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On the sustainment of optical power in twisted clad dielectric cylindrical fibers

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The paper presents an analytical investigation of twisted clad dielectric optical fiber with the emphasis on the propagation of power through the guide. The fiber structure assumes twists in the form of loading of conducting sheath helical windings with a particular value of pitch at the core–clad interface. It is assumed that the turns of right-handed helical windings are continuous in nature, but still insulated from each other. The dispersion relation is deduced for the structure followed by the evaluation of the propagation of power (in terms of power confinement factor) under different values of helix pitch angle. It has been found that the pitch plays determining role to control power confinement in the guide as the amount of power in the core section is found to be the maximum corresponding to 0° pitch and goes on decreasing with the increase in its value. It is presumed that such characteristics of the guide would be useful to device varieties of optical components wherein the need of tuning optical power propagation remains vital.

1. Introduction

During the last couple of decades, *complex* optical waveguides have been of prime focus of research among the R&D community owing to their exotic electromagnetic behavior leading to possible technological use.[1–5] There are several forms of complex waveguides wherein the complexities may appear either in the geometry or the materials which they are constructed of. As the transmission properties of optical waveguides essentially depend on the cross-sectional geometry [6,7] and the nature of materials [8–10] in the guide, lightwave propagation through varieties of guides (with varying cross-sections as well as materials that include metamaterials [11–13]) has been greatly dealt with in the literature. The relevant study constitutes a forefront research area of complex mediums. Choudhury and Singh described detailed studies of various forms of unconventional waveguide structures during the early 2000.[14]

Twisted clad fibers also fall into the category of complex mediums wherein the pitch angle of conducting sheath helix plays determining role to govern the dispersion characteristics of the guide.[15–18] As the helix pitch imposes great control over the propagation characteristics, such structures become greatly demanding in several optical applications.

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Within the context, it is noteworthy that conducting helical structures were first introduced in making traveling wave tubes (TWTs), which are used to amplify radio frequency.[19] TWTs are cylindrical in structure, and therefore, the idea of the use of conducting sheath helix may be amalgamated with the principle of operation of dielectric and/or metamaterial optical fibers. This essentially allows the option to tailor the dispersion characteristics of the guide for technological needs – the subject of increased interest among the optics/electromagnetics R&D community.

Authors have reported studies of circular [15] and elliptical [16,17] dielectric fibers loaded with conducting sheath helix structures. It must be remembered that the used helical wraps on the guide (introduced at the core–clad interface) remain continuous in nature along with the property of their being isolated from each other. Reference [15] illustrates some work done in this direction wherein the dielectric guides under consideration attain a cylindrical geometry. At even more complex stage, authors have reported investigations of the propagation characteristics of chiral fibers with sheath helical conducting structures.[20] Also, the studies of electromagnetic properties of helical clad elliptical dielectric fibers have revealed the band gap properties.[21]

Transmission characteristics of optical power through a fiber have been of great importance before a suitable application of fiber is thought about. In the case of aforesaid twisted clad fiber structures, power characteristics of guides have not yet been given enough attention. The present communication aims at studying the propagation of power in dielectric cylindrical guides loaded with conducting sheath helix structure. The work discusses the evolution of dispersion relation and the power confinement patterns in the core/clad sections of fiber under consideration. The study incorporates effects on the propagation characteristics due to variations in helix pitch angle. Results reveal that a change in pitch essentially has the property to alter the sustained power in the guide, which is essentially due to the changes in dispersion characteristics, thereby allowing to attain a controlled transmission in such guiding structures.

2. Theory

Figure 1 shows the schematic of twisted clad fiber under consideration with a cylindrical symmetry. We assume the fiber core has the radius a, and the clad region (with

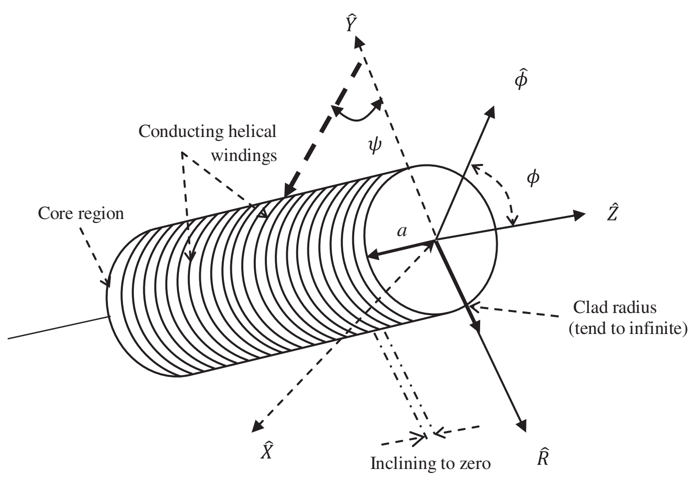

Figure 1. Schematic of a twisted clad fiber with helix pitch angle as ψ.

$r > a$) is taken to be infinitely extended. The core and the clad sections have the respective refractive index (RI) values as n_1 and n_2, and a right-handed sheath helix is introduced at the core–clad boundary with ψ as the helix pitch angle. Considering the propagation of time t- and the axis z-harmonic electromagnetic waves along the axial (i.e. z-) direction, the electric/magnetic fields will have forms as

$$E_Z = E(r,\phi)e^{j(\omega t-\beta z+v\phi)} \tag{1}$$

$$H_Z = H(r,\phi)e^{j(\omega t-\beta z+v\phi)} \tag{2}$$

For monochromatic light fields, each field of radian frequency ω and each mode that travels in the positive z-direction, because of the circular symmetry of the guide, must not change when the coordinate ϕ is increased by 2π. Therefore, each field mode trapped in waveguide has a time, length, and azimuthal dependence given by $e^{j(\omega t-\beta z+v\phi)}$ with β being the axial propagation constant and v being the mode index. Also, the free-space propagation constant may be considered as $k=2\pi/\lambda$; λ being the wavelength.

Now, considering the fields in the guide as being represented by Bessel $J_v(\bullet)$ and the modified Bessel $K_v(\bullet)$ functions, and their linear combinations, the use of Maxwell's field equations in the case of linear, homogeneous, and isotropic mediums will provide the radial and the tangential components of fields in the two regions (of the guide) as

$$E_{\phi 1} = -\frac{j}{U^2}\left[jA\frac{\beta v}{r}J_v(Ur) - BU\mu\omega J_v'(Ur)\right]e^{j(wt-\beta z+v\phi)} \tag{3a}$$

$$H_{\phi 1} = -\frac{j}{U^2}\left[jB\frac{\beta v}{r}J_v(Ur) + AU\omega\epsilon_1 J_v'(Ur)\right]e^{j(wt-\beta z+v\phi)} \tag{3b}$$

$$E_{\phi 2} = -\frac{j}{W^2}\left[jC\frac{\beta v}{r}K_v(Wr) - DW\mu\omega K_v'(Wr)\right]e^{j(wt-\beta z+v\phi)} \tag{4a}$$

$$H_{\phi 2} = -\frac{j}{W^2}\left[jD\frac{\beta v}{r}K_v(Wr) + CW\omega\epsilon_2 K_v'(Wr)\right]e^{j(wt-\beta z+v\phi)} \tag{4b}$$

$$E_{r1} = -\frac{j}{U^2}\left[A\beta UJ_v'(Ur) + jB\frac{\mu\omega v}{r}J_v(Ur)\right]e^{j(wt-\beta z+v\phi)} \tag{5a}$$

$$H_{r1} = -\frac{j}{U^2}\left[B\beta UJ_v'(Ur) - jA\frac{\omega\epsilon_1 v}{r}J_v(Ur)\right]e^{j(wt-\beta z+v\phi)} \tag{5b}$$

$$E_{r2} = -\frac{j}{W^2}\left[C\beta WK_v'(Wr) + jD\frac{\mu\omega v}{r}K_v(Wr)\right]e^{j(wt-\beta z+v\phi)} \tag{6a}$$

$$H_{r2} = -\frac{j}{W^2}\left[D\beta WK_v'(Wr) - jC\frac{\omega\epsilon_2 v}{r}K_v(Wr)\right]e^{j(wt-\beta z+v\phi)} \tag{6b}$$

In these equations, A, B, C, and D are arbitrary constants to be determined by implementing suitable boundary condition. Also, the suffixes 1 and 2, respectively, represent the situations in the core and the clad sections, prime represents the differentiation with respect to the arguments, and ε and μ are, respectively, the permittivity and the permeability of medium. Further, the quantities U and W are given as

$$U = \sqrt{n_1^2 k^2 - \beta^2} \tag{7}$$

$$W = \sqrt{\beta^2 - n_2^2 k^2} \tag{8}$$

Now, as stated before, considering ψ as the pitch angle of the introduced sheath helix at the core–clad interface, the boundary conditions [22] to be implemented can be written as

$$E_{Z1} \sin\psi + E_{\phi 1} \cos\psi = 0 \tag{9}$$

$$E_{Z2} \sin\psi + E_{\phi 2} \cos\psi = 0 \tag{10}$$

$$(H_{Z1} - H_{Z2}) \sin\psi + (H_{\phi 1} - H_{\phi 2}) \cos\psi = 0 \tag{11}$$

$$(E_{Z1} - E_{Z2}) \cos\psi - (E_{\phi 1} - E_{\phi 2}) \sin\psi = 0 \tag{12}$$

Using these boundary conditions and fields represented by Equations (3)–(6), we get a set of eight equations, which is not incorporated into the text. The coefficients in those equations allow to form an 8×8 determinant, which can be equated to zero, in order to have the dispersion relation for the waveguide structure. After some tedious mathematical steps, it can be shown that the dispersion relations will assume the form as

$$\begin{aligned}
&(1/(a^4 U^6 W^6)(jaW\mu\omega \cos\psi K_v'(aW)(U^2(aW^2 \cos\psi - \beta v \sin\psi) \times (W^2(\beta v \cos\psi \\
&\quad + aU^2 \sin\psi)^2 J_v^2(aU) - a^2 U^2 W^2 \mu\omega^2 \cos^2\psi \epsilon_1 \times (J_v'(aU))^2) K_v(aW) \\
&\quad + jaU^2 W\omega \cos\psi \epsilon_2 (-ja^2 U^3 W^2 \mu\omega \cos^2\psi J_v(aU) \times J_v'(aU) \\
&\quad - ja^2 U^3 W^2 \mu\omega \sin^2\psi J_v(aU) J_v'(aU)) K_v'(aW)) - (\beta v \cos\psi + aW^2 \sin\psi) K_v(aW) \\
&\quad \times (-U^2(-\beta v \cos\psi + aW^2 \sin\psi) \times (-ja^2 U^3 W^2 \mu\omega \cos^2\psi J_v(aU) J_v'(aU) \\
&\quad - ja^2 U^3 W^2 \mu\psi \sin^2\psi J_v(aU) J_v'(aU)) \times K_v(aW) - jaU^2 W\mu\omega \sin\psi (W^2(\beta v \cos\psi \\
&\quad + aU^2 \sin\psi)^2 J_v^2(aU) - a^2 U^2 W^2 \mu\omega^2 \cos^2\psi \epsilon_1 (J_v'(aU))^2) K_v'(aW))) \\
&\quad = 0
\end{aligned} \tag{13}$$

Now, by the use of Equations (3)–(6), the expressions of power [23] sustained in the two regions of the guide can be deduced as

$$P_{\mathrm{CO}} = \left(\frac{\pi DD^*}{U^4}\right) Re\left[\int_0^{2\pi}\int_0^{a}\left\{\left(\left(\frac{j\beta v}{r}\right)\Psi J_v(Ur) + \Pi\Psi U\omega\epsilon_1 J_v'(Ur)\right)^* - \left(\left(\frac{j\beta v}{r}\right)\Pi\Psi J_v(Ur) - \Psi U\ \mu\omega J_v'(Ur)\right)\left(\Psi\beta U J_v'(Ur) - \left(\frac{j\omega\epsilon_1 v}{r}\right)\Pi\Psi J_v(Ur)\right)^*\right\} r dr d\phi\right] \tag{14}$$

$$P_{\mathrm{CL}} = \left(\frac{\pi DD^*}{W^4}\right) Re\left[\int_0^{2\pi}\int_a^{\infty}\left\{\Omega\beta W K_v'(Wr) + \left(\frac{j\omega\mu v}{r}\right)K_v(Wr) \times \left(\left(\frac{j\beta v}{r}\right)K_v(Wr) + \Omega\omega\epsilon_2 W K_v'(Wr)\right)^* - \left(\left(\frac{j\beta v}{r}\right)\Omega K_v(Wr) - W\mu\omega K_v^*(Wr)\right) \times \left(\beta W K_v^*(Wr) - \left(\frac{j\omega\epsilon_2 v}{r}\right)\Omega K_v(Wr)\right)^*\right\} r dr d\phi\right] \tag{15}$$

In Equations (14) and (15), P_{CO} and P_{CL} are, respectively, power in the core and the clad sections. Also, the other symbols have their meanings as follows:

$$\Pi = \left[-j\frac{\mu\omega}{U}\cos\psi \Big/ \left\{\sin\psi + \frac{\beta v}{U^2 a}\cos\psi\right\}\right] \times \frac{J_v'(Ua)}{J_v(Ua)} \tag{16}$$

$$\Omega = \left[-j\frac{\mu\omega}{W}\cos\psi \Big/ \left\{\sin\psi + \frac{\beta v}{W^2 a}\cos\psi\right\}\right] \times \frac{K_v'(Wa)}{K_v(Wa)} \tag{17}$$

$$\Psi = -\frac{\left[\left\{\left(\frac{\left(\frac{j\mu\omega}{W}\right)\cos\psi}{\sin\psi + \left(\frac{\beta v}{W^2 a}\right)\cos\psi}\right) \times \left(-\cos\psi + \left(\frac{\beta v}{W^2 a}\right)\sin\psi\right)\right\} + \left(\frac{j\mu\omega}{W}\right)\sin\psi\right]}{\left[\left\{\frac{-\left(\frac{j\mu\omega}{U}\right)\cos\psi}{\sin\psi + \left(\frac{\beta v}{U^2 a}\right)\cos\psi} \times \left(\cos\psi - \left(\frac{\beta v}{U^2 a}\right)\sin\psi\right)\right\} - \left(\frac{j\mu\omega}{U}\right)\sin\psi\right]} \times \frac{K_v'(Wa)}{J_v'(Ua)} \tag{18}$$

In Equations (16) and (17), the previous use of boundary conditions determines the values of constants *A*, *B,* and *C* terms of only one constant *D*, as follows:

$$A = \Pi \times \Psi \times D \tag{19}$$

$$B = \Psi \times D \tag{20}$$

$$C = \Omega \times D \tag{21}$$

where the meanings of the used symbols are given in Equations (16)–(18) above. Now, the total power in the fiber will be given as the algebraic sum of Equations (14) and (15), and the ratio P_{CO} and P_{CL} to the total optical power will provide the power confinement factors in the respective regions of the guide. In this stream, in the following section, we use symbols Γ(Core) and Λ(Clad) to determine power confinement factors in the core and the clad sections, respectively.

3. Results and discussion

We now make an attempt to study the power confinement factors in the two regions of twisted clad fiber structure under consideration. For this purpose, we take the core radius a of the guide to be 5 μm and the operating wavelength λ to be 1550 nm. Also, the core/clad RI values are taken to be 1.462 and 1.4568, respectively, and the illustrative values of helix pitch angles are 0°, 15°, 30°, 45°, 60°, 75°, and 90°.The results in respect of confinement patterns are obtained corresponding to 0, 1, 2, and 3 modal indices and graphically represented through the plots of confinements against the allowed range of propagation constants, as determined by the guiding conditions. Within the context, it must be noted that a 0° pitch corresponds to the situation when the conducting helical windings are just perpendicular to the optical axis of the guide, and a 90° pitch makes the windings to be parallel to the direction of propagation. As such, the case of 90° pitch essentially triggers one to think of the situation when helical windings are almost eliminated, and the guiding structure becomes closely similar to an ordinary optical fiber.

Figures 2 and 3 are the illustrative plots of power confinement patterns against the allowed values of propagation constants in the core and the clad sections, respectively, corresponding to the helix pitch angle as 5° and the modes with $v = 0$, 1, 2, and 3. We observe that, corresponding to all the chosen values of modal indices, confinements in the fiber core generally increase with the increase in propagation constant (Figure 2), and the way it increases in the core section, it is simultaneously complemented in the clad region through gradual decrease in confinement, as shown in Figure 3. We further notice that the confinement exhibited by the meridianal mode is quite distinguished from those shown by higher order skew modes, which give power confinement values in the close proximity and higher than that presented by the meridianal modes.

In order to observe the effect due to helix pitch angle on the confinement factor, we illustrate Figures 4 and 5 to yield confinements in the core and the clad sections, respectively. Figure 4(a)–(d) represents the situations corresponding to v = 0, 1, 2, and 3, respectively, in the core section whereas those for the clad region are shown in

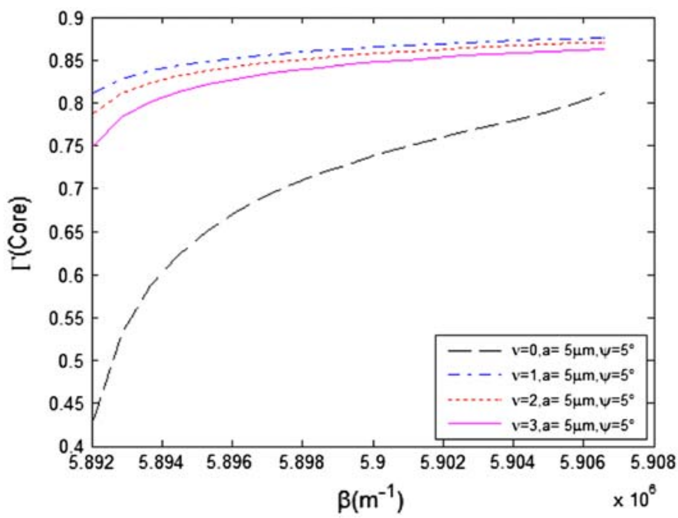

Figure 2. Power confinement factor in the twisted fiber core.

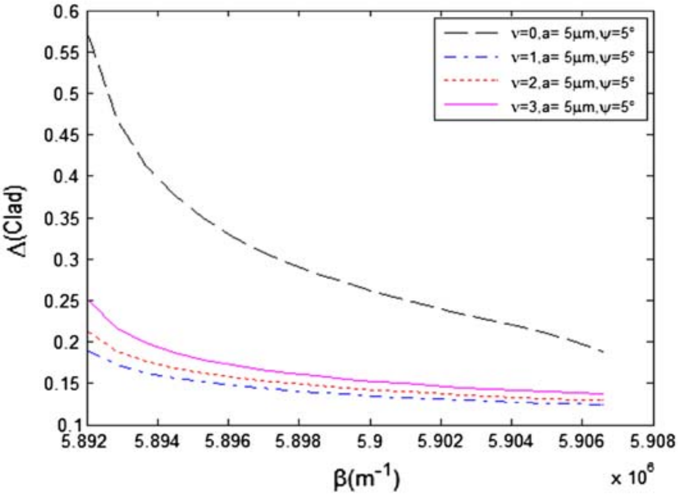

Figure 3. Power confinement factor in the twisted fiber clad.

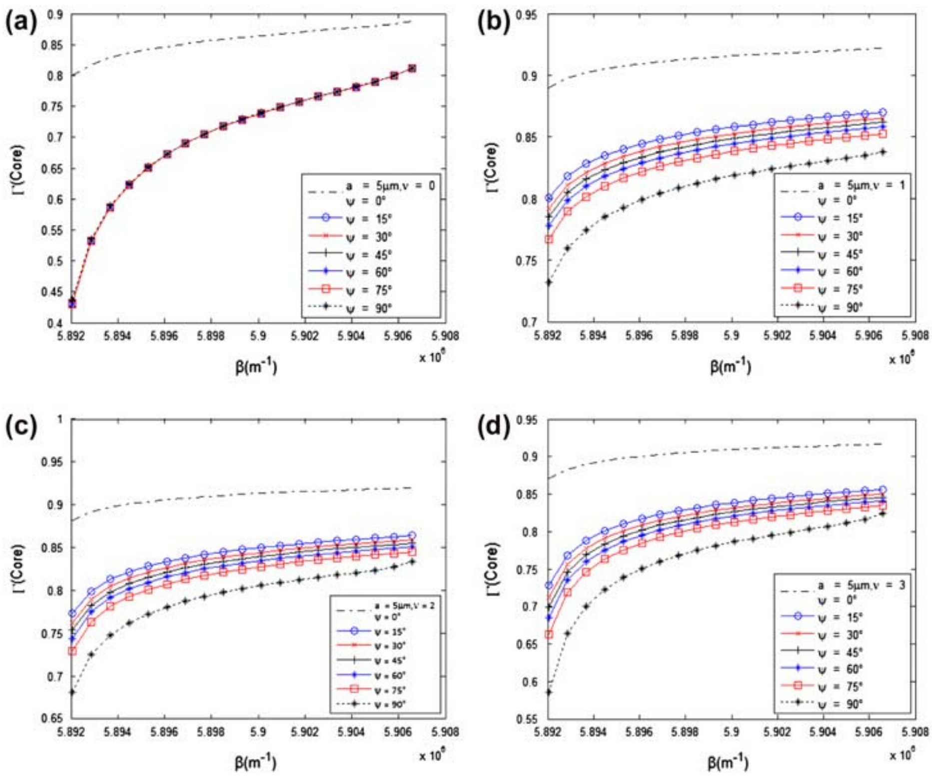

Figure 4. Power confinement factor in the twisted fiber core corresponding to (a) $v = 0$, (b) $v = 1$, (c) $v = 2$, and (d) $v = 3$.

Figure 5(a)–(d). From Figure 4, we observe that the confinement factors generally increase with the increase in propagation constant, and the increase is a little faster in the range of lower values of β. With the increase in β, confinements reach the situation close to saturation, which is more prominent when the windings possess 0° pitch. Corresponding to the meridianal mode (i.e. $v = 0$), all the different winding angles produce almost similar values of confinements with varying propagation constants (Figure 4(a)), except for the case when the windings are just perpendicular to the direction of propagation (i.e. $\psi = 0°$). This is attributed to the reason of mode being the meridianal one.

Further, we notice in Figure 4 that the maximum confinement is attained when the windings have 0° pitch. It is quite explicit from Figure 4 that the overall confinement by mode of any of the orders decreases with the increase in helix pitch, and the case of a 90° pitch exhibits a more marked decrease in confinement. We clearly observe that, corresponding to the modes of any particular order, there exists a marked difference between confinements presented in the situations of 0° and 90° values of pitch – the former one shows higher amount of power confinement factor. Corresponding to the other values of pitch, confinements lie in between. This clearly demonstrates the use of conducting helical windings at the core–clad interface of dielectric optical fibers. In the present case, we observe that the introduction of such helical structures makes the guide to amplify the signal propagating through it.

In Figure 4, if we consider confinements corresponding to particular values of pitch angle, say the two extreme values 0° and 90°, we observe that, for the entire range of the allowed β-values, power confinement generally remains the maximum for the mode with order $v = 1$, and the minimum for $v = 0$. Confinements corresponding to further higher modes become a little less, which states of lesser importance of exciting higher

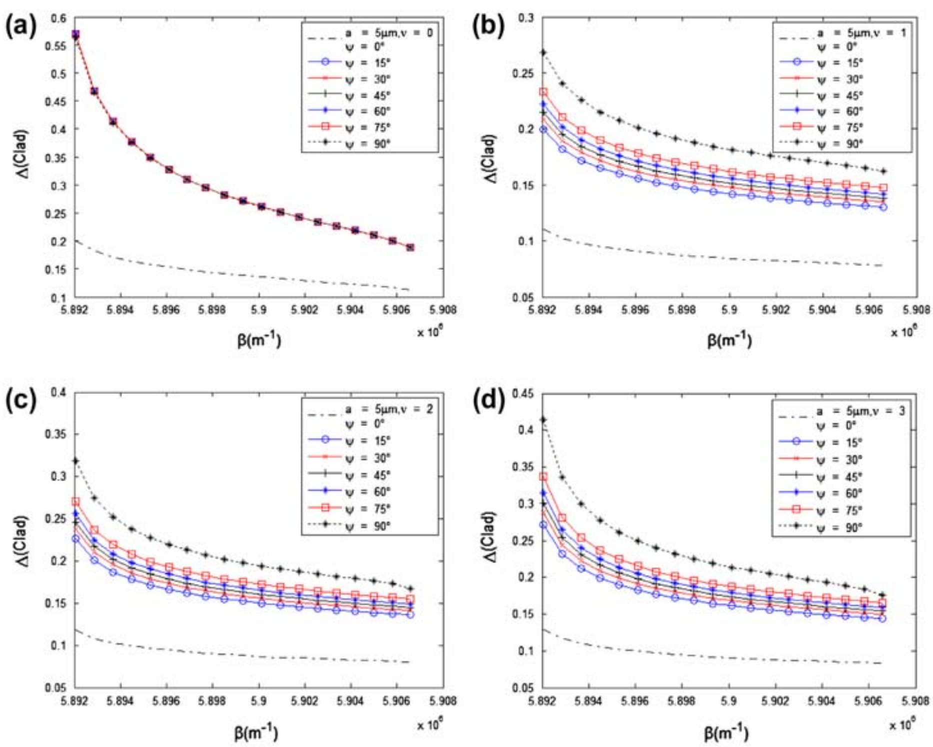

Figure 5. Power confinement factor in the twisted fiber clad corresponding to (a) $v = 0$, (b) $v = 1$, (c) $v = 2$, and (d) $v = 3$.

modes in the guide. However, in the present work, computations for higher modes are done for the sake of comparison only.

In Figure 5, as stated above, power confinement plots for the clad section are shown corresponding to the first four values of mode order. In all the figures, we observe that, for any particular mode order, the way power increases in the core section with increasing β-values, it simultaneously decreases in the clad region for the respective mode. In this region too we notice that, corresponding to modes of any of the orders, confinement remains the maximum for 90° pitch and minimum for 0° winding angle. This is very much justified and validates the analytical treatments performed. Also, we observe the decreasing trend of plots as getting almost toward saturation in the higher range of β-values – the way we observed the increasing trend toward saturation in the case of confinement factors in the core section.

4. Conclusion

Propagation of optical power in twisted clad dielectric optical fibers has been studied under the consideration of varying pitch angle of the conducting right-handed sheath helix loaded at the core–clad interface. It has been found that the pitch has great effect in governing the electromagnetic features of the guide. This is reflected from the feature that, for any particular mode order, the increase in pitch angle results in a decrease in optical power in the guide, which makes the guide to be possible for applications as power attenuators/amplifiers. As a 90° pitch corresponds to the situation almost of an ordinary optical fiber, the decrease in confinement factor with increasing helix pitch indicates the amplification of electromagnetic power in the guide.

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