Published in IET Communications
Received on 2nd March 2010
Revised on 10th June 2010
doi: 10.1049/iet-com.2010.0173

ISSN 1751-8628

Rate allocation games in multiuser multimedia communications

Q. Zhang   G. Liu

School of Electronic and Information Engineering, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China
E-mail: liugz@xjtu.edu.cn

Abstract: In this study, the authors study a game-theoretic framework for the problem of multiuser rate allocation in multimedia communications. The authors consider the multimedia users to be autonomous, that is, they are selfish and behave strategically. The authors propose a rate allocation framework based on a pricing mechanism to prevent the selfish users from manipulating the network bandwidth by untruthfully representing their demands. The pricing mechanism is used for message exchange between the users and the network controller. The messages represent network-aware rate demands and corresponding prices. The authors show that a Nash equilibrium can be obtained, according to which the controller generates allocations that are efficient, budget balanced and satisfy voluntary participation. Simulation results demonstrate the validity of the proposed framework.

1 Introduction

With the explosive growth of the internet and the rapid advance of compression techniques, delay-sensitive multimedia networking applications such as video conferencing, video on demand (VOD) or internet protocol TV (IPTV) get more and more popular. Therefore it is common among many video users to share network bandwidth over the same communication link and how to efficiently allocate the rate among them becomes more and more important.

The simplest multiuser rate allocation method is equally assigning the available network bandwidth to each user. A major problem of this method is that it does not consider the variable bit-rate characteristics of the video sequences. One method to overcome this disadvantage is that the controller collects the characteristics of all the video sequences and optimises a global objective function using conventional optimisation methods such as Lagrangian or dynamic programming [1]. For example, a commonly adopted method for the rate controller is to maximise the sum of the peak signal-to-noise ratio (PSNRs).

$$\max_{R_i} \sum_{i=1}^{N} \mathrm{PSNR}_i(R_i), \quad \text{s.t.} \sum_{i=1}^{N} R_i \le R \qquad (1)$$

where R is the available network bandwidth and PSNR_i is the PSNR of the ith user. However, this method depends on users to truthfully report their characteristics. If some users misrepresent their characteristics, the performance of the entire system may degrade considerably [2].

Recently, the fairness issue in multimedia applications has been considered. In [3], a max–min fairness allocation is presented using a combination of the bandwidth reservation and bandwidth borrowing to provide the network users, the required quality of service (QoS). As pointed out in [4], the max–min approach deals with the worstcast scenario, so it favours users with worse channels while sacrifices the system efficiency. In [5], a Nash bargaining solution is proposed to divide the available resources in order to achieve a utility-fair allocation, where the utility function for each user is defined as the inverse of the distortion. However, these proposed schemes assume that all users truthfully report their resource requirements. As pointed out in [6], this is not always true when the users are selfish. To guide users' behaviours, pricing-based rate allocation algorithms have been extensively investigated [7–9], where the price reflects the 'scarcity' of the bandwidth and the users adjust their demands based on the utility functions. However, the algorithms assume that the users are 'price-takers', that is, the users accept the price announced by the network controller and do not consider the effects of their actions on the price. If the users anticipate these effects, that is, behave strategically, the above algorithms will lead to an inefficient allocation [10].

In this paper, we provide a solution for multiuser multimedia rate allocation problem by explicitly considering the individual characteristics and the strategic behaviours of multimedia users. We propose a quality-rate (Q-R) model to model how the bit-rate impact the video quality of a user. To enforce users to declare their rate demands truthfully, we adopt a recently proposed pricing mechanism [11] for the controller to implement the rate allocation task. We analyse the features of the mechanism in the context of multiuser rate allocation, and show that the resulting allocations which are efficient, budget balanced and satisfy voluntary participation.

The rest of this paper is organised as follows. In Section 2, we describe the system model and formulate the rate allocation problem. In Section 3, we characterise the allocations for multimedia users by using the pricing mechanism. Finally, we present the simulation results in Section 4 and draw the conclusions in Section 5.

2 Multiuser rate allocation game description

In this paper, we consider the same system model as that used in [9], which is depicted in Fig. 1. In the system, it is assumed that there is a controller, N transmitters, $u_1, u_2, \ldots, u_N$, and N receivers, $r_1, r_2, \ldots, r_N$. User u_i transmits the video sequence v_i to the corresponding receiver r_i through a channel/link that is shared by other users $u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_N$. Since the channel has a limited bandwidth, it may not be able to satisfy the bandwidth requirements for all users. Therefore the controller takes charge of efficiently allocating the channel bandwidth to users $u_1, u_2, \ldots, u_N$.

2.1 Video Q-R model

In video compression, because of the quantisation process, there exists a tradeoff between the distortion (D) and the bit-rate (R). The distortion is defined as mean-squared error (MSE), and the bit-rate determines the channel bandwidth or storage space required to transmit or store the coded data. Generally, high bit-rate leads to small distortion whereas low bit-rate causes large distortion. Several models have been published in the literature to characterise this distortion-rate tradeoff for different video coders, such as MPEG-2 [12, 13], MPEG-4 [14, 15] and H.264 [16–18]. However, these models are usually used for rate control and cannot be used to model the distortion of an entire video coder for a given rate. In [19], a three-parameter model is proposed to feature the input–output behaviour of the video coder. For the convenience of mathematical derivation, a two-parameter model is employed in [9]. However, this model cannot describe the measured distortion-rate performance of a video for a given coder with sufficient accuracy.

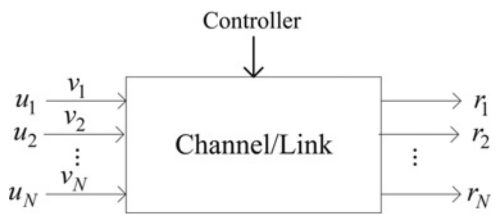

Figure 1 *System model from [9]*

We find that video distortion-rate characteristics can be modelled by a two-parameter model as follows

$$D(R) = aR^{-b} \tag{2}$$

where a and b are two positive parameters determined by the characteristics of the video content. Since PSNR is a measure more common than MSE in the video coding and communication community, we use PSNR to measure the video quality (Q), which is calculated by

$$Q = \text{PSNR} = 10\log_{10}\frac{255^2}{D} \tag{3}$$

By substituting (2) into (3), we obtain the Q-R function of user u_i

$$Q(R_i) = \alpha_i + \beta_i \ln R_i \tag{4}$$

where $\alpha_i = 10\log_{10}(255^2/a_i)$ and $\beta_i = 10b_i/\ln 10$. To verify the validity of this model, we compress a variety of video sequences using state-of-the-art H.264 JM14.2 [20]. By changing the quantisation parameter (QP), we generate a set of (Q_i, R_i). Fig. 2 shows that (4) approximates the Q-R characteristics of video sequences accurately.

2.2 Multiuser rate allocation game

In the rate allocation for video communications, each user generally desires to acquire as much of the network bandwidth as possible, since high bandwidth will leads to high video quality. To guide user's behaviour, we introduce a quasi-linear utility function of the form

$$U_i(R_i, \tau_i) = Q_i(R_i) + \tau_i \tag{5}$$

where τ_i represents the tax of user i, which depends on the employed mechanism and will be presented in greater detail in Section 3.2. In our framework, the controller utilises the tax to encourage truthful report of each user's demand.

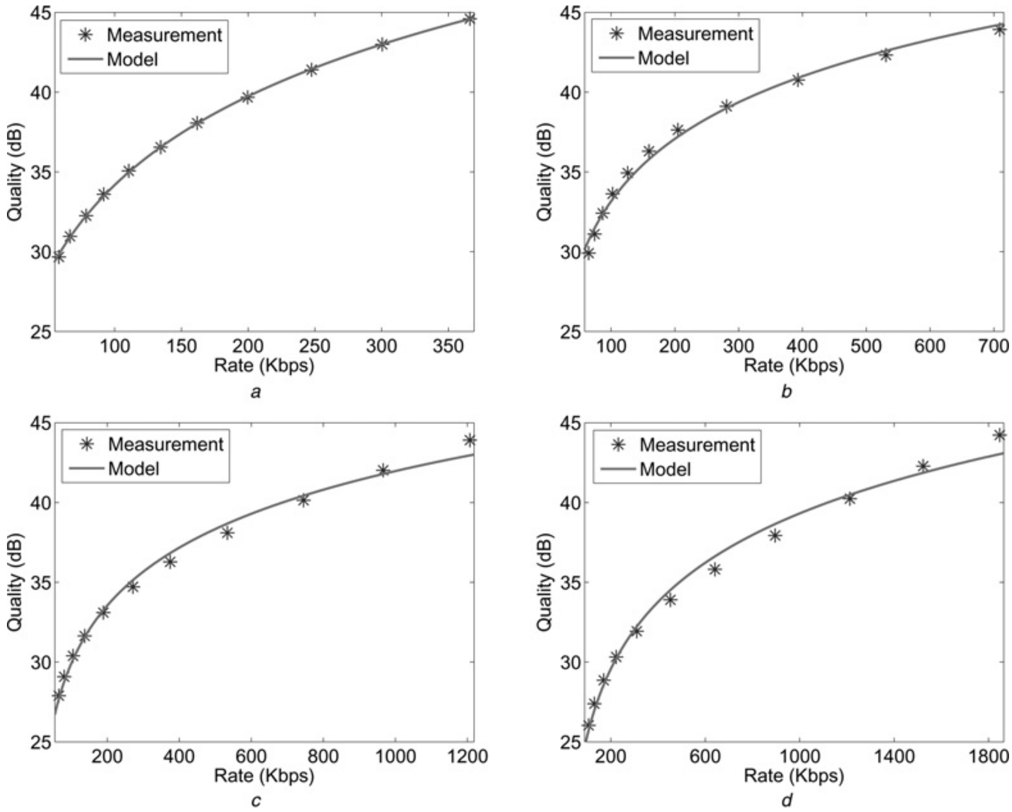

Figure 2 *Quality-rate curves for the test sequence*
a Silent
b Foreman
c Coastguard
d Mobile

By substituting (4) into (5), the utility function of user u_i becomes

$$U_i(R_i, \tau_i) = \alpha_i + \beta_i \ln R_i + \tau_i \tag{6}$$

We formulate the multiuser rate allocation problem as a game. In the game, there are N users/players, who share the available network bandwidth with each other. Each user u_i has his/her own utility function as shown in (6), and he/she also has a lowest desired quality constraint (lowest rate constraint R_i^{L}) and a highest satisfied quality constraint (highest rate constraint R_i^{H}). A user who requests to join in the game must report the R_i^{L} and R_i^{H} to the controller. The controller judges whether it has enough bandwidth available to accommodate a user's service, and then either accepts or rejects a user's request. Let $R_{\mathrm{sum}}^{\mathrm{L}}$ be the sum of the lowest rate constraints for all the users, which is calculated by $R_{\mathrm{sum}}^{\mathrm{L}} = \sum_{i=1}^{N} R_i^{\mathrm{L}}$, and $R_{\mathrm{sum}}^{\mathrm{H}}$ be the sum of the highest rate constraints for all the users, which is calculated by $R_{\mathrm{sum}}^{\mathrm{H}} = \sum_{i=1}^{N} R_i^{\mathrm{H}}$. Let R be the available bandwidth. If $R < R_{\mathrm{sum}}^{\mathrm{L}}$, it is impossible for the controller to afford all the users with an acceptable level of bandwidth for their services. In this case, the controller will reject some users' requests to guarantee that R is no less than a recalculated $R_{\mathrm{sum}}^{\mathrm{L}}$ for all the accepted users, therefore each user who joins in the game is guaranteed for a rate that is no less than R_i^{L}. If $R \geq R_{\mathrm{sum}}^{\mathrm{H}}$, the rate allocation problem is trivial since the controller just needs to allocate R_i^{H} to each user. If $R_{\mathrm{sum}}^{\mathrm{L}} \leq R < R_{\mathrm{sum}}^{\mathrm{H}}$, the available bandwidth is not able to satisfy all the users with R_i^{H} and the rate allocation problem becomes a challenge in this case. For this reason, we only consider the case of $R_{\mathrm{sum}}^{\mathrm{L}} \leq R < R_{\mathrm{sum}}^{\mathrm{H}}$ in this paper. The goal of the rate allocation game is to maximise the sum of the users' utilities subject to the constraint that the sum of the users' rate is not more than the available bandwidth. Therefore the game can be formulated as

$$\begin{aligned} &\max_{R_i,\tau_i} \sum_{i=1}^{N} U_i(R_i, \tau_i) = \sum_{i=1}^{N} (\alpha_i + \beta_i \ln R_i + \tau_i) \\ &\text{s.t.} \quad R_i^L \leq R_i \leq R_i^H, \quad \forall i = 1, 2, \ldots, N \\ &\sum_{i=1}^{N} R_i \leq R \end{aligned} \tag{7}$$

where R is the available network bandwidth. At the same time, from the users' point of view, they try to maximise their own utilities. Therefore a mechanism is expected to generate the rate allocation that satisfies (7), as well as maximise each user's utility.

doi: 10.1049/iet-com.2010.0173

3 Mechanism design for rate allocation

Note that while both [9] and this paper propose a game-theoretic framework to cope with the rate allocation problem in multiuser multimedia communication over a shared channel/link, there are two main differences. First, this paper proposes a Q-R model to model the characteristics of a video sequence, which is more accurate than that used in [9]. Second, Chen *et al.* [9] assumes that the users are 'price-takers' that is, the users accept the price announced by the controller and does not consider the effects of their actions on the price, whereas this paper explicitly considers the strategic behaviours of the users, and adopts a pricing mechanism that solves the problem in (7) using Nash equilibrium messages. In this section, we present the desired mechanism properties in Section 3.1. In Section 3.2, we describe the pricing mechanism. In Section 3.3, we discuss the overhead issue of the message exchanges.

3.1 Desired mechanism properties

To prevent users from untruthfully declaring their rate requirements, mechanism design introduces the concept of taxation, which is referred to as the currency and denoted by τ_i for user i. The tax integrated into each user's utility is designed to penalise the user by increasing his/her tax. Since each user aims to maximise his/her utility, the employment of tax effectively deters manipulation by users. Importantly, the actual value of a tax depends on the deployed mechanism.

Formally, a rate allocation mechanism formulates the mapping function $f(\cdot)$, the messages of the users $\boldsymbol{m}^u = (m_1^u, \ldots, m_N^u)$, the messages of the controller $\boldsymbol{m}^c = (m_1^c, \ldots, m_N^c)$ such that $f: (\boldsymbol{m}^u, \boldsymbol{m}^c) \mapsto (\boldsymbol{R}^*, \boldsymbol{\tau}^*)$, where $\boldsymbol{R}^* = (R_1^*, \ldots, R_N^*)$ and $\boldsymbol{\tau}^* = (\tau_1^*, \ldots, \tau_N^*)$ are the allocated rates and taxes, which achieve the properties designed by the designer. The mechanism framework can be deployed for video rate allocation as shown in Fig. 3. In the framework, the controller and the users iteratively exchange the messages until the message equilibrium is reached. The message exchange process will be described in detail in Section 3.2. After reaching the equilibrium, the controller assigns user u_i the rate R_i^* and charges him/her the tax τ_i^*. Lastly, each user u_i compresses his/her video sequence at the target rate R_i^* and transmits the compressed bitstream to the corresponding receiver.

Before we proceed to describe some of the desired properties of mechanisms, we present the Nash equilibrium concept generally used in a mechanism design.

Nash equilibrium: A message profile $\boldsymbol{m}^u \in \boldsymbol{M} = (M_1, \ldots, M_N)$ is said to be a Nash equilibrium message if

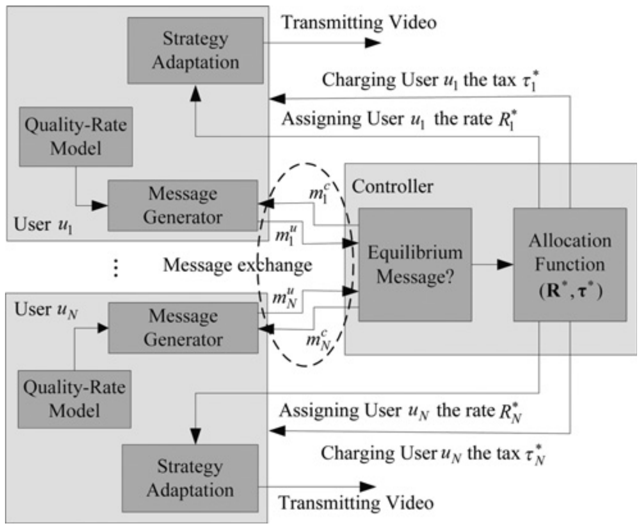

Figure 3 *Mechanism framework for the rate allocation in multiuser multimedia communications*

for every user i

$$U_i([f(m^c, (m_i^u, \boldsymbol{m}_{-i}^u))]_i) \geq U_i([f(m^c, (m_i^{u'}, \boldsymbol{m}_{-i}^u))]_i),$$
$$\forall m_i^u, m_i^{u'} \in M_i, \boldsymbol{m}_{-i}^u \in \prod_{j=1, j\neq i}^{N} M_j \quad (8)$$

where $\boldsymbol{m}_{-i}^u = (m_1^u, \ldots, m_{i-1}^u, m_{i+1}^u, \ldots, m_N^u)$ and M_i is the feasible message space for user i. In words, every user i receives an allocation that maximises his/her own utility when transmitting message m_i^u over any other possible message $m_i^{u'}$, given that the messages of other users are unchanged.

For the rate allocation problem considered in this paper, there are several properties that a mechanism design should possess.

Voluntary participation: A feasible allocation vector $(\boldsymbol{R}, \boldsymbol{\tau}) \in S$ is said to satisfy the property of voluntary participation if

$$U_i(R_i, \tau_i) > 0 \quad \forall i = 1, 2, \ldots, N \quad (9)$$

where $S = \{(\boldsymbol{R}, \boldsymbol{\tau}) | R_i^L \leq R_i \leq R_i^H, \sum_{i=1}^N R_i \leq R, \forall i = 1, \ldots, N\}$.

The voluntary participation of $(\boldsymbol{R}, \boldsymbol{\tau})$ can be interpreted as follows: after the rate allocation process, all users can be better than before in terms of the gained utilities.

Utility-maximising: A feasible allocation vector $(\boldsymbol{R}, \boldsymbol{\tau}) \in S$ is said to be utility-maximising if it satisfies

$$\sum_{i=1}^{N} U_i(R_i, \tau_i) \geq \sum_{i=1}^{N} U_i(R'_i, \tau'_i), \quad \forall(\boldsymbol{R}', \tau') \in S \quad (10)$$

doi: 10.1049/iet-com.2010.0173

Utility-maximising allocations are also called 'efficient' allocations [21].

Quality-maximising: A feasible allocation vector $(\boldsymbol{R}, \tau) \in S$ is said to be quality-maximising if it satisfies

$$\sum_{i=1}^{N} Q_i(R_i) \geq \sum_{i=1}^{N} Q_i(R'_i), \quad \forall(\boldsymbol{R}', \boldsymbol{\tau}') \in S \tag{11}$$

Budget balance: A feasible allocation vector $(\boldsymbol{R}, \tau) \in S$ is said to be budget balanced if it satisfies

$$\sum_{i=1}^{N} \tau_i = 0 \tag{12}$$

The property of budget balance states that the net value of the tax imposed by the controller for all users is zero.

3.2 Pricing mechanism implemented in nash equilibrium

In this subsection, we present a pricing mechanism that implements the problem in (7) using Nash equilibrium messages. The pricing mechanism [11, 22] is implemented in the considered multiuser rate allocation in three steps. Functionalities of all steps are listed in a flowchart as shown in Fig. 4, and we explicate the details of each step in what follows.

Step 1. Initialisation: At the first step of the mechanism, the controller determines an initial rate allocation R_i^0 and initial price p_i^0 for each user, where $R_i^0 = (R/R_{\text{sum}}^{\text{L}}) \cdot R_i^{\text{L}}$, $p_i^0 = 0$, $\forall i = 1, \ldots, N$.

Step 2. Message exchange: Note that at the first step of the mechanism, some users will have been assigned a rate larger than the optimal rate, whereas others will have been assigned a rate that is too small. At this step of the mechanism, the controller and the users iteratively exchange the pricing messages so as to guide each user to achieve the desirable amount of rate. The step is repeated until the message equilibrium is reached:

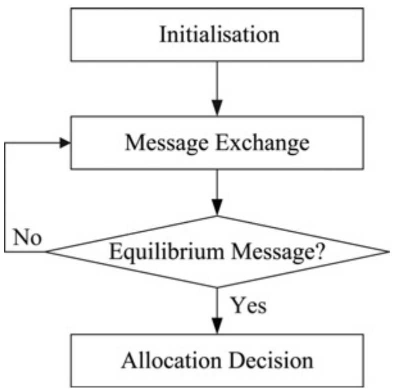

Figure 4 *Flowchart of the pricing mechanism implementation*

3.2.1 Messages of the controller: The controller conveys two variable, $m_i^c = (p_{-i}, d_i)$, to each user i as the message. Let p_i represents user i's valuation of the bandwidth. For the first iteration, we let $R_i = R_i^0$ and $p_i = p_i^0$. The messages of the controller $m_i^c = (p_{-i}, d_i)$ is defined by

$$p_{-i} = \frac{1}{N-1}\sum_{\substack{j=1 \\ j \neq i}}^{N} p_j \tag{13}$$

which represents the average price per unit of bandwidth from the other users and

$$d_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} R_j - R \tag{14}$$

is the excess bandwidth demand excluding the demand from user i. The messages from the controller are used by users to compute their tax.

3.2.2 Messages of the users: Given the message $m_i^c = (p_{-i}, d_i)$ transmitted by the controller, each user i maximises his/her utility by solving the following rate allocation problem

$$\begin{aligned} \max_{R_i, p_i} U_i(R_i, \tau_i) &= \max_{R_i, p_i}\{Q_i(R_i) + \tau_i(R_i, p_i, m_i^{\text{c}})\} \\ \text{s.t.} \quad & R_i^L \leq R_i \leq R_i^H \\ & \sum_{i=1}^{N} R_i \leq R \end{aligned} \tag{15}$$

The tax function in (15) is defined by

$$\begin{aligned} \tau_i(R_i, p_i, m_i^{\text{c}}) = &-(R_i - R_i^0)p_{-i} \\ &- \left[p_i - p_{-i}\left(1 + \frac{d_i + R_i}{R}\right) - \chi_+(d_i, R_i)\right]^2 \end{aligned} \tag{16}$$

where

$$\chi_+(d_i, R_i) = \max\left\{0, \frac{d_i + R_i}{R}\right\} \tag{17}$$

The first term in (16) represents the amount of currency user i pays/earns for buying/selling $(R_i - R_i^0)$ amount of bandwidth from/to the other users. The second term is the penalty that user i pays because of the mismatch of its price to the average price of the other users p_{-i}. The third term $\chi_+(\cdot)$ in (16) is introduced to prevent the solution from reaching an inefficient equilibrium such as (i) $p_i = 0$ for all the users and (ii) the total demand exceeds the available bandwidth, that is, $\sum_{i=1}^{N} R_i > R$.

IET Commun., 2011, Vol. 5, Iss. 3, pp. 396–407

doi: 10.1049/iet-com.2010.0173

By substituting (4) and (16) into (15), the optimisation problem can be written as

$$\max_{R_i,p_i} U_i(R_i, \tau_i) = \max_{R_i,p_i}\left\{\alpha_i + \beta_i \ln R_i - (R_i - R_i^0) \times p_{-i} - \left[p_i - p_{-i}\left(1 + \frac{d_i + R_i}{R}\right) - \chi_+(d_i, R_i)\right]^2\right\} \quad (18)$$

The optimisation in (18) can be decomposed into two subproblems by solving R_i and p_i independently

$$\max_{R_i}\{\alpha_i + \beta_i \ln R_i - (R_i - R_i^0) \times p_{-i}\} \quad (19)$$

and

$$p_i = p_{-i}\left(1 + \frac{d_i + R_i}{R}\right) + \chi_+(d_i, R_i) \quad (20)$$

Under the first constraint condition in (15), (19) is maximised at R_i, where

$$R_i = \max\left[R_i^{\mathrm{L}},\ \min\left(\frac{\beta_i}{p_{-i}}, R_i^{\mathrm{H}}\right)\right] \quad (21)$$

User i then submits his/her message to the controller, that is, $m_i^u = (R_i, p_i)$.

Step 3. Allocation decision: After reaching the equilibrium, the controller allocates the bandwidth based on the equilibrium message $(\boldsymbol{R}^*, \boldsymbol{p}^*)$, where $\boldsymbol{R}^* = (R_1^*, \ldots, R_N^*)$ and $\boldsymbol{p}^* = (p_1^*, \ldots, p_N^*)$. At equilibrium, the sum of every user's R_i^* equals to R and the squared term in (16) becomes zero. Hence, the tax for user i calculated by the controller is

$$\tau_i^*(R_i^*, p_i^*) = -(R_i^* - R_i^0) \times p_{-i}^* \quad (22)$$

The following theorem proves that even in the case when the users behave strategically, the allocations generated by the mechanism presented above are utility-maximising and budget balanced.

Theorem 1: The mechanism implemented in steps 1–3 generates the allocations in Nash equilibrium for multiuser video rate allocation problem in (7), and satisfies the property of budget balance.

Proof: See Appendix.

Lemma 1: The mechanism implemented in steps 1–3 satisfies voluntary participation.

Proof: Given the message (p_{-i}, d_i), user i maximises his/her own utility. To show that the allocation satisfies voluntary participation, we only need to show the maximum utility for user i is larger than 0.

$$\begin{aligned}
\max_{R_i,p_i} U_i(R_i, \tau_i) &= \max_{R_i,p_i}\left\{\alpha_i + \beta_i \ln R_i - (R_i - R_i^0) \times p_{-i} - \left[p_i - p_{-i}\left(1 + \frac{d_i + R_i}{R}\right) - \chi_+(d_i, R_i)\right]^2\right\} \\
&\geq \max_{p_i}\left\{\alpha_i + \beta_i \ln R_i^{\mathrm{L}} - (R_i^{\mathrm{L}} - R_i^0) \times p_{-i} - \left[p_i - p_{-i}\left(1 + \frac{d_i + R_i^{\mathrm{L}}}{R}\right) - \chi_+(d_i, R_i^{\mathrm{L}})\right]^2\right\} \\
&= \underbrace{\alpha_i + \beta_i \ln R_i^{\mathrm{L}}}_{>0} - \underbrace{(R_i^{\mathrm{L}} - R_i^0) \times p_{-i}}_{\leq 0} - \underbrace{\min_{p_i}[p_i - p_{-i}\left(1 + \frac{d_i + R_i^{\mathrm{L}}}{R}\right) - \chi_+(d_i, R_i^{\mathrm{L}})]^2}_{=0} \\
&> 0
\end{aligned} \quad (23)$$

Since the above inequalities hold for any given messages (p_{-i}, d_i), the allocations generated by the mechanism satisfy the voluntary participation property.

Lemma 2: The mechanism implemented in steps 1–3 satisfies the quality-maximising property.

Proof: For user i, given (R_i, τ_i), from (5) we have

$$Q_i(R_i) = U_i(R_i, \tau_i) - \tau_i \quad (24)$$

By summing (24) over all i, at the Nash equilibrium $(\boldsymbol{R}^*, \boldsymbol{\tau}^*)$ we have

$$\begin{aligned}
\sum_{i=1}^{N} Q_i(R_i^*) &= \sum_{i=1}^{N} (U_i(R_i^*, \tau_i^*) - \tau_i^*) \\
&= \sum_{i=1}^{N} U_i(R_i^*, \tau_i^*) - \sum_{i=1}^{N} \tau_i^* \\
&= \sum_{i=1}^{N} U_i(R_i^*, \tau_i^*)
\end{aligned} \quad (25)$$

where the third line of (25) follows from Theorem 1, the property of budget balances. In addition, from Theorem 1 we know that the sum of $U_i(R_i^*, \tau_i^*)$ over all i is maximising. This proves that the allocation generated by the mechanism satisfies the quality-maximising property.

3.3 Message exchange overhead

The message exchange overhead introduces additional delay, which is undesirable in real-time video communications. Therefore we quantify the message exchange overhead as the number of parameters exchanged by the users and the controller per message exchange iteration. From Section 3.2, the controller conveys to user i the average price and excess demand of all users except i, that is, $m_i^c = (p_{-i}, d_i)$. User i solves the optimisation problem in (18) and transmits the bandwidth demand and his/her valuation of the bandwidth back to the controller, that is, $m_i^u = (R_i, p_i)$. Hence, the number of messages per iteration is $(2 + 2)N$. Note that the message exchange overhead of the mechanism is independent of the employed Q-R model, and only depends on the number of iterations to reach the equilibrium and on the number of video users present in the communication system.

4 Simulation results

To evaluate the proposed game-theoretic multiuser rate allocation framework, we conduct simulations on eight video sequences. They are: Foreman, Carphone, Akiyo, Coastguard, Silent, Mobile, Football and Flower in QCIF format. Note that these video sequences include slow, medium and fast motion, as well as smooth or complex scene. In this paper, we use state-of-the-art H.264 JM14.2 video codec to encode the video sequence. By changing the QP or using the rate control feature, we are able to compress the video sequences at different bit-rate and achieve different quality requirements.

4.1 Parameter estimation

From Section 2, we can see that there are several parameters in our framework, α_i, β_i, R_i^{L}, and R_i^{H}. We can estimate α_i and β_i using offline training. For each video sequence, we first generate a set of (Q_i, R_i) by encoding the sequence using H.264 JM14.2 with different QP. Then, the optimal α_i and β_i can be computed by curve fitting tools, where the curve is featured by (4), and the data set needs to be fitted are the set of (Q_i, R_i). We have shown the results of Silent, Foreman, Coastguard and Mobile in Fig. 2. From the figure we can see that, with the optimal α_i and β_i, (4) can approximate Q-R characteristics well. Similar results are observed for other sequences.

After finding the optimal α_i and β_i, we derive the values for R_i^{L} and R_i^{H}. As in [9], we suppose the lowest desired quality constraint Q^{L} is 30 dB, and the highest satisfied quality constraint Q^{H} is 45 dB. According to (4), we have

$$R_i^{\mathrm{L}} = \exp\left(\frac{Q^{\mathrm{L}} - \alpha_i}{\beta_i}\right)$$
$$R_i^{\mathrm{H}} = \exp\left(\frac{Q^{\mathrm{H}} - \alpha_i}{\beta_i}\right) \tag{26}$$

The α_i, β_i, R_i^{L} and R_i^{H} for different sequences are shown in Table 1. From Table 1, we can see that the R_i^{L} and R_i^{H} for a test sequence could be very different to those for another test sequence, depending on the characteristics of the test sequence. For example, the R_i^{L} and R_i^{H} for Akiyo are much smaller than those for football. The reason is that Akiyo has low motion and smooth scene, whereas football has fast motion and complex scene.

4.2 Multiuser rate allocation

We compare the proposed framework with the approach maximising the sum of the PSNRs (MSPSNR), that is, the traditional optimisation-based approach shown in (1), where PSNR(R_i) is expressed by (4). Note that for MSPSNR, the allocated rate should be within $[R_i^{\mathrm{L}}, R_i^{\mathrm{H}}]$. Otherwise, we set it to be R_i^{L} or R_i^{H} and re-allocate the rest rate for other users. The proposed framework and MSPSNR are implemented by using Matlab. Given the video sequences to be transmitted, the available bandwidth R, the controller can figure out the rate allocated to each video sequence by using MSPSNR and the proposed

Table 1 α_i, β_i, R_i^{L} and R_i^{H} for different video sequences

Sequence	α_i	β_i	R_i^{L}, kbps	R_i^{H}, Kbps
Foreman	7.1390	5.6500	57.1793	813.2653
Carphone	6.7610	6.2600	40.9478	449.6468
Akiyo	−6.4310	10.4800	32.3378	135.3055
Coastguard	5.7910	5.2370	101.7666	1784.5382
Silent	−2.7590	8.0200	59.4218	385.6706
Mobile	−2.5110	6.0550	214.7078	2556.9433
football	−8.7750	7.0790	239.2394	1991.0289
flower	−5.5640	6.8390	181.3034	1625.3421

IET Commun., 2011, Vol. 5, Iss. 3, pp. 396–407
doi: 10.1049/iet-com.2010.0173

framework. Then, setting the allocated bit-rate as the target bit-rate, we compress the video sequence using the rate control feature in H.264 reference software JM14.2. Finally, each user transmits the compressed bitstream to the corresponding receiver. In the simulations, we consider eight users, $u_1, \ldots, u_8$. They transmit Foreman, Carphone, Akiyo, Coastguard, Silent, Mobile, football and flower to eight receivers, $r_1, \ldots, r_8$, respectively.

First, we verify the properties of the proposed framework (a) are quality-maximising, (b) are budget balanced, (c) are utility-maximising and (d) satisfy voluntary participation. We set R to be 3000 kbps. Table 2 shows the rate allocations and video qualities for MSPSNR, and the rate allocations, prices, taxes, utilities and video qualities for the proposed framework. Since video quality is a concave function, MSPSNR will generate the global optimal rate allocation, that is, be quality-maximising. From Table 2 we can see that, the sum of the video qualities generated by the proposed framework is equivalent to that generated by MSPSNR, which verifies that the allocations generated by the proposed framework are quality-maximising. We also see that the sum of the taxes of all the users equals to zero, which verifies the allocations generated by the proposed framework are budget balanced. Since the allocations are both quality-maximising and budget balanced, they are also utility-maximising. Finally, since all the users' utilities are positive, we conclude that the users voluntarily participate the game.

Next, we evaluate the impact of selfish behaviour of users. Note that there are two parameters in the Q-R model in (4): α and β. Given α, if we increase β, the quality will increase at the same bit-rate R. Therefore a selfish user may lie about his/her Q-R model by increasing β, so as to cheat the controller into allocating more rate to him/her. In this simulation, the available network bandwidth is set to be 2000 kbps.

In MSPSNR, we consider two cases: (i) all the users truthfully report their Q-R model; (ii) user 1 who transmits Foreman sequence lie about his/her Q-R model by using $\tilde{\beta} = 2\beta$, whereas other users are honest. Table 3 shows the rate allocations and video qualities for MSPSNR in the two cases. From Table 3 we can see that, by lying about his/her Q-R model, user 1 increases his/her final video quality more than 3 dB, whereas decreases the performances of all the other users except user 3 and also decreases the performance of the whole system 1.3870 dB. In this case, the selfish behaviour of user 1 does not incur any penalty for him/her. This verifies that MSPSNR depends on users to truthfully report their Q-R models, otherwise the performance of the entire system will degrade. Unfortunately, since a user can benefit his/her final video quality by lying about his/her Q-R model while without any penalty for him/her, MSPSNR has no means to deter users from lying about their Q-R models.

To evaluate our proposed framework, we also simulate two cases: (a) all the users strategically submit their rate demands and corresponding prices by maximising their utilities at each iteration; (b) user 1 lies about his/her Q-R model by using $\tilde{\beta} = 2\beta$, and submits his/her rate demand and corresponding price by maximising his/her fake utility at each iteration, and other users behave as in case (a). Table 4 shows the rate allocations, utilities and video qualities for the proposed framework in the two cases. From Table 4 we note that, although user 1 increases his/her final video quality more than 3 dB, the final utility of user 1 is actually reduced from 35.2837 to 33.2575. This is because user 1 does not submit the optimal rate demand at each iteration, he/she has to pay a much higher tax than when he/she strategically responds. In other words, our proposed framework enforces all the users to truthfully report their optimal rate demands and corresponding prices

Table 2 Rate allocation and video qualities of MSPSNR, and rate allocations, prices, taxes, utilities and video qualities of the proposed framework

	MSPSNR		Proposed framework				
	Allocated rate R_i, kbps	Video quality, dB	Allocated rate R_i, kbps	Price p_{-i}	Tax τ_i	Utility U_i	Video quality, dB
user u_1	377.4690	40.6632	377.3299	0.0150	−2.8789	37.7822	40.6611
user u_2	418.4206	44.5494	418.0681	0.0150	−4.2755	40.2686	44.5441
user u_3	135.3055	45.0000	135.3055	0.0150	−0.4588	44.5412	45.0000
user u_4	349.3571	36.4594	349.7481	0.0150	−0.3050	36.1602	36.4652
user u_5	385.6706	45.0000	385.6706	0.0150	−2.8951	42.1049	45.0000
user u_6	404.7330	33.8385	404.3774	0.0150	4.3505	38.1837	33.8332
user u_7	472.3656	34.8157	472.7643	0.0150	4.5154	39.3371	34.8217
user u_8	456.6787	36.3179	456.7361	0.0150	1.9476	38.2663	36.3188
summation	3000.0000	316.6441	3000.0000	–	0.0000	316.6441	316.6441

Table 3 Rate allocations and video qualities of MSPSNR when users truthfully report their Q-R models or not

	Users truthfully reporting their Q-R models		User 1 lying but other users truthfully reporting their Q-R models	
	Allocated rate R_i, kbps	Video quality, dB	Allocated rate R_i, kbps	Video quality, dB
user u_1	233.3794	37.9466	413.3739	41.1766
user u_2	258.8440	41.5430	229.9832	40.8029
user u_3	135.3055	45.0000	135.3055	45.0000
user u_4	215.9815	33.9409	191.0440	33.2984
user u_5	330.8050	43.7693	295.5834	42.8664
user u_6	250.3168	30.9291	222.2286	30.2084
user u_7	292.6056	31.4254	260.7254	30.6088
user u_8	282.7623	33.0395	251.7559	32.2451

Table 4 Rate allocations, utilities and video qualities of the proposed framework when users strategically play the game or not

	Users strategically playing the game			User 1 lying about his/her Q-R model but other users strategically playing the game		
	Allocated rate R_i, kbps	Utility U_i	Video quality, dB	Allocated rate R_i, kbps	Utility U_i	Video quality, dB
user u_1	233.3966	35.2837	37.9470	414.8661	33.2575	41.1970
user u_2	258.5952	37.4158	41.5370	229.8285	36.9453	40.7987
user u_3	135.3055	43.4136	45.0000	135.3055	43.2151	45.0000
user u_4	216.3360	34.0281	33.9495	192.2702	34.0758	33.3319
user u_5	331.2993	38.8650	43.7812	294.4448	38.3077	42.8354
user u_6	250.1268	36.0845	30.9245	222.3021	36.7741	30.2104
user u_7	292.4274	36.8384	31.4211	259.8971	37.5677	30.5863
user u_8	282.5132	35.6645	33.0334	251.0858	36.0434	32.2269

at each iteration. Our proposed framework also penalises the selfish users lying about their rate demands by imposing higher taxes.

Finally, we access the prices and convergence rates of the proposed framework at different available bandwidth. We test R at 1000, 2000, 3000, 4000 and 5000 kbps. The prices and convergence rates of the proposed framework are shown in Table 5. From the table we can see that, the price increases as the available bandwidth decreases. This matches the intuition that the scarcer the bandwidth, the higher the price. We also notice that the numbers of iterations at all bandwidth shown in Table 5 except 1000 kbps are around 20, which shows that the convergence rate of the proposed framework is quite fast. We find a larger number of iterations is needed to reach the equilibrium at 1000 kbps. Note that 1000 kbps is close to the $R^{\mathrm{L}}_{\mathrm{sum}}$, which equals to 926.9039 kbps (calculated from Table 1). Generally speaking, as the price increases, a user will reduce the rate demand so as to maximise his/her utility. On the other hand, the rate demand of a user is assumed always no less than his/her lowest rate constraint R_i^{L}, no matter what the price is. In other words, if a number of users reach R_i^{L} during iterations, increasing the price will only affect the remaining users' rate demands,

Table 5 Prices and convergence rates of the proposed framework at different available bandwidth

Bandwidth, kbps	Price	The number of iterations
1000	0.1203	128
2000	0.0242	17
3000	0.0150	18
4000	0.0102	20
5000	0.0077	18

IET Commun., 2011, Vol. 5, Iss. 3, pp. 396–407
doi: 10.1049/iet-com.2010.0173

therefore a larger number of iterations will be needed to reach the equilibrium. We find that in the case of 1000 kbps, five out of eight users reach R_i^L during iterations, that is why it takes a larger number of iterations to reach the equilibrium.

5 Conclusions

In this paper, we propose a game-theoretic framework for multiuser multimedia rate allocation, while explicitly considering the strategic behaviours of the users. This framework is based on mechanism design to deter the users from manipulating the rate allocation. By modelling the Q-R functions of video sequences, the proposed framework takes into account the unique characteristics of individual users. The framework allows the users to exchange with the controller a limited number of messages to reach Nash equilibrium. The resulting Nash equilibrium messages generate the optimal rate allocations. Our simulations verify that the allocations generated by the framework are efficient, budget balanced and satisfy voluntary participation. Our results show that the equilibrium price gracefully scales with varying network bandwidth conditions. The convergence rate is also discussed.

In our next step of this research, we shall extend the framework proposed in this paper to cope with the rate allocation problem in multiuser multimedia over more complicated topologies with multiple bottlenecks, cross traffic etc.

6 Acknowledgments

This work is supported in part by the National 973 project no. 2007CB311002, the National Natural Science Foundation of China (NSFC) project no. 60572045 and the Ministry of Education of China PhD Program Foundation project no. 20050698033.

7 References

[1] ORTEGA A., RAMCHANDRAN K.: 'Rate-distortion methods for image and video compression', *IEEE Signal Process. Mag.*, 1998, **15**, (6), pp. 25–50

[2] VAN DER SCHAAR M., ANDREOPOULOS Y., HU Z.: 'Optimized scalable video streaming over IEEE 802.11a/e HCCA wireless networks under delay constrainsts', *IEEE Trans. Mob. Comput.*, 2006, **5**, (6), pp. 755–768

[3] MALA A., EL-KADI M., OLARIU S., TODOROVA P.: 'A fair resource allocation protocol for multimedia wireless networks', *IEEE Trans. Parallel Distrib. Syst.*, 2003, **14**, (1), pp. 63–71

[4] HAN Z., JI Z., LIU K.J.R.: 'Fair multiuser channel allocation for OFDMA netowrks using nash bargaining solutions and coalitions', *IEEE Trans. Commun.*, 2005, **53**, (8), pp. 1366–1376

[5] PARK H., VAN DER SCHAAR M.: 'Bargaining strategies for networked multimedia resource management', *IEEE Trans. Signal Process.*, 2007, **55**, (7), pp. 3496–3511

[6] LUCKY R.W.: 'Tragedy of the commons', *IEEE Spectrum*, 2006, **43**, (1), p. 88

[7] KELLY F.: 'Charging and rate control for elastic traffic', *Eur. Trans. Telecommun.*, 1997, **8**, (1), pp. 33–37

[8] KELLY F., MAULLOO A., TAN D.: 'Rate control for communication networks: shadow prices, proportional fairness and stability', *Oper. Res. Soc.*, 1998, **49**, pp. 237–252

[9] CHEN Y., WANG B., LIU K.J.R.: 'Multiuser rate allocation games for multimedia communications', *IEEE Trans. Multimed.*, 2009, **11**, (6), pp. 1170–1181

[10] HAJEC B., YANG S.: 'Strategic buyers in a sum bid game for flat networks'. IMA Workshop 6: Control and Pricing in Communications and Power Networks, 2004

[11] STOENESCU T., LEDYARD J.: 'A pricing mechanism which implements a network rate allocation problem in Nash equilibria'. Proc. 45th IEEE Conf. Decision and Control, 2006

[12] DING W., LIU B.: 'Rate control of MPEG video coding and recording by rate-quantization modeling', *IEEE Trans. Circuits Syst. Video Technol.*, 1996, **6**, (1), pp. 12–20

[13] HANG H.M., CHEN J.J.: 'Source model for transform video coder and its application – Part i: fundamental theory', *IEEE Trans. Circuits Syst. Video Technol.*, 1997, **7**, (2), pp. 287–298

[14] CHIANG T., ZHANG Y.-Q.: 'A new rate control scheme using quadratic rate distortion model', *IEEE Trans. Circuits Syst. Video Technol.*, 1997, **7**, (1), pp. 246–250

[15] CORBERA J.R., LEI S.: 'Rate control in DCT video coding for low-delay communications', *IEEE Trans. Circuits Syst. Video Technol.*, 1999, **9**, (1), pp. 172–185

[16] CHEN Y., AU O.C.: 'Simultaneous RD-optimized rate control and video de-noising'. Proc. IEEE Int. Conf. Acoustics, Speech and Signal Process., 2008

[17] LIU Y., LI Z.G., SOH Y.C.: 'A novel rate control scheme for low delay video communication of H.264/AVC standard', *IEEE Trans. Circuits Syst. Video Technol.*, 2007, **17**, (1), pp. 68–78

[18] LI Z.G., PAN F., LIM K.P., RAHARDJA S.: 'Adaptive rate control for H.264'. Proc. IEEE Int. Conf. Image Process., 2004

[19] STUHLMULLER K., FARBER N., LINK M., GIROD B.: 'Analysis of video transmission over lossy channels', *IEEE J. Sel. Areas Commun.*, 2000, **18**, (6), pp. 1012–1032

doi: 10.1049/iet-com.2010.0173

[20] JVT Codec Reference Software. http://iphome.hhi.de/suehring/tml/download

[21] JACKSON M.: 'Mechanism theory' (Optimization and Operations Research in the Encyclopedia of Life Support Systems, Oxford, UK, 2003)

[22] FU F., STOENESCU T.M., VAN DER SCHAAR M.: 'A pricing mechanism for resource allocation in wireless multimedia applications', *IEEE J. Sel. Topics Signal Process.*, 2007, **1**, (2), pp. 264–279

[23] BOYD S., VANDENBERGHE L.: 'Convex optimization' (Cambridge Univ. Press, Cambridge, UK, 2004)

8 Appendix

In proving this theorem we proceed as follows. First, we present the necessary and sufficient conditions for the utility-maximising allocations of the problem in (7). Then we show that the Nash equilibria of mechanism presented in Section 3.2 satisfy the utility-maximising conditions. Finally, we show that the Nash equilibrium allocations are budget balanced. In order to determine an optimal solution of problem (7) we first write the Lagrangian function

$$L(R_i, \lambda, \kappa_i, \upsilon_i) = \sum_{i=1}^{N} (\alpha_i + \beta_i \ln R_i + \tau_i) - \lambda\left(\sum_{i=1}^{N} R_i - R\right) - \sum_{i=1}^{N} \kappa_i(R_i - R_i^{\mathrm{H}}) - \sum_{i=1}^{N} \upsilon_i(R_i^{\mathrm{L}} - R_i) \quad (27)$$

At an optimal allocation $(\boldsymbol{R}^*, \boldsymbol{\tau}^*) = (R_i^*, \tau_i^*)_{i=1,\ldots,N}$, the necessary and sufficient Karush–Kuhn–Tuker (KKT) conditions for optimality [23] are

$$\begin{aligned} &\frac{\beta_i}{R_i} - \lambda - \kappa_i + \upsilon_i = 0 \\ &\lambda\left(\sum_{i=1}^{N} R_i - R\right) = 0 \\ &\kappa_i(R_i - R_i^{\mathrm{H}}) = 0, \quad \forall i = 1, \ldots, N \\ &\upsilon_i(R_i^{\mathrm{L}} - R_i) = 0, \quad \forall i = 1, \ldots, N \\ &R_i^{\mathrm{L}} \le R_i \le R_i^{\mathrm{H}}, \quad \forall i = 1, \ldots, N \\ &\sum_{i=1}^{N} R_i \le R \\ &\lambda \ge 0, \kappa_i \ge 0, \upsilon_i \ge 0, \quad \forall i = 1, \ldots, N \end{aligned} \quad (28)$$

By solving the KKT conditions above, the optimal solution is

$$R_i = \max\left[R_i^{\mathrm{L}}, \min\left(\frac{\beta_i}{\lambda}, R_i^{\mathrm{H}}\right)\right] \quad (29)$$

where

$$\sum_{i=1}^{N} R_i = R \quad (30)$$

In order to show that the mechanism implements in Nash equilibria the rate allocation in (7), we need to show that the Nash allocations satisfy (29) and (30). Note that, given a fixed (p_{-i}, d_i), user i chooses R_i and p_i such that his/her individual utility function is maximised

$$\max_{R_i, p_i}\left\{\alpha_i + \beta_i \ln R_i - (R_i - R_i^0) \times p_{-i} - \left[p_i - p_{-i}\left(1 + \frac{d_i + R_i}{R}\right) - \chi_+(d_i, R_i)\right]^2\right\} \quad (31)$$

Equation (31) can be decomposed into two subproblems by solving R_i and p_i independently

$$\max_{R_i}\{\alpha_i + \beta_i \ln R_i - (R_i - R_i^0) \times p_{-i}\} \quad (32)$$

and

$$p_i = p_{-i}\left(1 + \frac{d_i + R_i}{R}\right) + \chi_+(d_i, R_i) \quad (33)$$

To solve (32), by defining $f(R_i)$ as

$$f(R_i) = \alpha_i + \beta_i \ln R_i - (R_i - R_i^0) \times p_{-i} \quad (34)$$

and taking the derivative of it over R_i, we have

$$\frac{\partial f(R_i)}{\partial R_i} = \frac{\beta_i}{R_i} - p_{-i} \quad (35)$$

Therefore $f(R_i)$ is maximised at R_i, where

$$R_i = \max\left[R_i^{\mathrm{L}}, \min\left(\frac{\beta_i}{p_{-i}}, R_i^{\mathrm{H}}\right)\right] \quad (36)$$

At a Nash equilibrium message, (31) is maximised for each user $i = 1, \ldots, N$, therefore both (36) and (33) are satisfied. Assume that $(\boldsymbol{R}^*, \boldsymbol{p}^*)$ is a Nash equilibrium message, then we have

$$R_i^* = \max\left[R_i^{\mathrm{L}}, \min\left(\frac{\beta_i}{p_{-i}^*}, R_i^{\mathrm{H}}\right)\right] \quad (37)$$

$$p_i^* = p_{-i}^*\left(1 + \frac{d_i^* + R_i^*}{R}\right) + \chi_+(d_i^*, R_i^*) \quad (38)$$

doi: 10.1049/iet-com.2010.0173

where

$$p^*_{-i} = \frac{1}{N-1}\sum_{\substack{j=1\\ j\neq i}}^{N} p^*_j \tag{39}$$

$$d^*_i = \sum_{\substack{j=1\\ j\neq i}}^{N} R^*_j - R \tag{40}$$

and

$$\chi_+(d^*_i, R^*_i) = \max\left\{0, \frac{d^*_i + R^*_i}{R}\right\} \tag{41}$$

Note that all the available bandwidth should be fully utilised, otherwise the allocations are not efficient. Therefore $(\boldsymbol{R}^*, \boldsymbol{p}^*)$ should satisfied that

$$\sum_{i=1}^{N} R^*_i = R \tag{42}$$

With (40)–(42), we have

$$d^*_i + R^*_i = \sum_{i=1}^{N} R^*_i - R = 0 \tag{43}$$

and

$$\chi_+(d^*_i, R^*_i) = 0 \tag{44}$$

This along with (38) and the definition of p^*_{-i}, implies that at Nash equilibrium

$$p^*_i = p^*_{-i}, \quad \forall i = 1, \ldots, N \tag{45}$$

$$p^*_i = p^*_j = p, \quad \forall i, j = 1, \ldots, N \tag{46}$$

Substituting (45) and (46) into (37), we have that

$$R^*_i = \max\left[R^{\mathrm{L}}_i,\ \min\left(\frac{\beta_i}{p}, R^{\mathrm{H}}_i\right)\right] \tag{47}$$

By letting $p = \lambda$ and together with (42), (29) and (30) of the KKT conditions are satisfied. This proves that the mechanism presented in Section 3.2 implements in Nash equilibria, the rate allocation problem in (7).

Now, we show that the Nash equilibrium messages generate allocations which satisfy (12) (i.e. are budget balanced)

$$\begin{aligned}\sum_{i=1}^{N}\tau^* &= \sum_{i=1}^{N}\Bigg\{-(R^*_i - R^0_i)\times p^*_{-i}\\ &\quad -\left[p^*_i - p^*_{-i}\left(1+\frac{d^*_i + R_i*}{R}\right) - \chi_+(d^*_i, R^*_i)\right]^2\Bigg\}\\ &= \sum_{i=1}^{N}\Bigg\{-(R^*_i - R^0_i)\times p\\ &\quad -\left[p - p\left(1+\frac{d^*_i + R^*_i}{R}\right) - \chi_+(d^*_i, R^*_i)\right]^2\Bigg\}\\ &= \sum_{i=1}^{N}\{-(R^*_i - R^0_i)\times p - [p-p]^2\}\\ &= 0\end{aligned} \tag{48}$$

where the first line of (48) follows from (16), the second line follows from (45) and (46), and the third line follows from (43) and (44). This proves that the Nash equilibrium allocations are budget balanced.