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Statistical analysis of H.264 video frame size distribution

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Abstract: H.264 video traffic is expected to account for the majority of multimedia traffic to be carried in future heterogeneous networks. Modelling video frame sizes is highly useful in simulation studies, mathematical analysis and generating synthetic video traces for the purpose of testing and compliance. In this study, a statistical analysis is performed to determine an appropriate distribution of video frame sizes generated by the popular H.264 video codec. The study makes use of a number of real video traces with the goal of evaluating and fitting their frame sizes with well-known distributions. In the literature, it is reported that the Gamma and Weibull distributions give the best fit for frame sizes in the most popular video codecs including H.264. Our statistical analysis shows that both Gamma and Weibull distributions are very close to each other in terms of goodness-of-fit results and they give the best fit. The authors also show that the Inverse Gaussian distribution is ranked second after Gamma and Weibull distributions. Finally, they show that the distributions of Pearson Type V and Lognormal are ranked third and fourth in terms of goodness-of-fit.

1 Introduction

Networks are moving towards the use of Internet Protocol technology for integrated voice, data and even video services. Multimedia applications and services have already possessed a major portion of today's traffic over computer and mobile communication networks. Among the various types of multimedia, video services (transmission of moving images and sound) are proven dominant in present and future broadband networks.

Modelling video traffic can be highly useful in mathematical analysis, simulation and in generating synthetic video traces for the purpose of performance, testing and compliance. In addition, traffic models can be used in several practical purposes including allocation of network resources, design of efficient networks in streaming services and delivery of certain quality of service (QoS) guarantees to end users.

Video traffic is produced by imaging devices in frames that contain both the audio and picture portions of video traffic. The sizes of the video frames vary both between and within video formats. After the advent of video coding, two main encoding schemes have been proposed and are still used: the constant bit rate (CBR) and the variable bit rate (VBR) modes. In the CBR mode the quantisation parameters are maintained constant for the encoding process. So, the deduced video quality is almost steadily sustained but the derived encoding bit rate fluctuates around a mean value. On the contrary, in the VBR mode, a rate-control algorithm dynamically alters the quantisation parameters according to the frame complexity to achieve the required target bit rate.

The most popular and widely used encoding algorithms are the ones developed by the Moving Picture Experts Group (MPEG) and the Video Coding Expert Group of the International Telecommunication Union. In 2003, these two organisations jointly developed a new codec, the H.264. Most new videoconferencing products now include the H.264 compression standard, and the older H.263 and H.261 standards. Although H.263 is primarily used in videoconferencing and H.261 is an Integrated Services Digital Network standard developed in 1990, H.264 supports video compression (coding) in videoconferencing and video-telephony applications.

With the new compression tools, the H.264 codec typically compresses video down to roughly half the average bit rate of the MPEG-4 Part 2 codec [1, 2]. The pervasive use of H.264/advanced video coding (AVC) and H.264 scalable video coding (SVC) in compression of networked videos is promoted by the widespread adoption of these encoding standards in DVB, ATSC, 3GPP, 3GPP2, Media FLO, DMB, DVD Forum (HD-DVD) and Blu-Ray Disc Association (BD-ROM). Owing to the advances in H.264, it is expected that it will prevail in future networks and mobile application systems, making traffic modelling and characterisation of H.264 video streams a useful tool for network managers and designers.

Among the various characteristics of video traffic, there are specific major interests including the distribution of frame sizes, the autocorrelation function (ACF) that captures the dependencies between frame sizes in VBR traffic and the self-similarity or long-range dependence (LRD) that describes the bursty (highly variable) of the video traffic over a wide range of timescales. Regarding ACF and LRD, several studies have been done to model the video traffic based on the ACF and to examine the existence of the LRD in such traffic [3–5]. Some other studies have focused on the evaluation of the impact of LRD on the performance of transferring the video traffic over the network [6, 7]. However, our study specifically focuses on analysing the H.264 video frame size distributions as a major characteristic of the video traffic.

This paper studies the video streams generated from an H.264 codec at the frame level. In particular, the work focuses on video traffic that is generated by VBR H.264 coding since it offers relatively constant quality and has less bandwidth and storage capacity requirements. Our study is based on the available traces in [8] that were initiated in May 2009. The purpose of this paper is to expand the knowledge generated in the literature by fitting several competing statistical distributions with the frame sizes from several video data sets generated by H.264 encoder.

The rest of the paper is structured as follows. Section 2 introduces the related work. In Section 3, we describe the methodology followed to evaluate the frame sizes. We also describe different ways to characterise encoded video and determine the dedicated distributions and video traces used in this paper. In Section 4, we present detailed results, analysis and modelling assessments. We provide our conclusions in Section 5.

2 Related work

The problem of modelling video traffic, in general, and videoconferencing, in particular, has been extensively studied in the literature. Heyman *et al.* [9, 10] and Xu and Huang [11] show that H.261 videoconference sequences generated by different hardware coders, using different coding algorithms, have Gamma marginal distributions and they used this result to build a discrete autoregressive (DAR) model of order one, which works well when several sources are multiplexed. Krunz and Hughes [12] modelled the frame sizes in MPEG-2 streaming (also known as H.262). In their research, a Lognormal distribution was found to best fit the frame sizes.

Fitzek and Reisslein [13] have presented an extensive public available library of frame size traces of MPEG-4, H.263 and H.263+ encoded video along with a detailed statistical analysis of the generated traces. In the same study, the use of movies as visual content lead to frame generation with a Gamma-like frame-size sequence histogram.

Lazaris *et al.* [14] have used four different long sequences of MPEG-4 encoded videos and they showed that the use of the Gamma and Lognormal distributions is not the most appropriate in MPEG-4 videoconference traffic. They showed that, for modelling single videoconference sources, the best choice among all the examined distributions is the Pearson Type V distribution.

Poon and Lo [15] have proposed using a normal mixture distribution as a method for fitting the sample histogram generated from H.261 and H.263 encoded videos. It has been proved that it performs better than the simple Gamma and Lognormal distributions. Ryu [16] has proposed using the Weibull instead of the Gamma density for the fit of the sample histogram in a model of videoconference traffic encoded by the ViC Intra-H261 encoder.

Koumaras *et al.* [17] have derived the density functions of the H.264 frame sizes and it has been shown that the sizes can be successfully represented by Gamma distributions. However, they have not presented the detailed results for the Gamma distribution such as the Kolmogorov–Smirnov (K–S) test, the Anderson–Darling (A–D) test or the chi-square goodness-of-fit test, nor the comparison graphs like quintile–quintile (Q–Q) plot to show the goodness-of-fit for the Gamma distribution. Furthermore, they have used only one film, ‘Spider Man II’, to generate frame size using the H.264 reference encoder to assist their findings.

Auwera *et al.* [3] and Dai *et al.* [4] have developed a modelling framework that is able to capture the ACF structure of H.264 and MPEG-4 VBR video traffic considering the co-existence of both LRD and short-range dependent (SRD) properties in the structure of single-layer and multi-layer video traffic. They used a wavelet transforms to model the distribution of I-frame sizes based on Gamma distribution and a time-domain linear model of P/B frame sizes based on intra-group of pictures (GOP) correlation. Other characteristics have been studied by Dai and Loguinov [18] including the bit rate distortion performance, bit rate variability and LRD of the H.264 codec. They found several distinct characteristics that distinguish the H.264 codec from the MPEG-4 one as stated in their work [18]. However, our study focuses on another major characteristic of the video traffic that is the distribution of frame sizes by evaluating several distributions that could be used to model the frame sizes.

Lazarisa and Koutsakisb [5] have investigated the possibility of modelling multiplexed traffic from H.264 videoconference streams with quite a few well-known distributions including Pearson Type V, exponential, Gamma, Lognormal and Weibull. They have concluded that the best fit among these distributions is the Pearson Type V. However, the study has focused on investigating the videoconference traffic; thus they have only examined two traces of low or moderate motion (Sony Demo and NBC News).

To the best of our knowledge, the subject of modelling H.264 video traffic has been addressed in the literature only in [5, 17, 19]. Koumaras *et al.* [17] have studied the density function in the video traffic generated from H.264 using only one trace that is ‘Spider Man II’ and the only examined distribution is the Gamma distribution. If other films were studied using common probability distributions, the results might have been quite different. On the other hand, Domoxoudis *et al.* [19] presented measurement and modelling results of H.264 encoded traces. They concluded that H.264 traffic can be reasonably represented by a D/G/∞ queue with deterministic arrivals according to the video frame rates and service times that can be fitted by a histogram-based model using a Gamma distribution. However, the study considered only the videoconference traffic generated by the H.264 encoder included in the videoconference software tools: VCON Vpoint HD [20] and Polycom PVX [21]. In addition, the contribution in our paper is a clear extension of [5, 17, 19]. Our research work presented in this paper examines more than one specific distribution (including Gamma, Weibull, Inverse Gaussian, Lognormal, LogLogistic and Pearson Type V distributions) done on more than one type of video traces (i.e. most of the available traces in [8]).

3 Methodology

Our study focuses on modelling video frames sizes generated by H.264 encoders. We use the video traces as a data source to be evaluated. In the literature, there are many video traces generated by H.264. We decided to select four traces that are available online at [8]. These four traces include three movies and one news channel: Star Wars IV, Silence of the Lambs, NBC news and Matrix III videos. The Star Wars IV movie, described as science fiction/action, was used popularly in the literature [3, 13, 18, 22, 23] to characterise the properties of the video traffic generated by MPEG-4 and/or H.264 encoders. The Silence of the Lambs movie, which is a drama/thriller, was used in [3, 18, 24] to study the performance of single-layer video traffic of the H.264 and MPEg-4 encoders. NBC News, described as news video, was also used in [3, 5, 24].

We follow the recommended approach by Law and Kelton [25] for fitting statistical distributions to a given data set that starts with hypothesising families of distributions that appear to be appropriate on the basis of their shapes. Then, for each distribution type, a first guess of parameters is made using maximum-likelihood estimator (MLE). Then, the fit is optimised using the Levenberg–Marquardt method (described in Section 4). Finally, the goodness-of-fit is measured in the optimised function to decide how well the sample data fit a hypothesised probability density function. This approach has been implemented using the MATLAB programming language in data analysis and to estimate parameters and to create graphs to visualise the data and distributional fits.

We investigate a variety of distributions including those that have been used often in video traffic modelling in the literature, like Gamma, Weibull, Inverse Gaussian, Exponential, Pareto, Lognormal, Pearson Type V and LogLogistic distributions. Table 1 presents the set of all distributions used in this study. The parameters for these distributions are estimated initially using the MLE. In fact, there are many other ways to specify the form of an estimator for a particular parameter of a given distribution [26]. However, we used the MLE for the following three reasons. First, MLE has several desirable properties not enjoyed by alternative methods of estimation, for example, method-of-moment (MOM). Second, the use of MLE turns out to be important in justifying the chi-square goodness-of-fit test. Third, the central idea of the MLE has a strong intuitive appeal as it is stated by Law and Kelton [25].

To test the goodness-of-fit, we used the common three tests discussed in the literature on data fitting [27, 28], which are the chi-square test, the K–S test and the A–D test. The chi-square test is the most common goodness-of-fit test. It can be used with any type of input data (raw sample data or frequency data) and any type of distribution (discrete or continuous). A weakness of the chi-square test is that there are no clear guidelines in selecting intervals that is, the numbers of classes. In some situations a different conclusion can be reached from the same data depending on how many intervals were specified. In contrast, K–S test does not depend on the number of intervals (which makes it more powerful than the chi-square test). The K–S can be used with any type of input data but cannot be used with discrete distribution functions. A weakness of the test is that it does not detect tail discrepancies very well since it gives the same weight to the difference between the actual data and the fitted distribution of all values of data. However, the A–D test is very similar to the K–S test but places more emphasis on tail values by highlighting the differences between the tails of the fitted distribution and requires the input data based on a weighted average of the squared difference between the observed and expected cumulative densities.

Another way of interpreting the results from the fitting process is to visually assess how well a distribution agrees with the input data. The statistical test was made with the use of Q–Q graphs, which is a powerful graphical goodness-of-fit procedure [9, 25]. The Q–Q is used to plot the percentile values of the fitted distribution against percentile values of input data. If the fit is good the points of the plot should fall approximately along a 45° reference line. In additional to the Q–Q plot, the probability–probability (P–P) plot is used in further assessment as it plots the empirical distribution of the input data against the fitted distribution. If the fit is good, the P–P plot will be nearly linear.

Table 1 Distributions used for fitting the frame size

Beta	Logistic	Pearson Type V
Erf	Log-Logistic	Rayleigh
Exponential	Lognormal	Student's t
Extreme Value	Normal	Triangular
Gamma	Pareto	Uniform
Inverse Gaussian	Pearson Type V	Weibull

4 Results of the analysis and modelling assessment

We begin our study by investigating a set of well-known distributions listed in Table 1. With these distributions, we have investigated the possibility of modelling the frame size generated by H.264 in the four chosen video traces described above. The initial results showed that among these distributions, only six distributions (Gamma, Weibull, LogLogistic, Pearson Type V, Inverse Gaussian and Lognormal) were valid to model all given traces, and they occupied the first six ranks among others according to the goodness-of-fit results. The Exponential and Pareto2 distributions were only valid fits in the Star Wars IV trace. Thus, in our study, we chose the first six distributions as fitting candidates in order to compare their results in the case of H.264 video streams.

After determining the candidate distributions, the values of their parameters were estimated from the given observational data using the MLE. As defined by Law and Kelton [25], the MLE of a distribution are the parameters of that function that maximise the likelihood of the distribution given a set of observational data. Given a set of observational data x, and a probability density function (PDF) f, the likelihood function is

$$l = \prod_{i=1}^{n} f(x_i, \text{parameters})$$

MLE tends to determine the values of the parameters that maximise the function l. We implemented the MLE using MATLAB programming language.

The Gamma distribution has two parameters, which are the shape parameter α (>0) and the scale parameter β (>0). Its density function is given by

$$f(x, \alpha, \beta) = \frac{e^{-x/\beta} x^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}}, \quad x > 0, \ \alpha > 0, \ \beta > 0$$

The MLE for estimating the parameters of the Gamma distribution can be obtained by satisfying the following two

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equations that are solved numerically by Newton's method

$$\hat{\beta} = \frac{\bar{X}}{\hat{\alpha}}$$

$$\ln \hat{\beta} + \psi(\hat{\alpha}) = \frac{\sum_{i=1}^{n} \ln X_i}{n}$$

where $\psi(\hat{\alpha})$ is the digamma function.

In the Weibull distribution with a shape parameter of α (>0) and a scale parameter of β (>0), the density function, f, and the corresponding parameters derived from the MLE are given by the following equations

$$f(x, \alpha, \beta) = \alpha\beta^{-\alpha}x^{\alpha-1}\mathrm{e}^{-(x/\beta)^{\alpha}}, \quad x > 0,\ \alpha > 0,\ \beta > 0$$

$$\frac{\sum_{i=1}^{n} X_i^{\hat{\alpha}} \ln X_i}{\sum_{i=1}^{n} X_i^{\hat{\alpha}}} - \frac{1}{\hat{\alpha}} = \frac{\sum_{i=1}^{n} \ln X_i}{n}$$

$$\hat{\beta} = \left(\frac{\sum_{i=1}^{n} X_i^{\hat{\alpha}}}{n}\right)^{-\hat{\alpha}}$$

where the first equation of the MLE can be solved for $\hat{\alpha}$ numerically by Newton's method and the second equation then gives $\hat{\beta}$ directly.

The Inverse Gaussian distribution has two parameters, which are the location parameter, μ, and the scale parameter λ. The distribution function, f, and the corresponding parameters derived from the MLE are given by the following equations

$$f(x, \mu, \lambda) = \sqrt{\lambda/2\pi x^3}\,\mathrm{e}^{-(\lambda(x-\mu)^2/2\mu^2 x)}$$

$$\hat{\mu} = \frac{\sum_{i=1}^{n} X_i}{n}, \qquad \frac{1}{\hat{\lambda}} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{X_i} - \frac{1}{\hat{\mu}}\right)$$

where X_i is the ith sample in the observational data.

The PDF of a Lognormal distribution is

$$f(x, \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\mathrm{e}^{-((\ln x-\mu)^2)/2\sigma^2}$$

where μ and σ are the scale and the shape parameters. The MLE parameters of the Lognormal distribution are

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \ln X_i}{n}, \qquad \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (\ln X_i - \hat{\mu})^2}{n}}$$

In the Pearson Type V, the density function is

$$f(x, \alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)}\frac{\mathrm{e}^{-\beta/x}}{(x/\beta)^{\alpha+1}}$$

where α is the shape parameter and β is the scale parameter.

However, there is no closed-form solution for estimating the α using the MLE. Thus, a numerical solution was proposed by Law and Kelton [25] using the Newton's method to approximate α and β as follows

$$\hat{\alpha} = \frac{3 - T + \sqrt{(T-3)^2 + 24T}}{12T}$$

$$T = \ln\left(n^{-1}\sum_{i=1}^{n} x_i\right) - n^{-1}\sum_{i=1}^{n} \ln(x_i)$$

$$\hat{\beta} = \frac{\hat{\alpha}}{\bar{X}(n)}$$

where $\bar{X}(n)$ is the mean of the given samples.

The LogLogistic distribution has two parameters, which are the shape parameter α (>0) and the scale parameter

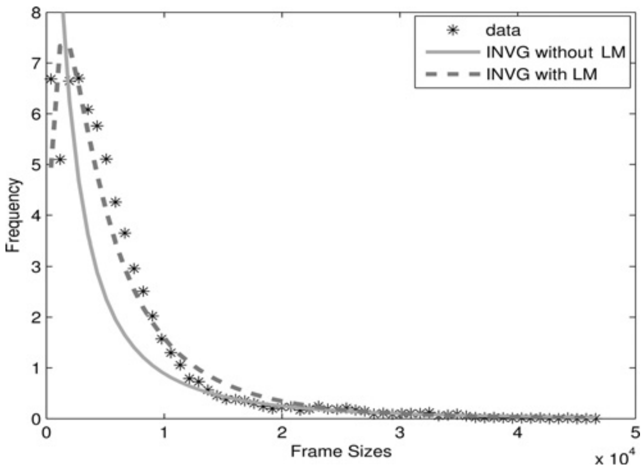

Fig. 1 *LM method optimisation for fitting Star Wars IV with Inverse Gaussian*

Chi-S	K-S	A-D
`E=cdf('lognormal',edges,p_fit);`	`y=cdf('lognormal',data ,p_fit);`	`y=cdf('lognormal',data ,p_fit);`
`Ed=diff(E);`	`K_s=kstest(d, [d y]);`	`ADsum = 0;`
`E=Ed.*N;`		`for jj = 1:N`
`S=histc(d, edges);`		`Index1 = jj;`
`O=S(1:end-1);`		`Index2 = N - jj + 1;`
`x2=sum((E-O).^2./E;`		`Q1 = log(Z(1,Index1));`
		`Q2 = log(1-Z(1,Index2));`
		`ADsum = ADsum + (2*jj-1)*(Q1 + Q2);`
		`end`
		`AD(1,ii) = -N - ADsum/N;`

Fig. 2 *MATLAB code to implement goodness-of-fit tests*

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β (>0). Its density function is given by

$$f(x, \alpha, \beta) = \frac{\alpha(x/\beta)^{\alpha-1}}{\beta[1+(x/\beta)^{\alpha}]^2}, \quad x > 0,\ \alpha > 0,\ \beta > 0$$

The MLE of the LogLogistic distribution can be obtained by solving the following two equations of $\hat{\alpha}$ and $\hat{b}$

$$\sum_{i=1}^{n} [1 + \mathrm{e}^{(\ln x_i - \hat{\alpha})/\hat{b}}] = \frac{n}{2}$$

$$\sum_{i=1}^{n} \left(\frac{\ln x_i - \hat{\alpha}}{\hat{b}}\right) \frac{1 - \mathrm{e}^{(\ln x_i - \hat{\alpha})/\hat{b}}}{1 + \mathrm{e}^{(\ln x_i - \hat{\alpha})/\hat{b}}} = n$$

Table 2 Goodness-of-fit statistics

Trace	Distributions	Rank | value			Distribution parameters	
		Chi-square	A–D	K–S		
Star Wars IV	Gamma	1	2	4	α	1.0485
		2650.8	93.2	0.0365	β	6581.5
	Weibull	2	1	6	α	31 266
		2984.6	80.7	0.04759	β	3.0779
	Inv-Gaussian	3	3	1	μ	7697.53
		5364	105.4	0.03313	λ	7183.09
	LogNormal	4	4	3	μ	7613.97
		5607	113.3	0.03410	σ	9869.69
	Pearson Type V	6	5	2	α	2.22
		8157	164.6	0.03379	β	11 448.25
	LogLogistic	5	6	5	γ	−66.35
		6620	182.1	0.03858	β	4385.39
					α	1.56
Matrix III	Gamma	3	2	5	α	0.95374
		11 491	570.4	0.083677	β	3310.9
	Weibull	1	1	3	α	3046.4
		9042.7	436.1	0.072881	β	0.93221
	Inv-Gaussian	2	3	1	μ	4146.80
		9576	603.2	0.05046	λ	2160.64
	LogNormal	5	5	4	μ	4079.03
		15 628	994.4	0.07373	σ	6355.85
	Pearson Type V	4	4	2	α	1.27
		13 032	706.0	0.05131	β	1985.78
	LogLogistic	6	6	6	γ	−837 729 086.00
		198 917	7102	0.2271	β	837 732 200.56
					α	356 264.91
Silence of the Lambs	Gamma	1	1	2	α	0.82285
		1621.5	467.8	0.065085	β	9092.4
	Weibull	2	2	1	α	7007.2
		1966.3	603.9	0.042265	β	0.87747
	Inv-Gaussian	4	5	4	μ	8089.42
		15 690	684.4	0.08308	λ	4526.19
	LogNormal	3	4	3	μ	9122.01
		13 802	662.0	0.07861	σ	19 643.04
	Pearson Type V	5	3	5	α	1.60
		22 693	660.2	0.09160	β	6931.95
	LogLogistic	6	6	6	γ	−18 46 211 858.90
		92 971	2225	0.1868	β	18 46 217 974.18
					α	445 655.22
NBC news	Gamma	4	4	4	α	9.9594
		6607.2	694.5	0.084221	β	2809.3
	Weibull	5	5	5	α	31 266
		20 616	1067.3	0.099981	β	3.0779
	Inv-Gaussian	1	1	3	μ	20 314.36
		2097	124.1	0.04119	λ	93 306.64
	LogNormal	2	3	2	μ	19 580.58
		2284	134.2	0.04110	σ	9633.06
	Pearson Type V	3	2	1	α	7.37
		2433	125.5	0.03552	β	149 710.00

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Then the MLE of the LogLogistic distribution are $\hat{\alpha} = 1/\hat{\alpha}$ and $\hat{\beta} = \mathrm{e}^{\hat{b}}$.

Once all the parameters associated to the candidate distributions have been estimated using MLE, as described above, the Levenberg–Marquardt (LM) method has been implemented to further optimise the parameters estimated by the MLE. The purpose of such optimisation is to maximise the goodness-of-fit between a data set and a distribution function. Simply stated, the LM method takes a first guess of the parameters of the distribution function (i.e. the MLE is the first guess), and then varies each parameter slightly until it finds a good fit using a least-square criterion for the convergence, as described by Levenberg [29]. To show the improvement resulting from using the LM method, Fig. 1 shows an example of fitting the Star Wars IV video with the Inverse Gaussian distribution, a case without using the LM method and the other one using the LM method. It is evident from the figure, how the LM method optimises the fitting process.

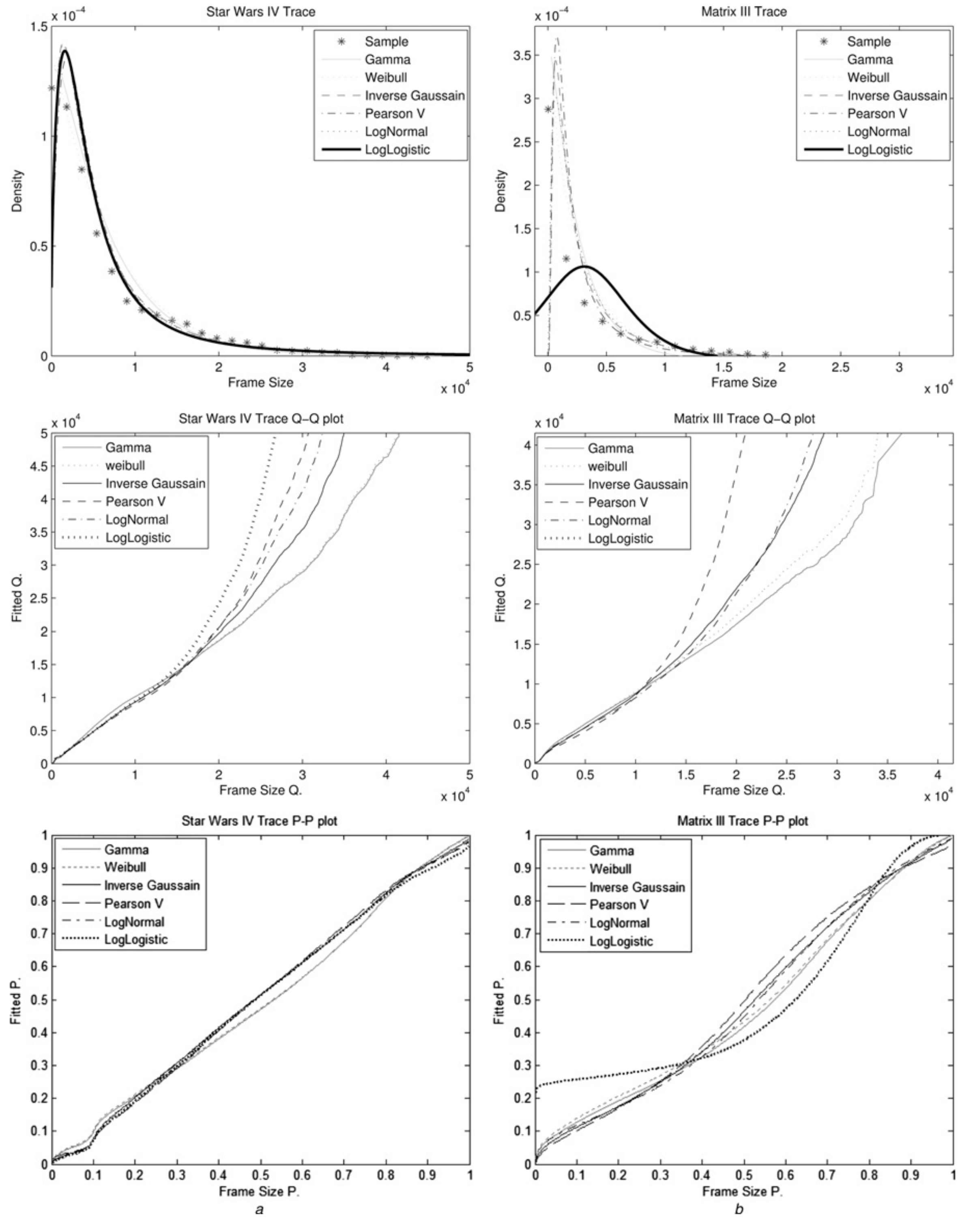

Fig. 3 *Frame-size histograms against fitted distributions along with their respective Q–Q and P–P plots*

a Star Wars IV, H.264
b Matrix III, H.264

We followed the approach described by Law and Kelton [25] to implement the goodness-of-fit tests discussed above in the Methodology section. The goodness-of-fit statistic tells how probable it is that a given distribution function, produces the data set. The goodness-of-fit statistic is normally used in a relative sense by comparing these values with the goodness-of-fit of other distribution functions.

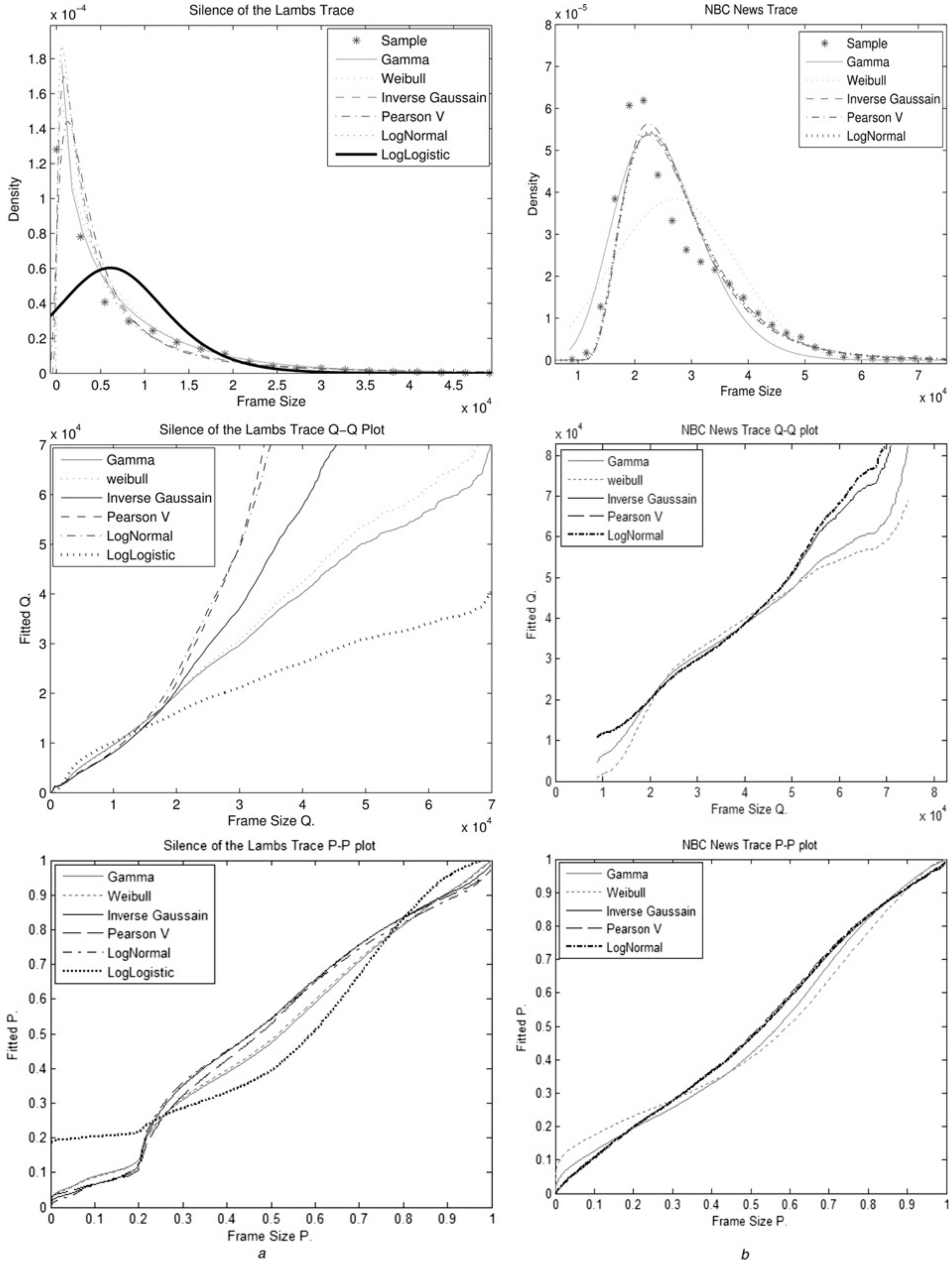

Fig. 4 *Frame-size histograms against fitted distributions along with their respective Q–Q and P–P plots*

a Silence of the Lambs, H.264

b NBC News, H.264

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The implementation of chi-square is straightforward as it is described in [25] as

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed probability value of a given histogram bar and E_i is the theoretical probability that a value will fall with the X range of the histogram bar.

The K–S statistic has been implemented as defined in [25]

$$D_n = \sup[F_n(x) - \hat{F}(x)]$$

where sup of a set of members A is the smallest value that is greater or equal to all members, $\hat{F}(x)$ is the cumulative distribution function of x and $F_n(x) = i/n$ in which i is the number of samples less than x.

The A–D Statistic is defined in by

$$A^2 = n\int_{-\infty}^{+\infty} [F_n(x) - \hat{F}(x)]^2 \psi(x) \hat{f}(x)\,\mathrm{d}x$$

where the weight function $\psi(x) = \{\hat{F}(x)[1 - \hat{F}(x)]\}^{-1}$.

Fig. 2 shows the implementation of these three tests in the MATLAB language. Table 2 shows the statistical results of the implemented goodness-of-fit tests (chi-square, A–D and K–S) along with the rank of the fits for all evaluated traces. The results show that both Gamma and Weibull distributions are very close to each other in terms of goodness-of-fit results and they give the best fit for all the evaluated traces except for the NBC news where the Inverse Gaussian ranks first. The results also show that the Inverse Gaussian distribution performed well in most of the traces and it ranks second after Gamma and Weibull distributions, among the other examined distributions, and this is true in all the evaluated traces except in the Silence of the Lambs trace as the Lognormal distribution outperforms the Inverse Gaussian distribution. On the contrary, the LogLogistic is the worst fit among the other fitted distribution examined in this paper since it ranks last in all the studied traces and moreover, it was invalid for fitting the NBC news video. The tests also show that the Lognormal distribution performed well in some of the traces and generally, among the analysed movies, it ranks after the Inverse Gaussian.

For further assessments of the results showed in Table 2, the Q–Q and P–P plots are presented in Figs. 3 and 4. The plots confirm our claims regarding the Gamma, Weibull and the LogLogistic distributions as it is evident from the Q–Q plots that the Gamma and Weibull have the best plots close to the a 45° reference line (see Figs. 3*a*, *b* and 4*a*, *b*) and the LogLogistic has more deviation from the reference line compared with the others as depicted clearly in Fig. 3*b*. In addition, the Q–Q plots show that all the examined distributions performed well at the lower tail of the evaluated samples and not fit well at the upper tail. The plots also showed that Gamma and Weibull behave similarly and they are close to each other in all Q–Q and P–P plots. The plots in Figs. 3*a*, 4*a* and *b* showed that the Pearson Type V is comparable with the Lognormal distribution in fitting the frame sizes generated by H.264 in the Star Wars IV, Silence of the Lambs and the NBC new videos.

5 Conclusion

In this paper, we have investigated modelling the frame sizes H.264 encoded video with well-known distributions. Modelling video traffic can be highly useful in mathematical analysis, simulation and in generating synthetic video traces for the purpose of performance, testing and compliance. In our study, we have considered different types of video sequences that included a science-fiction movie, action movie, drama and news. Our results show that the Gamma and Weibull distributions give appropriate statistical distributions of video frame sizes. The Inverse Gaussian distribution performs well in most of the traces but not as well as Gamma and Weibull distributions. The Pearson Type V was shown to be comparable with the Lognormal distribution of fitting the frame sizes. The LogLogistic distribution was found to be much less appropriate in modelling the frame sizes generated by the H.264 encoded video. As a future study, we plan to develop a software plugin that can be used in the popular NS2 simulation for the purpose of generating synthesised video traffic traces.

6 References

1 Auwera, G.V., David, P.T., Reisslein, M.: 'Traffic and quality characterization of single-layer video streams encoded with H.264/MPEG-4 advanced video coding standard and scalable video coding extension', *IEEE Trans. Broadcast.*, 2008, **75**, pp. 698–718

2 Schwarz, H., Marpe, D., Wiegand, T.: 'Overview of the scalable video coding extension of the H.264/AVC standard', *IEEE Trans. Circuits Syst. Video Technol.*, 2007, **17**, pp. 1103–1120

3 Auwera, G.V., David, P.T., Reisslein, M.: 'Traffic characteristics of H.264/AVC variable bit rate video', *IEEE Commun. Mag.*, 2008, **46**, (11), pp. 698–718

4 Dai, M., Zhang, Y., Loguinov, D.: 'A unified traffic model for MPEG-4 and H.264 video traces'. Presented at IEEE Transactions on Multimedia, 2009, pp. 1010–1023

5 Lazaris, A., Koutsakis, P.: 'Modeling multiplexed traffic from H.264/AVC videoconference streams', *Comput. Commun.*, 2010, **33**, (10), pp. 1235–1242

6 Beran, J., Sherman, R., Taqqu, M.S., Willinger, W.: 'Long-range dependence in variable-bit-rate video traffic', *IEEE Trans. Commun.*, 1995, **43**, (2/3/4), pp. 1566–1579

7 Ryu, B., Elwalid, A.: 'The importance of long-range dependence of VBR video traffic in ATM traffic engineering: myths and realities'. Proc. Conf. on Applications, Technologies, Architectures, and Protocols for Computer Communications, California, USA, August 1996, pp. 3–14

8 'H.264/AVC video trace library', available at: http://trace.eas.asu.edu

9 Heyman, D.P., Tabatabai, A., Lakshman, T.V.: 'Statistical analysis and simulation study of video teleconference traffic in ATM networks', *IEEE Trans. Circuits Syst. Video Technol.*, 1992, **2**, (1), pp. 49–59

10 Heyman, D.P., Lakshman, T.V., Tabatabai, A., Heeke, H.: 'Modeling teleconference traffic from VBR video coders'. Proc. IEEE Int. Conf. on Communications (ICC'94), New Orleans, LA, USA, May 1994, pp. 1744–1748

11 Xu, S., Huang, Z.: 'A Gamma autoregressive video model on ATM networks', *IEEE Trans. Circuits Syst. Video Technol.*, 1998, **8**, (2), pp. 138–142

12 Krunz, M., Hughes, H.: 'A traffic model for MPEG-Coded VBR streams'. Proc. ACM SIGMETRICS Conf., 1995, pp. 47–55

13 Fitzek, F., Reisslein, M.: 'MPEG-4 and H.263 video traces for network performance evaluation', *IEEE Netw.*, 2001, **15**, (6), pp. 40–54

14 Lazaris, A., Polychronis, K., Paterakis, M.: 'A new model for video traffic originating from multiplexed MPEG-4 videoconference streams', *Perform. Eval.*, 2008, **65**, (1), pp. 51–70

15 Poon, W.C., Lo, K.T.: 'A refined version of M/G/∞ processes for modelling VBR video traffic', *Comput. Commun.*, 2001, **24**, (11), pp. 1105–1114

16 Ryu, B.: 'Modeling and simulation of broadband satellite networks – Part II: traffic modeling', *IEEE Commun. Mag.*, 1999, **37**, pp. 72–79

17 Koumaras, H., Skianis, C., Gardikis, G., Kourtis, A.: 'Analysis of H.264 video encoded traffic'. INC 2005 Fifth Int. Network Conf., Samos Island, Greece, July 2005

18 Dai, M., Loguinov, D.: 'Analysis and modeling of MPEG-4 and H.264 multi-layer video traffic'. Proc. 24th Annual Joint Conf. IEEE Computer

doi: 10.1049/iet-com.2010.0868

and Communications Societies (INFOCOM '05), Miami, FL, USA, March 2005, vol. 4, pp. 2257–2267

19 Domoxoudis, S., Kouremenos, S., Drigas, A., Loumos, V.: 'Frame-based modeling of H264 constrained videoconference traffic over an IP commercial platform'. Proc. IEEE Second Int. Conf. on Testbeds & Research Infrastructures for the Development of Networks and Communities, July 2006, pp. 6–221

20 VCON Vpoint HD, http://www.vcon.com

21 Polycom PVX, http://www.polycom.com

22 Sarkar, U.K., Ramakrishnan, S., Sarkar, D.: 'Modeling full-length video using markov-modulated gamma-based framework', *IEEE/ACM Trans. Netw.*, 2003, **11**, pp. 1979–1983

23 Auwera, G., David, P., Reisslein, M.: 'Video traffic analysis of H.264/AVC and extensions: single layer statistics'. Technical report, ASU, Department of Electrical Engineering, Tempe, AZ, USA, February 2007

24 Reisslein, M., Lassetter, J., Ratman, S., Lotfallah, O., Fitzek, F., Panchanathan, S.: 'Traffic and quality characterization of scalable encoded video: a large-scale trace-based study – part 1: overview and definitions'. Technical report, Arizona State University, Tempe, AZ, USA, December 2003

25 Law, A.M., Kelton, W.D.: 'Simulation modeling and analysis' (McGraw-Hill, New York, NY, 1991, 2nd edn.)

26 Gupta, R.D., Kundu, D.: 'Generalized exponential distributions: different methods of estimation', *J. Stat. Comput. Simul.*, 2001, **69**, (4), pp. 315–338

27 Knuth, D.E.: 'The art of computer programming – seminumerical algorithms' (Addison-Wesley, Reading, MA, 1998, vol. 2, 3rd edn.), pp. 45–52

28 Bock, R.K.: http://rkb.home.cern.ch/rkb/AN16pp/node143.html

29 Levenberg, K.: 'A method for the solution of certain non-linear problems in least squares', *Q. Appl. Math.*, 1944, **2**, (2), pp. 164–168

doi: 10.1049/iet-com.2010.0868