IET Communications

Research Article

Journals
The Institution of Engineering and Technology

Optimal power allocation for multiple input single output cognitive radios with antenna selection strategies

ISSN 1751-8628
Received on 30th September 2014
Revised on 30th January 2015
Accepted on 21st February 2015
doi: 10.1049/iet-com.2014.0939
www.ietdl.org

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Abstract: The opportunistic spectrum access technology is one of the most promising methods for alleviating the spectrum scarcity problem, which enables a secondary user (SU) to utilise a primary user spectrum band that is detected idle. However, the throughput achieved by cognitive radios is limited by the interference constraint imposing on the SU. Multiple input single output antenna techniques and antenna selection (AS) techniques are exploited to combat the interference constraint and improve the achievable average throughput of the SU. The optimal power allocation strategy is proposed to maximise the achievable average throughput. Performance analyses for the achievable average throughputs are performed under the maximum channel gain AS strategy, the minimum interference channel gain AS strategy and the ratio AS strategy. It is proved that the optimal transmitted AS strategy is the ratio AS strategy when the optimal power allocation strategy is used. The optimal sensing parameters are designed to further improve the maximum average throughput. Extensive simulation results are conducted to verify this analysis.

1 Introduction

Spectrum scarcity problem becomes severer because of increasing requirements for the higher data rate and capacity. Cognitive radios (CR) is one of the most promising technologies that provides a solution to alleviate the spectrum scarcity problem [1]. In CR, there are several operation models, such as opportunistic spectrum access and spectrum sharing [2]. According to opportunistic spectrum access, a secondary user (SU) is able to access a primary user (PU) spectrum band only when the PU is detected idle [3]. Spectrum sharing allows the SU to coexist with the PU as long as the interference causing to the PU is tolerable [4]. Since CR applications in television spectrum band have adopted the opportunistic spectrum access model instead of the spectrum sharing model [5], in this paper, we focus on the opportunistic spectrum access model.

In CR, spectrum sensing is required to make a decision between the absence and the presence of the PU. Since spectrum sensing is the precondition of implementation of CR, many spectrum sensing schemes have been proposed, such as energy detection, eigenvalue-based detection, match filter detection, cyclostationary detection, as reported in [6]. Recently, we have proposed an efficient spectrum sensing algorithm based on Cholesky decomposition [7]. However, because of the practical communication environment including fading and the limitation of spectrum sensing techniques, spectrum sensing is an imperfect function, where the PU may be detected idle by mistake. Therefore the interference constraint is imposed on the SU to protect the PU. However, the interference constraints limit the throughput achieved by the CR system [8].

Multiple input multiple output (MIMO) techniques are promising to improve the achievable throughput [9, 10]. However, the hardware complexity and high cost are the main drawbacks of multiple antenna techniques because of the requirement for radio frequency chains. Antenna selection (AS) techniques have been extensively studied to overcome that shortage [11–15]. In [11], several AS strategies, namely, minimum interference strategy, maximum data channel gain strategy, maximum sum capacity strategy and the maximum signal-to-leak interference power ratio strategy are proposed for the multiple input single output (MISO) cognitive radio system, where the spectrum sharing model is used. In [12], an AS method termed difference selection is proposed. However, the difference selection requires to set a parameter, which is difficult to obtain in practice. To maximise CR data rates and satisfy interference constraints on the SU, two solutions to the problem of joint transmit-receive AS in MIMO CR are presented [13]. In [14], authors develop a novel optimal AS based on Chernoff-bound to minimise an upper bound on the symbol error probability at the SU receiver. The exact performance analysis of MIMO CR using transmitted AS is performed [15].

Recently, the optimal power allocation strategy has been considered to protect PUs and improve the throughput of a CR system [16–21]. In [16, 17], it shows that constraints on the average interference power and on the transmitted power can better protect the PUs and achieve higher throughput compared with constraints on the peak interference power and on the peak transmitted power. The optimal power allocation strategy for CR system based on spectrum sensing enhancement is proposed [18]. In [19], authors propose a fast and efficient parallel-shift water-filling algorithm for the power allocation strategy. In [20], the continuous power allocation strategy is obtained based on the sensing statistics, which is different from the traditional power allocation strategy. The optimal power allocation strategy is designed to reduce the energy consumption for energy efficient cognitive radio network [21].

Although the optimal power allocation strategy for CR based on spectrum sharing has been well studied in many works, that strategy is rarely analysed under the operation model of opportunistic spectrum access. In this paper, we extend the throughput trade-off problem proposed in [22] into a MISO CR system. More specially, the optimal power allocation strategy for the CR system with the opportunistic spectrum access model is proposed under the average interference power constraint. In addition, the achievable average throughputs are analysed when the minimum interference channel gain AS strategy, the maximum data channel gain AS strategy and the ratio AS strategy are used, respectively. To further improve the achievable average throughput of the MISO CR system, the optimal sensing parameters are designed.

The rest of this paper is organised as follows. Section 2 presents the system model and the optimisation problem. The optimal power allocation strategy is proposed in Section 3. Section 4 performs performance analyses for the achievable average throughputs under the three AS strategies. The optimal sensing parameters are designed for further improving the average throughput in Section 5. Section 6 conducts numerical simulation results to evaluate our analysis. The paper concludes with Section 7.

2 System model and problem formulation

As shown in Fig. 1, a CR system with the model of opportunistic spectrum access is considered, which has one PU and one SU. The SU can opportunistic access the PU spectrum band only when the spectrum band is detected idle. The SU transmitter (SU-Tx) has M transmitted antennas and the corresponding receiver (SU-Rx) has one receive antenna. The PU receiver (PU-Rx) has one receive antenna. Considering that spectrum sensing is an imperfect function, the SU may impose interference on the PU. Therefore the interference constraint is required to impose on the SU to protect the PU.

2.1 System model and frame structure

Let x_s denote the transmitted signal from the SU-Tx. Using the ith transmitted antenna, the received signal of the SU-Rx and the interference signal from the SU-Tx causing to the PU-Rx when the absence of the PU is detected by mistake are denoted by $y_{s,i}$ and $y_{p,i}$, respectively, given as

$$y_{s,i} = \sqrt{P_s g_i} x_s + n_s \quad (1)$$

$$y_{p,i} = \sqrt{P_s h_i} x_s \quad (2)$$

where n_s is the circular symmetric complex additive white Gaussian noise with mean zero and variance N_0. The transmitted power of the

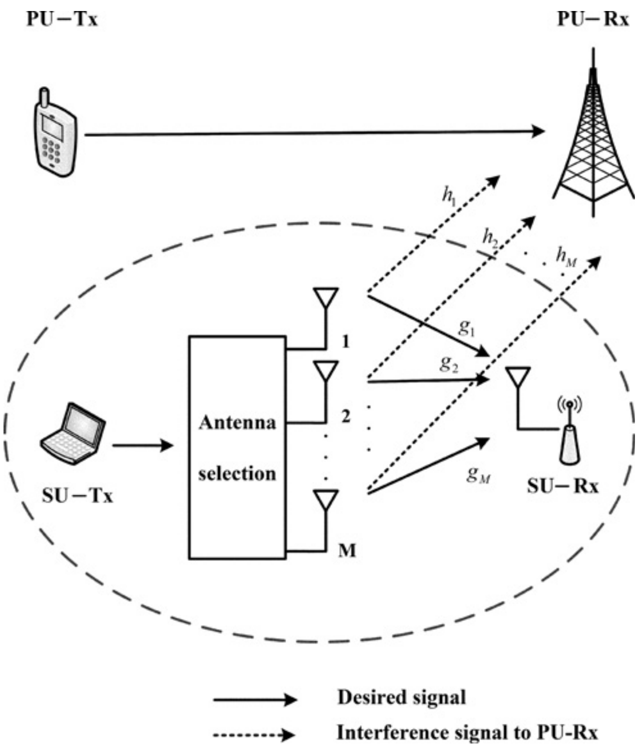

Fig. 1 *System model*

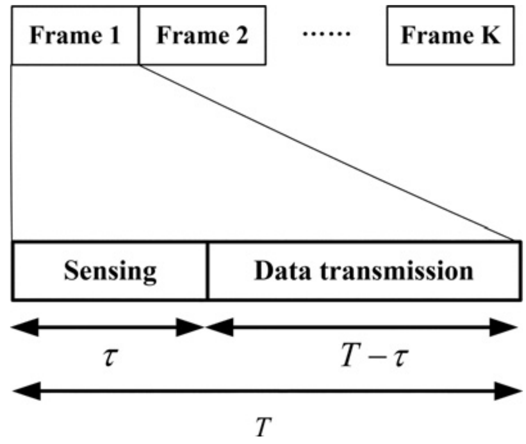

Fig. 2 *Frame structure of the cognitive radio network with the opportunistic spectrum sensing model (τ: sensing slot duration; $T-\tau$: data transmission slot duration)*

SU-Tx is denoted by P_s. The instantaneous channel power gains from the ith SU-Tx to the SU-Rx and the PU-Rx, namely, g_i and h_i. $i \in \{1, 2, \dots, M\}$, are modelled as flat fading and independent exponential distribution with means being $1/\lambda_s$ and $1/\lambda_p$, respectively.

According to Liang *et al.*, [22] Pei *et al.*, [23] Stotas *et al.*, [24], the frame structure of the CR with the opportunistic spectrum access model is based on the classical two-phase model, consisting of a sensing slot and a data transmission slot, as depicted in Fig. 2. As considered in [22–24], the duration of each frame is fixed and denoted by T. At the beginning of each frame, the SU requires to spectrum sensing and makes a decision whether the PU is absent or not. This is the sensing stage and the time used to spectrum sensing is the sensing slot, denoted by τ. In the remaining duration of each frame, namely, data transmission slot, the SU transmits data if the PU is detected idle, otherwise, the SU keeps silent. Thus, the duration of the data transmission slot is $T-\tau$.

2.2 Problem formulation

According to Zhang and Liang [10], the average interference power constraint can better protect the PU compared with the peak interference power constraint. Therefore in this paper, the constraint on the average interference power is taken into consideration. In the transmitted phase, the SU-Tx selects one of the M transmitted antennas based on a certain AS strategy to transmit data. Since the quality of service (QoS) of the PU should be protected, a high target detection probability is required. According to IEEE 802.22 WRAN standard [3], the target detection probability is required to be higher than 90% for the signal-to-noise ratio (SNR) as low as −20 dB related to the PU signal at the SU detector. Based on the analysis in [23], in the worst case, the interference causing from the PU to the SU can be ignored. In this paper, the worst case is focused.

Thus, for the objective of maximising the achievable average throughput, the optimisation problem for obtaining the optimal power allocation strategy and a certain AS strategy under the constraint on the average interference power is formulated as

$$\underset{\{\tau, P_s, s\}}{\text{maximise}} \quad R(\tau, P_s, s)$$
$$= \frac{T-\tau}{T}\mathbb{E}\left\{[p_0(1-p_f(\tau)) + p_1(1-p_d)]\log_2\left(1+\frac{g_s P_s}{N_0}\right)\right\} \quad (3a)$$

subject to

$$\frac{T-\tau}{T}\mathbb{E}[p_1(1-p_d)h_s P_s] \le \Gamma_{av} \quad (3b)$$

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$$P_s \geq 0, \quad 0 \leq p_d \leq 1 \tag{3c}$$

where p_1 and p_0 are the probabilities for the presence or the absence of the PU, respectively. p_d denotes the target detection probability. $p_f(\tau)$ is the false alarm probability, which is a function of the sensing time. Γ_{av} represents the prescribed average interference power threshold. s denotes the selected transmitted antenna based on a certain criterion. $\mathbb{E}\{\cdot\}$ denotes the expectation operator. $R(\tau, P_s, s)$ represents the achievable average throughput.

Equations (3a)–(3c) show that effects of the parameters of spectrum sensing on the power allocation strategy are taken into consideration. Note that the optimisation problem is dependent on the probability distributions of channel gains of g_s and h_s under a certain AS strategy. In addition, P_s is also dependent on these probability distributions. It is seen that the optimisation problem is intractable. A three-step method is proposed to solve the optimisation problem. In the first step, the optimal power allocation strategy is proposed. In the second step, the optimal AS strategy is analysed on the basis of the optimal power allocation strategy. Finally, the optimal sensing time is designed. It is noted that solutions obtained by the three-step method may not be optimal. However, the three-step method largely simplifies the optimisation problem and makes the optimisation problem be tractable in practice.

3 Optimal power allocation strategy

In this paper, energy detection is used as the spectrum sensing scheme because of its low complexity and wide application in practice. According to Liang *et al.*, [22], the false alarm probability is given for a target detection probability as

$$p_f(\tau) = Q\left(\sqrt{2\gamma+1}Q^{-1}(p_d) + \sqrt{\tau f_s}\gamma\right) \tag{4}$$

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian. $Q^{-1}(\cdot)$ is the inverse function of $Q(\cdot)$. The received SNR related to the PU signal at the SU detector is denoted by γ. f_s represents the sampling frequency.

Equations (3a) and (3b) show that the optimisation problem is a non-linear and non-convex function with respect to sensing time τ, which has no efficient methods of the optimisation over the sensing time. However, similar to [23], the optimal sensing time is able to obtain by using one-dimensional exhaustive search because of it lying with the interval (0, T). Therefore the optimal power allocation strategy is the focus of this paper.

It is seen that the optimisation problem is a convex function with respect to P_s from (3a). Therefore the Lagrangian related to the transmitted power P_s for a given sensing time and a target detection probability is expressed as

$$\begin{aligned} L(P_s, u) = \frac{T-\tau}{T}\mathbb{E}\left\{[p_0(1-p_f(\tau)) + p_1(1-p_d)]\log_2\left(1+\frac{g_s P_s}{N_0}\right)\right\} \\ - u\left\{\frac{T-\tau}{T}\mathbb{E}[p_1(1-p_d)h_s P_s] - \Gamma_{av}\right\} \end{aligned} \tag{5}$$

where $u \geq 0$ is the Lagrange multiplier.

The Lagrange dual optimisation problem is given as

$$\underset{u\geq 0}{\text{minimise}} \quad \psi(u) \tag{6}$$

where $\psi(u)$ is the Lagrange dual function given by

$$\psi(u) = \sup_{P_s} \quad L(P_s, u) \tag{7}$$

Since the optimisation problem in (3a) is a convex function with affine inequality constraints related to P_s, the primal optimisation problem in (3a) is equivalent to the dual optimisation problem in (6). Therefore according to the KKT conditions [25], the optimal power allocation P_s is given as

$$P_s = \left(\frac{[p_0(1-p_f(\tau)) + p_1(1-p_d)]}{up_1(1-p_d)h_s \ln 2} - \frac{N_0}{g_s}\right)^+ \tag{8}$$

where $(x)^+$ denotes max (0, x). u is obtained by using the subgradient-based method as

$$u_{i+1} = u_i - \beta\left[\Gamma_{av} - \frac{T-\tau}{T}\int_0^\infty\int_0^\infty \left[p_1(1-p_d)y\widetilde{P}_{s,i}\right] f_{g_s}(x) f_{h_s}(y)\mathrm{d}x\mathrm{d}y\right] \tag{9}$$

where $f_{g_s}(x)$ and $f_{h_s}(y)$ are the probability density functions (PDFs) for channel gains from the SU-Tx to the SU-Rx and the PU-Rx under a certain AS strategy, respectively. β is a small positive step size. $\widetilde{P}_{s,i}$ and u_i are the optimal power allocation and the Lagrange multiplier for the ith iteration, respectively. Thus, the average throughput of the CR based on the proposed optimal power allocation strategy is given as

$$\begin{aligned} R(\tau, P_s, s) = \frac{(T-\tau)}{T\ln 2}\mathbb{E}\{[p_0(1-p_f(\tau)) + p_1(1-p_d)] \\ \times \ln\left\{\frac{[p_0(1-p_f(\tau)) + p_1(1-p_d)]}{N_0 up_1(1-p_d)\ln 2}\frac{g_s}{h_s}\right\}\right\} \end{aligned} \tag{10}$$

Equation (10) shows that the average throughput increases with the ratio of the channel gains. The maximum channel gain AS strategy selects the transmitted antenna which has the maximum channel gain between the SU-Tx and the SU-Rx, whereas the minimum interference channel gain AS strategy chooses the transmitted antenna which causes the minimum interference on the PU. However, neither of the two AS strategies chooses the transmitted antenna that obtains the maximum channel gains ratio, namely, g_s/h_s. Since the ratio AS strategy selects the transmitted antenna having the maximum channel gains ratio, the optimal AS strategy is the ratio selection scheme.

Corollary 1: Under the average interference power constraint, the optimal AS strategy is the ratio selection scheme for the MISO CR system with the proposed optimal power allocation strategy.

4 Average throughput under three antenna selection strategies

In this section, the achievable average throughputs under the three AS strategies are analysed. The distributions of channels gains, g_s, h_s and the ratio of these channel gains under the three AS strategies are derived.

4.1 Average throughput under the maximum channel gain AS strategy

In CR, to obtain more throughput, it is reasonable that the SU selects the transmitted antenna with the maximum channel gain between the SU-Tx and the SU-Rx as long as the interference imposed on the PU is tolerable. This transmitted AS strategy is defined as the maximum channel gain AS strategy. Since the data channel gain is independent of the interference channel gain, the maximum channel gain AS strategy has no effect on the interference channel gain. Thus, the probability distribution of the interference gain with the maximum channel gain AS strategy is equal to that of the interference gain with random selection AS strategy.

According to (9) and (10), to analyse the achievable average throughput, it is necessary to derive PDFs for channel gains, g_s, h_s and the ratio of these channel gains. Let G_{MAX} denote the channel gain between the SU-Tx and the SU-Rx with the maximum channel gain AS strategy, namely, $G_{\text{MAX}} = \max\{g_1, g_2, \ldots, g_M\}$. Since g_i follows an exponential distribution with mean $1/\lambda_s$, where $i = 1, 2, \ldots, M$, one has

$$f_{g_i}(x) = \lambda_s e^{-\lambda_s x}, \quad x \ge 0 \tag{11}$$

$$F_{g_i}(x) = 1 - e^{-\lambda_s x}, \quad x \ge 0 \tag{12}$$

where $f_{g_i}(x)$ and $F_{g_i}(x)$ denote the PDF and the cumulative distribution function (CDF) of g_i, respectively. Based on the maximum channel gain AS strategy, one has the lemma 1, namely,

Lemma 1: The CDF and the PDF of the channel gain with the maximum channel gain AS strategy, denoted by $F_{G_{\text{MAX}}}(x)$ and $f_{G_{\text{MAX}}}(z)$, respectively, are given as

$$F_{G_{\text{MAX}}}(x) = (1 - e^{-\lambda_s x})^M, \quad x \ge 0 \tag{13}$$

$$f_{G_{\text{MAX}}}(z) = M\lambda_s e^{-\lambda_s x}(1 - e^{-\lambda_s x})^{M-1}, \quad x \ge 0 \tag{14}$$

Proof: See Appendix 1. □

As stated before, the maximum channel gain AS strategy has no effect on the interference gain, h_i, which follows an exponential distribution with mean $1/\lambda_p$. Thus, the CDF and PDF of the interference channel gain, h_s, denoted by $F_{h_s}(y)$ and $f_{h_s}(y)$, respectively, are given as

$$F_{h_s}(y) = 1 - e^{-\lambda_p y}, \quad y \ge 0 \tag{15}$$

$$f_{h_s}(y) = \lambda_p e^{-\lambda_p y}, \quad y \ge 0 \tag{16}$$

Lemma 2: Let Z_{MAX} denote the ratio of the data channel gain to the interference channel gain with the maximum channel gain AS strategy, namely, $Z_{\text{MAX}} = G_{\text{MAX}}/h_s$. The PDF of Z_{MAX}, denoted by $f_{Z_{\text{MAX}}}(z)$, is given as

$$\begin{aligned} f_{Z_{\text{MAX}}}(z) &= \int_0^\infty f_{G_{\text{MAX}}}(yz) f_{h_s}(y) y \,\mathrm{d}y \\ &= \int_0^\infty M\lambda_s e^{-\lambda_s yz}(1 - e^{-\lambda_s yz})^{M-1} \lambda_p e^{-\lambda_p y} y \,\mathrm{d}y, \quad z \ge 0 \end{aligned} \tag{17}$$

Thus, based on (10) and (17), the achievable average throughput is given as

$$\begin{aligned} R(\tau, P_s, s) = \frac{a}{\ln 2}\int_0^\infty \ln\left\{\frac{a}{N_0 ub \ln 2} z\right\} \\ \times \int_0^\infty M\lambda_s e^{-\lambda_s yz}(1 - e^{-\lambda_s yz})^{M-1} \lambda_p e^{-\lambda_p y} y \,\mathrm{d}y \mathrm{d}z \end{aligned} \tag{18a}$$

$$a = \frac{T - \tau}{T}[p_0(1 - p_f(\tau)) + p_1(1 - p_d)] \tag{18b}$$

$$b = \frac{T - \tau}{T}[p_1(1 - p_d)] \tag{18c}$$

where u can be obtained by using the proposed algorithm based on the subgradient-based method, given in (9). Although, the achievable average throughput is in integral form, it can be obtained readily by using standard numerical computation of the integral. The integral is well behaved having a strictly positive integrand.

4.2 Average throughput under the minimum interference channel gain AS strategy

The SU can benefit from the maximum channel gain AS strategy achieved by the diversity gain. However, the maximum channel gain AS strategy has no influence on the interference channel because of their independence. In CR, the priority is to protect the QoS of the PU. Thus, from the perspective of the PU, the transmitted AS strategy should select the transmitted antenna with the minimum interference channel gain between the SU-Tx and the PU-Rx, which is defined as the minimum interference channel gain AS strategy.

Lemma 3: Let $Y = \min(h_1, h_2, \ldots, h_M)$, representing the minimum interference channel gain. The CDF and the PDF of Y, denoted by $F_Y(y)$ and $f_Y(y)$, respectively, are given as

$$F_Y(y) = 1 - e^{-M\lambda_P y}, \quad y \ge 0 \tag{19}$$

$$f_Y(y) = M\lambda_P e^{-M\lambda_P y}, \quad y \ge 0 \tag{20}$$

Proof: See Appendix 2 □

Since the data channel gain is independent of the interference channel gain, the minimum interference channel gain AS strategy has no effect on the data channel gain. Thus, the probability distribution of the data channel gain with the minimum interference channel gain AS strategy is equal to that of the data channel gain with random AS strategy, given in (11).

Lemma 4: Let Z_{MIN} denote the ratio of the data channel gain to the interference channel gain with the minimum interference channel gain AS strategy, namely, $Z_{\text{MIN}} = g_s/Y$. The CDF and the PDF of Z_{MIN}, denoted by $F_{Z_{\text{MIN}}}(z)$ and $f_{Z_{\text{MIN}}}(z)$, respectively, are given as

$$F_{Z_{\text{MIN}}}(z) = \frac{\lambda_s z}{\lambda_s z + M\lambda_P} \tag{21}$$

$$f_{Z_{\text{MIN}}}(z) = \frac{M\lambda_s\lambda_P}{(\lambda_s z + M\lambda_P)^2} \tag{22}$$

The Lemma 4 can be easily proved by solving the PDF of the ratio of two independent random variables with exponential distributions, namely

$$f_{Z_{\text{MIN}}}(z) = \int_0^\infty f_{g_s}(zy) f_Y(y) y \,\mathrm{d}y \tag{23}$$

where $f_{g_s}(y)$ is the PDF of the data channel gain with the minimum interference channel gain AS strategy, given in (11).

Thus, according to (10) and (22), the achievable average throughput with the minimum interference channel gain AS strategy is given as

$$R(\tau, P_s, s) = \frac{a}{\ln 2}\ln\left[\frac{(\lambda_s ubN_0 \ln 2 + Ma\lambda_P)}{N_0 ub \ln 2}\right] \tag{24}$$

where a and b are given in (18b) and (18c), respectively. u can be obtained by using the subgradient-based method, given in (9). It is seen that the achievable average throughput increases with the number of transmitted antennas (M). The reason is that the transmitted power of the SU can increase because of the fact that the interference channel gain decreases with the increase in the number of transmitted antennas.

Proof: See Appendix 3. □

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4.3 Average throughput under the ratio AS strategy

The maximum channel gain AS strategy improves the achievable average throughput by choosing the transmitted antenna with the maximum channel gain. However, the interference channel gain does not decrease because of the independence of the data channel gain with the interference channel gain. Although, the minimum interference channel gain AS strategy decreases the interference channel gain by selecting the transmitted antenna with the minimum interference channel gain to transmit the SU signal, whereas the data channel gain can not benefit from this strategy. Those two AS strategies only take the data channel gain or the interference channel gain into consideration. As stated before, the ratio AS strategy not only takes the data channel gain into consideration, but considers the interference channel gain, which is proved the optimal AS strategy.

To analyse the average throughput achieved by using the ratio AS strategy, it is necessary to derive PDFs for channel gains, g_s, h_s and the ratio of these channel gains under the ratio AS scheme

Lemma 5: Let

$$Z = \arg\max_i \left\{ \frac{g_1}{h_1}, \frac{g_2}{h_2}, \ldots, \frac{g_i}{h_i}, \ldots, \frac{g_M}{h_M} \right\}$$

Owing to the independence among those ratios of channel gains, the PDF and the CDF for Z are derived, denoted by $f_Z(z)$ and $F_Z(z)$, respectively, as

$$f_Z(z) = \frac{M\lambda_s^M \lambda_p z^{M-1}}{(\lambda_s z + \lambda_p)^{M+1}} z \geq 0 \tag{25}$$

$$F_Z(z) = \left(\frac{\lambda_s z}{\lambda_s z + \lambda_p} \right)^M, \quad z \geq 0 \tag{26}$$

Proof: See Appendix 4. □

Lemma 6: Let $f_g(x)$ and $F_g(x)$ denote the PDF and the CDF of the data channel gain between the SU-Tx and the SU-Rx with the ratio AS strategy, respectively. $f_g(x)$ and $F_g(x)$ are given as

$$f_g(x) = Mx^{M-1}\lambda_s^M \Gamma(2-M, \lambda_s x), \quad x \geq 0 \tag{27}$$

$$F_g(x) = \gamma(2, \lambda_s x) + (\lambda_s x)^M \Gamma(2-M, \lambda_s x), \quad x \geq 0 \tag{28}$$

where $\gamma(\cdot,\cdot)$ and $\Gamma(\cdot,\cdot)$ are the lower and upper incomplete gamma functions given by [26, eq. (8.350.1)] and [26, eq. (8.350.2)].

Proof: See Appendix 5 □

Lemma 7: The PDF and CDF of the interference channel gain between the SU-Tx and the PU-Rx with the ratio AS strategy, h_s, denoted by $f_h(y)$ and $F_h(y)$, respectively, are given as

$$F_h(y) = M(M-1)\sum_{i=0}^{M-2} \binom{M-2}{i} \frac{1}{2+i} (-1)^i \left[\left(\lambda_p y\right)^{2+i} \Gamma\left(-1-i, \lambda_p y\right) - e^{-\lambda_p y} + 1 \right] \tag{29}$$

$$f_h(y) = M(M-1)\sum_{i=0}^{M-2} \binom{M-2}{i} (-1)^i \lambda_p^{2+i} y^{1+i} \Gamma(-1-i, \lambda_p y), \ y \geq 0 \tag{30}$$

Proof: The proof for the results is similar to the derivation given in Appendix 5.

According to (10) and (25), the closed-form expression for the average throughput is given as

$$R(\tau, P_s, s) = A\left\{ \sum_{k=1}^{M} \binom{M}{k} (-1)^{k+1} \frac{\lambda_p^k {}_2F_1\left(k, k, k+1, -\lambda_p/\lambda_s B\right)}{(\lambda_s B)^k k} \right\} \tag{31a}$$

$$A = \frac{(T-\tau)\left[p_0(1-p_f(\tau)) + p_1(1-p_d)\right]}{T\ln 2} \tag{31b}$$

$$B = \frac{N_0 u p_1 (1-p_d)\ln 2}{\left[p_0(1-p_f(\tau)) + p_1(1-p_d)\right]} \tag{31c}$$

where $F_1(a, b, c, d)$ denotes the hypergeometric function given by [26, eq. (9.14.2)].

Proof: See Appendix 6. □

The average throughputs achieved by using those three AS strategies are derived. Equation (18a), (24) and (31a) show that the average throughput increases with the number of transmitted antennas, which proves that multiple antennas technologies can improve the average throughput because of the diversity gain. It is also seen that the sensing parameters have an effect on the achievable average throughput. Thus, the optimal sensing parameters are designed in the following section to further improve the average throughput of the CR.

5 Optimal sensing parameters design

The optimal power allocation strategy is proposed to improve the achievable average throughput of the CR. As sated before, Sensing parameters of spectrum sensing have an effect on the average throughput. Thus, the designs of the optimal sensing parameters are of importance.

Equation (18a), (24) and (31a) show that the achievable throughput increases with the target detection probability. It can be explained by the fact that the interference imposed on the PU can be decreased with the increase in the target detection probability. Thus, the transmitted power of the SU can be improved and the average throughput increases.

Since the probability of detection is the prescribe target, the other sensing parameter that can be designed is the sensing time. It is seen that the interference time can be decreases with the increase in the sensing time because of the mis-detection of the PU. Meantime, the increase in the sensing time can improve the sensing performance and thus can better protect the PU. However, the transmitted time of the SU decreases when the sensing time increases for a given frame time, T. As stated before, the optimal sensing time can be obtained by using the exhaustive search method because of it within in the interval $(0, T)$.

Finally, the scheme that obtains the optimal sensing time τ_{opt} and optimal power allocation strategy for the MISO CR system with the opportunistic spectrum access model is given as in Fig. 3.

6 Simulation evaluations

In this section, simulation results are given to confirm the high accuracy of the theoretical analysis. We also give numerical results to illustrate the proposed optimal power allocation strategy for the MISO CR system. In all simulations, it is assumed that $\lambda_s = \lambda_p = 1$. The channel is assumed to be block fading. The sampling frequency is set to be 6 MHz. The frame duration of the MISO CR system is fixed and set to be $T = 100$ ms. The noise variance

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Scheme: Obtain the optimal sensing time and optimal power allocation strategy for MISO CR system with the opportunistic spectrum access model.

⊵ For $\tau = 0 : T$

1) Initialize u.

2) Repeat:

– calculate P_s by using Eq. (8),

– update u by using subgradient-based method,

3) Until u converge.

⊵ End.

4) optimal sensing time and optimal power allocation: $\tau_{opt} = \arg\max R(\tau, P_s, s)$, $[P_s]_{\tau=\tau_{opt}}$.

Fig. 3 *Scheme: Obtain the optimal sensing time and optimal power allocation strategy for MISO CR system with the opportunistic spectrum access model*

equals to unit, $N_0 = 1$ and the SNR related to the PU signal for spectrum sensing is $\gamma = -20$ dB. The probability that the PU is absent is set to be $p_0 = 0.8$. The number of the SU transmitted antennas is set several values.

Figs. 4–7 illustrate that the empirical CDFs match very well with the theoretical results. In these figures, simulation results are achieved by using 10^7 Monte Carlo simulations when different numbers of transmitted antennas are set. Fig. 4 shows the theoretical results and the empirical CDFs of the data channel gains between the SU-Tx and the SU-Rx with the maximum channel gain AS strategy. It is seen that the data channel gain increases with the number of the transmitted antennas. Fig. 5 shows the theoretical results and the empirical CDFs of the interference channel gains between the SU-Tx and the PU-Rx with the minimum interference channel gain AS strategy. It is clear that the interference channel gain decreases with the increase in the number of the transmitted antennas. The theoretical results and the empirical CDFs of the data channel gains between the SU-Tx and the PU-Rx with the ratio AS strategy are given in Fig. 6, whereas Fig. 7 shows the corresponding theoretical results and the empirical CDFs of the interference channel gain between the

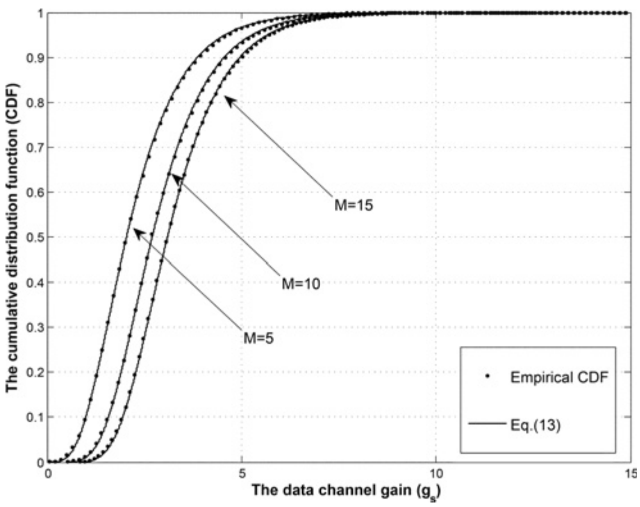

Fig. 4 *Theoretical CDFs and the empirical CDFs of the channel gains between the SU-Tx and the SU-Rx for different numbers of transmitted antennas with the maximum channel gain AS strategy*

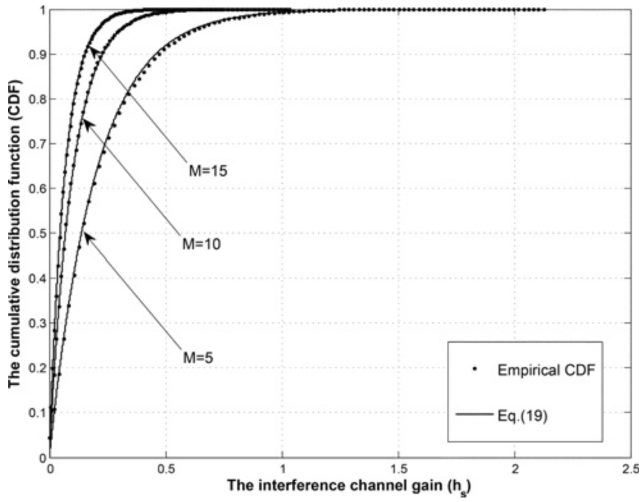

Fig. 5 *Theoretical CDFs and the empirical CDFs of the interference channel gains between the SU-Tx and the PU-Rx for different numbers of transmitted antennas with the minimum interference channel gain AS strategy*

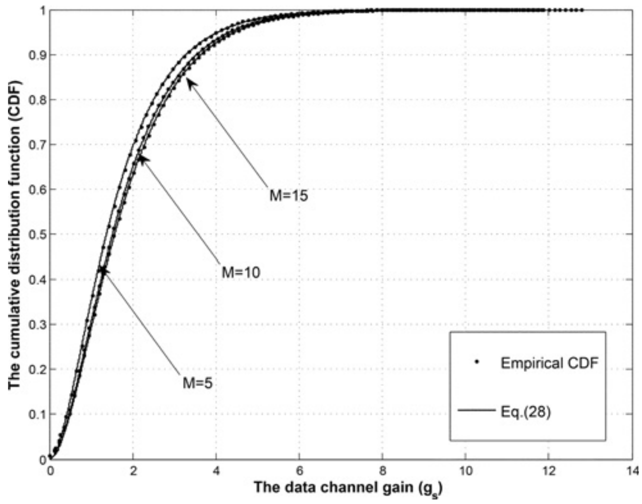

Fig. 6 *Theoretical CDFs and the empirical CDFs of the channel gains between the SU-Tx and the SU-Rx for different numbers of transmitted antennas with the ratio AS strategy*

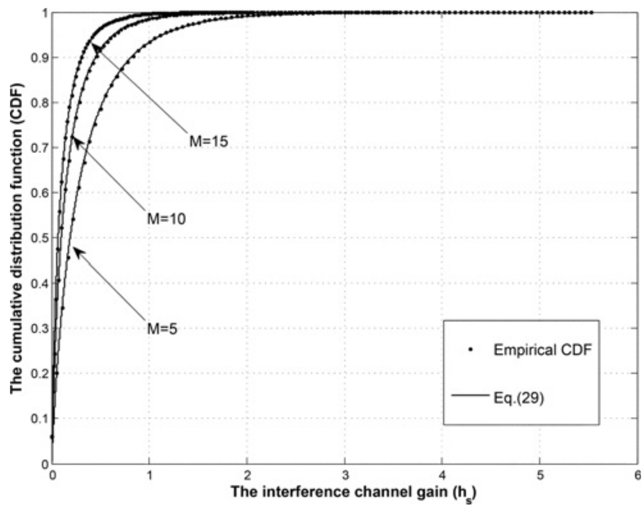

Fig. 7 *Theoretical CDFs and the empirical CDFs of the interference channel gains between the SU-Tx and the PU-Rx for different numbers of transmitted antennas with the ratio AS strategy*

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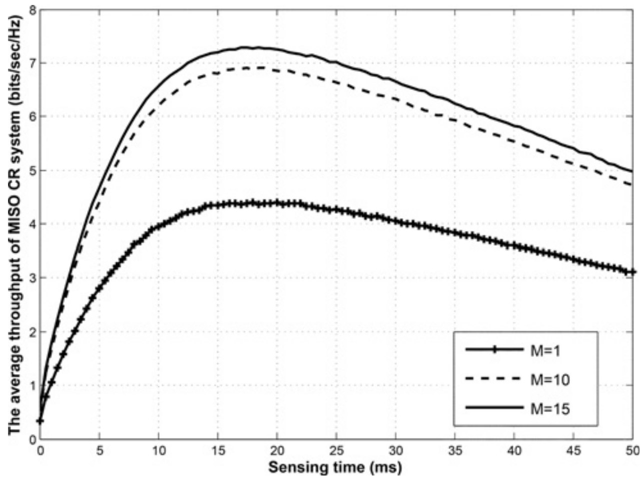

Fig. 8 *Achievable average throughput against the sensing time for different numbers of transmitted antennas with the ratio AS strategy*

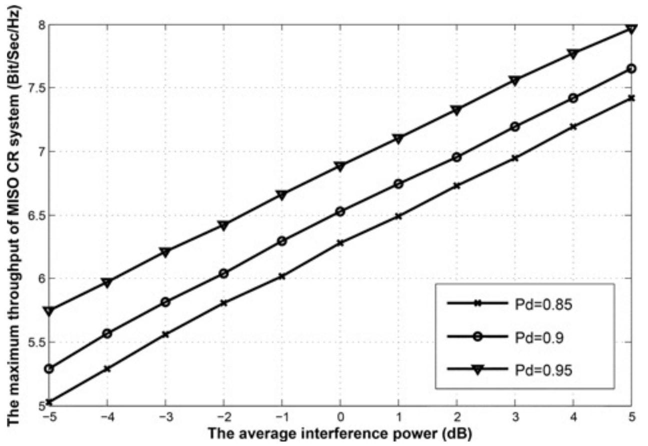

Fig. 10 *Achievable maximum average throughput against the average interference power for different detection probabilities with the ratio AS strategy*

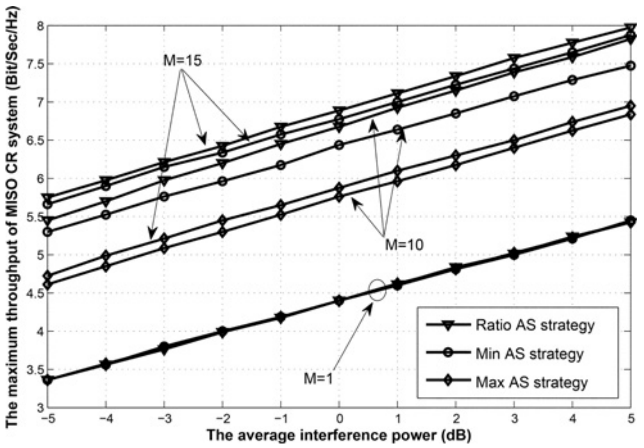

Fig. 9 *Maximum average throughput against the average interference power for different numbers of transmitted antennas and different transmitted AS strategies*

SU-Tx and the PU-Rx. It is seen that the empirical results match very well with the theoretical results.

Fig. 8 shows the achievable average throughput of the MISO CR system against the sensing time τ with the ratio transmitted AS strategy when different numbers of the transmitted antennas are set. In all simulations, the target detection probability and the average interference power are set as $p_d = 0.95$ and $\Gamma_{av} = 0$ dB, respectively. It is observed that there exists an optimal sensing time for the MISO CR system. In addition, the optimal sensing slightly increases with the number of the transmitted antennas. The reason is that the ratio of the channel gain g_s to the interference channel gain h_s increases with the number of the transmitted antennas and thus the sensing time is able to increase to improve the performance of spectrum sensing and better protect the PU. In other words, multiple antenna techniques have the ability of combating the interference constraint and improving the performance of the CR system.

Fig. 9 shows the achievable maximum average throughputs against the average interference power constraint with different numbers of transmitted antennas and different transmitted AS strategies. The target detection probability is set to be $p_d = 0.95$. Fig. 9 illustrates that the ratio AS strategy is the optimal AS strategy based on the optimal power allocation strategy. It is also seen that the maximum average throughput is improved when the constraint on the average interference power becomes loose. The reason is that the transmitted power can be improved when the constraint on the average interference power becomes loose. Furthermore, it is seen that the average throughput achieved by using the minimum interference channel gain AS strategy is larger than that achieved by using the maximum channel gain AS strategy. It can be explained by the fact that the minimum interference channel gain AS strategy chooses the transmitted antenna with the minimum interference channel gain and thus the transmitted power can be improved, whereas the maximum channel gain AS strategy has no effect on the interference channel gain and thus the improvement in average throughput is dependent on the diversity gain.

The achievable maximum average throughputs against the average interference power for different detection probabilities are given in Fig. 10. The ratio AS strategy is used in the simulation. It is seen that the achievable maximum average throughput benefits from the performance of spectrum sensing. It is explained by the fact that the PU is better protected when the performance of spectrum sensing is higher. Therefore the SU is able to increase the transmitted power and achieve higher throughput.

7 Conclusion

Multiple antenna techniques and transmitted AS techniques were exploited to combat the interference constraint imposed on the SU. The optimal power allocation strategy was proposed under the average interference power constraint. The CDFs and the PDFs of the data channel gains and the interference channel gains with the three AS strategies were derived. The theoretical analyses for the achievable average throughputs with those three AS strategies were outlined. It was proved that the optimal transmitted AS strategy is the ratio AS strategy with the optimal power allocation strategy. The optimal sensing time was designed to further improve the average throughput for the MISO CR with the opportunistic spectrum access model. Numerical simulation results show that the optimal sensing time for the MISO CR slightly increases with the number of the transmitted antennas.

8 Acknowledgment

The research reported in this article was supported by the Natural Science Foundation of China (grant nos. 61072070, 61301179 and 61102058).

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10 Appendix

10.1 Appendix 1: Proof of Lemma 1

The channel gains, $g_1, g_2, \ldots, g_M$, are independent. Since $G_{\mathrm{MAX}} = \max\{g_1, g_2, \ldots, g_M\}$, one has

$$\begin{aligned} F_{G_{\mathrm{MAX}}}(x) &= \Pr(G_{\mathrm{MAX}} \le x) \\ &= \Pr(g_1 \le x, g_2 \le x, \ldots, g_M \le x) \\ &= \prod_{i=1}^{M} F_{g_i}(x) \\ &= (1 - e^{-\lambda_s x})^M, \quad x \ge 0 \end{aligned} \tag{32}$$

The PDF of G_{MAX} is the derivation of $F_{G_{\mathrm{MAX}}}(x)$.

10.2 Appendix 2: Proof of Lemma 3

The interference channel gains, $h_1, h_2, \ldots, h_M$, are independent. Since $Y = \min(h_1, h_2, \ldots, h_M)$, one has

$$\begin{aligned} F_Y(y) &= \Pr(Y \le y) \\ &= 1 - \Pr\{\min\{h_1, h_2, \ldots, h_M\} > y\} \\ &= 1 - \Pr\{h_1 > y, h_2 > y, \ldots, h_M > y\} \\ &= 1 - \left(1 - \Pr\{h_i \le y\}\right)^M \\ &= 1 - e^{-M\lambda_P y} \end{aligned} \tag{33}$$

$f_Y(y)$ is the derivation of $F_Y(y)$.

10.3 Appendix 3

Based on (10) and (22), the achievable average throughput can be derived as

$$\begin{aligned} R(\tau, P_s, s) &= \frac{a}{\ln 2}\mathbb{E}\left[\ln\left(\frac{a}{N_0 ub \ln 2}\frac{g_s}{h_s}\right)\right] \\ &= \frac{a}{\ln 2}\int_{z \ge N_0 ub \ln 2/a} \ln\left(\frac{a}{N_0 ub \ln 2} z\right)\frac{M\lambda_s\lambda_P}{(\lambda_s z + M\lambda_P)^2}\mathrm{d}z \\ &= \frac{a}{\ln 2}\left[\ln\left(\frac{a}{N_0 ub \ln 2}\right)\frac{aM\lambda_P}{(\lambda_s ub N_0 \ln 2 + Ma\lambda_P)}\right. \\ &\quad \left. + \int_{z \ge N_0 ub \ln 2/a} \ln(z)\frac{M\lambda_s\lambda_P}{(\lambda_s z + M\lambda_P)^2}\mathrm{d}z\right] \\ &= \frac{a}{\ln 2}\ln\left[\frac{(\lambda_s ub N_0 \ln 2 + Ma\lambda_P)}{N_0 ub \ln 2}\right] \end{aligned} \tag{34}$$

10.4 Appendix 4: Proof of Lemma 5

According to Kang *et al.* [17], for Rayleigh fading, the PDF for the ratio of channel gains, $Z_i = g_i/h_i$, is given as

$$f_{Z_i}(x) = \frac{\lambda_s\lambda_p}{\left(\lambda_s x + \lambda_p\right)^2}, \quad x \ge 0 \tag{35}$$

where $f_{Z_i}(x)$ denotes the PDF of Z_i, $i = 1, 2, \ldots, M$. Thus, the derivation for the CDF of Z

$$Z = \arg\max_i\left\{\frac{g_1}{h_1}, \frac{g_2}{h_2}, \ldots, \frac{g_i}{h_i}, \ldots, \frac{g_M}{h_M}\right\}$$

is given as

$$\begin{aligned} F_Z(z) &= \Pr(Z \le z) \\ &= \Pr\left(\frac{g_1}{h_1} \le z, \frac{g_2}{h_2} \le z, \ldots, \frac{g_M}{h_M} \le z\right) \\ &= \prod_{i=1}^{M}\int_0^z \frac{\lambda_s\lambda_p}{(\lambda_s y + \lambda_p)^2}\mathrm{d}y \\ &= \left(\frac{\lambda_s z}{\lambda_s z + \lambda_p}\right)^M \end{aligned} \tag{36}$$

The PDF of Z is the derivation of $F_Z(z)$.

10.5 Appendix 5: Proof of Lemma 6

Let $X_i = g_i$, $Y_i = h_i$. $f_g(x)$ is given by

$$f_g(x) = \int_0^\infty f_{X_i/Z_i=z}(x|z) f_Z(z)\mathrm{d}z \tag{37}$$

where $f_{X_i/Z_i=z}(x|z)$ is the conditional PDF, which is given based on the Bayes rule as

$$f_{X_i/Z_i=z}(x|z) = \frac{f_{X_i,Z_i}(x, z)}{f_{Z_i}(z)} \tag{38}$$

where $f_{X_i,Z_i}(x, z)$ is the joint PDF of X_i and Z_i. To obtain the joint PDF, the corresponding joint CDF of X_i and Z_i is given as

$$F_{X_i,Z_i}(x, z) = \int_0^\infty \Pr(X_i \le \min(x, yz)|y) f_{h_i}(y)\mathrm{d}y \tag{39}$$

where $f_{h_i}(y)$ is the PDF of h_i given in (16). According to (39), the joint CDF and PDF of X_i and Z_i are given as

$$F_{X_i,Z_i}(x, z) = \left(\frac{z}{z + \lambda_p/\lambda_s}\right)\left[1 - e^{-\left(\lambda_s + \lambda_p/z\right)x}\right], \quad x \ge 0, \ z \ge 0 \tag{40}$$

$$f_{X_i,Z_i}(x, z) = \frac{\lambda_p \lambda_s x}{z^2} e^{-\left(\lambda_s + \lambda_p/z\right)x}, \quad x \ge 0, \ z \ge 0 \tag{41}$$

Thus, based on (35), (38) and (41), the conditional PDF, $f_{X_i/Z_i=z}(x|z)$ is given as

$$f_{X_i/Z_i=z}(x|z) = \left(\frac{\lambda_s x + \lambda_p}{z}\right)^2 x e^{-\left(\lambda_s + \lambda_p/z\right)x} \tag{42}$$

Based on (25), (37) and (42), the CDF and PDF of g_s with the ratio AS strategy can be obtained.

10.6 Appendix 6

Substituting (31b) and (31c) into (10), the achievable average throughput is given as

$$\begin{aligned} R(\tau, P_s, s) &= A\mathbb{E}\left\{\ln\left(\frac{1}{B}\frac{g_s}{h_s}\right)\right\} \\ &= A\int_B^\infty \ln\frac{z}{B} f_Z(z)\mathrm{d}z \\ &= A\left\{\int_B^\infty \ln z f_Z(z)\mathrm{d}z - \ln B\left[1 - \left(\frac{\lambda_s B}{\lambda_s B + \lambda_p}\right)^M\right]\right\} \end{aligned} \tag{43}$$

where the integral form can be obtained as

$$\int_B^\infty \ln z f_Z(z)\mathrm{d}z = \ln B\,(1 - F_Z(B)) + \int_B^\infty \frac{1}{z}[1 - F_Z(z)]\mathrm{d}z \tag{44}$$

and the following results can be obtained

$$\begin{aligned} \int_B^\infty \frac{1}{z}[1 - F_Z(z)]\mathrm{d}z &= \int_B^\infty \frac{1}{z}\left[1 - \left(\frac{\lambda_s z}{\lambda_s z + \lambda_p}\right)^M\right]\mathrm{d}z \\ &= \int_B^\infty \frac{1}{z}\left[1 - \left(1 - \frac{\lambda_p}{\lambda_s z + \lambda_p}\right)^M\right]\mathrm{d}z \\ &= \sum_{k=1}^M \binom{M}{k}(-1)^{k+1}\int_b^\infty \frac{1}{z}\left[\left(\frac{\lambda_p}{\lambda_s z + \lambda_p}\right)^k\right]\mathrm{d}z \end{aligned} \tag{45}$$

According to [26, eq. (3.194.2)], one has

$$\begin{aligned} \int_B^\infty \frac{1}{z}[1 - F_Z(z)]\mathrm{d}z = \sum_{k=1}^M \binom{M}{k}(-1)^{k+1} \\ \times \frac{\lambda_p^k\, {}_2F_1\left(k, k, k+1, -\lambda_p/\lambda_s b\right)}{(\lambda_s b)^k k} \end{aligned} \tag{46}$$

After substituting (44) and (46) into (43), those results given in (31) can be obtained. The proof is finished. □