Robust multiple signal classification algorithm based on the myriad covariation matrix

A.M. Hmidat, B.S. Sharif and W.L. Woo

Abstract: A robust version of the multiple signal classification (MUSIC) bearing estimation algorithm based on robust statistics is developed for a direct sequence-code division multiple access impulsive noise channel. The proposed subspace algorithm is computed by using the antenna array covariance matrix, which is derived from the robust maximum likelihood estimator of location. Each element of the robust covariance matrix is computed as the sample myriad of a window of the received observations. The MUSIC antenna array scheme is jointly used to mitigate the effects of multipath and impulsive noise. Simulation results demonstrate that the proposed scheme significantly outperforms the other linear and nonlinear schemes.

1 Introduction

Spatial signal processing has received considerable interest during the last two decades, as a technique that has proven very useful. Antenna array together with spatial signal processing have become very important tool in increasing the system capacity and in mitigating the multipath effects.

An essential technique in array signal processing is estimating the direction-of-arrival (DOA) of the signals impinging the array. DOA estimation techniques can be classified into two main categories: spectral-based method and parametric (maximum likelihood (ML)) method. Spectral-based method has two different approaches, which can be sub-divided into conventional techniques and subspace-based techniques. Conventional techniques are based on classical beamforming and require a large number of elements to achieve a high resolution. Subspace-based approaches are sub-optimal techniques that exploit the eigenstructure of the received signal correlation matrix and result in higher resolution [1]. One of the most widely used subspace-based methods is the multiple signal classification (MUSIC) algorithm proposed by Schmidt [2].

In direct sequence-code division multiple access (DS-CDMA) system, MUSIC bearing estimation begins first by despreading the received signal collected from all antenna elements with the desired user's spreading code. These collected snapshots are then used to form the covariance matrix that is used in turn by the MUSIC algorithm to estimate the bearings.

The majority of antenna array techniques assume that the antenna array operate in additive white Gaussian noise, that is, Gaussian distribution, which is the favourite noise model commonly employed in wireless communication mainly because it often leads to tractable solution. However, noise sources in communication environments are decidedly non-Gaussian and impulsive in nature. These sources range from naturally occurring sources such as ice cracking in the arctic region and lightening to man-made such as multiuser interference, car ignition and switching transit [3–5]. The main feature of impulsive noise is that it contains large amplitude pulses called outliers that results in heavy tail distribution (non-Gaussian). The main disadvantage of non-Gaussian distributions is that they have infinite variance. Hence all of the processing techniques that are based on the second order moments, which exhibits finite variance, will fail to operate properly in the case of non-Gaussian processes and will give misleading results. Many statistical and empirical noise models have been proposed to model impulsive noise. Among the various models, the family of alpha stable distributions [5] arises under very general assumptions and describes a broad class of impulsive noise. Alpha stable is preferred over other impulsive noise models as it is a physical model, which takes the underlying physical properties of the noise into consideration and is defined only by four parameters. In addition, it satisfies the generalised central limit theorem and obeys the stability property [5, 6].

In the context of subspace-based DOA techniques, a good estimation of the covariance matrix is required in order to obtain a reliable performance. In the presence of impulsive noise, the conventional covariance matrix employing second order statistics is not optimal and its performance degrades dramatically under impulsive noise. This highlights the essential need to derive other nonlinear and robust subspace-based techniques. Many nonlinear subspace DOA finding methods are proposed in this literature. Algorithms based on ML estimators (M-estimators) have been proposed in Yardimci *et al.* [7] and Ollila and Koivunen [8] in which M-estimators are used to estimate the noise and signal subspace matrix. Arce and Li [9] proposed a robust subspace technique, which is based on median correlation and is optimised for Laplacian distribution. Methods based on fractional lower order moments (FLOM) have been proposed [10–13]. Among these methods is the robust covariation-based MUSIC (ROC-MUSIC) developed by Taskalides and Nikias [10] and is found to provide robust performance under impulsive noise. ROC-MUSIC is developed from the covariation matrix, which is derived from the FLOM where the

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covariation coefficients function of X and Y are are computed [10] as

$$\chi_{X,Y} = \frac{E[XY^{\langle p-1 \rangle}]}{E[|Y|^p]} \tag{1}$$

where $E[\cdot]$ denotes statistical expectation, $0 < p \leq 2$ and the convention

$$|Y|^{\langle p \rangle} = |Y|^{\langle p-1 \rangle} Y^* \tag{2}$$

with the superscript * denoting complex conjugate.

It is obvious that when $p = 2$, the ROC-MUSIC algorithm is equivalent to the linear MUSIC. However, when $p < 1$ the performance of the algorithm is not robust and often lead to unreliable result, because the pth order moments for values of $p < 1$ are infinite [11, 12, 14].

In this paper, a robust DOA technique that is reliable and consistent under a wide range of impulsiveness levels is proposed for a DS-CDMA system. The proposed MUSIC algorithm is computed using the sample myriad that can be optimised for a wide range of stable processes ranging from the Gaussian up to severely impulsive noise. The performance of the proposed scheme is subsequently evaluated and compared to the linear MUSIC and the nonlinear ROC-MUSIC algorithms where the achieved performance is very promising.

2 Myriad covariation matrix

ML estimators of location play a very important role in statistical estimation theory [3, 4]. Important types of the ML estimators are the sample mean and the sample median, which are derived from sets of identically independent distributed (i.i.d.) samples obeying Gaussian and Laplacian distributions, respectively. Another robust estimator that has been introduced recently by Gonzalez and Arce [15] is the sample myriad, which is the ML estimator of location of data following Cauchy distribution and is given by the following

$$\widehat{\beta} = \arg\min_{\beta} \prod_{i=1}^{N} [\eta^2 + (x_i - \beta)^2] \tag{3}$$

where η is a nonlinearity parameter that controls the robustness of the estimator, that is, the lesser the value η the more is the resistance of the estimator to outliers.

In the case of independent but not identically distributed observations, the sample mean, median and myriad can be extended to a more general model. Let the sample set $x_1, x_2, \ldots, x_n$ be independent but not identically distributed, that is, they follow the same distribution but with a different variance. If the distribution is Gaussian, the ML estimator of location in this case can be shown to the value $\bar{\beta}$ [15] that minimises the following expression

$$M_G(\beta) = \sum_{i=1}^{n} \frac{1}{\sigma_i^2} (x_i - \bar{\beta})^2 \tag{4}$$

where σ_i^2 is the variance of the ith sample of the set and the value $\bar{\beta}$ that minimises (4) is the normalised weighted average given by

$$\bar{\beta} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \tag{5}$$

where $w_i = 1/\sigma_i^2 > 0$.

If we let $w_i = y_i$ then the estimate in (5) become

$$\bar{\beta} = \frac{\sum_{i=1}^{n} y_i x_i}{\sum_{i=1}^{n} y_i} \tag{6}$$

The solution in (6) can also be considered as the solution to least weighted squares sum

$$\bar{\beta} = \arg\min_{\beta} \sum_{i=1}^{N} y_i \cdot (x_i - \beta)^2 \tag{7}$$

On the other hand, if the distribution is Cauchy the ML estimator of location is the value of $\widehat{\beta}$ that minimises the function

$$M_C(\beta) = \sum_{i=1}^{n} \log[\eta^2 + w_i (x_i - \beta)^2] \tag{8}$$

It is noted that the weights in (6)–(8) are constrained to take positive values only as they are the inverse of the variance. The constraint can be removed by following the same approach introduced in Kalluri and Arce [16] in which the samples (weights) can take on real values and by modifying (7) such that the signs of the weights are coupled with the samples x as follows

$$\bar{\beta} = \arg\inf_{\beta} \sum_{i=1}^{n} |w_i| \cdot (\mathrm{sign}(w_i) x_i - \beta)^2 \tag{9}$$

In addition, the solution to the weighted least sum given by (6) will be modified in the same way which leads to

$$\bar{\beta} = \frac{\sum_{i=1}^{n} |y_i| \cdot \mathrm{sign}(y_i) x_i}{\sum_{i=1}^{n} |y_i|} = \frac{\sum_{i=1}^{n} y_i \cdot x_i}{\sum_{i=1}^{n} |y_i|} \tag{10}$$

The same approach can be adopted in deriving the same expression for the sample myriad such that the ML estimate model given by (8) is modified to an expression that admits negative weights as follows

$$\widehat{\beta} = \arg\inf_{\beta} \sum_{i=1}^{n} \log[\eta^2 + |w_i| \cdot (\mathrm{sign}(w_i) x_i - \beta)^2] \tag{11}$$

$$= \mathrm{Myr}(|w_i| \circ \mathrm{sign}(w_i) x_i |_{i=1}^{n}) \tag{12}$$

It is well known that second order correlation has provided the foundation of statistical signal modelling and processing. Given a set of observation pairs $\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$ drawn from the joint random variables $\boldsymbol{x}$ and $\boldsymbol{y}$, the sample correlation [11] is given by the following

$$\boldsymbol{R}_{xy} = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{x}_i \boldsymbol{y}_i \tag{13}$$

Comparing (13) and (6), we can conclude that the correlation estimate given in (13) is proportional to $\bar{\beta}$ as

$$\boldsymbol{R}_{xy} = \bar{y}_a \bar{\beta} \tag{14}$$

where the scaling factor $\bar{y}_a$ in (14) is the average magnitude of y_i.

Replacing the fixed set of weights w_i in (11) by the correlating sample y_i, we obtain

$$\widehat{\beta} = \arg\inf_{\beta} \sum_{i=1}^{n} \log[\eta^2 + |y_i| \cdot (\mathrm{sign}(y_i) x_i - \beta)^2] \tag{15}$$

As in (13), the cross-correlation estimate is found by scaling $\widehat{\beta}$ as

$$\boldsymbol{R}_{xy} = \bar{y}_a \widehat{\beta} \tag{16}$$

leading to the following definition:

Definition 1 (sample myriad covariation): Given a pair of observations $\{\boldsymbol{x}_i, \boldsymbol{y}_i\}$ that are drawn from two jointly random variables that are following Cauchy distribution, the sample myriad correlation matrix can be given by

$$\boldsymbol{R}_{xy} = \left(\frac{1}{N}\sum_{i=1}^{N}|y_i|\right)\mathrm{Myr}\left(|y_i| \circ \mathrm{sign}(y_i)x_i|_{i=1}^{N}\right) \tag{17}$$

$$= \bar{y}_i \cdot \mathrm{Myr}\left(|y_i| \circ \mathrm{sign}(y_i)x_i|_{i=1}^{N}\right) \tag{18}$$

As the distribution is Cauchy, the average magnitude term $\bar{y}$ in the above equation will give a misleading result. To obtain a consistent estimate, the average magnitude term $\bar{y}$ will be replaced with the sample myriad and is given by:

$$\boldsymbol{R}_{xy} = \mathrm{Myr}\left(|y_i||_{i=1}^{N}\right)\mathrm{Myr}\left(|y_i| \circ \mathrm{sign}(y_i)x_i|_{i=1}^{N}\right) \tag{19}$$

Equation (19) is referred to as the myriad covariation and is more robust estimator than (17).

3 Nonlinear music array

A coherent synchronous DS-CDMA system is considered providing wireless access to K simultaneous users each user $k \in \{1, \ldots, K\}$ is assigned a different discrete signature waveform that has unit energy and is modulated by binary phase-shift keying with a power level that remains constant over a symbol period. At the receiver, a uniform linear array (ULA) of M elements is employed. We assume a narrow band model, that is, the propagation delay between antenna elements is assumed to be small, relative to the inverse of the transmission bandwidth, so that the received signal at the M baseband array output are identical within a complex constant. The continue-time received signal at the antenna array can be modelled as

$$\boldsymbol{r} = \sum_{i}\sum_{k=1}^{K}\boldsymbol{a}(\theta_k)A_k b_k(i)s_k(t-iT) + \boldsymbol{n}(t) \tag{20}$$

Where T is the symbol period. With respect to the kth user, $\boldsymbol{a}(\theta_k) = [a_{1,k}, \ldots, a_{M,k}]$, A_k, $b_k(i) \in \{\pm 1\}$, $\{s_k(t); 0 < t \leq T\}$ denote, the array response vector of a broadside signal vector of θ_k, the received amplitude, ith information bit and normalised signature waveform, respectively. The component of the array response vector $a_{m,k}$ is the complex gain of the kth user to the mth sensor. A ULA with half-wavelength spacing is assumed, the array response is given by

$$a_{m,k} = \exp\left[j(m-1)\pi\sin(\theta_k)\right] \tag{21}$$

The direct sequence spread spectrum signature waveform is defined as

$$s_k(t) = \sum_{j=1}^{N}\phi_j^k p(t-jT), \quad t \in [0, T] \tag{22}$$

Where N is the processing gain, $\phi_j^k \in \{\pm 1\}$ is a signature sequence assigned to the kth user and $p(t)$ is a rectangular waveform of duration T. $\boldsymbol{n}(t) = [n_1(t), \ldots, n_M(t)]^{\mathrm{T}}$ is a vector of the spatio-temporal noise, which is usually assumed to be an i.i.d. random variable with zero-mean. In this paper the noise is modelled with the commonly used symmetric α sable that has a characteristics function of the form

$$\phi(t) = \exp\{-\gamma|t|^{\alpha}\} \tag{23}$$

where $0 < \alpha \leq 2$ is the characteristics exponent that controls the impulsiveness level of the distribution and $\gamma > 0$ is the dispersion.

The received signal is passed through a matched filter bank where each filter is matched to a specific user, which is then sampled at a sampling rate $1/T$ to yield a discrete signal model at the ith symbol that is given by

$$\boldsymbol{y}_k(i) = \int_{iT}^{(i+1)T} r(t)s_k(t)\,\mathrm{d}(t) \quad k = 1, \ldots, K \tag{24}$$

This can be defined in vector form as

$$\boldsymbol{y}_k(i) = [y_{1,k}(i), \ldots, y_{M,k}(i)]^{\mathrm{T}} \tag{25}$$

The myriad covariation matrix will be used to robustify the linear MUSIC algorithm. In this section we will propose a robust nonlinear version of the linear MUSIC algorithm that is derived using the myriad covariation matrix.

The proposed nonlinear algorithm is called myradi-MUSIC (MYR-MUSIC) and it can be summarised into the following steps

1. Collect q sample vectors $\boldsymbol{y}$, which is given by (25), where q is the window size and $\boldsymbol{y}_{i+1}$ is one sample delayed version of $\boldsymbol{y}_i$.
2. Calculate the myriad covariation between the q sample vectors

$$\boldsymbol{R}_{p,i} = \mathrm{Myr}\left(|y_i||_{i=1}^{q}\right) \cdot \mathrm{Myr}\left(|y_{i,j}| \circ \mathrm{sign}(y_{i,j})y_{p,j}|_{i=1}^{q}\right), \quad i,j,p = 1, 2, \ldots, M \tag{26}$$

Where each element in this matrix is the myriad of two vectors of length $q = M$ and $y_{i,j}$ is the jth sample of vector $\boldsymbol{y}_i$.
3. Construct a Toeplitz matrix by performing the operation

$$\boldsymbol{R}_{i-1} = \frac{(\boldsymbol{R}_{1,i} + \boldsymbol{R}_{i,1})}{2}, \quad i = 0, 1, \ldots, q-1 \tag{27}$$

4. Perform the eigendecomposition on $\boldsymbol{R}$ such that

$$\boldsymbol{R} = \widehat{\boldsymbol{U}}_s\widehat{\boldsymbol{\Lambda}}_s\widehat{\boldsymbol{U}}_s^H + \widehat{\boldsymbol{U}}_n\widehat{\nabla}_n\widehat{\boldsymbol{U}}_n^H \tag{28}$$

where $\widehat{\boldsymbol{U}}_s$ is the signal eigenvector and $\widehat{\boldsymbol{U}}_n$ is the noise eigenvector that is orthogonal to the array steering vectors $\boldsymbol{A}$.
5. The directions of signals are found by selecting the steering vectors that are orthogonal to the noise sub-space by using the following expression

$$P(\theta) = \frac{\boldsymbol{a}^*(\theta)\boldsymbol{a}(\theta)}{\boldsymbol{a}^*(\theta)\widehat{\boldsymbol{U}}_n\widehat{\boldsymbol{U}}_n^H\boldsymbol{a}(\theta)} \tag{29}$$

The orthogonality between $\boldsymbol{a}(\theta)$ and $\widehat{\boldsymbol{U}}_n$ will minimise the denominator and hence the estimates of the DOA are given by the positions of the spectral peaks where the K largest peaks in the spectrum correspond to the DOAs of signals impinging the array.

4 Simulation results

The performance of the robust MYR-MUSIC is compared to the linear MUSIC and the nonlinear ROC-MUSIC algorithms. A CDMA system serving five users is considered, each user is employing 31-chip Gold code spreading sequences. All of users are being served by a base station that incorporate antenna array of eight elements ($M = 8$) and each user has a single path. The DOAs of all users are $\lfloor 120°, 25°, 65°, 105°, 75° \rfloor$ where the first user is the desired one. The myriad linearity parameter is computed using this formula: $\eta = \sqrt{(\alpha/(2-\alpha))}\gamma^{1/\alpha}$ [15].

4.1 Beampattern analysis

Figs. 1 and 2 illustrate a performance comparison between the MYR-MUSIC, linear MUSIC and the ROC-MUSIC. As expected all of the MUSIC arrays perform well under Gaussian noise channel as it can be seen from Fig. 1 that the beampattern of all of the MUSIC arrays are very close and steered towards the DOA of the impinging signal. In the case of impulsive noise ($\alpha = 1.5$), Fig. 2 shows the spectral plot of the beampatterns of the different arrays where the MYR-MUSIC algorithm exhibits the best performance and provided the most significant gain, where the linear MUSIC failed to steer towards the DOA, meanwhile the ROC-MUSIC provided a robust performance.

4.2 Root mean squared error analysis

Another performance criterion we have adopted is the root mean squared error (RMSE) [17] to compare the performance of the different schemes

$$\mathrm{RMSE} = \sqrt{\sum_{i}^{q} \frac{(\widehat{\theta}_k(i) - \theta_k)^2}{q}} \tag{30}$$

where q is the number of the data symbols which is taken as 200 and $\widehat{\theta}_k(i)$ (in degree) is the bearing estimate of the line-of-sight (LOS) path of the first user at the ith symbol and θ_k is the actual angle.

For a range of a geometric signal-to-noise (G-SNR) [18] Figs. 3 and 4 demonstrate the RMSE difference between the actual and the estimated DOA of all of the MUSIC schemes. Fig. 3 shows that all of the array schemes exhibits similar performance under Gaussian noise, as it can be seen from Fig. 3 that the RMSE plot of all schemes almost overlapped and the error between the actual and estimated angles is very small. On the contrary, Fig. 4 illustrate the performance of the MUSIC arrays in terms of RMSE under impulsive noise $\alpha = 1.5$. It can be seen from the plots in Fig. 4 that the linear MUSIC exhibits the highest error plot and the difference between the actual and the estimated angle is high for the whole G-SNR range which concludes that the performance of the linear MUSIC does not improve much with the increase of G-SNR. On the other hand, Fig. 4 shows that the MYR-MUSIC array has provided the lowest RMSE plot. Fig. 4 shows that in the case of MYR-MUSIC the difference between the actual and the estimated angle is less than 1° for a G-SNR of more than 5 dB, which infer that the performance of the MYR-MUSIC is reliable and more robust for the whole range of the G-SNR. In addition, MYR-MUSIC has outperformed the ROC-MUSIC by a gain of more than 2 dB.

Further to the RMSE performance, we also included the RMSE results in terms of the characteristics exponent. Fig. 5 illustrates the performance of the MUSIC schemes

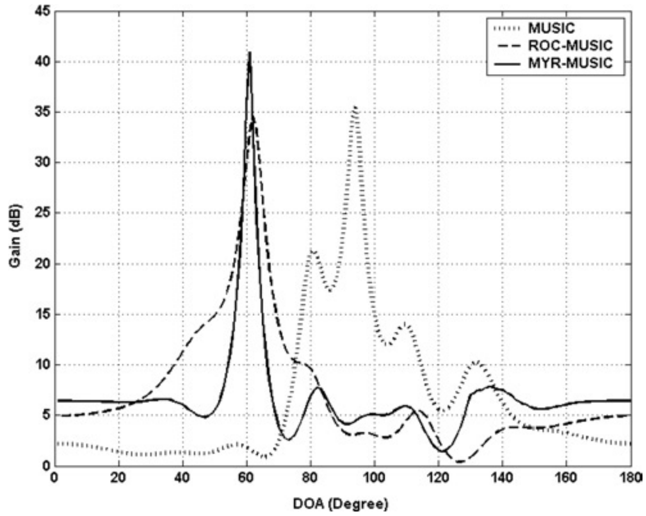

Fig. 2 *Array beampattern for MUSIC, ROC-MUSIC and MYR-MUSIC algorithms under impulsive noise, $\alpha = 1.5$*

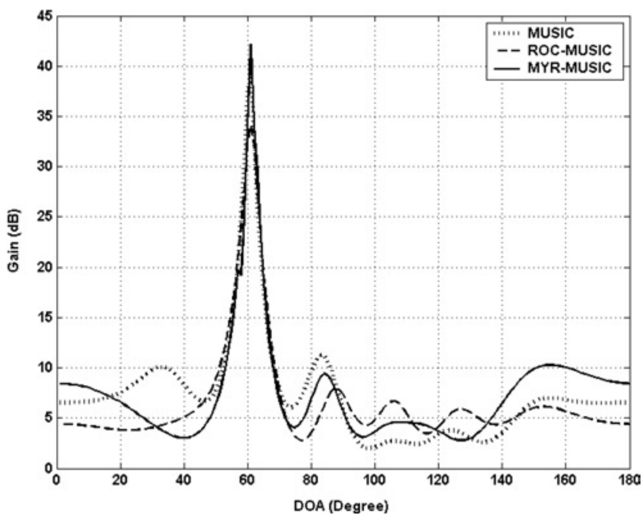

Fig. 1 *Array beampattern for MUSIC, ROC-MUSIC and MYR-MUSIC algorithms under Gaussian noise, $\alpha = 2$*

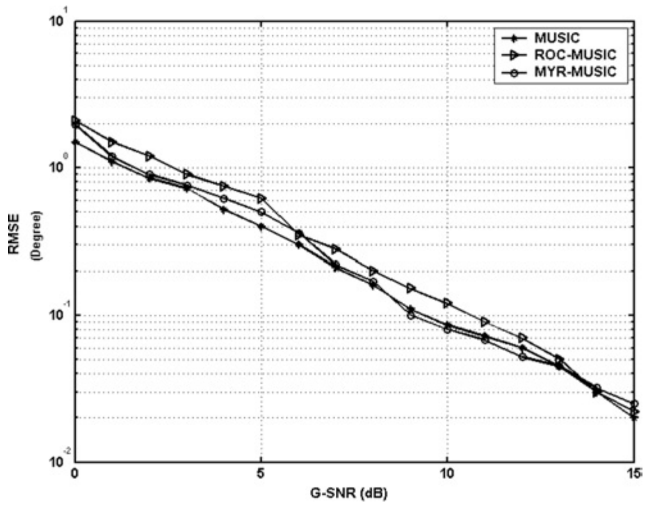

Fig. 3 *RMSE performance comparisons between MUSIC, ROC-MUSIC and MYR-MUSIC under Gaussian channel noise, $\alpha = 2$*

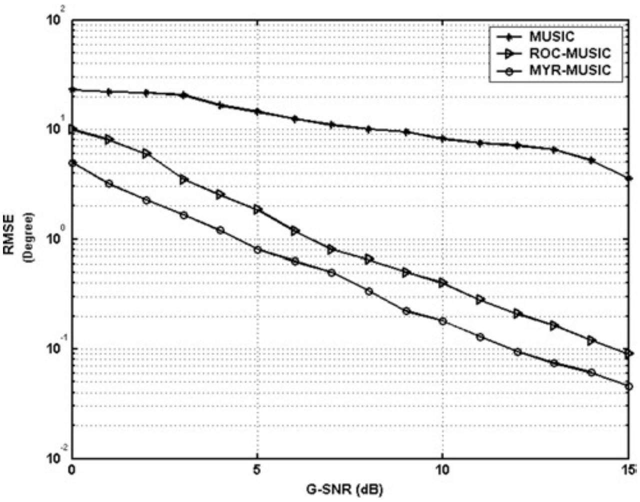

Fig. 4 *RMSE performance comparisons between MUSIC, ROC-MUSIC and MYR-MUSIC under impulsive channel noise, $\alpha = 1.5$*

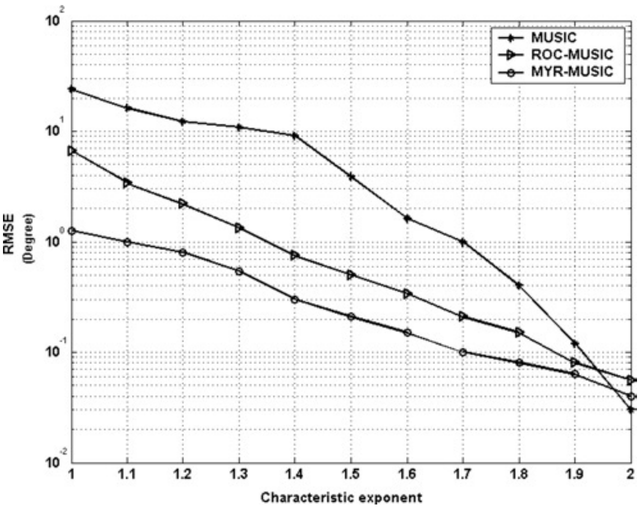

Fig. 5 *RMSE performance comparisons between MUSIC, ROC-MUSIC and MYR-MUSIC as a function of the characteristic exponent (α)*

for a different values of the characteristic exponent that range from $\alpha = 1$ to $\alpha = 2$. Comparing from the plots in Fig. 5, we conclude that the MYR-MUSIC and ROC-MUSIC arrays plots exhibit the smallest slope and the plot of linear MUSIC is the highest. In other words the figure shows that the performance degradation of the MYR-MUSIC as the value of α decreases is less than that of the ROC-MUSIC whiles the RMSE values obtained by the linear MUSIC increases dramatically with the decrease of α.

The achieved performance is obtained at the expenses of an affordable increment in complexity. Comparing with the linear scheme, $O(9M^2)$ operations have been added where M is the number of antenna elements. While comparing with the ROC-MUSIC, only $O(4M^2)$ operations have been added.

5 Conclusion

In this a paper, a robust covariation matrix using the sample myriad is derived from the ML location estimator under stable process distribution. Myriad covariation matrix is used to derive a nonlinear version of the well-known MUSIC algorithm that is referred to as MYR-MUSIC. The proposed scheme is inherently robust to impulsive noise and its robustness is demonstrated through Monte Carlo simulation where its performance in DS-CDMA system is compared to the linear MUSIC and the nonlinear ROC-MUSIC algorithms. The simulation results show that the proposed algorithm performs more robustly than the nonlinear ROC-MUSIC algorithm under impulsive noise and very comparable to the linear MUSIC under Gaussian noise. The achieved performance is obtained with moderate increase in complexity that is readily within the capability of modern processing.

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