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Multimedia approach in teaching mathematics – example of lesson about the definite integral application for determining an area

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This article presents the importance of using multimedia in the math classes by an example of multimedia lesson about definite integral and the results of the research carried out among the students of the first years of faculty, divided into two groups of 25. One group had the traditional lecture about the definite integral, while the other one had the multimedia method. The main information source in multimedia lectures was the software created in Macromedia Flash, with definitions, theorems, examples, tasks as well as in traditional lectures but with emphasized visualization possibilities, animations, illustrations, etc. Both groups were tested after the lectures. Students from the multimedia group showed better theoretical, practical and visual knowledge. Besides that, survey carried out at the end of this research clearly showed that students from multimedia group were highly interested in this way of learning.

Keywords: multimedia learning; multimedia lessons; definite integral; area

1. Introduction

Definite integral is one of the basic terms in mathematical analysis, and is therefore included in the lecture programmes in the majority of high schools and faculties, which makes this area one of the most important segments of mathematics teaching at all the levels of education.

In teaching mathematics, it is also important to avoid too much of the so-called 'knowledge/information adoption'. Students often solve problems mechanically, by following the algorithm steps without real awareness of their actual meaning. In the example of the definite integral, some of the common problems are the mechanical calculation of its value without understanding its definition, the meaning of upper and lower limits or relation between the definite integral and the size of an area or a volume, etc. One of the main aims in modern approach in teaching is also the knowledge transfer by giving the facts and explaining the relations between them.

Mathematics teachers show great interest in visualization of the mathematical terms and emphasize that visualized lectures are of great help in abstract thinking in mathematics [1,2]. Tall [3] believes that it is of the major importance to connect the

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existing pictures that students have on certain terms in order to develop them further and to enable students to accept further knowledge. Therefore, it is necessary to combine the picture method and the definition method in order to improve the existing knowledge and to enlarge it with the new facts, which is on of the points of the cognitive theory of multimedia learning [4,5].

Nowadays, usage of different kinds of multimedia is largely included in the education because it allows the wider spectrum of possibilities in teaching and learning. Visualization is very useful in the process of explaining mathematical ideas, abstract terms, theorems, problems, etc.

Experience in work with students showed that they are highly interested in modern methods in learning, which include all kinds of multimedia, such as educational software and internet.

Modern methods in multimedia approach to learning include the whole range of different possibilities applicable in mathematics lectures for different levels of education and with different levels of interactivities [6–13]. The authors usually work on suggestions on using different kinds of software in education, especially in the field of mathematics: geometry, algebra, numerical analyses, etc. [7–13], as well as the definite integrals [6].

All the above-mentioned resulted in an idea of making applicative software which would be helpful in a modern and more interesting approach to the field of teaching mathematics and raising the students' knowledge from the scope of definite integrals to a higher level. The aim of this article is to recognize the importance of multimedia in the teaching process as well as to examine the students' reaction to this way of learning and teaching.

2. Multimedia presentation of the area calculation

The problem of area calculation in the field of integration calculations led us directly to the definition of the definite integral. The basis for the defining and calculation of the definite integrals was made by Archimedes[1] and his quadrature of the parabola. Because of that, we decided to use the modern, multimedia approach in explaining Archimedes' quadrature of the parabola to the students. In order to use PC as a teaching aid, we were led by suggestions of Tall [14], who emphasized the importance of PC in the teaching because of its great possibilities in the scope of visual presentations.

The quadrature of the parabola problem is formulated as follows: *For any given parabola* $y = x^2$ *and rectangle with nodes* $A(0,0)$, $B(a,0)$, $C(a,a^2)$ *and* $D(0,a^2)$, $(a > 0)$ *the parabola divides the rectangle in two zones of which the area of one is twice bigger than the area of the other one*.

Given problem is shown in Figure 1.[2] The area of the rectangle is

$$P = a \cdot a^2 = a^3. \tag{1}$$

If we mark the zone under the parabola – limited by sides AB and BC of the rectangle and the arc AC – with S, and the other one with P, our next task is to prove the following equation:

$$S : (P - S) = 1 : 2, \text{ i.e. } S = \frac{1}{3}P = \frac{1}{3}a^3. \tag{2}$$

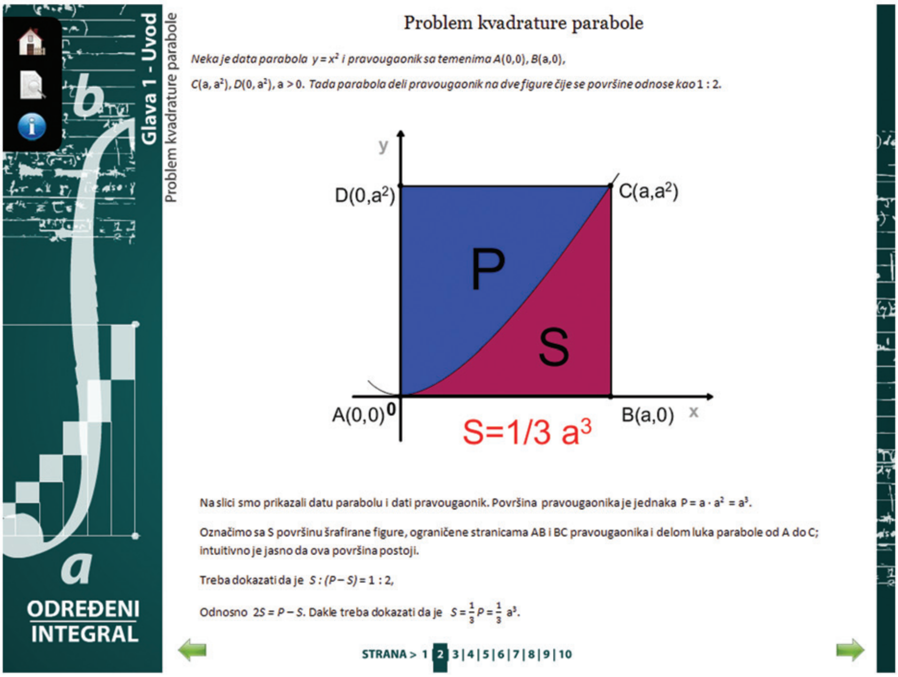

Figure 1. Illustration of given problem taken from the multimedia lesson about the definite integrals.

The next step is to divide the interval $[0, a]$ of the Ox-axis in n equal parts of length $\frac{a}{n}$, where n is a natural number, which should be shown to the students by animation (step-by-step), as shown in Figure 2(a). Within each of these intervals, we construct two rectangles: 'circumscribed' one, with upper right vertex on the parabola (the animation of this step is shown in Figure 2b), and 'inscribed' one, with upper left vertex on the parabola (shown in Figure 2d). It is obvious that the first part has no inscribed rectangle. The heights of these rectangles are shown in Figure 2(b), and their areas are as follows: $\frac{a}{n} \cdot \left(\frac{a}{n}\right)^2, \frac{a}{n} \cdot \left(2\frac{a}{n}\right)^2, \ldots, \frac{a}{n} \cdot \left((n-1)\frac{a}{n}\right)^2, \frac{a}{n} \cdot \left(n\frac{a}{n}\right)^2$. (Part of this animation is shown in Figure 2c.)

Area of each inscribed rectangle is the difference between the area of adjoining circumscribed rectangle and the area of 'added' rectangle (part of this animation is shown in Figure 2d and e).

It is obvious that

$$P_u < S < P_O. \tag{3}$$

Therefore, $\frac{a^3}{3} - \frac{a^3}{2n} + \frac{a^3}{6n^2} < S < \frac{a^3}{3} + \frac{a^3}{2n} + \frac{a^3}{6n^2}$, that is: $-\frac{a^3}{2n} + \frac{a^3}{6n^2} < S - \frac{a^3}{3} < \frac{a^3}{2n} + \frac{a^3}{6n^2}$. These inequalities are correct for any given natural number n.

Since $\lim_{n\to\infty}\left(-\frac{a^3}{2n} + \frac{a^3}{6n^2}\right) = \lim_{n\to\infty}\left(\frac{a^3}{2n} + \frac{a^3}{6n^2}\right) = 0$, we can conclude that

$$S = \frac{1}{3}P = \frac{1}{3}a^3, \text{ i.e.} \tag{2}$$

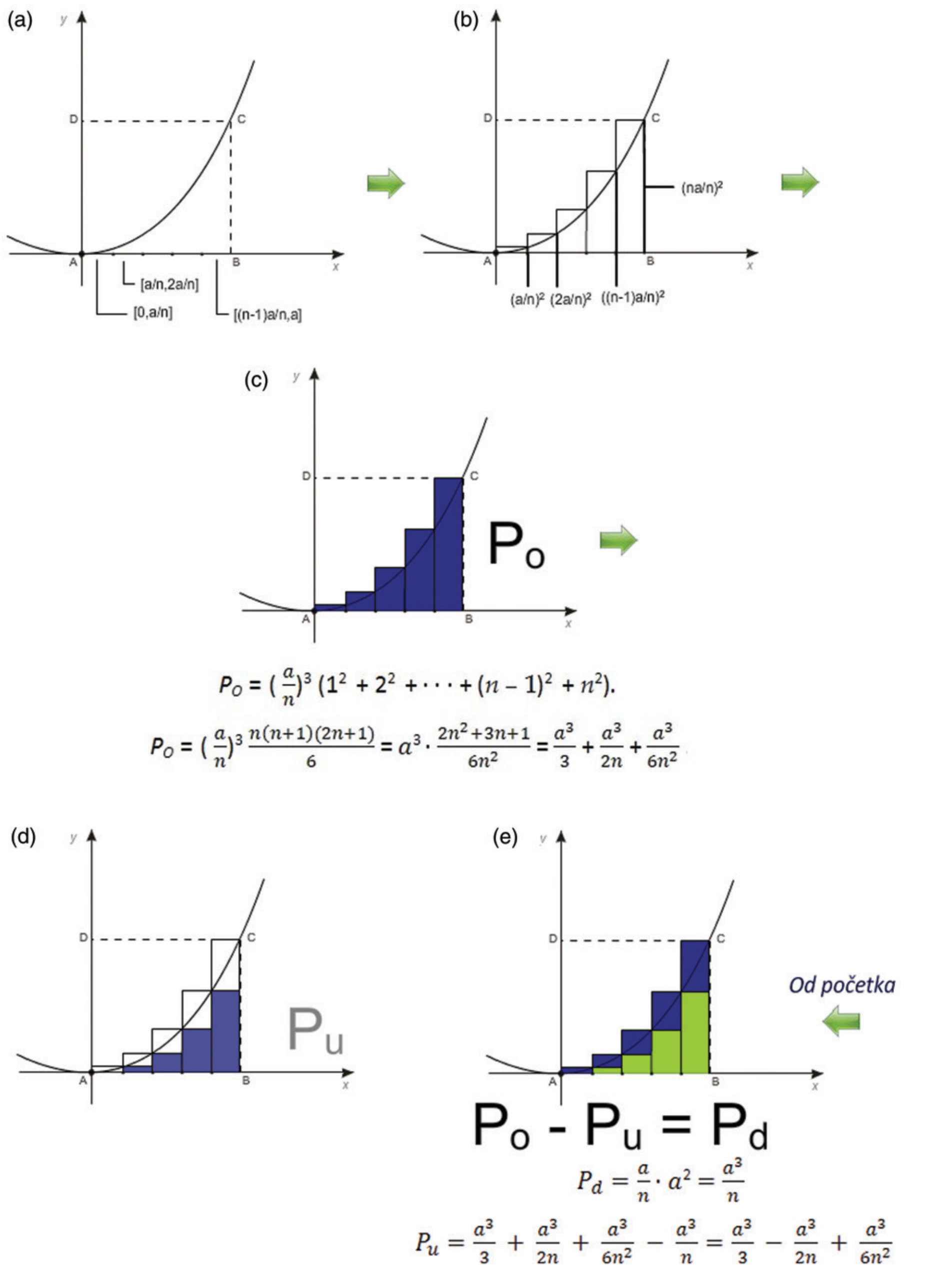

Figure 2. The first illustration of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step).

This was the solution of the given problem via numerical method, but we can offer much more by using the multimedia lessons. In animation shown in Figure 3, students can clearly see that with increasing *n*, i.e. the number of circumscribed and inscribed rectangles, these areas will get closer and closer, until they, according to our intuition and visual perception, both become equal to the area *S*.

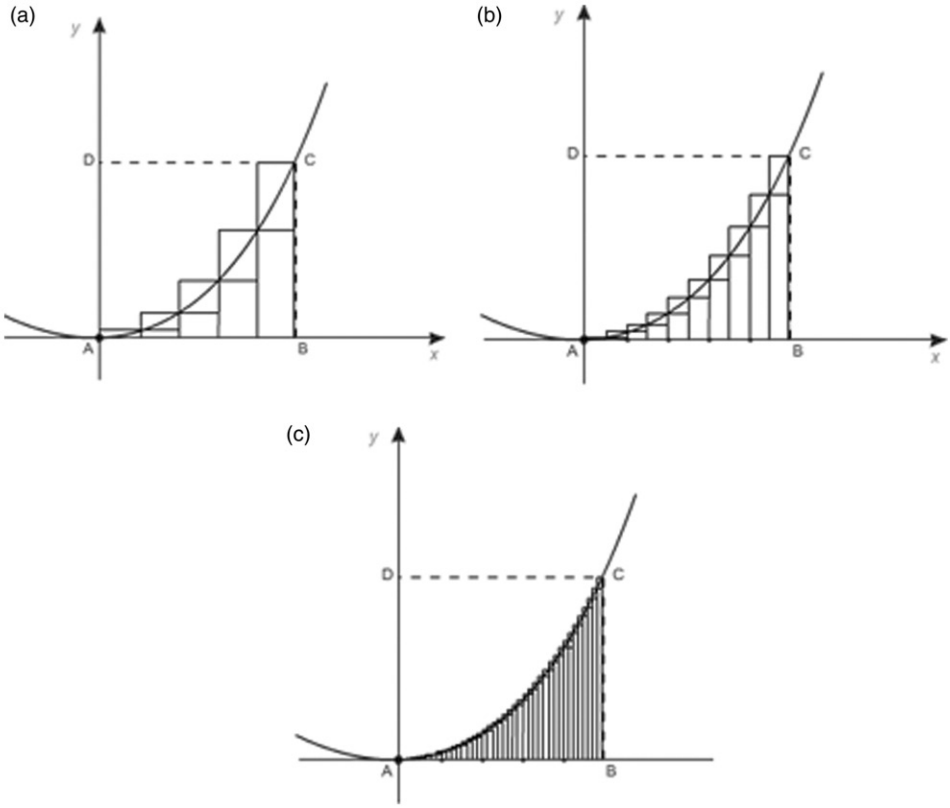

Figure 3. The second illustration of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step).

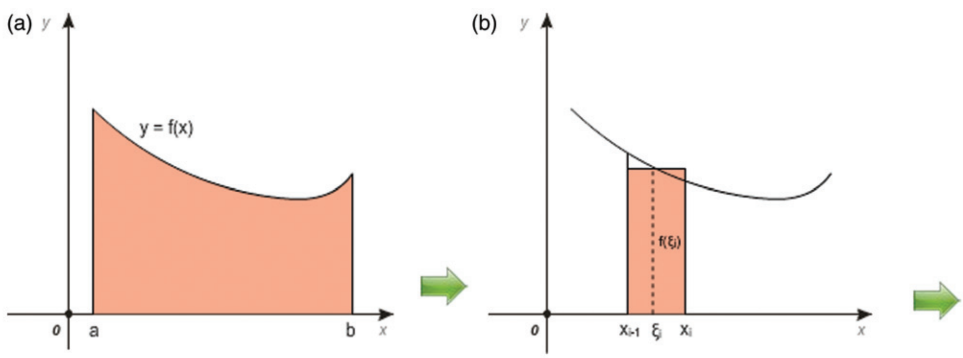

Figure 4. The first part of solution of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step).

Led by the similar idea as in previous example, we will try to calculate the area of the curvilinear trapezium (students will see it in animation, as shown in Figure 4a and b, etc.) formed by the graph of the function $y = f(x), x \in [a, b]$, the abscissa's segment [a, b] and the two segments of the lines $x = a$ and $x = b$ making the figure

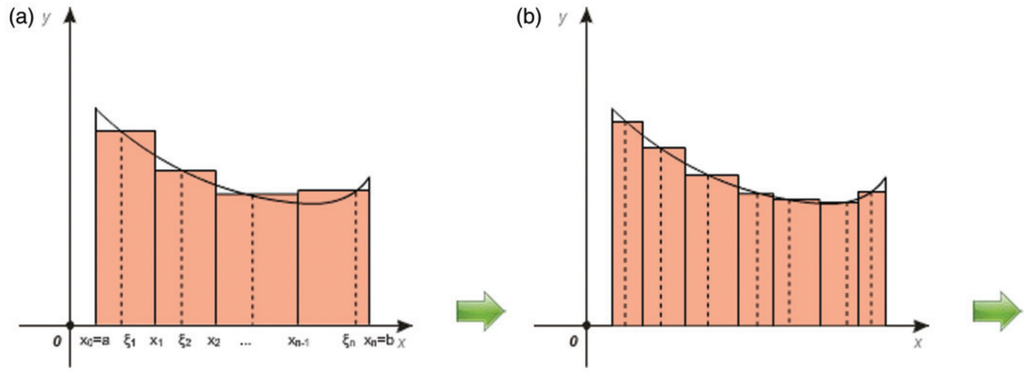

Figure 5. The second part of solution of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step).

closed (Figure 4a). If the values $x_0, x_1, \ldots, x_{n-1}, x_n$ define points on the Ox-axis as follows:

$$a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b,$$

these points divide interval $[a, b]$ into n sub-intervals:

$$[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n].$$

Therefore, we can name the $(n+1)$-plet $(x_0, x_1, \ldots, x_{n-1}, x_n)$ as *division of interval* $[a, b]$. For simplification, we will mark it as $\Pi = (x_0, x_1, \ldots, x_{n-1}, x_n)$.

If we choose any of these sub-intervals (Figure 4b), for example (x_{i-1}, x_i), and if ξ_i is arbitrary value within that sub-interval, the area of the rectangle whose basis is sub-interval $[x_{i-1}, x_i]$ and height is $f(\xi_i)$ can be calculated as:

$$P_i = f(\xi_i)(x_i - x_{i-1}). \tag{4}$$

If we do the same with every sub-interval $[x_{i-1}, x_i]$, $i = 1, 2, \ldots, n$, we will get the series of rectangles – figure S – with total area:

$$P(S) = \sum_{i=1}^{n} P_i = f(\xi_i)(x_i - x_{i-1}). \tag{5}$$

For a given curvilinear trapezium – i.e. for given interval $[a, b]$ and given function $f(x)$– the shape of figure S depends on division $\Pi = (x_0, x_1, \ldots, x_{n-1}, x_n)$and on choice of values $\xi_i \in [x_{i-1}, x_i]$, $i = 1, 2, \ldots, n$. Let us mark this n-plet of choices as $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$. If all the sub-intervals $[x_{i-1}, x_i]$, $i = 1, 2, \ldots, n$ are 'small', the shape of figure S will be 'very' similar to the curvilinear trapezium F (which are shown in Figures 5 and 6a).

If we mark the value of $\Delta x_i = x_i - x_{i-1}, i - 1, 2, \ldots, n$, then set $\{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\}$ is a finite set of positive numbers, and consequently has the largest element, which we will mark as d;

$$d = d(\Pi) = \max\{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\}.$$

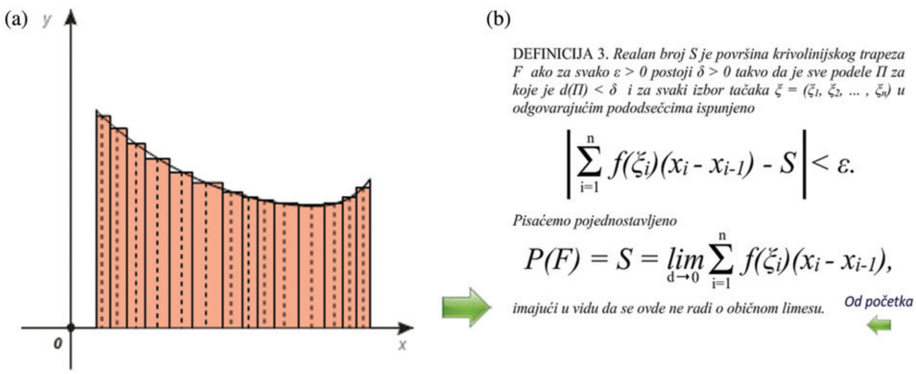

Figure 6. *Definition 1*: Area of the curvilinear trapezium F – visualization method.

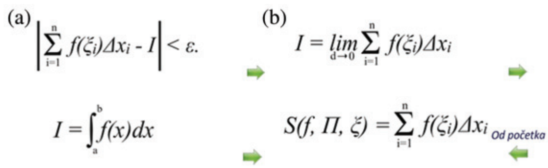

Figure 7. *Definition 2*: Area of the curvilinear trapezium F – visualization method.

If the value of d is a small enough natural number, it means that sub-intervals are smaller and the division Π is 'fine'. If we introduce new breaking points, d gets smaller and smaller, so the division gets finer. Consequently – and according to our intuition and multimedia presentation – figure S will get more and more similar to the curvilinear trapezium, so we can conclude that the following definition of the area of the curvilinear trapezium F is valid:

Definition 1: Real number S is the area of the curvilinear trapezium F if for every $\varepsilon > 0$, there exists $\delta > 0$, such that for every division Π for which $d(\Pi) < \delta$ and for any chosen set of values $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ in correspondent sub-intervals:

$$\left|\sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1}) - S\right| < \varepsilon. \tag{6}$$

Or it is simplified as

$$P(F) = S = \lim_{d\to 0}\sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1}),\ \text{i.e.} \tag{5}$$

Definition 2: Let the real-valued function f be defined on interval $[a, b]$. The real number I is definite integral of the function f on the interval $[a, b]$ if for every $\varepsilon > 0$, there exists $\delta > 0$, such that for every division $\Pi = (x_0, x_1, \ldots, x_{n-1}, x_n)$, $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$ for which $d = d(\Pi) = \max\{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\} < \delta$ and for any chosen set of values $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$, $\xi_i \in [x_{i-1}, x_i]$, $i = 1, 2, \ldots, n$ (animation, Figure 7 shows further development step-by-step).

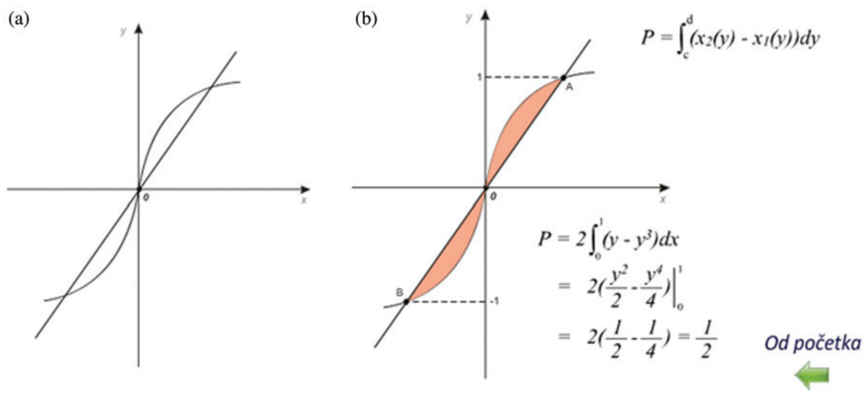

Figure 8. Animation parts, which represents the graphs of the given task and solution in determining the area of a plane figure.

With numerous visual presentations, animations, illustrations and examples, we can also introduce and explain *integrability*, *integral sum*, *integrand*, *limits of integration*, *Newton–Leibniz formula*, *applications of integrals*, etc.

Example: Determining the area of plane figure.

Task: Determine the area of the figure in the xOy-plane bounded by the curves $x - y = 0, x - y^3 = 0$.

Solution: Animation shows the graphs of given curves (Figure 8a) and their intersection points $A(1,1), O(0,0), B(-1,1)$ (with numerical and graphic presentation). We can see that the given figure consists of two identical parts. The next step in the animation is solving the given problem step-by-step. The final result is illustrated in Figure 8(b).

3. Research methodology

3.1. *Aim and questions of the research*

Thanks to the experiences of some previous researches and results, some of the questions during this research were as follows:

(1) Are there any differences between results of the first group of students, who had traditional lectures (control group – *traditional group*) and the second group, who had multimedia lectures (experimental group – *multimedia group*)? Where were these differences the most obvious?
(2) What do students from the experimental group think about multimedia lectures? Do they prefer this or traditional way and why?
(3) Do students think it is easier to understand and learn the matter individually and during the classes by multimedia lectures?

3.2. *Participants of the research*

The research was conducted on two groups of 25 students of the first year at the Faculty of the Architecture of the UNION University, Belgrade, Serbia.

The first group had traditional lectures and the second one had multimedia lectures. Groups were formed randomly, so the previous knowledge needed for the lectures about limited integrals was practically the same, which was confirmed by test. The pre-test included tasks from the area of the continuous functions (analysis and graph-drawing) as well as the tasks about the basic figures in analytic geometry (circle, ellipse, parabola, etc.). Average score of this pre-test was practically equal in these groups (I: 72.35, II: 71.25 out of 100).

3.3. *Methods, techniques and apparatus*

Lectures in both groups included exactly the same information on the finite integrals, i.e. axioms, theorems, examples and tasks. It is important to emphasize that the lecturer and the number of classes were the same, too. The main information source for the multimedia group was software created in Macromedia Flash 10.0, which is proven to be very successful and illustrative for creating multimedia applications in mathematics lectures [15]. Our multimedia lecturing material was created in accordance with methodical approach, i.e. cognitive theory of multimedia learning [4,5], as well as with principles of multimedia teaching and design based on researches in the field of teaching mathematics [16,17]. This material includes a large number of dynamic and graphic presentations of definitions, theorems, characteristics, examples and tests from the area of the finite integrals based on step-by-step method with accent on visualization. An important quality of making one's own multimedia lectures is the possibility of creating a combination of traditional lecture and multimedia support in those areas we have mentioned as the 'weak links' (finite integral definition, area, volume, etc.) [18–20].

After the lectures were finished, students had the same test:

(1) Use Archimedes' method to determine the area of plane figure bounded by the Ox-axis, line $x = a$, and part of the curve $y = x^3$ for $0 \leq x \leq a$.
(2) Write the finite integral definition.
(3) Determine the finite integral $\int_0^{\pi/4} \sin x \cdot \cos^2 x \mathrm{d}x$.
(4) Determine the area of the plane figure F bounded by the Oy-axis, graph of the function $y = x^2$ and the tangent on this graph in point $(1, 1)$.

Scoring was within the interval from 0 to 100 (25 points per task).

Results were analysed with *Student's t-test* for independent samples using *SPSS* (version 10.0) software. The result was considered significant if the probability p was less than 0.05.

3.4. *Results*

Average score in the multimedia group was 76.24 with standard deviation 16.19, and in traditional group, average score was 63.96 with standard deviation 19.7 (Figure 9). Statistical comparison with t-test for two independent samples showed that multimedia group had remarkably higher scores in comparison with the traditional group, with statistical significance of $p < 0.05$.

Test scores by tasks for both groups are given in Figure 10. It is evident that students from multimedia group were more successful in tasks 1, 2 and 4, compared

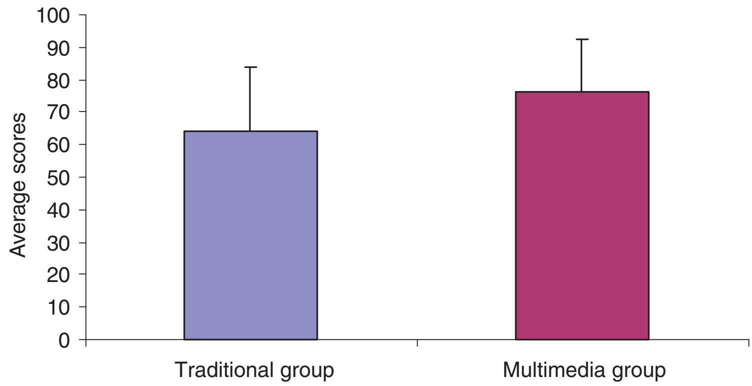

Figure 9. Total average test scores for traditional and multimedia groups.

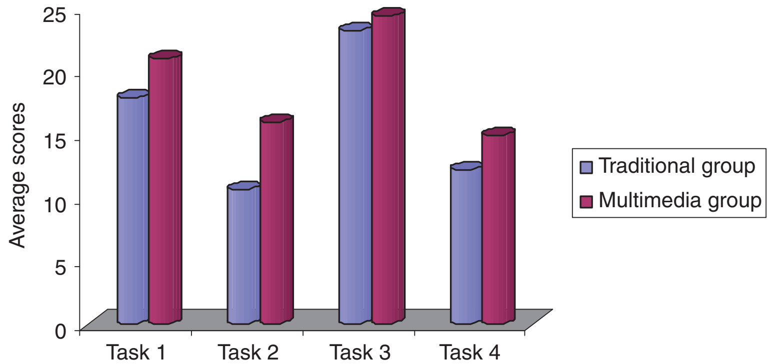

Figure 10. Average test scores by single tasks for traditional and multimedia groups.

to the traditional group. On the other hand, the average score in the third task was similar in both groups.

When asked whether they prefer classical or multimedia way of learning, 12% (3 students) answered classical and 82% (22 students) answered multimedia, explaining it with the following reasons:

- 'It is much easier to see and understand some things, and much easier to comprehend with the help of step-by-step animation'.
- 'Much more interesting and easier to follow, in opposite to traditional monotonous lectures with formulas and static graphs'.
- 'More interesting and easier to see, understand and remember'.
- 'I understand it much better this way and I would like to have similar lectures in other subjects, too'.
- 'This enables me to learn faster and easier and to understand mathematical problems which demand visualization'.
- 'Quite interesting, although classical lectures can be interesting – depending on teacher'.

When asked whether it was easier for them to learn, understand and solve problems after having lectures and individual work with multimedia approach, students answered the question as shown in Figure 11.

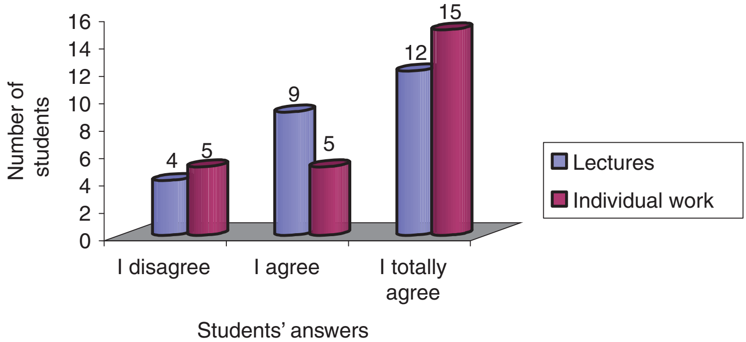

Figure 11. Students' answers to the question: *Should PC be used in teaching and learning mathematics?*

4. Discussion and conclusions

During past few years, multimedia learning has become very important and interesting topic in the field of teaching methodology. Mayer and Atkinson's [4,5,16] researches resulted in establishing the basic principles of multimedia learning and design, which were confirmed in our research too. Our multimedia lessons about the finite integrals, created in accordance with these principles, proved to be successful. According to the students' reactions, highly understandable animations from multimedia lessons are the best proof that a picture is worth a thousand words. Their remark, and consequently one of this research's conclusions, was that there should be much more of this kind of lessons in education, made – of course – in accordance with certain rules and created in the right way.

Many research works in different scientific fields, including mathematics, have proven that multimedia makes learning process much easier.

The test of adopted knowledge conducted during this research showed that students from multimedia group had much higher average scores (the difference was 12.28 points) in comparison with students from traditional group, which also corresponds to the results of some other authors [6].

Researches on learning the finite integrals with software packages Mathematica and GeoGebra have shown that students who had used PC in the learning process had higher scores on tests. Although this research was conducted with different multimedia teaching tools for the same subject – the finite integral as one of the most important areas in mathematical analyses – our results only proved the universality of multimedia in the process of teaching mathematics.

According to Figure 10, which shows average scores in single tasks, we can conclude that students from multimedia group were remarkably more successful in problems which demand visual comprehension (tasks 1, 2 and 4), while the average score in the third task was practically the same on the both groups.

Wishart's [21] research included analyses of comments on how much multimedia approach affects teaching and learning processes. Teachers emphasized that multimedia lectures have made their work easier and have proved to be motivating for students, while students said that multimedia lessons, in comparison with traditional methods, have offered better visual idea about the topic. As shown in Figure 11, a great number of them insisted that multimedia tools enabled easier understanding, learning and implementation of knowledge.

Their remark, and consequently one of this research's conclusions, was that there should be more multimedia lessons, i.e. that multimedia is an important aspect of teaching and learning process.

One of this research's conclusions can be put in the way one student did it during the survey (by answering the question: What is multimedia learning): 'Multimedia learning is use of multimedia as an addition to the traditional way of learning. Multimedia enables us to have better understanding of many mathematical problems and to experiment with them.'

5. Guidelines for further researches

During our research, several new questions appeared that should be solved in the future: (a) In which scientific fields does the multimedia approach give the best results? (b) For which areas of mathematics (geometry, analyses, etc.) would the multimedia approach be the most successful? (c) How much success of the multimedia approach depends on an individual student's ability and how much on a teacher's skills? (d) How can we improve the understanding of lectures by using the multimedia approach, because our aim is learning and understanding, not the multimedia *per se*.

Notes

1. *Archimedes* of Syracuse (Greek: Ἀρχιμήδης; c. 287–212 BC) was a Greek mathematician, physicist, engineer, inventor and astronomer.
2. Figure 1 is taken from the multimedia lesson about the definite integrals.

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