IET Microwaves, Antennas & Propagation

Research Article

Patch size reduction of rectangular microstrip antennas by means of a cuboid ridge

ISSN 1751-8725
Received on 15th August 2014
Revised on 12th June 2015
Accepted on 2nd July 2015
doi: 10.1049/iet-map.2014.0559
www.ietdl.org

Alireza Motevasselian[1] ✉, William G. Whittow[2]

[1]*Research and development, LEAX Arkivator TELECOM, Torsgatan 5B, Gothenburg, 41104, Sweden*
[2]*School of Electronic, Electrical and Systems Engineering, Loughborough University, University Road, Loughborough, LE11 3TU, UK*
✉ *E-mail: alireza.motevasselian@ee.kth.se*

Abstract: An effective approach to reduce the patch size in rectangular patch microstrip antennas is presented. The proposed approach is based on inductively loading the patch using a cuboid ridge. A theoretical background of the approach using the transmission line model has been provided. A prototype of the proposed antenna is fabricated and measured. The results, advantages and limitations of the proposed approach are presented and discussed.

1 Introduction

The widespread use of portable RF communication devices has increased the demands for low profile antennas. One of the most promising candidates for radiating element in modern wireless communication systems has been microstrip antennas. This is due to their well-known attractive characteristics such as low weight, low cost, possible conformity, ease of fabrication and simple design principle [1]. Microstrip antennas have been intensively developed during the past few decades and many of their shortcoming have been overcome.

In a conventional rectangular patch microstrip antenna, the length of the conducting patch is required to be of the order of a half wavelength to achieve a reasonable radiation performance. Such a patch size may be too large for some practical applications. The high demands for compact size antennas have made the patch miniaturisation one of the key challenges in micro strip antenna design. A variety of techniques have been proposed for patch size miniaturisation in microstrip antennas. High dielectric constant material [2, 3], shorting walls [4, 5], shorting pin [6], folded patch [7], loading the patch with an inductive notch [8, 9] and irregular ground planes [10] have been some of those techniques.

In this paper, we present a new approach to reduce the patch size in rectangular patch microstip antennas. This provides an uncomplicated, easily manufacturable low cost technic for patch miniaturisation. In this approach a flange cuboid ridge is placed in the middle of the patch to create a non-uniform height substrate in the radiating face of the antenna. In [11], numerical results indicate a downward shift in the resonance frequency of a rectangular patch microstrip antenna with a ridge in the middle. The present paper is a continuation of the study in [11], by a systematic investigation on the effect of a flange cuboid ridge on rectangular patch microstrip antennas. Furthermore, the present paper gives a complete physical understanding of the miniaturisation phenomena achieved by a flange discontinuity using the transmission line theory together with a parameter investigation of the approach.

In this paper the ridge is included in the transmission line model of the patch antenna using two different electromagnetic perspectives, as a series short-circuited stub or as a piece of transmission line with higher characteristic impedance. A prototype of the patch antenna with ridge is fabricated and measured. The measured and simulated results are presented and compared. The results indicate a significant reduction in physical dimension of the patch size. Many emerging applications have specific space requirements and therefore this work will disseminate new information and facilitate a new degree of freedom for antenna design engineers by providing at least three additional parameters characterising the antenna. These parameters are the height, width and location of the ridge. The proposed antenna provides a symmetric broadside radiation pattern. The simulation results show no substantial degradation in antenna gain as compared with a conventional patch antenna on a similar substrate and operating in the same frequency band.

Traditionally, microstrip patch antennas have been manufactured by etching a double sided printed circuit board. This paper demonstrates that the frequency can be reduced with a 3D substrate by manually combining separate substrates. However, this work has particular relevance to emerging manufacturing techniques and the consequent applications. For example, 3D substrates with curved or complex shapes can be easily created using 3D printing. Furthermore, wearable antennas are rapidly growing area of research [12]. In this case the patch antennas are generally assembled additively rather than by using destructive techniques and hence there are no fabrication disadvantages to altering the substrate height locally.

2 Configuration of the antenna

The configuration of a rectangular patch microstrip antenna with a cuboid ridge is illustrated in Fig. 1. The geometry consists of a grounded, square cross section dielectric substrate of dimension W and thickness h with a dielectric cuboid ridge of height h_r and width of g on the radiating face. The ridge is located between the two conducting sections of the patch with lengths of l_1 and l_2. These two sections are electrically connected through the conducting faces on top and sides of the ridge.

The substrate and the ridge are assumed to be homogeneous materials characterised by the permittivity ε_{r1} and ε_{r2}, and the permeability μ_{r1} and μ_{r2}, respectively. The antenna is fed by a coaxial line placed a distance d from the edge of the patch and on the antenna symmetry plane.

3 Transmission line model

A conventional rectangular patch mirostrip antenna can be approximately modelled by its basic transmission line circuit as shown in Fig. 2 [13, 14]. In this illustration, the resonant length of the patch is $L_t = L_1 + L_2$, where L_1 and L_2 are the distances of the feed point from each of the radiating slots at the edges and along the length of the patch. In this model each of the radiating slots is represented as a parallel RC circuit with a conductance G and a

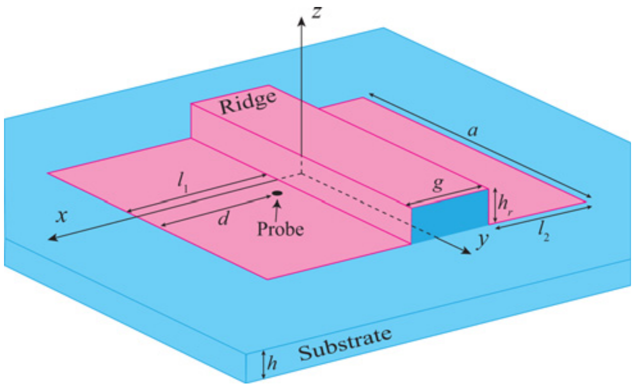

Fig. 1 *Configuration of a rectangular patch antenna with a cuboid ridge. The patch is attached to a square substrate of dimension W*

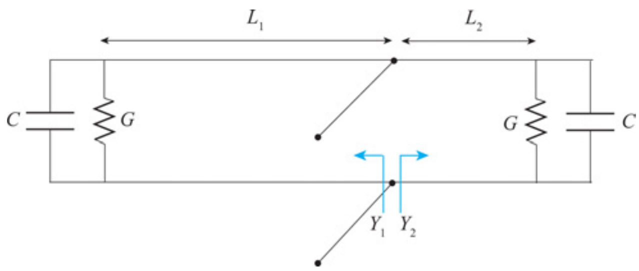

Fig. 2 *Basic circuit representation of a rectangular patch microstrip antenna*

susceptance jωC. The patch connects the two RC admittances as a transmission line. The value of the capacitance C is proportional to the effective patch width divided by the thickness of the substrate ($C \propto a_e/h$ [1]). The effective width of the patch, a_e, is a function of the physical width of the patch, the thickness and dielectric constant of the substrate. It means for a wide patch on a thin substrate with a relatively high dielectric constant, the impedance of the radiating slots would be highly capacitive.

The length of the patch is usually about a half-wavelength at its resonance frequency. The location of the feed is such that $L_1 > \lambda/4$. Hence the highly capacitive admittance of the radiating slot is transformed to an inductive admittance Y_1 at the fed point. Similarly, since $L_2 < \lambda/4$, the transformed admittance Y_2 remains capacitive but with lower susceptance. If L_1 and L_2 have been carefully chosen, the susceptance seen from each side at the feed point cancel each other out [15] and the input admittance becomes purely real and the patch resonance. In other words, the length of the patch as a transmission line should be long enough to add sufficient value of inductance to the highly capacitive slot admittances to provide matching at the feed point.

By introducing a cuboid ridge in a rectangular patch microstrip antenna, the patch length required to obtain resonant matching at the feed point is reduced. This size reduction can be described based on the transmission line model of the patch antenna. Here, we investigate the ridge from two different perspectives, as a series flange stub and as a part of the transmission line with different characteristic impedance.

3.1 Ridge as a series stub

The flange discontinuity created by the ridge at one side of the feed point can be modelled as a short-circuited series stub in the transmission line model of the rectangular patch microstrip antenna.

The input impedance of a short-circuit terminated lossless transmission line seen looking toward and at a distance d_s from the termination is given by [16]

$$Z_{\text{in}} = \text{j}Z_0 \tan \beta d_s, \tag{1}$$

where β is the propagation constant and Z_0 is the characteristic impedance of the transmission line.

Since βd_s can vary from $-\infty$ to $+\infty$, the input impedance of a short circuited lossless line can be either purely inductive for $\tan \beta d_s > 0$ or purely capacitive, for $\tan \beta d_s < 0$. However, for a transmission line with a small electrical length, $\beta d_s \ll 1$, $\tan \beta d_s \simeq \beta d_s$ and the input impedance is purely inductive and can be approximately determined by

$$Z_{\text{in}} \simeq \text{j}Z_0 \beta d_s. \tag{2}$$

This is a pure inductive impedance and hence, a short-circuited series stub in the patch transmission line model behaves as a series inductor L as shown in Fig. 3. By adding this series inductive impedance to the transmission line model of the patch, the required inductance to provide matching at the feed point is partly achieved and the remaining part can be achieved by a shorter length of the patch.

When the patch width is much larger than the substrate thickness ($a \gg h$) the patch and ground plane can be approximately considered as a parallel plate waveguide. A brief formulation of a parallel plate waveguide is presented in the Appendix at the end of this paper. By assuming the ridge as a parallel plate waveguide, the impedance of this stub can be determined by substituting (9) and (10) into (2) as

$$\begin{aligned} Z_{\text{scpp}} &\simeq \text{j}\frac{\eta d_p}{w_p} d_s \omega \sqrt{\mu\varepsilon} \\ &= \text{j}\sqrt{\frac{\mu}{\varepsilon}}\frac{d_p}{w_p} d_s \omega \sqrt{\mu\varepsilon} = \text{j}\omega\mu \frac{d_p d_s}{w_p}. \end{aligned} \tag{3}$$

Hence, for the ridge shown in Fig. 1, as a short stub, the contributed series inductive impedance can be approximately determined as

$$Z_{\text{ridge}} \simeq \text{j}\omega\mu \frac{g h_r}{a}. \tag{4}$$

This equation shows that the impedance of a short ridge is a function of frequency, permeability of the material under the ridge, the patch width and the cross sectional area of the ridge. Hence for $h_r/\lambda \ll 1$, the dielectric constant of the medium under the ridge has no contribution in the ridge impedance.

The independence of the inductance provided by a short ridge from the dielectric constant of the medium under the ridge is also verified by our numerical simulations in CST Microwave Studio.

3.2 Ridge as a transmission line with different characteristic impedance

The ridge is a flange stub transformer in the transmission line model of the patch. Assuming the whole patch and the ground plane as a transmission line, the area under the ridge is part of this transmission line with different characteristic impedance than the

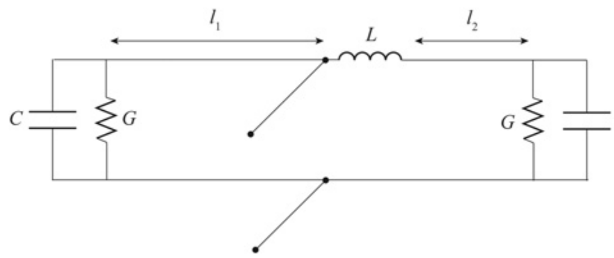

Fig. 3 *Circuit representation of the patch antenna with ridge shown in Fig. 1. The ridge is modelled as a series stub*

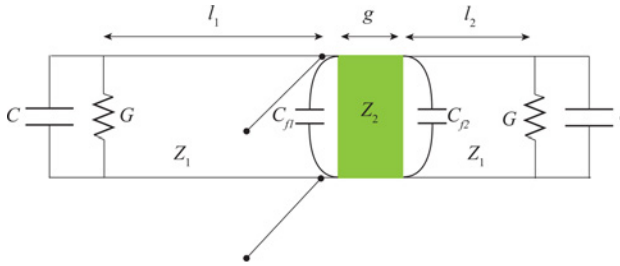

Fig. 4 *Circuit representation of the patch antenna with ridge shown in Fig. 1. The ridge is modelled as a transmission line with different characteristic impedance*

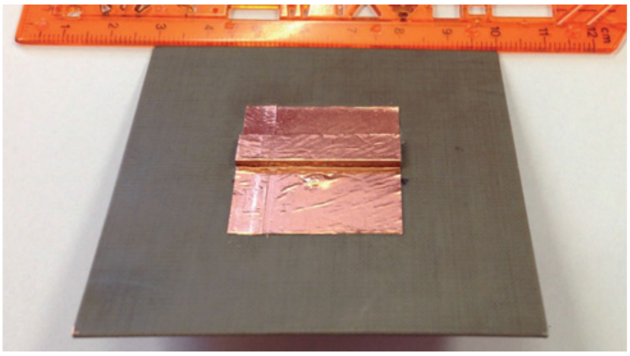

Fig. 6 *Fabricated antenna prototype*

rest of the patch. This is due to the larger separation between the conductors in the area under the ridge. The discontinuities across the junctions at the edges of the ridge can be modelled as shunt capacitive susceptances at the junctions [17–19] as shown in Fig. 4.

A lossless transmission line is described by two circuit elements, a series inductance L and a shunt capacitance C; where $LC = \mu\varepsilon$ [20]. For a parallel plate transmission line (see Fig. 12), $C \propto 1/d_p$. Therefore, by increasing the separation between the plates in a parallel plate waveguide, the capacitance of the guide is reduced and consequently its inductance increases.

By adding the ridge to the patch, part of the transmission line which is located under the ridge has a greater separation distance between the plates as compared with other parts of the patch. Therefore, an increase in the inductance and a decrease in the capacitance associated to the transmission line model of the patch is expected. However, the reduced capacitance is compensated by the shunt conditional capacitive admittances due to the discontinuities at the junctions. Therefore, the overall capacitance of the antenna system remains unchanged, while the overall inductance has been increased.

The circuit model of a patch antenna filled with a hypothetical dispersion-free and lossless material is shown in Fig. 5. The resonance frequency of the antenna network can be determined by

$$f_{r0} = \frac{1}{2\pi\sqrt{L_0 C_0}}, \quad (5)$$

where L_0 and C_0 are the equivalent inductor and capacitor associated with the patch as a resonator.

According to (5), the resonance frequency of the antenna network can be lowered by inductively or capacitively loading the antenna. As explained, the ridge increases the overall inductance of the antenna network in (5). Such an increase, leads to a reduction in the resonance frequency. The size of the antenna can then be reduced to keep the operation frequency unchanged.

4 Results

To verify the proposed approach in miniaturisation of rectangular patch antennas, a prototype of the proposed antenna is designed, fabricated and measured. The fabricated prototype is depicted in Fig. 6. This is built on a square cross sectional, low loss Taconic laminate substrate of dielectric constant $\varepsilon_{r1} = 2.2$ with the lateral dimension of 70 mm. The substrate is 0.8 mm thick (h) and is grounded on the bottom side. The two split parts of the patch of dimensions $(a,\ l_1) = (29,\ 15.5)$ mm and $(a,\ l_2) = (29,\ 9.5)$ mm are etched on the top side of the substrate. There is a gap of 7 mm width between these two parts. A cuboid laminate of dimensions $29 \times 7 \times 1.6$ mm and dielectric constant of $\varepsilon_{r2} = 2.2$ is located in the gap area and affixed using two small drops of superglue. This cuboid laminate acts as a ridge of height $h_r = 1.6$ mm. The parameters of the antenna prototype are summarised in Table 1. It should be noted that according to our simulation results, the ridge is most effective if it is located at the centre of the patch ($l_1 = l_2$). However, to provide a space for the feeding, a small offset from the centre has been considered.

The antenna is fed by a coaxial line at a point $d = 12.5$ mm away from the edge of the larger part of the patch. The whole area of the two parts of the patch and the top and sides of the cuboid ridge are covered by a copper tape with conductive adhesive to provide a uniform electrically conductive area.

The simulated and measured $|S_{11}|$ response of the antenna to a 50 Ω port, as a function of frequency, are shown in Fig. 7. The antenna resonant at 2.35 GHz and the graphs indicate a good agreement between the simulated and measured results. The difference between measured and simulated bandwidth can be due to the fabrication of the prototype using lossy copper tape that is not considered in the simulated model.

To compare the proposed antenna with the conventional flat patch antennas, a rectangular patch antenna operating at the same frequency band is also designed using the same dielectric material substrate. The dimensions of the rectangular patch are 42 mm and

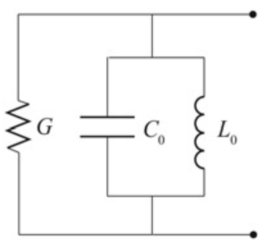

Fig. 5 *Circuit network model of an ideal patch antenna*

Table 1 Design parameters for the fabricated prototype. The dimensions are in millimetres

W	l_1	l_2	h	h_r	d	a	g	ε_{r1}	ε_{r2}
70	15.5	9.5	0.8	1.6	12.5	29	7	2.2	2.2

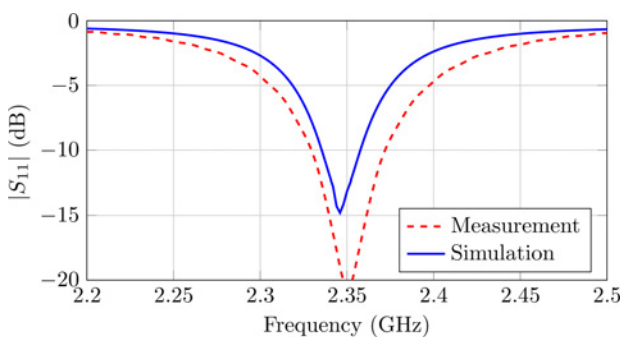

Fig. 7 *Simulation and measured results for reflection coefficient*

38 mm for the resonant length and impedance length (patch width) respectively. The probe feed is located 15 mm away from the edge of the patch aligned the resonant length and at the centre of the impedance length. By comparing the dimensions of the two antennas, it is observed that by adding a 7 mm wide and 1.6 mm thick ridge, a 24% reduction in the resonant length of the antenna has been achieved. The comparison between this flat patch and the ridge patch antenna is summarised in Table 2. CST microwave studio [21] is the primary simulation tool in this paper, however, EMPIRE XCcel FDTD [22] is used to further corroborate the results in this paper. It should be noted that the resonant length can be reduced even further by using a wider or thicker ridge. Note the dimensions of the prototype are chosen due to material availability and just to verify the validity of the approach.

The measured and simulated realised gain of the proposed design are plotted in Fig. 8. The measured and simulated data are in agreement around the resonance frequency and illustrate the peak realised gain of 6 dBi in both cases. The difference between the broad side realised gain at frequencies upper and lower resonance can be due to spurious radiations from feeding network and fabrication uncertainties. As compared with the designed conventional patch antenna, no considerable difference is observed in the realised gain of the proposed antenna.

The measured and simulated radiation patterns of the proposed antenna at 2.35 GHz at *E*-plane and *H*-plane are plotted in Figs. 9*a* and *b* respectively. The measured and simulation radiation patterns are in agreement. It is observed that the antenna produces a fairly symmetric, broadside radiation pattern.

The presented miniaturisation approach using cuboid ridge may be looked similar to antennas reported in [23, 24], where a single step discontinuity is introduced in patch antenna to affect the resonant frequency and radiation efficiency. However, the phenomenon of variation in resonance frequency is completely different. The present paper provides a more general form of discontinuities, the flange ridge containing two step discontinuities, which reduces the resonant frequency by inductively loading the antenna equivalent circuit while in [24] by only one step, there will be no inductance loading. Furthermore, the present approach gives more free parameters for the antenna design, is more effective for patch size reduction and also leads to a more compact design in terms of total volume of the antenna.

Table 2 Comparison of the conventional patch with the ridge patch prototype

	Flat patch	Ridge patch
resonant length, mm	42	32
CST f_r, GHz	2.4	2.34
empire f_r, GHz,	2.4	2.35
measured f_r, GHz	2.4	2.35
CST realized gain, dBi	6.5	6.0
empire realized gain, dBi	5.7	5.2
measured realized gain, dBi	7.5	6.0

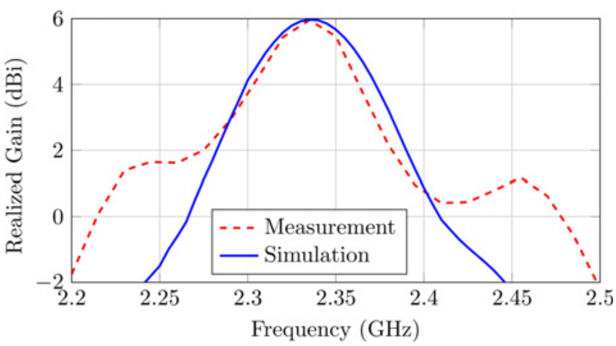

Fig. 8 *Simulation and measured results for realized gain*

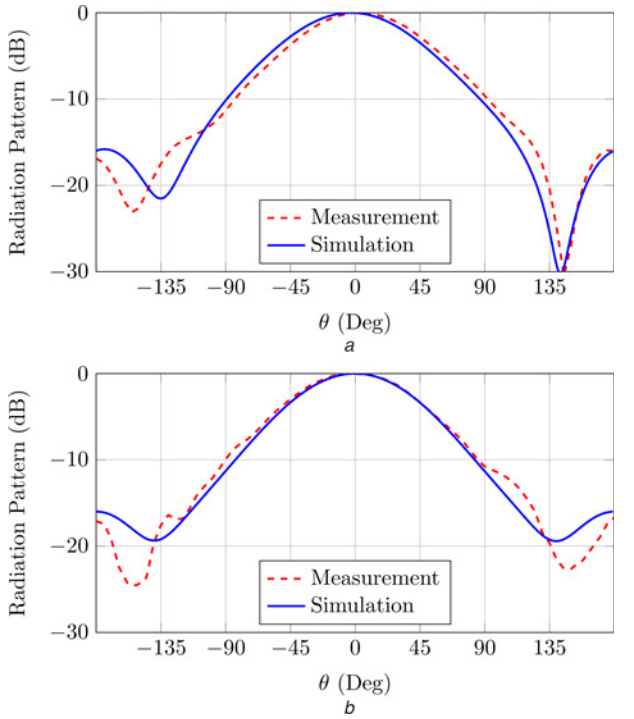

Fig. 9 *Simulation and measured results for radiation pattern at 2.35 GHz*

a $\phi = 90^\circ$ cut plane
b $\phi = 0^\circ$ cut plane

5 Design consideration

The proposed technique for size reduction of a rectangular patch microstrip antenna is based on inductively loading of the antenna network. As explained in Section 3, a cuboid ridge in a rectangular patch microstrip antenna loads the antenna network with an inductance. The value of this loaded inductance can be approximately determined as

$$L_{\text{ridge}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{g}{a\omega}\tan(\omega\sqrt{\mu\varepsilon}h_r). \quad (6)$$

The value of the inductance of the loaded antenna is then

$$L \simeq L_0 + L_{\text{ridge}}, \quad (7)$$

where L_0 is the inductance of the unloaded antenna.

Substituting (6) and (7) into (5) gives an approximate relation between the resonance frequency of an inductively loaded patch antenna with ridge and the resonance frequency of an unloaded conventional patch antenna with the same dimensions, f_{r0}

$$f_r \simeq \frac{f_{r0}}{\sqrt{1 + \sqrt{(\mu/\varepsilon)}(g/a\omega L_0)\tan(\omega\sqrt{\mu\varepsilon}h_r)}}. \quad (8)$$

In (8), the ridge height h_r contributes as a term in the tangent argument. This implies that the rate of resonant frequency reduction by means of a cuboid ridge is faster for smaller values of the ridge height. As the ridge height increases the rate of resonant frequency reduction decreases.

The tangent term in (8) has $1/L_0$ as a factor. Since $L_0 \propto h$, it is implied that a ridge with a specific height is more efficient to reduce the resonant frequency of a patch antenna on a thinner substrate.

These characteristics are verified by simulation results shown in Fig. 10. In this figure the variation of resonance frequency of the patch antenna is presented as a function of ridge height, for two

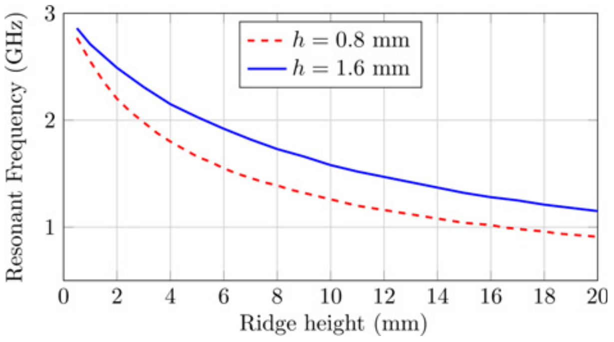

Fig. 10 *Variation of the resonant frequency with the height of the ridge (h_r) for two different values of substrate thickness*

different values of substrate thickness, $h = 0.8$ and $h = 1.6$ mm. For this numerical investigation, regardless of the substrate thickness and ridge height, all other parameters are the same as presented in Table 1. It is observed in the figure that the curves are steeper for smaller values of the ridge height. In other words, although the effect of ridge increase with its height growth, further increase in the ridge height is less effective but increase the volume of the antenna. It is also shown in Fig. 10 that a ridge with certain height is more effective in resonance frequency reduction for the patch antenna with a thinner dielectric substrate.

Another parameter that contributes to the ridge effect is its width, g. An investigation has been carried out on the variation of resonance frequency with respect to the ridge width. In this investigation, a rectangular patch micristrip antenna with patch size of $a = 146$ mm and $l_1 + l_2 + g = 116$ mm, where $l_1 = l_2$, has been considered. The substrate is a 250×250 mm square cross section dielectric slab with $\varepsilon_{r1} = 2.2$ and thickness of $h = 1$ mm. Three different values of ridge height are considered. The variation of resonance frequency as a function of ridge width, g, is plotted in Fig. 11 for the three values of ridge height. It is observed that for all the three cases the resonance frequency is reduced as the width of the ridge starts to grow. This reduction in resonance frequency continues up to the limit $g \simeq l_1 + l_2$. However, further increase in the length of the patch leads to an increase in resonance frequency. Hence, the maximum reduction in resonance frequency achieved by a ridge with specific height occurred when the length of the ridge is roughly equal to the a half of the total resonance length of the patch. As the width of the ridge approaches the total patch length, the resonant frequency approaches the value for the unloaded regular patch, as expected.

One of the advantages of the presented approach for patch antenna miniaturisation is the simple design procedure and a high degree of flexibility for the designer. In contrast to other miniaturisation approach such as shorting pin which is very sensitive to the location of the pin and requires an optimisation process to design, the proposed approach provides a very simple design procedure and is relatively robust to the distortion of design parameters.

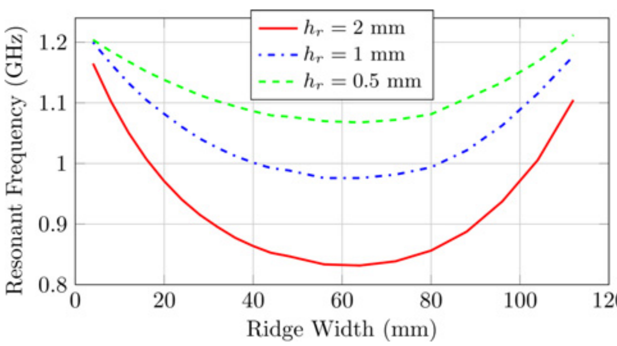

Fig. 11 *Variation of the resonant frequency of the proposed type of antennas with the length of the ridge (g) for three values of the ridge height*

The proposed technique can be used for the patch size reduction in microstrip antennas. Nevertheless, in most of the applications a smaller ground plane is also desired. In other words, the miniaturisation of the patch and keeping the ground plane size the same can be insufficient for some applications. In the comparison between the conventional patch antenna and the proposed antenna carried out in Section 4, the ground plane size was chosen the same for both of the antennas. This provides a fair gain comparison between two antennas. However, for an ordinary case, the minimum ground plane size for the proposed antenna with cuboid ridge can be much smaller than the smallest ground plane size required for the conventional patch on a similar substrate and operating in the same frequency. A simulation based investigation indicates that reducing the ground plane size up to ∼1.1 times of the maximum patch dimension keeps the antenna performance for the both conventional patch and the cuboid ridge patch antennas. In both of the cases to reduce the ground plane size leads to reducing the broadside gain of the antenna and increasing the side-lobe level. A very small variation in resonant frequency is also observed.

6 Conclusion

An effective approach to reduce the patch size in rectangular patch microstrip antennas has been proposed. The approach is based on inductively loading the patch using a cuboid ridge. The cuboid ridge is included in the transmission line model of the patch antenna and a theoretical background of the approach has been explained. The concept was numerically and experimentally verified. The results, advantages and limitations of the proposed approach were presented and discussed. Design considerations and limitations have been investigated for the proposed antenna.

7 Acknowledgment

This work was funded by EPSRC under grant no. EP/K011383/1. In line with EPSRC guidelines, this paper is available Open Access. The supporting simulation files for this paper can be accessed at http://www.dx.doi.org/10.17028/rd.lboro.2001255.

8 References

1 Garg, R., Bhartia, P., Bahl, I., *et al.*: 'Micristrip antenna design handbook' (Boston, Artech House, 2000)
2 Lee, B., Harackiewicz, F.: 'Miniature microstrip antenna with a partially filled high-permittivity substrate', *IEEE Trans. Antennas Propag.*, 2002, **50**, (8), pp. 1160–1162
3 Lo, T., Ho, C.-O., Hwang, Y., *et al.*: 'Miniature aperture-coupled microstrip antenna of very high permittivity', *Electron. Lett.*, 1997, **33**, (1), pp. 9–10
4 Pinhas, S., Shtrikman, S.: 'Comparison between computed and measured bandwidth of quarter-wave microstrip radiators', *IEEE Trans. Antennas Propag.*, 1988, **36**, (11), pp. 1615–1616
5 Chair, R., Lee, K.F., Luk, K.M.: 'Bandwidth and cross-polarization characteristics of quarter-wave shorted patch antennas', *Microw. Opt. Technol. Lett.*, 1999, **22**, (2), pp. 101–103
6 Waterhouse, R.: 'Small microstrip patch antenna', *Electron. Lett.*, 1995, **31**, (8), pp. 604–605
7 Luk, K., Chair, R., Lee, K.-F.: 'Small rectangular patch antenna', *Electron. Lett.*, 1998, **34**, (25), pp. 2366–2367
8 Reed, S., Desclos, L., Terret, C., *et al.*: 'Patch antenna size reduction by means of inductive slots', *Microw. Opt. Technol. Lett.*, 2001, **29**, (2), pp. 79–81. Available at: http://dx.doi.org/10.1002/mop.1089
9 Cheng, X., Wu, J., Blank, R., *et al.*: 'An omnidirectional wrappable compact patch antenna for wireless endoscope applications', *IEEE Antennas Wirel. Propag. Lett.*, 2012, **11**, pp. 1667–1670
10 Wang, D., Wong, H., Chan, C.H.: 'Small patch antennas incorporated with a substrate integrated irregular ground', *IEEE Trans. Antennas Propag.*, 2012, **60**, (7), pp. 3096–3103
11 Whittow, W.: 'Microstrip patch antennas with 3-dimensional substrates'. Antennas and Propagation Conf. (LAPC), 2012 Loughborough, 2012, pp. 1–5
12 Kellomaki, T., Whittow, W., Heikkinen, J., *et al.*: '2.4 GHz plaster antennas for health monitoring'. Third European Conf. on Antennas and Propagation, 2009. EuCAP 2009. 2009, pp. 211–215
13 Munson, R.: 'Conformal microstrip antennas and microstrip phased arrays', *IEEE Trans. Antennas Propag.*, 1974, **22**, (1), pp. 74–78

14 Balanis, C.A.: 'Antenna theory: analysis and design' (Wiley-Interscience, New Jersey, 2005, 3rd edn.)
15 Nascimento, D.C., da, J.C., Lacav, S.: 'Design of low-cost probe-fed microstrip antennas' (Online Open Access, New York, 2011)
16 Collin, R.E.: 'Foundations for microwave engineering' (McGraw-Hill, New York, 1992, 2nd edn.)
17 Collin, R.E., 'Field theory of guided waves' (Wiley-Interscience, New York, 1991, 2nd edn.)
18 Whinnery, J., Jamieson, H.W.: 'Equivalent circuits for discontinuities in transmission lines', *Proc. IRE*, 1944, **32**, (2), pp. 98–114
19 Macfarlane, G.G.: 'Quasi-stationary field theory and its application to diaphragms and junctions in transmission lines and wave guides', *Electr. Eng. –IIIA: Radiolocation, J. Inst.*, 1946, **93**, (4), pp. 703–719
20 Pozar, D.M.: 'Microwave engineering' (Addison Wesley, 2000)
21 Available at: https://www.cst.com
22 Available at: http://www.empire.de
23 Lee, C., Lee, C.S., Lakshmanan, A.: 'Radiation efficiency of a microstrip antenna with height discontinuity (mahd)'. Antennas and Propagation Society Int. Symp. (APSURSI), 2010 IEEE, 2010, pp. 1–4
24 Lee, C., Lee, C.S.: 'Electrically small microstrip antenna with height discontinuity'. Antennas and Propagation Society Int. Symp., 2009. APSURSI '09. IEEE, 2009, pp. 1–4

9 Appendix

9.1 Parallel plate waveguide

The geometry of a parallel plate waveguide is shown in Fig. 12. It contains two parallel conducting plates separated by a dielectric medium of permittivity ε and permeability μ. Ideally, the plates are infinitely wide. However, the solution can well approximate the characteristics of plates of finite width provided that $w \gg d_p$.

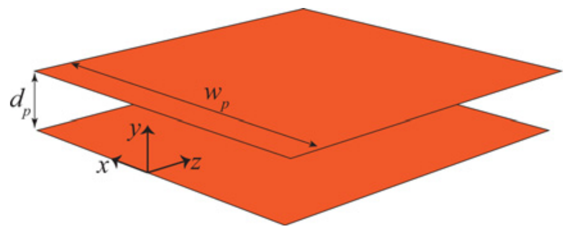

Fig. 12 *Geometry of a parallel plate waveguide*

The fundamental mode in a parallel plate waveguide is the TEM mode. The characteristic impedance for the TEM mode [20] is

$$Z_0 = \frac{\eta d_\mathrm{p}}{w_\mathrm{p}}, \tag{9}$$

where d_p is the separation distance between two strips, w_p is the strip width and $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the medium between the parallel plates.

The propagation constant of the TEM wave is

$$k = \omega\sqrt{\mu\varepsilon}, \tag{10}$$

where, ω is the angular frequency.

Higher order modes in a parallel plate waveguide are TM_n modes and TE_n modes. The propagation constant for these modes is given by

$$\beta = \sqrt{k^2 - k_\mathrm{c}^2}, \tag{11}$$

where k_c is the cutoff wave-number given by [20]:

$$k_\mathrm{c} = \frac{n\pi}{d_\mathrm{p}}, \quad n = 0, 1, 2, 3 \ldots. \tag{12}$$

The cutoff frequency of the TM_n and TE_n mode is the frequency that makes $\beta = 0$, and can be expressed as

$$f_\mathrm{c} = \frac{k_\mathrm{c}}{2\pi\sqrt{\mu\varepsilon}} = \frac{n}{2d_\mathrm{p}\sqrt{\mu\varepsilon}}. \tag{13}$$

Waves with $f > f_\mathrm{c}$ propagate with phase constant β, and waves with $f \le f_\mathrm{c}$ are evanescent.