Mathematical and Computer Modelling of Dynamical Systems 1387-3954/02/0802-199$16.00
2002, Vol. 8, No. 2, pp. 199–211

A Priority Discrete Queueing Model for Multimedia Multiplexers

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ABSTRACT

Multiplexers have been extensively modeled as discrete time queueing systems. In this article, we model a multimedia multiplexer handling traffic of two classes. One class represents real-time traffic, e.g., packets of live audio or video transmissions, and the other nonreal-time traffic, e.g., packets of file transfer transmissions. These packets arrive into the multiplexer in batches. In each time slot, one batch of each class arrive. The multiplexer gives service priority to class-1 packets over class-2. The demands of each class are in conflict with that of the other, and thus they are treated by the multiplexer differently.

The multiplexer is thus modeled as a (preemptive) priority discrete queueing system with simultaneous batch arrivals and geometric service time. The system occupancy is analyzed and the joint probability generating function (PGF) of the number of packets of each class is derived. From this PGF, marginal PGFs of interest are obtained. The results for deterministic service time, most suitable for ATM purposes, are readily obtainable as a special case from the results of this article.

Keywords: multimedia ATM multiplexer, preemptive priority, discrete queueing, system occupancy.

1. INTRODUCTION

Modern communications networks, such as the Broadband Integrated Services Digital Network (ISDN), use Asynchronous Transfer Mode (ATM) in their operation [1]. This mode is characterized by encapsulating the traffic in small, fixed-size (53 bytes) packets. Multiplexers are used in these networks to save on communications channels when the traffic is bursty [2].

Suppose, for example, that a bank branch is to be connected to the bank headquarters. The branch has N tellers, each with a computer terminal. The headquarter has a central computer having a master data base. It is required to connect the terminals to the computer, so that the tellers can access the data base. There are two approaches to attain this connection.

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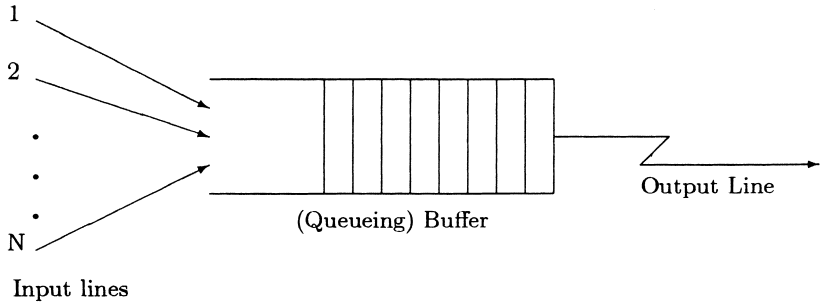

Fig. 1. Multiplexer with *N* input lines.

The first approach is to use *N* leased lines, one for each terminal. But this solution is uneconomical, given that the traffic of each terminal is bursty. Specifically, a terminal will actually transmit every once in a while, leaving the costly line idle most of the time. The second approach, which is economically better, is to use a multiplexer and only *one* leased line. As shown in Figure 1, the multiplexer has *N* input lines, a buffer (waiting room), and one output line. The *N* terminals will then be connected to the *N* input lines, and the leased line to the output line. Packets transmitted by the terminals will first go to the multiplexer, which queues them in its buffer, and ultimately transmits them one at a time off the output line. It should be noted that at the receiving end, there is supposed to be a demultiplexer, a device with one input line, *N* output lines, and no buffer, which reverses the action of the multiplexer. Namely, it redistributes the packets arriving on its input line onto its *N* output lines.

When multiplexers operate in a multimedia environment, e.g., a Broadband ISDN [1], they face a challenging problem. In such environments, the traffic is of two classes. Class-1 traffic is made up of packets of real time communications, e.g., live audio and video. Class-2 traffic, on the other hand, is made up of packets of nonreal time communications, e.g., file transfer. Each of these two classes should be treated by the multiplexer differently, since their properties are different. Namely, class-1 traffic is loss-insensitive but delay-sensitive, which means that packets of this type should be served rapidly even if some are lost as a result. On the other hand, class-2 traffic is delay insensitive but loss-sensitive, which means that no packet of this traffic should be lost even if some, or all, of the packets are delayed as a result.

The solution to this problem is to use a priority scheme in the multiplexer [3]. In this scheme class-1 packets are assigned higher service priority over class-2 packets. That is, if it contains packets of both classes in its buffer, the multiplexer will serve class-1 packets first.

One of two disciplines may be used if a priority scheme is adopted by the multiplexer, concerning what happens if a class-1 packet arrives while a class-2 packet is in service. In the *preemptive* discipline, the arriving packet enters service immediately in the next slot, ejecting the class-2 packet to the buffer. When the multiplexer has no more class-1 packets to serve, the ejected class-2 packet enters service again. In the *nonpreemptive* discipline, on the other hand, the arriving packet waits until the class-2 packet finishes service and then takes its place in the server.

One of two options may be used if the preemptive discipline is chosen, concerning how the ejected packet is served after it goes back to the server. In the *resume* option, the packet is served from the point it was ejected. In the *repeat* option, the packet is served from the start.

Multiplexers handling uniclass traffic have received much research attention, since the advent of digital communications in the seventies. Buffered, they have typically been modelled as a discrete time queueing system. A large number of these models are available in the literature (see, e.g., [4–8]). The differences among the models of these works are usually in the assumptions, but sometimes are in the solution technique.

Multiplexers handling biclass traffic, on the other hand, have gained research attention in recent years due to the proliferation of multimedia traffic over communications networks. They have typically been modelled as a discrete *priority* queueing system. A number of these models are available in the literature (see e.g., [9–13]).

In this article, we model a multimedia multiplexer operating under the assumptions given in Section 2. These assumptions can be thought of as the union of the three sets of assumptions given in [9], [10] and [11]. The multiplexer is modeled as a (preemptive) priority discrete time queueing system.

The article is organized as follows. We start by formally introducing the model assumptions in Section 2. In Section 3, we derive the joint PGF of the output multiplexer occupancy. In Section 4, we apply the results to a special case, and in the last Section we draw conclusions.

2. MODEL ASSUMPTIONS AND NOTATIONS

First of all, it is assumed that the multiplexer operates in a discrete time manner. That is, the time axis is divided into slots, each exactly equal to the transmission time of one packet. Nonnegative integers $k = 0, 1, \ldots,$ are assigned to the individual slot boundaries. Time interval $(k, k+1)$ is referred to as slot $k+1$.

In the following, we formally state the multiplexer assumptions, which are largely reflected by the diagram in Figures 1 and 2.

1. The multiplexer has a single buffer of infinite capacity to host arriving packets in the form of a queue.

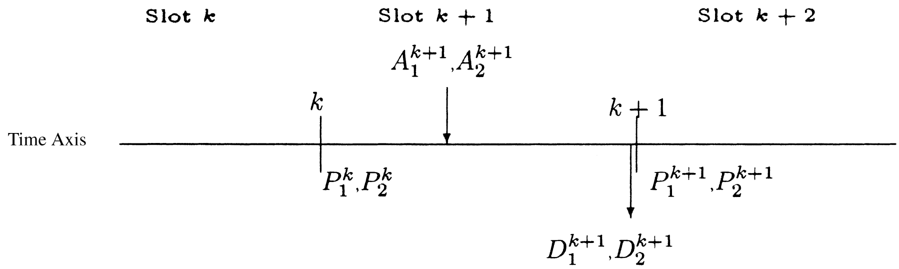

Fig. 2. System occupancy evolution.

2. In each slot k, two *independent* batches *must* arrive into the multiplexer, one containing $A_1^k = 0, 1, \ldots,$ class-1 packets and one containing $A_2^k = 0, 1, \ldots,$ class-2 packets. This may look as a restrictive assumption, but the fact that the batches could be of zero size shows that there can be slots with no arrivals. The A_1^k are independent and identically distributed (iid) RVs, and so are the A_2^k. Let r_1, and r_2 denote the arrival *rates* (packets per slot) at the multiplexer of class-1 packets and class-2 packets, respectively. If r denotes the packet arrival rate regardless of class, then r is related to r_1 and r_2 through the relation $r = r_1 + r_2$.
3. A packet may enter queue or service only at the beginning of a slot. This implies that if a packet arrives into the multiplexer in a given slot, it enters either queue or service (if the queue is empty) at the beginning of the next slot. That is, the packet is not regarded to be in the multiplexer throughout its arrival slot.
4. The multiplexer has a single server, e.g., a register, to host the packet under transmission. Service time is the time the packet spends in this register.
5. A packet may end service only at the end of a slot.
6. In each slot k, either $D_1^k = 0, 1$ class-1 packets or $D_2^k = 0, 1$ class-2 packets are served. The D_1^k are independent and identically (Bernoulli) distributed (iid) RVs, and so are the D_2^k. Let s denote the service *rate* (packets per slot) of the multiplexer. As the D_1^k and D_2^k are Bernoulli distributed, a packet being served in a certain slot will end service by the end of that slot with probability s and will not with probability $\overline{s} = 1 - s$. This implies that the service time is geometrically distributed with expectation $\frac{1}{s}$.
7. Class-1 packets have service priority over class-2 packets. That is, no class-2 packet can start service while a class-1 packet is in the multiplexer. Thus we can look at the multiplexer as having *two* logical queues, one of class-1 packets and one of class-2 packets. With the priority scheme adopted, no class-2 packet can enter service unless the class-1 queue is empty.
8. An arriving batch of a given class is placed at the end of its appropriate queue on a first come first serve (FCFS). As for the packets inside that batch, they are placed in

the queue in random order. The packets in each queue then enter service on a FCFS basis.

9. The priority discipline adopted is that of the preemptive type. Note that the option of preemption (resume or repeat) is irrelevant here, due to the memoryless property of the geometric distribution of service time.

One of the features of this article is assuming geometric service time. Most of the published works consider deterministic time of one slot, with the perception that in ATM applications the packet service time is just its transmission time (i.e., one slot). But it is very likely that prior to transmission the packet will undergo some processing, such as error encoding and decoding or encryption and decryption. In this case service time is not just transmission time. Also, if the multiplexer does not dispose of the packet before an acknowledgement is returned from the receiving end in the next slot, retransmitting the packet at the end of that slot, service time is not just transmission time. In fact, in this latter case, retransmission can go on and on indefinitely, making the geometric distribution assumption perfect for service time. Additionally, if we were modelling a fileserver, rather than a multiplexer, where the requests for (geometrically long) files arrive from N work stations, get stored in a queue, and are dismissed only when the requested files have been transmitted, the geometric distribution assumption becomes perfect for service time.

In the sequel we will analyze the occupancy of the multiplexer under the above assumptions. Most of the variables in the analysis are random variables (RVs), all of which are nonnegative and integral valued.

3. SYSTEM OCCUPANCY

Due to the biclass nature of our system, we may identify three types of system occupancy. First, class-1 system occupancy refers to the number of class-1 packets in system at an arbitrary slot. Second, class-2 system occupancy refers to the number of class-2 packets. Third, system occupancy refers to the number of packets in system, regardless of class. We will obtain results for all three types in steady state.

Let $P_1^k = 0, 1, \ldots$, be a RV denoting the class-1 system occupancy at slot k, i.e., the number of class-1 packets in system at the end of slot k. In a similar manner let $P_2^k = 0, 1, \ldots$, be a RV denoting the class-2 system occupancy at slot k. Since D_1^k is the number of class-1 packets served in slot k, it clearly depends on P_1^k with the following conditional distribution

$$\Pr[D_1^{k+1} = n | P_1^k = i] = \begin{cases} s & \text{if } n = 1, i > 0 \\ \bar{s} & \text{if } n = 0, i > 0 \\ 1 & \text{if } n = 0, i = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Similarly, since D_2^k is the number of class-2 packets served in slot k, it clearly depends on P_1^k and P_1^k with the following conditional distribution

$$\Pr[D_2^{k+1} = n | P_1^k = i, P_2^k = j] = \begin{cases} s & \text{if } n = 1, i = 0, j > 0 \\ \bar{s} & \text{if } n = 0, i = 0, j > 0 \\ 1 & \text{if } n = 0, i = 0, j = 0 \\ & \text{or } n = 0, i > 0, j \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Figure 2 reflects the evolution of the system occupancy. Referring to it, we can easily write the following two stochastic recursive equations.

$$P_1^{k+1} = P_1^k - D_1^{k+1} + A_1^{k+1} \tag{3}$$

and

$$P_2^{k+1} = P_2^k - D_2^{k+1} + A_2^{k+1}. \tag{4}$$

Let $p_{i,j}^k$ be the joint distribution of P_1^k and P_2^k. That is $p_{i,j}^k = \Pr\left[P_1^k = i,\, P_2^k = j\right]$. And let $P^k(z_1, z_2)$ be the PGF of $p_{i,j}^k$. That is

$$P^k(z_1, z_2) \triangleq \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p_{i,j}^k z_1^i z_2^j = E\left[z_1^{P_1^k} z_2^{P_2^k}\right]. \tag{5}$$

where the operator $E[\cdot]$ indicates the expectation of the expression between brackets.

Now utilizing (3) and (4), and recognizing the independency of A_1^{k+1} and A_1^{k+1} from one another and from the RVs P_1^k, D_1^{k+1}, P_1^k, and D_1^{k+1}, we write

$$\begin{aligned} P^{k+1}(z_1, z_2) &= E\left[z_1^{P_1^{k+1}} z_2^{P_2^{k+1}}\right] \\ &= E\left[z_1^{P_1^k - D_1^{k+1} + A_1^{k+1}} z_2^{P_2^k - D_2^{k+1} + A_2^{k+1}}\right] \\ &= A_1(z_1)\, A_2(z_2) E\left[z_1^{P_1^k - D_1^{k+1}} z_2^{P_2^k - D_2^{k+1}}\right] \end{aligned} \tag{6}$$

where

$$A_i(z) \triangleq \sum_{n=0}^{\infty} \Pr\left[A_i^k = n\right] z^n = E\left[z^{A_i^k}\right], \quad i = 1, 2. \tag{7}$$

are the common PGFs of the A_i^k. Note from this definition that the class-1 and class-2 arrival rates r_1 and r_2 are related to the $A_i(z)$ through the relation

$$r_i = A_i'(1) \triangleq \frac{d}{dz} A_i(z)|_{z=1}, \qquad i = 1, 2. \tag{8}$$

Now, using (1) through (7) and with some algebraic and probabilistic (especially conditional and unconditional) manipulation, we can find that

$$E\left[z_1^{P_1^k - D_1^{k+1}} z_2^{P_2^k - D_2^{k+1}}\right] = \frac{P^k(z_1, z_2)(s + \bar{s}z_1)}{z_1} + P^k(0, z_2)\frac{s(z_1 - z_2)}{z_1 z_2} + p_{0,0}^k \frac{s(z_2 - 1)}{z_2} \tag{9}$$

Substituting in (6), we get

$$P^{k+1}(z_1, z_2) = A_1(z_1)A_2(z_2)\left(\frac{P^k(z_1, z_2)(s + \bar{s}z_1)}{z_1} + P^k(0, z_2)\frac{s(z_1 - z_2)}{z_1 z_2} + p_{0,0}^k \frac{s(z_2 - 1)}{z_2}\right) \tag{10}$$

For system stability, the total packet arrival rate r should be strictly less than the packet service rate s. If this condition is met, the system will reach, steady state after a sufficiently large number of slots (i.e., as $k \to \infty$), in which case the $P^k(z_1, z_2)$ will converge to a common PGF $P(z_1, z_2)$. As a result, (12) in steady state yields

$$P(z_1, z_2) = \frac{A_1(z_1)A_2(z_2)s\big(P(0, z_2)(z_1 - z_2) + z_1(z_2 - 1)p_{0,0}\big)}{z_2(z_1 - A_1(z_1)A_2(z_2)(s + \bar{s}z_1))}. \tag{11}$$

where $p_{0,\,0}$ denotes the steady state probability that the system is empty.

Since the probability that a queueing system, regardless of whether uniclass or biclass, is empty is always the complement of the probability that it is busy, and since the latter is known (see e.g., [14]) to be just the utilization ρ, defined as the ratio of packet arrival rate to packet service rate, then in our case we have

$$p_{0,0} = \frac{s - r}{s}. \tag{12}$$

Thus (11) becomes

$$P(z_1, z_2) = \frac{A_1(z_1)A_2(z_2)(P(0, z_2)(z_1 - z_2) + z_1(z_2 - 1)(s - r))}{z_2(z_1 - A_1(z_1)A_2(z_2)(s + \bar{s}z_1))} \tag{13}$$

Equation (13) does not give the final form of $P(z_1, z_2)$, since $P(0, z_2)$ is still unknown. However, it can be used to obtain the PGF of several interesting subsystem occupancies.

For example, the marginal PGF $P_1(z)$ of class-1 system occupancy P_1 can be obtained from (13) as follows:

$$\begin{aligned} P_1(z) &= P(z, 1) \\ &= \frac{A_1(z)(z - 1)\,P(0, 1)}{z - A_1(z)(s + \bar{s}z)}. \end{aligned}$$

We note that $P(0,1) = p_{1_0}$ can be obtained using the normalization condition $P_1(1) = 1$. Doing that, we find $P(0,1) = s - r_1$, and therefore

$$P_1(z) = \frac{A_1(z)(z-1)(s-r_1)}{z - A_1(z)(s+\bar{s}z)}. \tag{14}$$

This result is identical to that obtained in [14] in a uniclass context. This identity is natural since we are assuming here preemptive priority for class-1, which makes the class-1 system occupancy indifferent to class-2 packets.

Also, (13) can be used to obtain the PGF $P(z)$ of the total system occupancy $P = P_1 + P_2$, disregarding the class of the packets, as follows:

$$P(z) = P(z,z) = \frac{A_1(z)A_2(z)(z-1)(s-r)}{z - A_1(z)A_2(z)(s+\bar{s}z)} \tag{15}$$

Note that we are adopting polymorphism as far as function notation is concerned. So, the P in $P(z)$ is different from the P in $P(z,z)$.

Finally, (13) can be manipulated to yield the system occupancy of the equivalent uniclass system, i.e., the system with all our assumptions except that all the arriving batches become of one class of size $A = A_1 + A_2$ having PGF $A(z)$. The occupancy P of that system would have PGF $P(z)$ that can be obtained from (13) as follows:

$$P(z) = P(z,z)|_{A=A_1+A_2} = \frac{A(z)^2(z-1)(s-r)}{z - A(z)(s+\bar{s}z)} \tag{16}$$

This result is identical to that in [15], which is obtained for a uniclass system.

Going back to the unknown function $P(0,z_2)$ in (13), we can show, using Rouché's theorem [16], that the factor

$$z_1 - A_1(z_1)A_2(z_2)(s+\bar{s}z_1) = 0$$

in the denominator has exactly one zero on the unit disk $z_1 \leq 1$. Denoting this zero by ξ, it can be found (as a function of z_2) by Lagrange's theorem [16] as follows:

$$\xi = \sum_{i=1}^{\infty} \frac{[A_2(z_2)]^i}{i!} \frac{d^{i-1}}{dz_1^{i-1}} [A(z_1)(s+\bar{s}z_1)]^i \bigg|_{z_1=0}.$$

But since $P(z_1,z_2)$ is a PGF, it should have no poles on the unit disk [17], and therefore ξ must also be a zero of the numerator of (13). That is, if we substitute $z_1 = \xi$ in the numerator of (13), we should get a 0, which enables us to find $P(0,z_2)$ to be

$$P(0,z) = \frac{\xi(z_2-1)(s-r)}{z_2 - \xi}, \quad z_2 \neq \xi \tag{17}$$

Substituting from (17) into (13), yields

$$P(z_1, z_2) = \frac{A_1(z_1)A_2(z_2)(z_2 - 1)(z_1 - \xi)(s - r)}{(z_2 - \xi)(z_1 - (s + \bar{s}z_1)A_1(z_1)A_2(z_2))}, \tag{18}$$

which is the final form of the joint PGF of class-1 and class-2 occupancies.

The marginal PGF $P_2(z)$ of class-2 system occupancy can be obtained from (22) as follows:

$$\begin{aligned} P_2(z) &= P(1, z) \\ &= \frac{A_2(z)(z - 1)(1 - \xi)(s - r)}{(z - \xi)(1 - A_2(z))}. \end{aligned} \tag{19}$$

The PGFs derived above can be used to obtain the expected values of the corresponding occupancies. For example, the expected class-1 occupancy can be obtained as follows:

$$\begin{aligned} E[P_1] &= P_1'(1) \\ &= A_1'(1) + \frac{A_1''(1) + 2A_1'(1)\bar{s}}{2\big(s - A_1'(1)\big)}. \end{aligned} \tag{20}$$

This result is identical to that obtained in [14] for a uniclass system.

Also, the expected class-2 occupancy can be obtained as follows:

$$\begin{aligned} E[P_2] &= P_2'(1) \\ &= \frac{(s - r)\big(2r_2[r_2(\xi - 1) + \xi'] + \big[2(1 - \xi')r_2 + (1 - \xi)A_2''(1)\big]\big)}{2(1 - \xi)r_2^2}. \end{aligned} \tag{21}$$

where $\xi' = \frac{d}{dz_2}\xi\Big|_{z_2=1}$.

4. SPECIAL CASE

In this section, we consider the case where the service time of each packet is deterministically one slot, typical of normal ATM applications, and where the arriving batches of class-1 and class-2 packets have Poisson distributed sizes. This implies that $s = 1$, which, by the way, means that there is no difference between preemptive and nonpreemptive priority. It also implies that A_1 has distribution $\Pr\big[A_i^k = n\big] = \frac{r_1 e^{-r_1}}{n!}$ and the PGF $A_1(z) = e^{r_1(z-1)}$, and that A_2 has distribution $\Pr\big[A_i^k = n\big] = \frac{r_2 e^{-r_2}}{n!}$ and the PGF $A_2(z) = e^{r_2(z-1)}$. Clearly, then, $A_1'(1) = r_1$, $A_1''(1) = r_1^2$, $A_2'(1) = r_2$ and $A_2''(1) = r_2^2$.

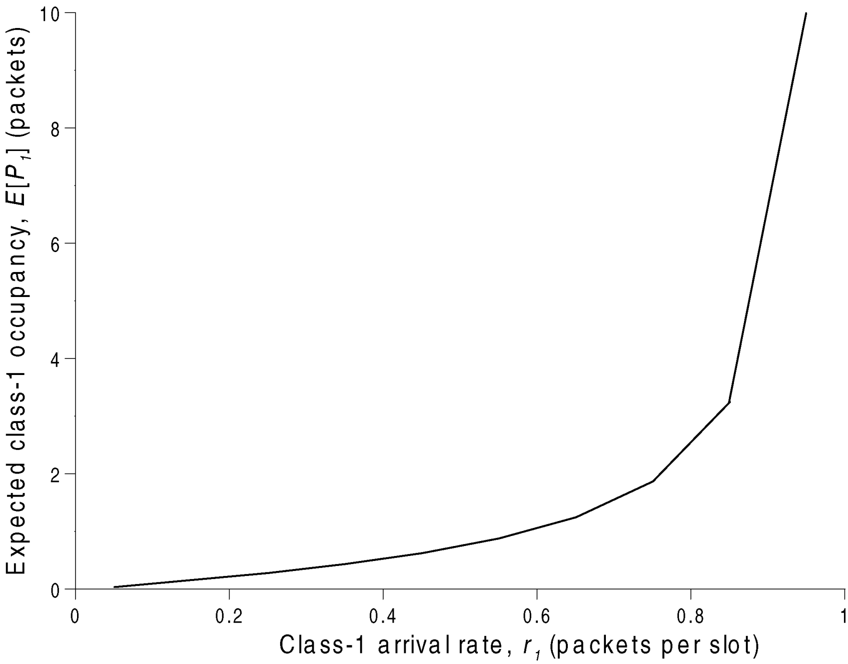

Fig. 3. Expected class-1 occupancy versus class-1 arrival rate r_1 for $s = 1$.

Substituting for $A_1'(1)$, $A_1''(1)$ and $s = 1$ in (20), we get the expected class-1 occupancy as follows:

$$E[P_1] = \frac{r_1(2 - r_1)}{2(1 - r_1)} \tag{22}$$

That is identical to (4.85) in [14] when $s = 1$. Equation (22) shows that $E[P_1]$ is not affected by r_2, which is assuring since class-2 packets are invisible to class-1 packets when it comes to service. In Figure 3, we plot $E[P_1]$ against r_1. The Figure shows what we expect – the occupancy increases as the arrival rate increases.

The PGF $P(z)$ of the system occupancy regardless of the packet class, i.e., of the RV $P_1 + P_2$, can be obtained by substituting for s, $A_1(z)$, $A_2(z)$, $A_1'(1)$ and $A_2'(1)$ in (15), to get

$$P(z) = \frac{e^{(r_1+r_2)(z-1)}(z - 1)(1 - r_1 - r_2)}{z - e^{(r_1+r_2)(z-1)}} \tag{23}$$

The expectation $E[P_T]$ can be obtained as follows:

$$E[P] = P^{'}(1) = \frac{r(2 - r)}{2(1 - r)}. \tag{24}$$

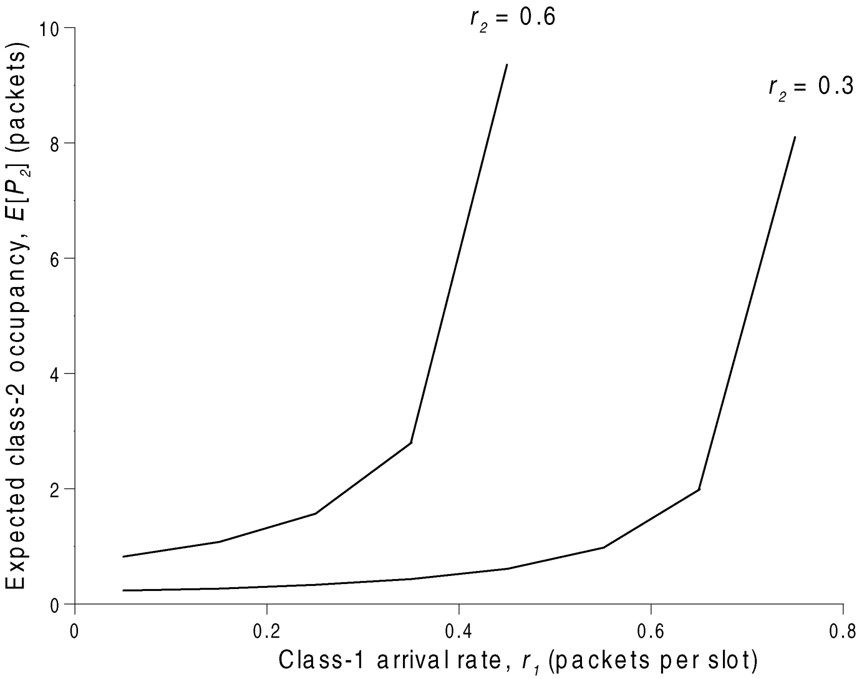

Fig. 4. Expected class-2 occupancy versus class-1 arrival rate r_1 for $s = 1$, and for two values of r_2.

From this result, we can get the expected class-2 occupancy as follows:

$$\begin{aligned} E[P_2] &= E[P] - E[P_1] \\ &= \frac{r(2-r)}{2(1-r)} - \frac{r_1(2-r_1)}{2(1-r_1)} \end{aligned} \tag{25}$$

In Figure 4, we plot $E[P_2]$ against r_1 for two values of r_2. The figure shows again what we expect – the class-2 occupancy increases as the class-2 arrival rate increases. More interesting is that the class-2 occupancy increases as the class-1 arrival rate increases, which is also to be expected, because class-1 packets have service priority over class-2 packets.

5. CONCLUSION

In this article, we have modeled a multimedia multiplexer handling biclass traffic as a priority, discrete time, single server, batch arrival queueing system with geometric service. We have obtained the joint PGF of the system occupancy, and used it to derive some useful marginal PGFs. We have applied the model to a special case suitable for ATM environments. Namely, we assumed a deterministic service time of one slot and Poisson batch sizes. The numerical results indicate that the expected class-1 occupancy is not affected by the class-2 arrival rate, while the expected class-2 occupancy is affected by the class-1 arrival rate (in addition, of course, to the class-2 arrival rate). This is intuitively clear, since the class-2 packets are invisible as far as the class-1 packets are concerned, thanks to the preemptive priority of the latters over the formers.

This work can be extended in various ways. For example, the buffer may be assumed finite, instead of infinite, the service time general, instead of geometric, and the arrival process renewal, instead of Bernoulli. Also, two different service rate s_1 and s_2, instead of one, may be assumed.

ACKNOWLEDGEMENTS

This research was carried out under the auspices of the Math Department, College of Science, Suez Canal University. The authors wish to thank the Department Head, Dr. Sherif Zaki, together with his colleagues for their help and encouragement.

REFERENCES

1. Prycker, M.D.: Asynchronous Transfer Mode: Solution for Broadband ISDN. Chichester, *Telecommunication Systems (MASCOTS'94)*, 1994, pp. 405–406.
2. Eddie, K. and Gracia, A.: ATM Multiplixer and Output Controllers With Distributed Control and Flexible Queueing Disciplines. *Proccedings- IEEE- INFOCOM* 2 (1996), San Francisco, CA, pp. 663–670.
3. Jing, R. and Johnny, W.: Dynamic Priority Queueing Approach to Traffic Regulation and Scheduling in B-ISDN. *IEEE Global Telecommunicatons Conference*, 1994, pp. 612–618.
4. HSU, J.: Buffer Behavior in Computer Communication Systems. *IEEE Trans. Commun.* 22 (1974), pp. 1940–1941.
5. Kobayashi, H. and Konheim, A.G.: Queueing Models for Computer Communications System Analysis. *IEEE Trans. Commun.* Com-25 (1977), pp. 1–27.
6. Gracia, J. and Casals, O.: Discrete Time Queueing Model to Study the Cell Delay Variation in an ATM Network. *Performance Evaluation* 21-1-2 (1994), pp. 3–22.
7. Steyaert, B. and Xiong, Y.: Analysis of a Discrete-Time Queue With General Three-State Markovian Traffic Sources. *Performance Evaluation* 27 (1996), pp. 277–294.
8. Cidon, I. and Sidi, M.: Dispersed Messages in Discrete-Time Queues: Delay, Jitter and Threshold Crossing. *Performance Evaluation* 29 (1997), pp. 85–104.
9. Takine, B. and Hasegawa, T.: An Analysis of a Discrete-Time Queue for Broadband ISDN With Priorities Among Traffic Classes. *IEEE Trans. On Commun.* 42 (1994), pp. 1837–1843.
10. Lee, Y. and Sung, D.: Priority Queueing System with Fixed-Length Packet-Train Arrivals. *IEEE Proc. Commun.* 145 (1998), pp. 331–336.
11. Laevens, K. and Bruneel, H.: Discrete-Time Multiserver Queues with Priorities. *Performance Evaluation* 33 (1998), pp. 249–275.

12. Takemori, K. and Sasase, I.: Performance Analysis as Input and Output Queueing Packet Switch With Multiple Priority Classes. *Electronics and Commun.* 79(6) (1996), pp. 38–50.
13. Lee, J. and Un, C.: Performance Analysis of Space-Priority Mechanisms in an Input and Output Queueing ATM Switch. *IEE Proc. In Commun.* 144(4) (1997), pp. 229–236.
14. Woodward, M.: *Communication and Computer Networks*. IEEE Comput. Society Press, Los Alamitos, CA, 1994.
15. Bruneel, H.: Performance of Discrete-Time Queueing Systems. *Comput. Oper. Res.* 20(3) (1993), pp. 303–320.
16. Takagi, H.: *Queueing Analysis* 1. Elsevier Science Publishers B.V., 1991.
17. Bruneel, H. and Kim, B.: *Discrete-Time Models for Communication Systems Including ATM*. Kluwer Academic Publishers, Boston, MA, 1993.