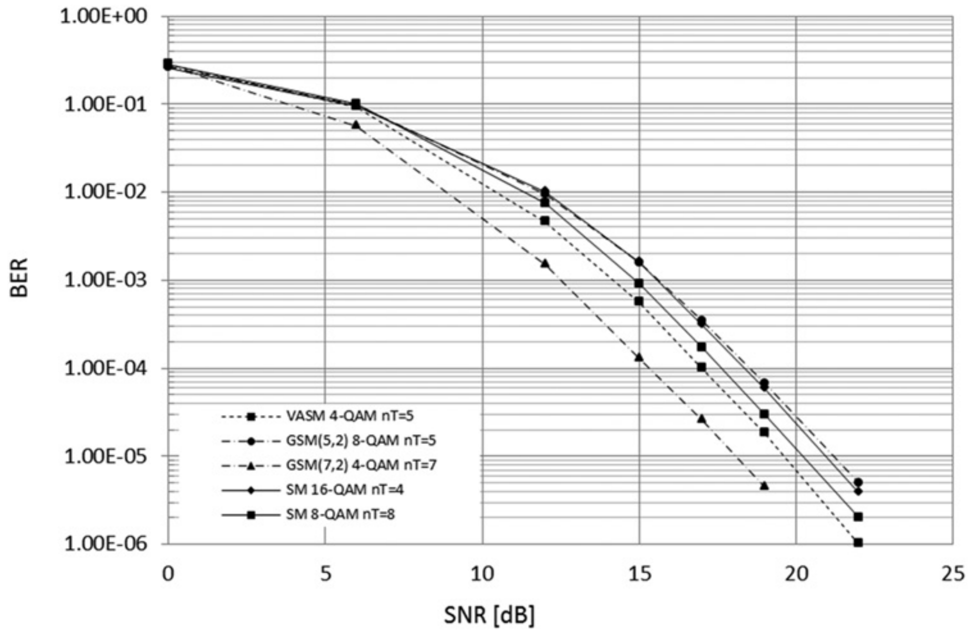

Fig. 2 *BER performance of VASM, GSM, and SM at k = 6 bit/s/Hz in uncorrelated Rayleigh fading channel*

is better than SM configurations. The first configuration for GSM is (5,2), which indicates five transmit antennas and two number of active antennas, $u^1 = 3$ bits and $u^2 = 3$ bits results in 8-QAM which has the worst BER performance. The second configuration for GSM is (7,2), here $u^1 = 4$ bits and $u^2 = 2$ bits results in 4-QAM which has the best BER performance. VASM, GSM (5,2), and SM for $n_T = 4$ are comparable because their antenna configurations are very close to each other. In these antenna configurations, VASM has the best BER performance. For $n_T = 7$ and 8, there is no possible antenna selection method for VASM and, GSM (8,2) has better performance than SM 8-QAM. BER performance of the VASM scheme provides SNR gain of 1.4 dB over the similar configurations, GSM (5, 2) and SM 16-QAM for $k = 6$ bit/s/Hz, respectively.

Second, $k = 8$ bit/s/Hz spectral efficiency is considered and BER performances are shown in Fig. 3. Configurations for SM are $n_T = 4$ for 64-QAM and $n_T = 8$ for 32-QAM. Configurations for VASM are $n_T = 4$ for 32-QAM, $n_T = 6$ for 8-QAM, and $n_T = 7$ for 4-QAM. Configurations for GSM are (7,2) for 16-QAM and (8,4) for 4-QAM. The best BER performances are GSM (8,4) for 4-QAM and VASM for $n_T = 7$. Antenna configurations and BER performances of these schemes are very close to each other and they provide 2.3 dB SNR gain over SM for $n_T = 8$. If we compare the BER performances with respect to exactly the same antenna configurations, VASM for $n_T = 7$ provides SNR gain of 1.5 dB over GSM (7,2).

Third, $k = 10$ bit/s/Hz spectral efficiency is considered and BER performances are shown in Fig. 4. Configurations for SM are $n_T = 4$ for 256-QAM, $n_T = 8$ for 128-QAM, and $n_T = 16$ for 64-QAM. Configurations for VASM are $n_T = 4$ for 128-QAM, $n_T = 7$ for 16-QAM, and $n_T = 9$ for 4-QAM. Configurations for GSM are (7,2) for 64-QAM and (9,4) for 16-QAM. For similar antenna configurations, VASM schemes have better BER performances than the other methods. VASM for $n_T = 9$ provides SNR gain of 3 dB over GSM (9,4), 4.2 dB over SM with $n_T = 16$, and 8 dB over SM with$n_T = 8$. For $n_T = 7$, VASM provides only 1 dB SNR gain over GSM (7,2).

4 Antenna codeword design for exponentially correlated channels: VASMi

In VASM scheme, the number of transmit antennas strictly depends on the number of input bits to the VASM mapper, $n_T = n(u^1) + 1$. To make the system flexible and less sensitive to correlated channels, more number of transmit antennas can be used in VASMi schemes. To analyse the VASMi scheme in correlated channels, spatial correlation channel matrix model [18] is considered as $H_{corr} = R_r^{1/2} H \left(R_t^{1/2}\right)^H$, where R_r is $n_R \times n_R$ and R_t is $n_T \times n_T$ spatial correlation matrices whose elements are indicated by $r_{i,j}$. Here, exponential correlation matrix model is used where $r_{i,j}$ is the correlation coefficient between ith and jth antennas and takes value as $r_{i,j} = r_{j,i}^* = r^{|i-j|}$ and $|r| < 1$ [19]. This model is physically reasonable in the sense that the correlation decreases with increasing distance between receive antennas and it also corresponds to some realistic physical configurations.

In contrast to SM and similar to GSM, the linear combination of the channel coefficients occurs in the received signal. It is expected that, this makes VASM and GSM schemes more sensitive to correlation than SM scheme. To reduce the effect of spatial correlation, the best antenna selection method should be derived from the analytical BER calculation.

4.1 Analytical BER calculation for VASMi

The analytical BER performance of VASMi is estimated using the well-known union bounding technique [20] and BER for GSM [12]. The average BER for VASMi

$$P_b \le E\left[\sum_{x \ne \hat{x}} \frac{d(u_x, \hat{u}_{\hat{x}}) P[x \to \hat{x}]}{2^k}\right] \quad (8)$$

where $d(u_x, \hat{u}_{\hat{x}})$ is the number of bits in error between u_x and $\hat{u}_{\hat{x}}$. $P[x \to \hat{x}]$ indicates the probability of deciding on $\hat{x}$ given that x is transmitted. The probability of $P[x \to \hat{x}]$ can be computed as

$$P[x \to \hat{x}] = P\left[\sum_{j=1}^{n_R} \left|D_j(x)\right|^2 > \sum_{j=1}^{n_R} \left|D_j(\hat{x})\right|^2\right] \quad (9)$$

where the difference

$$D_j(x) = y_j - s h_j \quad (10)$$

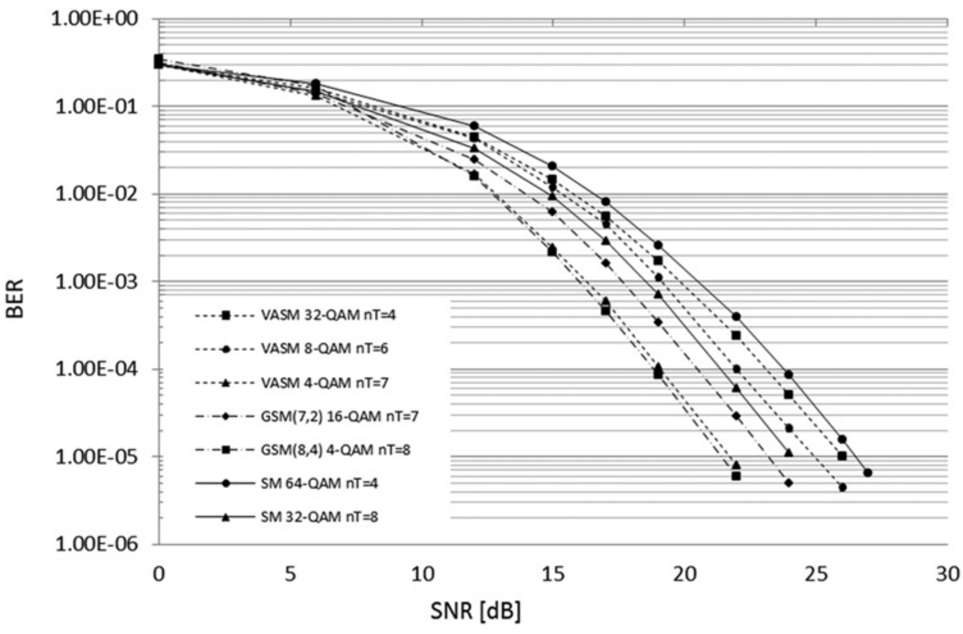

Fig. 3 *BER performance of VASM, GSM, and SM at k = 8 bit/s/Hz in uncorrelated Rayleigh fading channel*

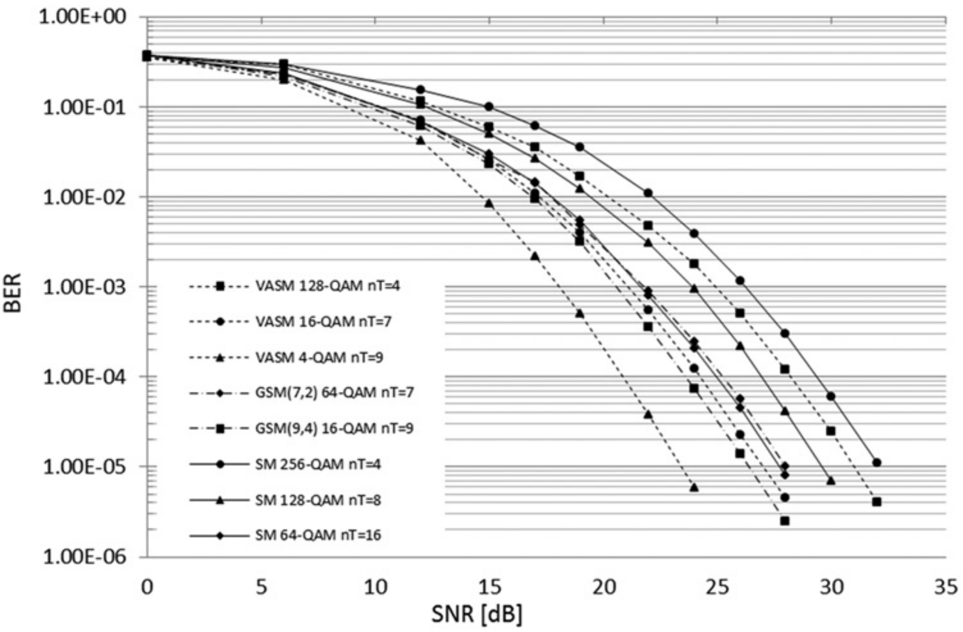

Fig. 4 *BER performance of VASM, GSM, and SM at k = 10 bit/s/Hz in uncorrelated Rayleigh fading channel*

where $\boldsymbol{h}_j$ is the jth column of $\boldsymbol{H}$ and

$$y_j = s\boldsymbol{h}_j + \boldsymbol{\eta}_j \tag{11}$$

then

$$D_j(\boldsymbol{x}) = \boldsymbol{\eta}_j \tag{12}$$

where $\boldsymbol{\eta}_j \sim CN(0, \sigma_n^2)$. $D_j(\hat{\boldsymbol{x}})$ can be given as

$$D_j(\hat{\boldsymbol{x}}) = \boldsymbol{s}\boldsymbol{h}_\mathrm{j} + \boldsymbol{\eta}_j - \hat{\boldsymbol{s}}\boldsymbol{h}_\mathrm{j} = (\boldsymbol{s} - \hat{\boldsymbol{s}})\boldsymbol{h}_j + \eta_j \tag{13}$$

where $\boldsymbol{h}_j$, the channel information, is assumed to be perfectly known in the receiver. $D_j(\hat{\boldsymbol{x}}) \sim CN(0, \sigma_{D_{\hat{x}}}^2)$ and $\sigma_{D_{\hat{x}}}^2$ can be obtained as

$$\begin{aligned}\sigma_{D_{\hat{x}}}^2 &= |s|^2 n(A - \hat{A}) + |\hat{s}|^2 n(\hat{A} - A) \\ &\quad + |s - \hat{s}|^2\left(n_{\mathrm{C}_x} - n(A - \hat{A})\right) + 2|s - \hat{s}|^2 \sum_{\substack{i,j \in A_\mathrm{c} \\ i \neq j}} r_{i,j} \\ &\quad + 2|s|^2\left(\sum_{\substack{i \in A_\mathrm{c} \\ j \in d_A}} r_{i,j} + \sum_{\substack{i,j \in d_A \\ i \neq j}} r_{i,j}\right) + 2|\hat{s}|^2\left(\sum_{\substack{i \in A_\mathrm{c} \\ j \in d_{\hat{A}}}} r_{i,j} + \sum_{\substack{i,j \in d_{\hat{A}} \\ i \neq j}} r_{i,j}\right) \\ &\quad - 2\mathrm{Re}\{s\hat{s}^*\}\left(\sum_{\substack{i \in A_\mathrm{c} \\ j \in d_A}} r_{i,j} + \sum_{\substack{i \in A_\mathrm{c} \\ j \in d_{\hat{A}}}} r_{i,j} + \sum_{\substack{i \in d_A \\ j \in d_{\hat{A}}}} r_{i,j}\right)\end{aligned} \tag{14}$$

where A and $\hat{A}$ are the set of active antennas in $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$, respectively, $n(A - \hat{A})$ is the number of antennas in the set of A difference $\hat{A}$, $n(\hat{A} - A)$ is the number of antennas in the set of $\hat{A}$ difference$A$, A_c is the set of common antennas in A and $\hat{A}$, d_A is the set of antennas of A difference $\hat{A}$, $d_{\hat{A}}$ is the set of antennas of $\hat{A}$ difference$A$, and n_{C_x} is the number of active antennas in $\boldsymbol{x}$.

Let, $\xi_{D_x} = \sum_{j=1}^{n_\mathrm{R}} \left|D_j(\boldsymbol{x})/\sigma_n/\sqrt{2}\right|^2$ and $\xi_{D_{\hat{x}}} = \sum_{j=1}^{n_\mathrm{R}} \left|D_j(\hat{\boldsymbol{x}})/\sigma_{D_{\hat{x}}}/\sqrt{2}\right|^2$ be the summation of n_R squared complex Gaussian r.v. with mean zero and unit variance, thus ξ_{D_x} and $\xi_{D_{\hat{x}}}$ are central chi-squared r.v. with $2n_\mathrm{R}$ degrees of freedom [20]. Thus (9) becomes

$$\boldsymbol{P}[\boldsymbol{x} \rightarrow \hat{\boldsymbol{x}}] = \boldsymbol{P}\left[\frac{\sigma_n^2}{2}\xi_{D_x} > \frac{\sigma_{D_{\hat{x}}}^2}{2}\xi_{D_{\hat{x}}}\right]. \tag{15}$$

Let $\psi = \xi_{D_{\hat{x}}}/\xi_{D_x}$, which has F-distribution with degrees of $\upsilon_1 = \upsilon_2 = 2n_\mathrm{R}$, then (15) can be rewritten as cumulative distribution function (cdf) of F-distribution

$$\boldsymbol{P}[\boldsymbol{x} \rightarrow \hat{\boldsymbol{x}}] = \boldsymbol{F}_\psi\left(\frac{\sigma_n^2}{\sigma_{D_{\hat{x}}}^2}; \upsilon_1 = 2n_\mathrm{R}, \upsilon_2 = 2n_\mathrm{R}\right). \tag{16}$$

We are concerning about how can the bit error probability be decreased, so a simple formula is needed for (8) or for $\boldsymbol{P}[\boldsymbol{x} \rightarrow \hat{\boldsymbol{x}}]$. Thus, chi-square shrinkage factor approximation is used [21]

$$\boldsymbol{F}_\psi\left(\frac{\sigma_n^2}{\sigma_{D_{\hat{x}}}^2}; \upsilon_1 = 2n_\mathrm{R}, \upsilon_2 = 2n_\mathrm{R}\right) \simeq \boldsymbol{G}\left(\lambda\upsilon_1\frac{\sigma_n^2}{\sigma_{D_{\hat{x}}}^2}; \upsilon_1\right) \tag{17}$$

where $\boldsymbol{G}(.)$ is cdf of chi-square distribution and λ is the shrinkage factor

$$\lambda = \frac{6n_\mathrm{R} + \frac{2}{3}n_\mathrm{R}\left(\sigma_n^2/\sigma_{D_{\hat{x}}}^2\right) - 2}{4n_\mathrm{R} + \frac{8}{3}n_\mathrm{R}\left(\sigma_n^2/\sigma_{D_{\hat{x}}}^2\right)}. \tag{18}$$

Let $\tau = \lambda\left(\sigma_n^2/\sigma_{D_{\hat{x}}}^2\right)$, the upper bound of this cdf can be obtained by using Chernoff bounds as

$$\boldsymbol{G}\left(\tau 2n_\mathrm{R}; 2n_\mathrm{R}\right) \leq \left(\tau e^{1-\tau}\right)^{n_\mathrm{R}} \quad \text{for } 0 < \tau < 1. \tag{19}$$

Substituting (19) in (8) gives

$$P_\mathrm{b} \leq E\left[\sum_{\boldsymbol{x} \neq \hat{\boldsymbol{x}}} \frac{d(\boldsymbol{u}_x, \hat{\boldsymbol{u}}_{\hat{x}})\left(\lambda\left(\sigma_n^2/\sigma_{D_{\hat{x}}}^2\right)\mathrm{e}^{1-\lambda\left(\sigma_n^2/\sigma_{D_{\hat{x}}}^2\right)}\right)^{n_\mathrm{R}}}{2^k}\right]. \tag{20}$$

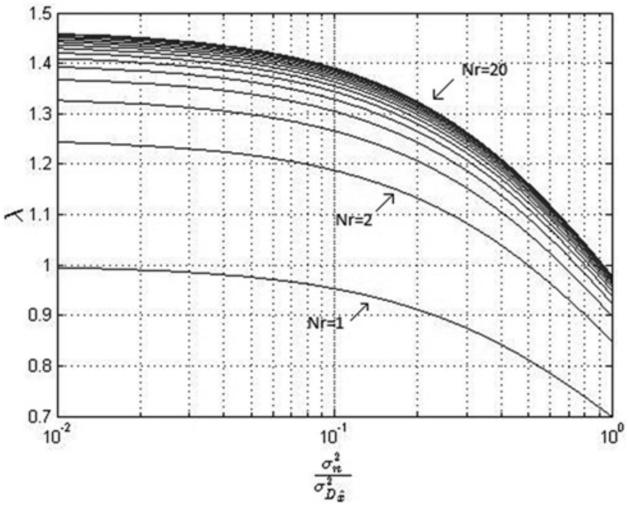

Fig. 5 *Variation of the shrinkage factor λ for different number of receive antennas, N_r*

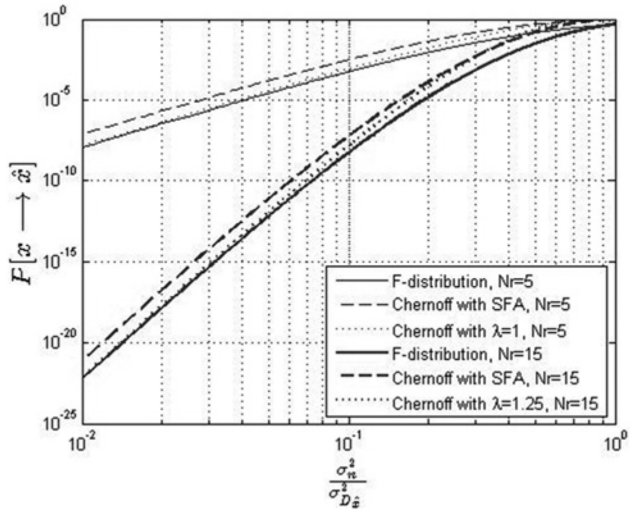

Fig. 6 *PEP variation with respect to the term $\sigma_n^2/\sigma_{D_{\hat{x}}}^2$*

First of all, the changing of the shrinkage factor λ is analysed according to n_R and $\sigma_n^2/\sigma_{D_{\hat{x}}}^2$. It is found that λ is varying slowly in the very limited interval for constant n_R as shown in Fig. 5. Therefore, and for simplicity, assumption about λ is made for $10^{-2} < \sigma_n^2/\sigma_{D_{\hat{x}}}^2 < 1$, which gives the closest upper bound to the cdf of F-distribution as shown in Fig. 6

$$\lambda \simeq \begin{cases} 1 & \text{for} \quad 2 \le n_R \le 5 \\ 1.25 & \text{for} \quad 6 \le n_R \le 15 \\ 1.5 & \text{for} \quad n_R > 15 \end{cases}$$

From (20), the improvement of the bit error probability is directly related with decreasing the term $\sigma_n^2/\sigma_{D_{\hat{x}}}^2$. Therefore, $\sigma_{D_{\hat{x}}}^2$ should be increased by developing an antenna selection method.

4.2 Antenna selection method: antenna codeword design

Antenna codeword design criterions are derived for exponentially correlated channel model by maximising $\sigma_{D_{\hat{x}}}^2$. We only deal with the antenna selection method and ignore the signal constellation. Therefore, using normalised signal power, the analysis for antenna codeword design is carried out under the assumption of $s = \hat{s} = \sqrt{\rho/\mu}$. Thus, (14) becomes

$$\sigma_{D_{\hat{x}}}^2 = \frac{\rho}{\mu}\left[n\left(A - \hat{A}\right) + n\left(\hat{A} - A\right) + 2\left(\sum_{\substack{i,j \in d_A \\ i \ne j}} r_{i,j} + \sum_{\substack{i,j \in d_{\hat{A}} \\ i \ne j}} r_{i,j} - \sum_{\substack{i \in d_A \\ j \in d_{\hat{A}}}} r_{i,j} \right) \right]. \tag{21}$$

In this variance calculation, in the case of uncorrelated case, the number of different antennas in codewords determines the result. However, in the case of correlated case, correlation coefficients between antennas become important. To maximise (21), first of all μ should be small which means less number of active antennas in the whole code set should be selected. Then, first and second summations should be increased and finally, the third summation should be decreased.

In exponentially correlated channel model, correlation coefficients are bigger for close antennas. Therefore, in the first summation, the antenna group in d_A should be side by side, and for the second summation, the antenna group in $d_{\hat{A}}$ should also be side by side. To minimise the third summation, antennas in d_A and $d_{\hat{A}}$ should be as much as away from each other to decrease the correlation coefficients.

Table 2 Output vectors of VASMi and GSM mappers for 4 bits input

$\boldsymbol{u}^1$	$\boldsymbol{S}$ VASMi	$\boldsymbol{S}$ GSM (7,2)
(0,0,0,0)	(s,0,0,0,0,0,0)	(s,s,0,0,0,0,0)
(0,0,0,1)	(0,s,0,0,0,0,0)	(0,s,s,0,0,0,0)
(0,0,1,0)	(0,0,s,0,0,0,0)	(0,0,s,s,0,0,0)
(0,0,1,1)	(0,0,0,s,0,0,0)	(0,0,0,s,s,0,0)
(0,1,0,0)	(0,0,0,0,s,0,0)	(0,0,0,0,s,s,0)
(0,1,0,1)	(0,0,0,0,0,s,0)	(0,0,0,0,0,s,s)
(0,1,1,0)	(0,0,0,0,0,0,s)	(s,0,s,0,0,0,0)
(0,1,1,1)	(s,s,0,0,0,0,0)	(0,s,0,s,0,0,0)
(1,0,0,0)	(0,s,s,0,0,0,0)	(0,0,s,0,s,0,0)
(1,0,0,1)	(0,0,s,s,0,0,0)	(0,0,0,s,0,s,0)
(1,0,1,0)	(0,0,0,s,s,0,0)	(0,0,0,0,s,0,s)
(1,0,1,1)	(0,0,0,0,s,s,0)	(s,0,0,s,0,0,0)
(1,1,0,0)	(0,0,0,0,0,s,s)	(0,s,0,0,s,0,0)
(1,1,0,1)	(s,0,s,0,0,0,0)	(0,0,s,0,0,s,0)
(1,1,1,0)	(0,0,0,0,s,0,s)	(0,0,0,s,0,0,s)
(1,1,1,1)	(0,0,0,s,0,s,0)	(s,0,0,0,s,0,0)

According to the analysis given above, antenna codeword design criterions can be summarised as below:

(i) Total number of active antennas in the codeword set should be minimised.
(ii) For exponentially correlated channel model, to maximise the first and the second summations in (21), active antennas in all antenna codewords should be side by side, or placed close to each other.
(iii) To minimise the third summation in (21), active antennas in different codewords should be placed away from each other.

Therefore, in antenna codeword design, first n_T codewords consist of one active antenna. If more codewords are needed, new codewords are constituted of two active antennas and first side by side antennas are chosen. It should be noted that active antennas in different codewords should be placed away from each other. In Table 2, VASMi and GSM output vectors are given for $n(u^1) = 4$ bits and $n_T = 7$.

5 BER performance in exponentially correlated Rayleigh fading channels

In this section, simulation results of the VASMi scheme for different correlation coefficients are given and comparisons with VASM, GSM, and SM are made. For $k = 6$, 8, and 10 bit/s/Hz spectral efficiencies, BER performances are obtained using computer simulation for four receive antennas, $n_R = 4$. Quasi-static Rayleigh

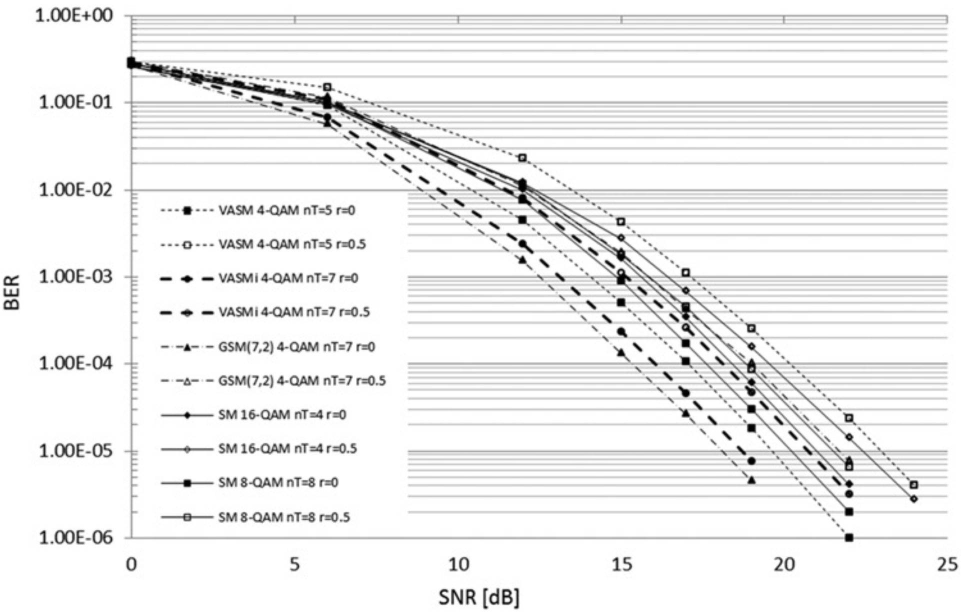

Fig. 7 *BER performance of VASM, VASMi, GSM, and SM at k = 6 bit/s/Hz in Rayleigh fading channel for r = 0 and 0.5*

fading channels with $CN(0,1)$ distribution and exponentially correlated channel model for $r = 0$, 0.5, 0.7, and 0.9 are considered.

In Fig. 7, BER performance of VASMi, VASM, GSM, and SM at $k = 6$ bit/s/Hz in Rayleigh fading channel for $r = 0$ and 0.5 are shown. To compare the systems with each other, similar antenna configurations are determined. Number of transmit antennas are $n_T = 5$ for VASM, $n_T = 7$ for VASMi and GSM, and $n_T = 4$ and 8 for SM. In uncorrelated case, GSM has quite better BER performance than SM and VASM, and 0.6 dB better performance than VASMi. In correlated case, VASMi has the best BER performance and it provides 1 dB SNR gain over GSM.

In Fig. 8, BER performance of VASM, VASMi, GSM, and SM at $k = 8$ bit/s/Hz in Rayleigh fading channel for $r = 0$, 0.5, and 0.7 are shown. Similar antenna configurations are determined to compare the systems. Number of transmit antennas are $n_T = 7$ for VASM, VASMi, and GSM (7,2), $n_T = 8$ for GSM (8,4) and SM. In uncorrelated case, BER curves of VASM, VASMi, and GSM are close to each other better than SM's. In correlated case of $r = 0.5$, VASMi has the best BER performance and it provides 1.1 dB SNR gain over VASM, 1.4 dB SNR gain over SM, and ∼2.3 dB SNR gain over GSM (8,4) and (7,2). In the case of $r = 0.7$, BER curves of VASMi and SM are close to each other. VASMi provides 1.2 dB SNR gain over VASM and ∼3 dB SNR gain over GSM (8,4) and (7,2).

In Fig. 9, BER performance of VASM, VASMi, GSM, and SM at $k = 10$ bit/s/Hz in Rayleigh fading channel for $r = 0$, 0.7, and 0.9 are shown. Similar antenna configurations are determined to compare the systems. Number of transmit antennas are $n_T = 9$ for VASM, VASMi, and GSM (9,4), $n_T = 8$ for SM. In uncorrelated case, BER curves of VASM and VASMi are very close to each other and provide ∼3.5 dB SNR gain over GSM (9,4), 6.5 dB over SM. In correlated case of $r = 0.7$, VASMi provides 1.5 dB SNR gain over VASM, 2.5 dB over SM, and 6 dB over GSM (9,4). In the case of $r = 0.9$, BER curve of VASMi is very close to SM below the BER of 10^{-5}. VASMi provides ∼1.6 dB SNR gain over VASM and 6 dB SNR gain over GSM (9,4).

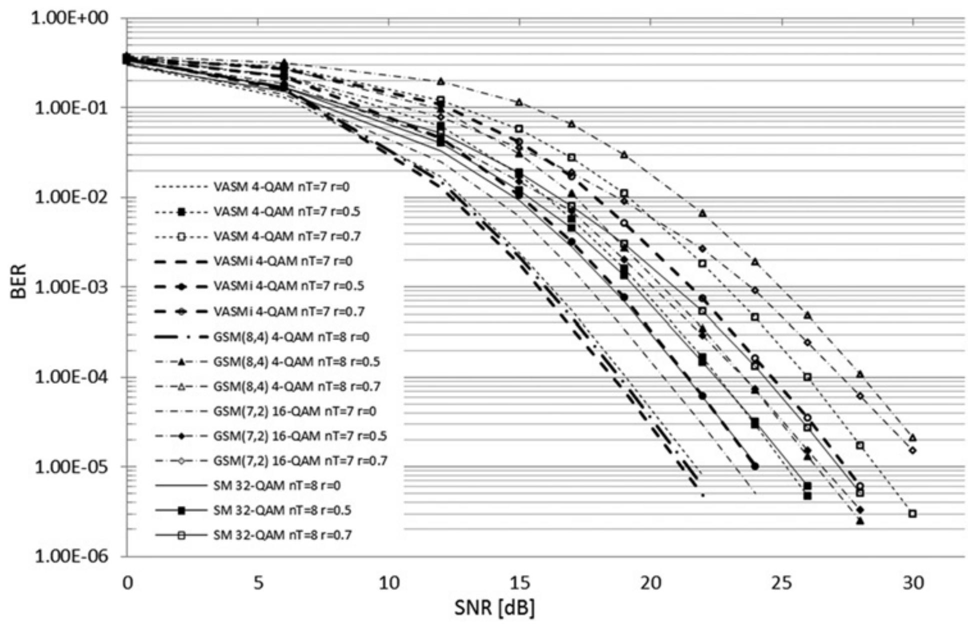

Fig. 8 *BER performance of VASM, VASMi, GSM, and SM at k = 8 bit/s/Hz in Rayleigh fading channel for r = 0, 0.5, and 0.7*

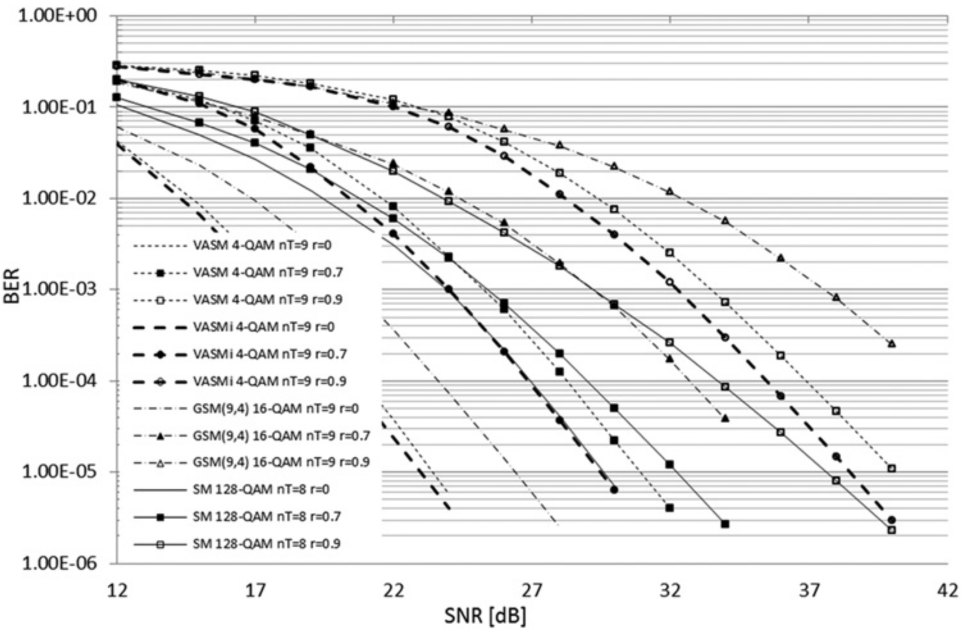

Fig. 9 *BER performance of VASM, VASMi, GSM, and SM at k = 10 bit/s/Hz in Rayleigh fading channel for r = 0, 0.7, and 0.9*

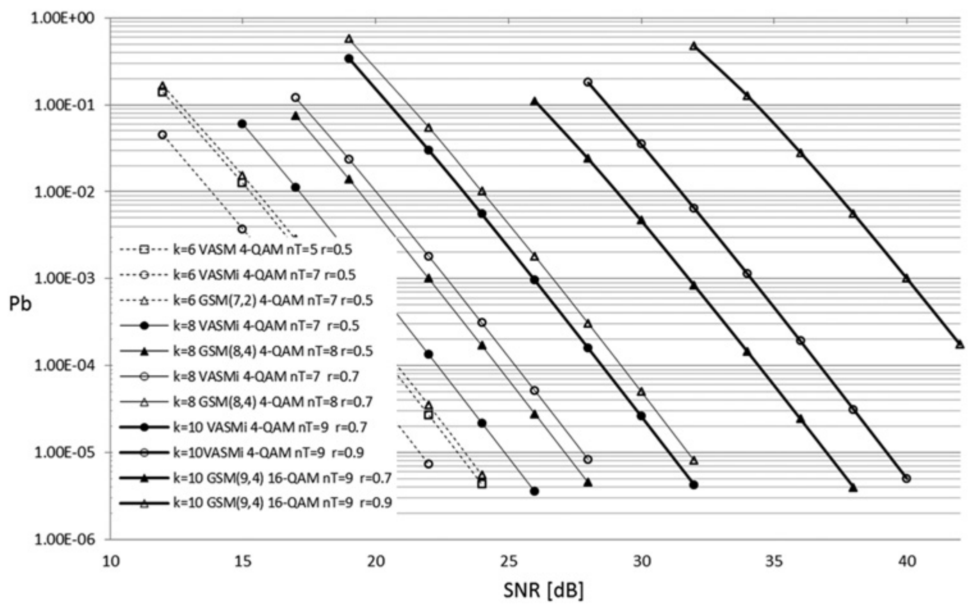

Fig. 10 *Theoretical bit error probability curves of VASM, VASMi, GSM, and SM at k = 6, 8, and 10 bit/s/Hz in Rayleigh fading channel for r = 0.5, 0.7, and 0.9*

It should be stated that SM is less sensitive to channel correlation than VASMi, VASM, and GSM because of using only one active antenna in each transmit interval. However, VASMi has the best BER performance where channel correlation factor is <0.9 for k = 10 bit/s/Hz (e.g.).

Theoretical curves of bit error probabilities according to (20) and (14) are given in Fig. 10 for VASM, VASMi, and GSM at $k = 6$, 8, and 10 bit/s/Hz in Rayleigh fading channel for $r = 0.5$, 0.7, and 0.9. It can be said that theoretical curves constitute a tight upper bound for simulation results.

6 Conclusions

In this paper, a novel MIMO transmission method referred as VASM and VASMi that allows selecting any number of antennas at the same signalling interval is introduced. Bit error probability and PEP are derived for uncorrelated and correlated channel models. Antenna selection criterions are determined for exponentially correlated channels. Antenna codewords are determined according to these criterions for $k = 6$, 8, and 10 bit/s/Hz and BER performances are compared with SM and GSM for various channel correlations, r = 0, 0.5, 0.7, and 0.9. Theoretical bit error probability curves are given for VASM, VASMi, and GSM at $k = 6$, 8, and 10 bit/s/Hz for $r = 0.5$, 0.7, and 0.9.

VASM uses the last antenna in only one code, which means that the antennas are not used homogeneously, thus VASM uses one less antenna as its given number of antennas. Therefore, VASMi outperforms VASM even in uncorrelated channels.

VASM, VASMi, and GSM have close BER performances for $k = 6$ and 8 bit/s/Hz and, for $k = 10$ bit/s/Hz in uncorrelated case, VASMi gives the best BER performance that provides 3.5 dB SNR gain over GSM (9,4) and 6.5 dB over SM. In exponentially correlated channels, BER performance of SM degrades slower than other methods, thanks to activating only one antenna at each transmit interval. Even if the case is high correlation between communication paths, VASMi gives similar BER performances to SM while GSM's degrades much more than VASMi's.

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