Hwang, W.-Y., Chen, N.-S., Dung, J.-J., & Yang, Y.-L. (2007). Multiple Representation Skills and Creativity Effects on Mathematical Problem Solving using a Multimedia Whiteboard System. *Educational Technology & Society*, 10 (2), 191-212.

Multiple Representation Skills and Creativity Effects on Mathematical Problem Solving using a Multimedia Whiteboard System

Wu-Yuin Hwang
National Central University, Taiwan

Nian-Shing Chen
Department of Information Management, National Sun Yat-sen University, Kaohsiung, Taiwan
Tel: +886-7-5252510 // Fax: +881-7-5254799 // nschen@cc.nsysu.edu.tw

Jian-Jie Dung
National Central University, Taiwan

Yi-Lun Yang
National Kaohsiung Normal University, Taiwan

ABSTRACT

The aim of this study is to explore student multiple representation skills and creativity in solving mathematical problems when supported by a multimedia whiteboard system. The subjects were 6th grade primary school students that were tested and selected as excellent students in mathematics. Twenty-one numerical and geometry problems were given to the students in the experiment. The learning activities including problem solving, peer criticizing and response improvement facilitated by the designed multimedia whiteboard system. The findings of this study are that student multiple representation skills are the keys to successful mathematical problem solving. Students with high elaboration ability can take better advantage from peer interactions and teacher guidance to generate more diversified ideas and solutions in mathematical problem solving. In contrast, students with low elaboration ability would have great difficulty in representation skills. We conclude that elaboration ability in creativity is a critical factor that affects student's multiple representation skills. The study suggests that teachers could design mathematical problem solving activities supported by a multimedia whiteboard system to improve student multiple representation skills.

Keywords

Representation, Creativity, Mathematical Problem Solving, Multimedia Whiteboard System

1. Introduction

Recent educational reform in mathematics education emphasizes that students should learn mathematical theory and calculation and also how to develop their reasoning and critical thinking abilities for problem solving. In the new educational reform for the 9-year Consistent Curriculum Syllabus in Taiwan passed in 1999, the main learning objective is for students "to learn how to solve real application problems" in the mathematics domain. Constructivists also claimed that students should discover and build upon their own knowledge. Therefore, the mathematics learning process should be active and meta-cognitive. Teachers should bridge the gaps between learning mathematical knowledge and solving real world problems. Solving a math problem is more than just filling in a blank in a test. Students need to form good expressions in elaborating their solutions. Some researches pointed out that good student representation skills are the key to acquiring a successful solution in problem solving (Gagne, 1985; Mayer, 1992).

While solving a mathematical application problem, students need to observe and find out specific patterns or rules inside the problem. That is, students need to formulate a concrete application problem into an abstract mathematical problem. In the formulation process, students must have multiple representation skills to articulate the same problem in different forms or views. However, some researchers pointed out that most students fail to grasp the importance of the connections between different types of representations (Ainsworth, 1999). Lesh (1987) proposed a three steps procedure for problem solving. The first step is translation of verbal or vocal to mathematical pattern, the second step is transforming the mathematical pattern into arithmetic symbol. The final step is explaining the solution by verbal writing or oral speaking. Lesh emphasized the importance of student transformation ability among multiple representations in solving application problems.

Students in mathematics class usually acquire knowledge by listening to their teachers and then taking some quizzes. There is not enough time for all students to elaborate their problem solutions using multiple representations in class. It is also not easy for some students to speak out their detailed explanations due to face-to-face pressure in class. As for quizzes, many mathematics teachers prefer a simpler test form like multiple choice and filling in blank questions rather than giving complex problem solving. Therefore, most students are trained to acquire knowledge through the first and second steps in problem solving proposed by Lesh (1987). They seldom have the chance to explain their solutions using verbal or written problem solving. However, without the final step, teachers cannot truly determine if a particular student misunderstands the problems solving process.

Cultivating better creative thinking ability in students has become an important trend in educational revolution. The mathematical problem solving process involves divergent thinking with many tools and requires more effort and time. However, it is not easy to cultivate student creative thinking ability in traditional classroom. This is because most students simply apply the formulas they have learnt to solve problems, but do not necessarily understand the real concepts or principles behind the formulas (Baer, 1993; Forbes, 1996).

To better understand student learning obstacles and cultivate creative thinking ability, teachers need to assess student solution procedures in detail, especially the multiple representations for their solutions, including formulas, graphs and language such that teachers can determine if students misunderstand a certain concept or are stuck at a specific point. Teachers can then provide more effective guidance to students. Sung also pointed out that the peer evaluation processes should be employed to help students understand the problem properties and reflect on their own solutions (Sung, et al., 2005). This can then strengthen student creative thinking ability and creativity.

According to the premises stated above, this study explores how primary school students use multiple representations including text, graphs, symbols, rules, and formulas in mathematical problem solving; and how a multimedia whiteboard system can be used to support students in doing multiple representations. This study also wants to examine the relationship between student creativity ability and multiple representation skills and the impact on mathematical problem solving. The main research questions are described as follows:

1. How does student multiple representation skills affect mathematical problem solving using a multimedia whiteboard system?
2. How does student elaboration ability in creativity affects their multiple representation skills in mathematical problem solving?
3. What are the advantages and disadvantages of using a multimedia whiteboard in mathematical problem solving?

2. Literature Review

2.1 Representation

The meaning of representation can be different in different contexts. There are external representations (real world) and internal representations (mind). In psychology, representation means the process of modeling concrete things in the real world into abstract concepts or symbols. Jonassen (2000) also interpreted mental models as complex mental representations composed of numerous kinds of mental components including metaphorical, visual-spatial, and structural knowledge.

In mathematical psychology, it means the description of the relationship between objects and symbols. Lesh, Post & Behr (1987 pointed out five outer representations used in mathematics education including real world object representation, concrete representation, arithmetic symbol representation, spoken-language representation and picture or graphic representation. Among them, the last three are more abstract and a higher level of representations for mathematical problem solving (Johnson, 1998; Kaput, 1987; Lesh, 1987; Shiau, 1993; Zhang, 1997; Milrad, 2002):

1) Language representation skill – The skill of translating observed properties and relationships in mathematical problems into verbal or vocal representations.

2) Picture or graphic representation skill – The skill of translating mathematical problems into picture or graphic representations.
3) Arithmetic symbol representation skill – The skill of translating mathematical problems into arithmetic formula representations.

Some students favor visual or concrete representations, while others favor symbolic or abstract representations. Normally, students with good problem solving abilities are those that can skillfully manipulate their language translation and representations (vocal), picture representation (picture, graphic) and formal representation (sentence, phrase, rule and formula). On the other hand, students with low problem solving abilities are always having difficulty with translation and representation in problem solving. Furthermore, different students have different learning styles for acquiring knowledge. It is better for teachers to adopt different teaching strategies to promote students performing multiple representations in class, thereby enhancing learning performance (Cai & Hwang, 2002).

To support students in performing multiple representations for problem solving, ICT tools can be used to better facilitate the learning process. Hwang (Hwang et. al., 2006) proposed using a multimedia whiteboard system for students to express their thoughts (internal model) with text, image or oral. Most students were satisfied with the usefulness and ease of using the multimedia whiteboard system (average score higher than 4.2 in Likert 5 point scale) in this study.

Multi-modal learning simply means using many ways to learn. Multi-modal learning promotes the use of new media and methods designed and offered by communication and computer technology. (Jeffery, 1993; Pilgrim, 1996) When solving mathematical problems, students first figure out some clues to solve the problem by trying different approaches like using formula, graphic reasoning or textual explanation. They then translate their ideas and methods into multi-representation solutions using various tools supported by the multimedia whiteboard system as shown in Figure 1.

Problem solving is the highest level of knowledge in the Bloom cognitive taxonomy (1956). Anderson et al (2001) revised Bloom cognitive taxonomy into a new learning process which including ‘remembering’, ‘understanding’, ‘applying’, ‘analyzing’, ‘evaluating’ and ‘creating’; among them creativity is listed at the highest level. Therefore, we argue that creativity would have a profound effect on multiple representation skills in the learning process. We will discuss more about this issue in the following section.

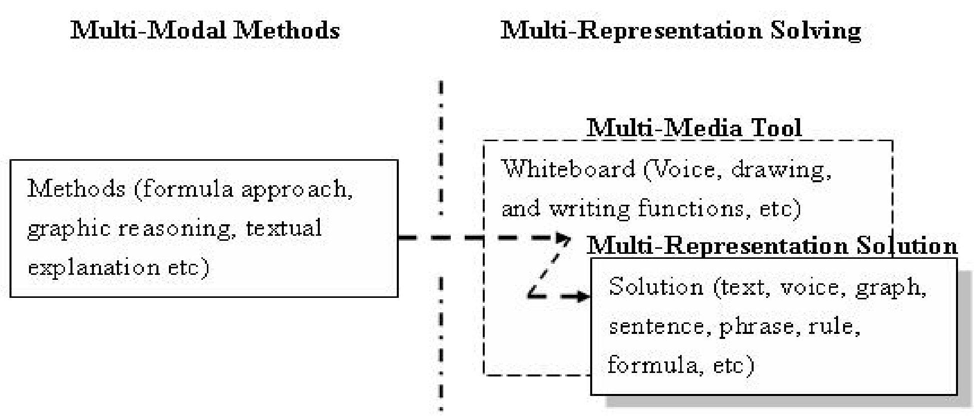


Figure 1. Translation from multi-modal methods into multiple representation solutions

2.2 Creativity

Creativity means the cognitive skill of proposing a solution to a problem or making something useful or novel from ordinary. Guilford cited the paucity of creativity and imagination among all students, and encouraged the study of creativity. Creativity requires six distinct but interrelated resources: intellectual abilities, knowledge, styles of thinking, personality, motivation, and environment (Gerard, 1999; Lubart & Sternberg, 1996; Mayer, 1992; Tennyson, 2002).

Williams (1971) also proposed the definition of creativity based on Guilford's multiple intelligence theory (1969) and emphasized the importance of creativity on learning. According to Williams's definition of creativity, it is composed of six cognitive factors, fluency, openness, flexibility, originality, elaboration and title as well as four affective factors, curiosity, imagination, challenge, and risk-taking. Sternberg (1999) also articulated that cognitive abilities and affective factors will influence the extent of creative problem solving.

The most prevalent creative problem solving process was developed by Isaksen, Dorval, and Treffinger (2000). It consists of four components:

1) Understanding the problem
2) Generating ideas
3) Preparing for action
4) Planning the approach

The first component has three stages (a) Constructing the opportunities, in which the problem is presented in a mess; (b) Exploring the data, in which the background of the problem is investigated; (c) Framing the problems, in which the problems is explicitly identified. The second component has only one stage; (d) Generating ideas, in which many relative ideas are gathered and generated. The third component, preparing for action, has two stages; (e) Developing the solution, in which the solutions are narrowed and fleshed out; (f) Building acceptance, in which the solution step details are identified. The final component, planning the approach, has two stages; (g) Appraising tasks, which assesses the appropriateness of the method, and (h) Designing process, in which the solution method details are accomplished (Canady, 1982; Firestien & Treffinger, 1983; Howe, 1996; Jennifer, 2003; Osborn, 1953; Parker, 1978).

Wallas (1926) proposed that creativity involves four stages: preparation, incubation, illumination, and verification. During the preparation stage, one must be conscious of the problem and gather related information or draw upon previous experience. During the incubation stage, the person will consider all possible solutions, but he or she may fail. During the illumination stage, one innovative idea may suddenly come to the person, often in an unconscious situation. During the verification stage, the solution will be executed and verified.

Osborn (1953) also provided four guidelines for creative problem solving for teachers to apply in the classroom environment to encourage more ideas, to accept strange ideas and to promote ideas, but not criticize ideas immediately after the students presented them (Johanna, 2003). Creativity seems a natural talent of people. However, creativity is a cognitive skill for proposing a solution to a problem, or making something useful and novel from the ordinary (Anderson et al, 2001). That means creativity is a skill that can be cultivated through the problem solving process. As the literature described above, several systematic procedures could promote creativity in solving problem. However, the assessment of student performance cannot be just either right or wrong for the solutions. More creative evaluations should be used, for example, using open-ended assessments can help students to reveal their thinking, reasoning and strategic used in the solving process.

2.3 QCAI Evaluation

Silver (1989) conducted a project called QUASAR (Quantitative Understanding Amplifying Student Achievement and Reasoning). Its attempt is to promote different learning methods to cultivate student problem solving and creative thinking abilities that can help all levels of students to learn math well. Under this premise, teachers need to adopt multiple materials and instructional strategies to encourage students to do discussion, interpretation and innovation. In this way, students will be trained to thinking about math instead of memorizing math.

Several evaluation instruments were developed in QUASAR projects (Ford Foundation, 2004; Lane, et al, 1995; Lane, 1999; Lane & Wang, 1996; NCTM, 1989; Moskal & Leydens, 2000; Parke, 2002)... The most famous and prevalent one is QUASAR Cognitive Assessment Instrument, which is also called QCAI with many open-ended questions. QCAI can evaluate student mathematical problem solving, reasoning, communication and representation skills, etc. The QCAI specification includes 4 components: mathematical contents, cognitive processes, representation mode (text, picture, graph, table) and task context (embedded in real world or not).

The general QCAI scoring rubric is specified for each of 5 score levels. Five score levels are used to capture various levels of student understanding. The sixth and seventh grade version of the QCAI consists of 36 open-ended tasks that are distributed into four forms, each containing 9 tasks (Lane, Stone, Ankenmann, & Liu, 1994). Based on the specified criteria at each score level of the general rubric, a specific rubric was developed for each task. The emphasis on each component for a specific rubric is dependent on the cognitive demands of the task. The criteria specified at each score level for each specific rubric are guided by theoretical views of the acquisition of mathematical knowledge and processes assessed by the task, and the examination of actual student responses to the task. The examination of the student responses ensures that the rubrics reflect the various representations, strategies, and ways of thinking (Lane, et al, 1996). This study focus on the types of multiple representation skills (text, voice, picture, graph, symbol and rule) in QCAI and use the 5 levels scoring rubric to evaluate multiple representation skills as shown in part A of Table 1.

3. Research Method

3.1 Setting & Procedure

The subjects were 25 six-grade primary school students in different classes who were tested and selected as excellent students in mathematics in the school. They participated in the mathematical problem solving learning activity using multimedia whiteboard system (Hwang etc., 2006) on the K12 Digital School, http://ds.k12.edu.tw/.

The designed multimedia whiteboard system has drawing tools, voice recording tools and editing functions. Students write down and modify their solutions and explanations on their own whiteboards, which then stores this information in the form of a discussion forum on a web site. The multimedia whiteboard system has the following functions: the drawing tools include the line, circle, rectangle and text. The editing functions including copy, paste, cut, move, undo and redo and the voice recording function, as shown in Figure 2.

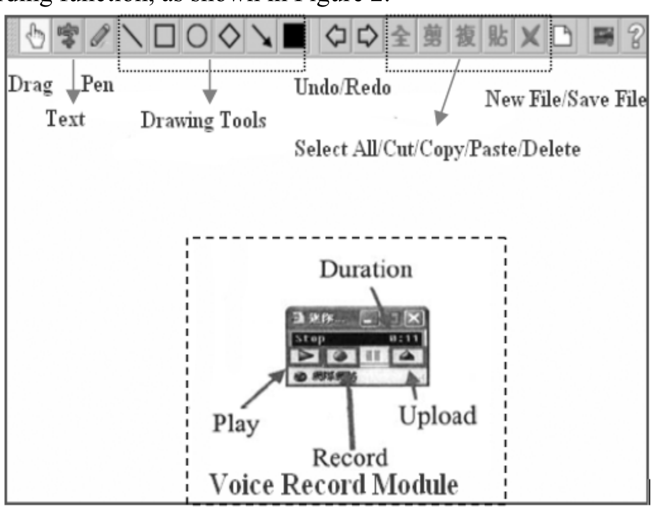


Figure 2. The user interface of voice recording in the designed multimedia whiteboard system

Students were given 21 problems by the teacher including 16 numeral problems and 5 geometry problems. Numeric problems contain the concepts about arithmetic series, geometric shapes, factors, multiple items and number applications, etc. Geometric problems contain the concepts of volume, area of surface, etc. Most students have computers and Internet connections at home, allowing them to work on the problem in class or after school.

The period for conducting the experiment was one semester (about 4 months). The students participated in two math class sessions for a total of 80 minutes every week. In the experiment, students were first given two week tutorials to learn how to use the multimedia whiteboard system. After that, one week is given for solving problems followed by another week for mutual criticizing and response activities until the end of the semester. The math teacher supervised and guided the students in the class learning activities using the multimedia whiteboard system. This included solving problems, criticizing other students' solutions and responding to comments made by other peers. Figure 3 shows examples of three kinds of activities, problem solving, criticizing and responding made by the student and teacher respectively.

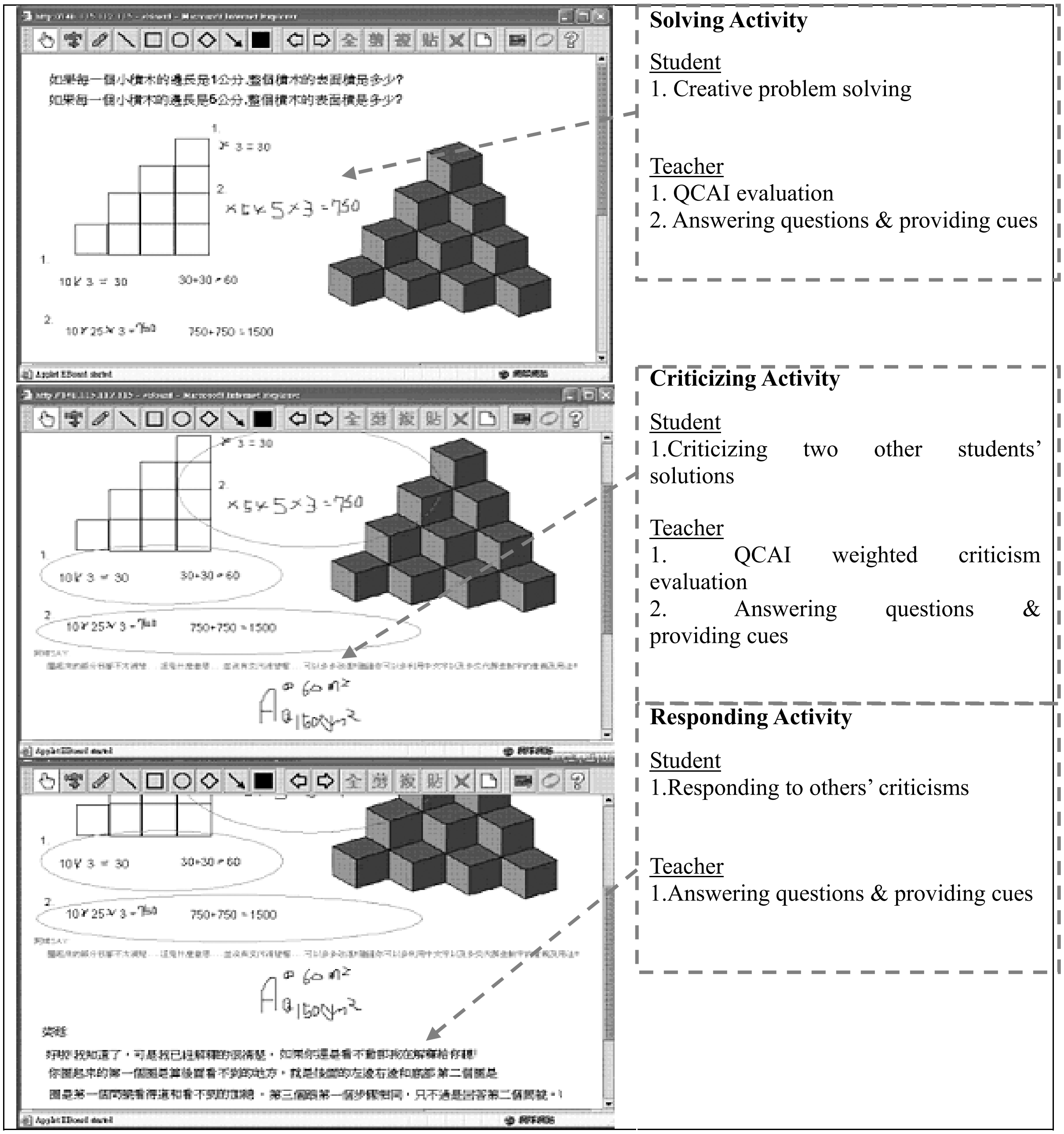


Figure 3. Three examples using multimedia whiteboard for mathematical problem solving

3.2 Evaluation Criteria

During the problem solving process, students were able to revise their solutions many times. The research involved how many kinds of solutions (quantity grade) students could create but also the quality of their solutions (quality grade). The student solutions were classified and evaluated into three types of representations: Text or Voice

Representation, presented as ‘T’; Graph or Symbol Representation, presented as ‘G’; Rule or Formula Representation, presented as ‘R’. Each representation was marked with a quantity grade and a quality grade respectively.

For quantity grade, the assessments were based on how many solutions provided. Two math teachers reviewed student solutions and devised a consensus grade for every individual student after discussion. The assessments were based on how good the solutions were. The teachers evaluated student solutions according to Solution Quality Evaluation Criteria, which revised by the researchers based on QCAI evaluation concept. The score was ranked into 5 categories (Level 1 ~ Level 5), shown in part A of Table 1.

The students were asked to solve problems and also criticize two other students’ solutions. Referring to revised Bloom’s Taxonomy (1956), criticism requires a higher level of cognitive ability. In this research, the teachers evaluated each student’s criticism content according to the Criticism Evaluation Criteria shown in part B of Table 1. The criticism performance was not evaluated in isolation but corresponding to the evaluated solution performance. For example, if a solution was quite good, it was not easy to give critical criticism on that. Therefore each student’s criticism grade needed to be weighted on the basis of the solution grades of the student being criticized. The weighted criticism grade could then be obtained by adding the solution grade of the student being criticized according to part A of Table 1. The original criticism grade was obtained according to part B of Table 1 (Weighted Criticism Grade = Criticism Grade + Solution Grades of the Student being Criticized).

Table 1. QCAI Evaluation Criteria

Part A: Solution Quality Evaluation Criteria
Level 5：Correct, the arithmetic calculation and verbal or graphic explanation are both correct and complete.
Level 4：Correct, the arithmetic calculation and verbal or graphic explanation are both correct but incomplete.
Level 3：Correct, the arithmetic calculation is correct but no verbal or graphic explanation.
Level 2：Not correct, the mathematical reasoning sounds ok but answer is incorrect. Or, the answer is correct but no arithmetic calculation process.
Level 1：Trying to solve problems.
Part B: Criticism Evaluation Criteria
Level 5 : Criticism is mathematical relevant and correct. The arithmetic calculation and verbal or graphic explanation are both correct and complete.
Level 4 : Criticism is mathematical relevant and correct. The arithmetic calculation and verbal or graphic explanation are both correct but incomplete.
Level 3 : Criticism is mathematical relevant and correct. The arithmetic calculation is correct but no verbal or graphic explanation.
Level 2 : Criticism is mathematical relevant but incorrect. The mathematical reasoning sounds ok but answer is incorrect. Or, the answer is correct but no arithmetic calculation process.
Level 1：Criticism is mathematical irrelevant or no criticism.

The teachers also conducted the Williams creative test – a Test of Divergent Thinking in the middle of the semester to collect various aspects of student creativity. At the end of the experimental period all students took a final exam with 10 problems. The final exam scores were for evaluating the learning effect. The Williams’s Creativity Assessment Packet (CAP), for ages from 9 to 17, contains two types of tests; a test of divergent thinking and test of divergent feeling. This research adopted only the test of divergent thinking which Lin and Wang (1994) had modified to a Chinese version. Six major abilities were measured in this test including fluency, openness, flexibility, originality, elaboration and title. The test’s half-reliability is .41 ~ .92; Cronbach alpha is .40 ~ .87; test-retest reliability is .49 ~ .81 (Guidance In Jiouhu, 2005; Psychology Press, 2005).

3.3 Research framework

To explore the effects of multiple representation skills and creativity on mathematical problem solving, the students were classified into different groups according to their solution representation styles. T test and One-way ANOVA were used to analyze the differences in solution, criticism and academic achievement. Pearson Correlation analysis was conducted for representation skills and creativity. The research framework is shown in Figure 4.

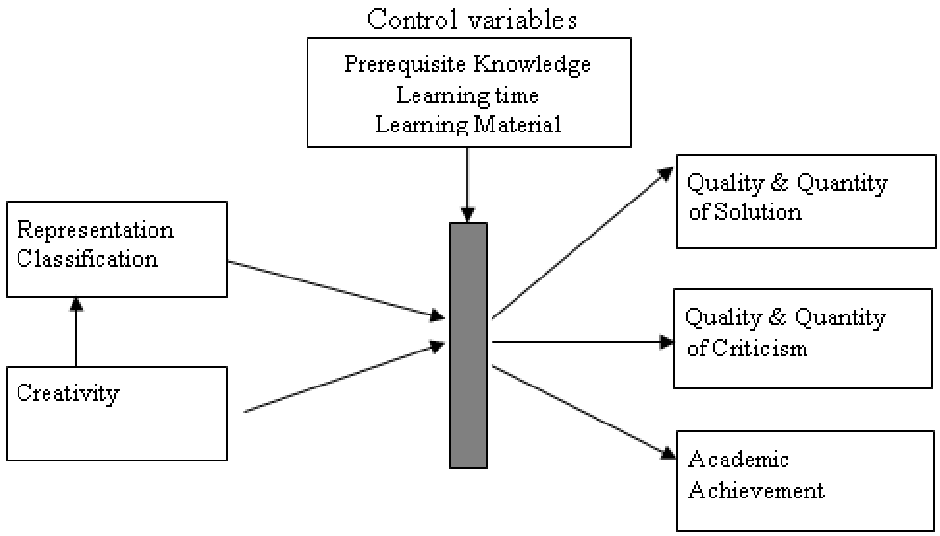


Figure 4. The Research Framework

4. Result & Discussion

4.1 Solution Type Analysis

Two teachers evaluated the student solutions according to the QCAI Evaluation Criteria presented in Table 1. In the Pearson Product-Moment correlation test, 14 out of 16 problem grades were correlated significant between two teacher evaluations. The coefficient is .01 at the alpha level, which validates evaluation reliability.

In the analysis of the students' representation performance, R (Rule or Formula) was better than T (Text or Voice) and G (Graph or Symbol) on both the quantity and quality scores. The T grade distribution was more centralized than the other two representations. The detailed data is shown in Table 2. This result matches with one teacher's observation from the student problem solving process. She made the following comment:

"During the solving activity, most students did not have ideas about how to creatively solve the problem, but simply applied their remembered formula to get an answer."

Table 2. Grades Comparison among Three Different Representation Skills

	Text or Voice Representation (T)		Graph or Symbol Representation (G)		Rule or Formula Representation (R)	
	M	SD	M	SD	M	SD
Numeral-Quantity (9)	7.82	1.22	6.64	2.08	8.52	0.88
Numeral-Quality (45)	26.12	6.36	22.84	9.16	31.82	4.39
Geometry–Quantity (5)	3.12	1.67	3.18	1.53	4.86	0.45
Geometry–Quality (25)	11.92	7.29	12.32	6.53	22.08	3.43

One student also said that it was not easy for him to use verbal, text, or graph to explain the meaning of his solutions.
"Giving answers to the Arithmetic series problems was easy for me. But I had great difficulty in giving further oral explanations and writing down texture description about my solutions."

However, once students can successfully explain their solutions, they would become more comprehensive about the problem.

4.2 Representation Skill Analysis

Student representation performance, both in quality and quantity were recorded and analyzed respectively. One-way ANOVA analysis and Scheffe's test were used to compare the representative scores for T, G, and R in numerical, geometry and the total. The result was divided into four representation types according to the students' representation performance:

1) Representation T was significantly lower than G and R, denoted as type $\boxed{G}\boxed{R}$.
2) Representation G was significantly lower than T and R, denoted as type $\boxed{T}\boxed{R}$.
3) Both representations T and G were significantly lower than R, denoted as type $\boxed{R}$.
4) No significant difference among three representations, denoted as blank (NULL).

The result shows that most students got type $\boxed{R}$ or type NULL; while only a few students got type $\boxed{T}\boxed{R}$ or type $\boxed{G}\boxed{R}$ in Numeral, Geometry or Total. All students' types are shown in Table 3.

Table 3. Student's Representation Types

Student	Numeral Quantity/Quality	Geometry Quantity/Quality	Total Quantity/Quality	Student	Numeral Quantity/Quality	Geometry Quantity/Quality	Total Quantity/Quality
01	$\boxed{T}\boxed{R}$ / $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$	14	/	$\boxed{R}$ /	$\boxed{R}$ / $\boxed{R}$
02	$\boxed{R}$ /	$\boxed{R}$ / $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$	15	/ $\boxed{R}$	/ $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$
03	/	/ $\boxed{R}$	/ $\boxed{R}$	16	/ $\boxed{R}$	/	$\boxed{R}$ / $\boxed{R}$
04	/	/	/	17	$\boxed{T}\boxed{R}$ / $\boxed{T}\boxed{R}$	$\boxed{R}$ / $\boxed{R}$	$\boxed{T}\boxed{R}$ / $\boxed{R}$
05	/	/	/	18	/	/	/
06	/	/	/	19	/	/	/
07	/	/ $\boxed{R}$	/	20	/	/	/ $\boxed{R}$
08	$\boxed{T}\boxed{R}$ / $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$	$\boxed{T}\boxed{R}$ / $\boxed{R}$	21	/	/	/
09	$\boxed{T}\boxed{R}$ / $\boxed{T}\boxed{R}$	/ $\boxed{R}$	$\boxed{T}\boxed{R}$ / $\boxed{T}\boxed{R}$	22	/	/	/
10	/	$\boxed{G}\boxed{R}$ / $\boxed{G}\boxed{R}$	$\boxed{R}$ / $\boxed{G}\boxed{R}$	23	/	/ $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$
11	/	$\boxed{R}$ / $\boxed{R}$	/ $\boxed{R}$	24	/ $\boxed{T}\boxed{R}$	$\boxed{R}$ / $\boxed{R}$	$\boxed{T}\boxed{R}$ / $\boxed{T}\boxed{R}$
12	/	/	/	25	/	/	/
13	/	/ $\boxed{R}$	$\boxed{R}$ / $\boxed{R}$				

Students of the types $\boxed{R}$, $\boxed{G}\boxed{R}$, or $\boxed{T}\boxed{R}$ seemed to favor using one or two representations in problem solving or criticism. Those students were classified as "Unbalanced Style" in our study. Those students that used all representations fairly were classified as "Balanced Style". The representation styles and the corresponding symbols are shown in Figure 5.

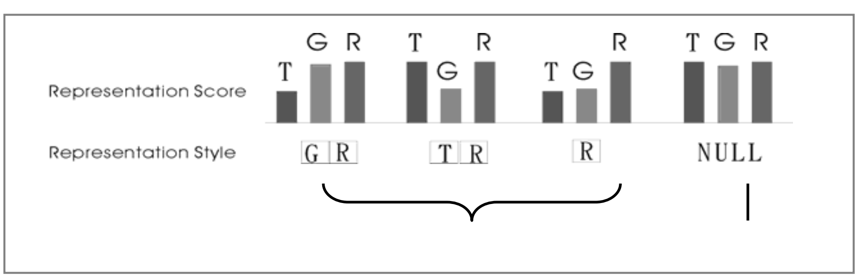


Figure 5. Representation Styles and Symbols

4.3 Performance Analysis of Different Representation Styles

Based on the revised Bloom's taxonomy, the higher level learning processes such as 'analyzing', 'evaluating' and 'creating' need various perspectives for problem solving. Moreover, according to the creativity theory, generating new ideas and thinking are very crucial to finding a good solution. The researchers, therefore, assume the Balanced Style students have higher creativity power than Unbalanced Style students. The Balanced Style students performed better than the Unbalanced Style students in problem solving.

To properly investigate students' favorite representation styles, both quality and quantity performances were taken into account. The researchers classified 13 students (ID 01, 02, 08, 09, 10, 11, 13, 14, 15, 16, 17, 23, 24) as Representation Unbalanced Group and the other 12 students (ID 03, 04, 05, 06, 07, 12, 18, 19, 20, 21, 22, 25) as Representation Balanced Group. Independent sample T test is used on creativity, problem solving, weighted criticism, and academic achievement. The results are shown in Table 4.

Table 4. T test of Representation Styles between Balanced and Unbalanced Groups

		Balanced Group (N=12)		Unbalanced Group (N=13)		t value	
		M	SD	M	SD		
Creativity	Fluency	23.50	1.24	24.00	0.00	-1.39	
	Flexibility	13.92	2.42	14.46	2.22	-.57	
	Originality	28.25	6.78	30.92	7.67	-.92	
	Openness	57.25	5.70	54.38	9.78	.88	
	Title	39.83	8.26	37.38	7.05	.80	
	Elaboration	27.25	7.94	19.46	8.31	2.39	*
Problem Solving	Total-Quantity	74.75	5.99	62.69	9.02	3.90	**
	Total-Quality	286.25	46.21	225.00	43.31	3.42	*
	Numeral-Quantity	24.13	2.21	21.89	3.72	1.81	
	Representation T	7.63	1.23	8.00	1.24	-.76	
	Representation G	8.03	.73	5.31	2.05	4.58	***
	Representation R	8.42	.82	8.62	.96	-.55	
	Numeral-Quality	88.54	15.37	73.89	16.80	2.27	*
	Representation T	27.46	6.57	24.89	6.15	1.01	
	Representation G	28.54	5.31	17.58	8.91	3.77	***
	Representation R	32.25	4.34	31.42	4.57	.46	
	Geometry-Quantity	13.25	1.90	9.46	2.16	4.63	***
	Representation T	4.33	1.07	2.00	1.29	4.89	***
	Representation G	4.13	1.09	2.31	1.38	3.64	***
	Representation R	4.71	.62	5.00	.00	-1.63	
	Geometry-Quality	54.58	12.34	38.62	10.09	3.55	**
	Representation T	17.13	5.79	7.12	4.86	4.69	***
	Representation G	16.00	5.50	8.92	5.61	3.18	**
	Representation R	21.54	3.99	22.58	2.90	-.75	
Weighted Criticism		125.86	11.30	114.70	8.76	2.39	*
Academic Achievement	Pre-test	95.31	3.35	95.58	2.39	-0.22	
	Post-test	95.63	3.08	94.39	3.36	0.97	

(* p < .05, ** p < .01, *** p < .001)

The Balanced Group students performed better than the Unbalanced Group in the Elaboration item of Williams's creativity package (t=2.39*). No significant difference was exhibited in the other five items, Fluency, Flexibility, Originality, Openness, and Title. For the problem solving solutions analysis, the Balanced Group students performed significantly better than the Unbalanced Group students in Total–Quantity (t=3.9**) and Total-Quality (t=3.42*) scores.

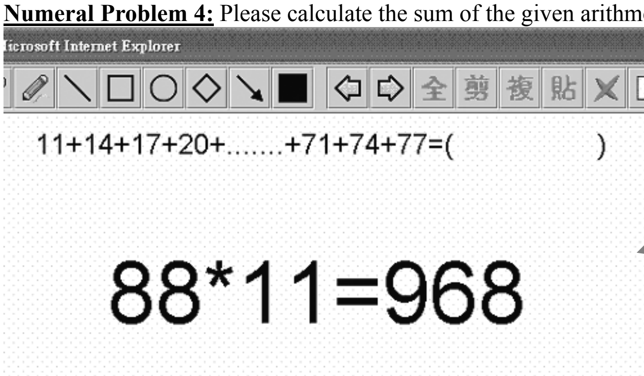
The representation T and G were the critical factors that caused different performance. For the Numeral problem section, the Balanced Group had higher representation G scores than those of the Unbalanced Group both in quantity (t = 4.58***) and quality (t=3.77***). Moreover, for the Geometry problem section, the Balanced Group also had higher representation G scores than the Unbalanced Group both in quantity (t=3.64***) and quality (t =3.18**). Furthermore, the T representation scores showed the same situation between the two groups in both quantity (t=4.89***) and quality (t= 4.69***).

As for weighted criticism scores, the performance of the Balanced Group was better than the Unbalanced Group (t=2.39*). In summary, the T and G representation skills were the key for students in acquiring higher performance, regardless in Numeral or Geometry problem solving and peer assessment.

As for representation R, there was no difference between the two groups. This situation could be explained by the teacher's observation, of which most students in the Unbalanced Group often applied their pre-remembered formulas to solve problems without thoroughly comprehending the problems. Those students were used to memorizing mathematical formulas and employing the formulas directly to solve given mathematical problems. In this case, students could not brainstorm and think deeply and broadly. Thus they had little chance to find good solutions in creative problem solving.

As for no significant difference in the academic achievement between the two groups, the possible reason could be the 25 students participated in our research were all math-talented students in the school. They always got very high marks. This can be supported by the average score close to 95. This phenomenon is called the ceiling effect.

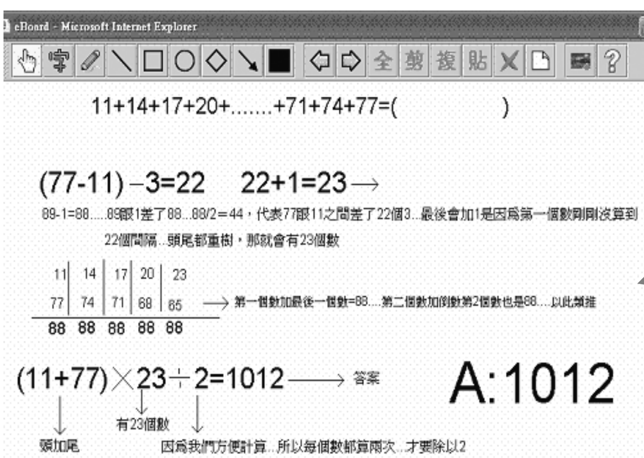
Numeral Problem 4: Please calculate the sum of the given arithmetic series problem.



Teacher's comments on the two solutions for the same Student ID24

First trial

The student solved the problem for the first time and the answer was wrong.
He applied only straight forward thinking to solve this problem without any description and explanation.



Second trial

After the student participated in peer interaction using the multimedia whiteboard system, he learned how to solve this problem. He then solved the problem again after one month, with the correct answer and a proper description and explanation for his solution.
Note: There is a typo error in the equation (77-11) - 3 = 22, the – 3 should be ÷3.

Figure 6. Students with high elaboration ability can perform multiple representations well through interactions

4.4 Effect of Elaboration Ability on Representation Skills

Because the two group students had significantly different performances on T and G representation and the creativity test elaboration item (M=10.16, SD=4.24), the relationship between representation skills and creativity should be further investigated. Pearson correlation analysis was conducted. The results showed that elaboration in creativity was significantly correlated with representation T, and G in several given problems. In Numeral problem section, the quality scores of both T (r = .479*) and G (r = .406*) are correlated to elaboration. Besides, in the total scores, representation T is also significantly correlated with elaboration in quantity (r = .437*) and quality (r = .472*), (* stands for $p < .05$).

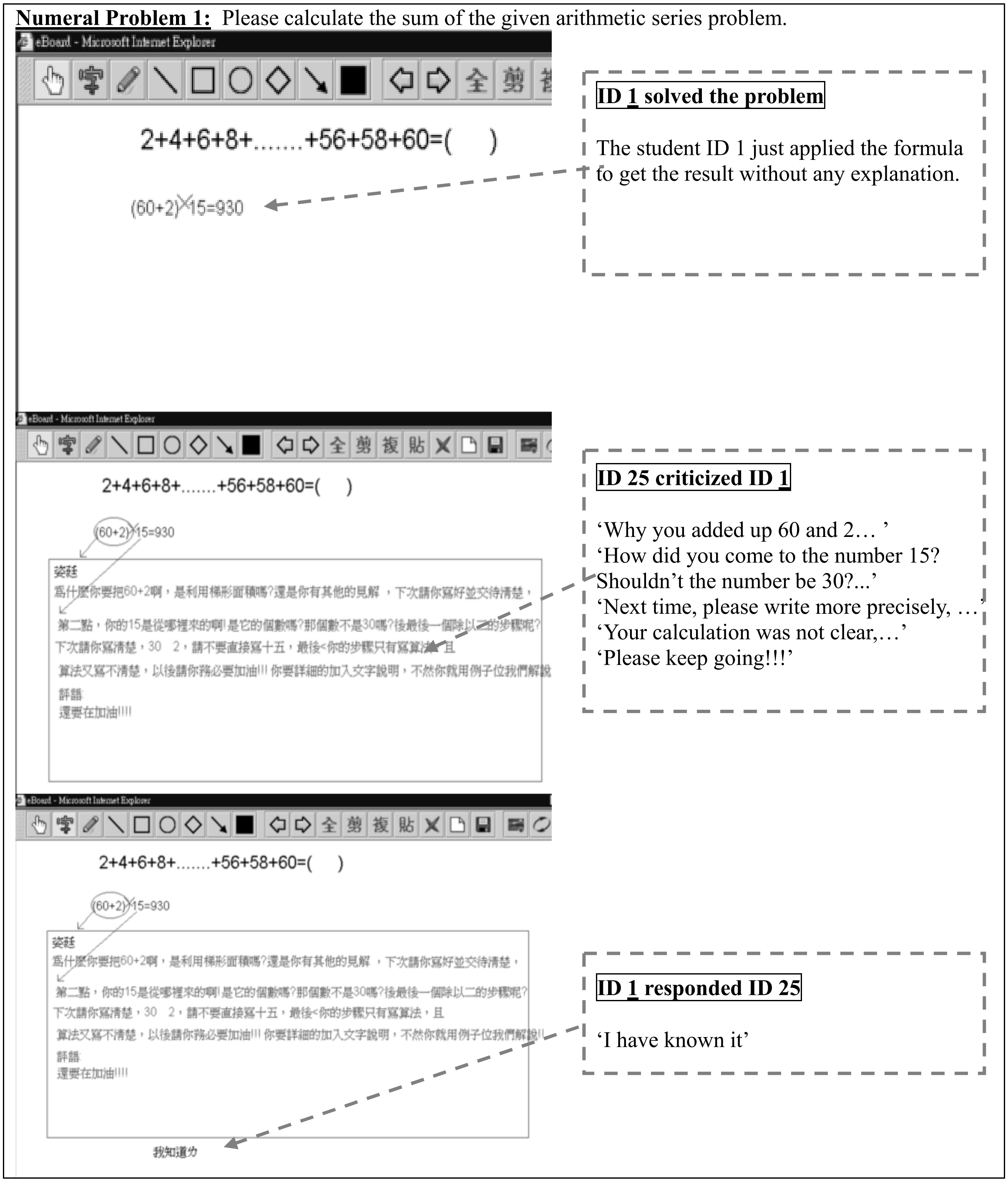
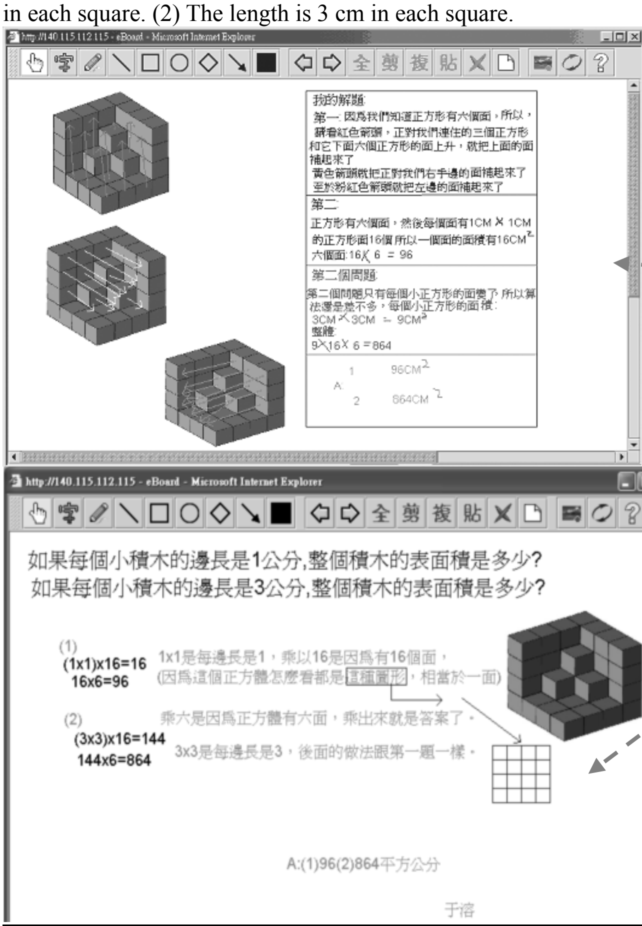


Figure 7. Students with low elaboration ability cannot manipulate multiple representations well

The meaning of elaboration is the ability to work a problem out using extra material, illustration or detail clarification. Therefore, the role of elaboration in solving and criticizing activities is the ability of students to elaborate their own solutions or to criticize others' solutions using various methods and perspectives. Elaboration is the critical factor affecting student's ability to use T and G representation in creative problem solving. It is really worthwhile to stimulate or cultivate a student's elaboration ability using different representations to improve mathematical problem solving.

Let us examine two examples to see how elaboration affects multiple representation skills. In Figure 6, the student ID 24 gave two solutions to a numeral problem using the multimedia whiteboard system. The student got a very high elaboration score, 15 points, which is much higher than the mean elaboration score. The student gave a wrong solution for the first trial (Upper half part of Figure 6), only an arithmetic equation (representation R) was given without any explanation. However, after several weeks of interaction with peers and the teacher, a correct answer and detailed textual explanations (representation T) were given in the second trial (Lower half part of Figure 6). Therefore, elaboration can promote student T and G problem solving representation skills, which can be improved and acquired by peer interaction with the teacher's help. In contrast, the student ID 1 with low elaboration could not perform well in representation when explaining his solution, as shown in Figure 7. The student got only 8 elaboration score points, which is lower than the average. He gave only one solution to a similar numeral problem using the multimedia whiteboard (Upper part of Figure 7). Although other students gave good suggestions and asked him to give detailed explanations (Middle part of Figure 7), 'I have known it' was only written in his response (Lower part of Figure 7). These two examples showed that elaboration is one of the critical factors affecting a student's T and G problem solving representation skills.

Geometric Problem 5: Please calculate the total surface area of the shape in two sub-questions. (1) The length is 1cm in each square. (2) The length is 3 cm in each square.



ID 25's Solution

The student realized that even though the front surface is not flat, if he lifted up every cube toward the boundary the surface will be a complete flat one. The surface area for the three visible dimensions were 3 4x4 squares and the invisible surface area for the other three dimensions were also 3 4x4 squares. Therefore, the total surface area is 4x4x6=96 cm^2 for the first sub-question.
He then got the surface area of 864 cm^2 using the same method for the second sub-question.

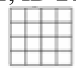
ID 21's Solution

To see is to believe. The student got a figure of 16 squares sharp from 6 different sides. She then calculated the surface as 16 X 6 = 96 cm^2 for the first sub-question. She then got surface area of 864 cm^2 using the same method for the second sub-question.

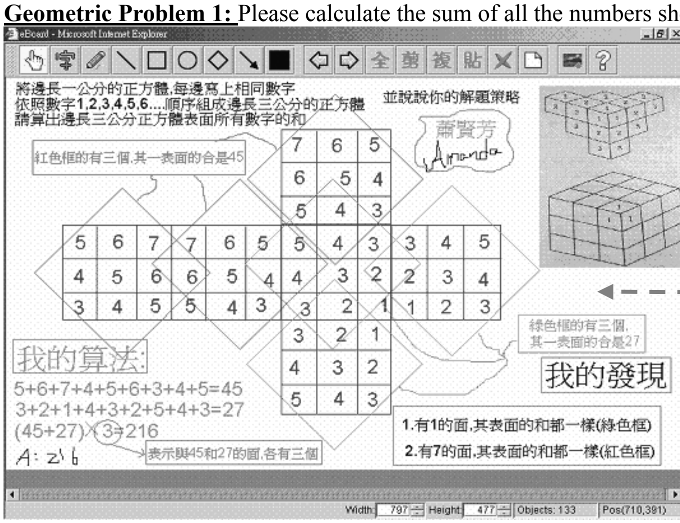
Figure 8. Various aspects and explanations for the same equation

4.5 Problem Solving Analysis

Mathematical problem solving using the web-based multimedia whiteboard system was a new experience for the students that participated in this study. In comparison with face-to-face environments, the web-based system has several advantages like whole solutions and criticizing content can be recorded automatically by the system and reviewed by the teachers and students anytime and anywhere. The students can communicate with each other such that more creative solutions can be stimulated through interaction, discussion and criticism learning activities. Students can modify and improve their solutions many times after receiving feedback from teachers or other students.

For example, Figure 8 shows an example of various aspects and explanations for the same equation. In the first sub-question for geometric problem 5, both students ID 25 and ID 21 got the correct solution: 16 X 6 =96, but gave different strategies and explanations. ID 25 found there were 3 flat surfaces and 3 irregular surfaces in the irregular shape. He figured that each cube in the irregular surfaces could be pulled out to make an irregular surface into a virtual flat one. The irregular shape then becomes a big square with the same surface area as the irregular surface. Therefore, the total surface area 6 X 16 can be easily obtained. However, ID 21 investigated each surface area from 6 different directions in 3 dimensions, and she got the same surface area  in each direction. She then calculated the total surface area as 6 X 16.

Geometric Problem 1: Please calculate the sum of all the numbers shown on the 3x3 cube.

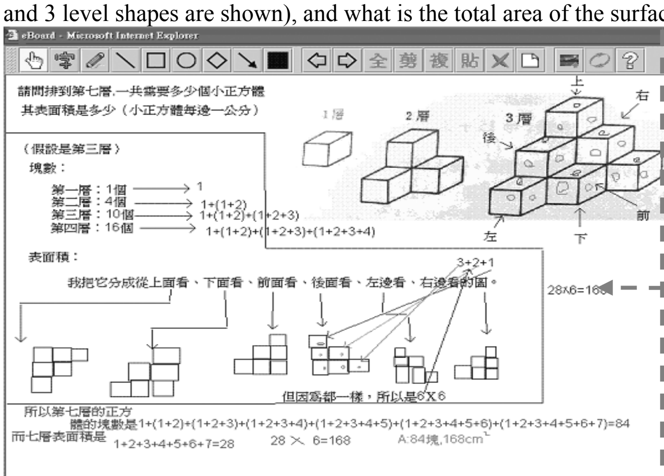


Teacher's comments on the three creative solutions.

Most students can only calculate those numbers that can be seen on the cube. However, they have great difficulty on calculating the numbers that are hiding behind the cube.

This genius student expanded the cube into a two dimensional shape such that all six surfaces can be seen clearly with the corresponding numbers on it. The student even figured out that one group of three surfaces actually has the same numbers and the other group of three surfaces are the same. So, he only needed to calculate two equations and then multiply the sum by 3 to get the final answer. This was really very creative.

Geometric Problem 3: Please calculate how many cubes are needed to form a similar shape with 7 levels (only 1, 2 and 3 level shapes are shown), and what is the total area of the surfaces in this shape.



For those students with less spatial sense and reasoning abilities, they will draw other shapes for levels 4, 5, 6 and 7 and then solve the problem using the 7 level shapes. This way is prone to error as there are too many cubes that cannot be seen.

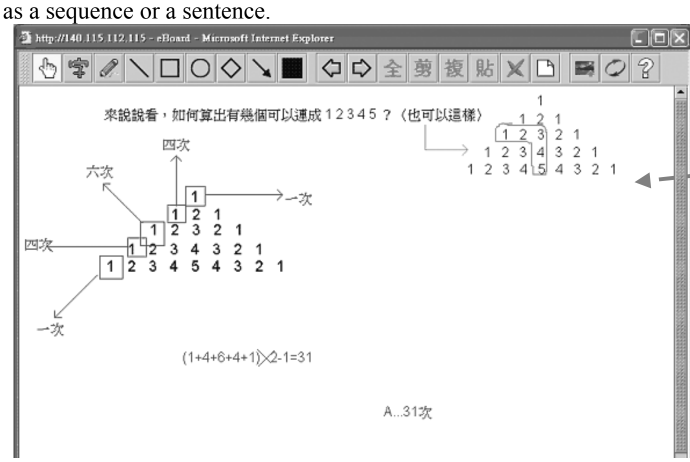
This smart student tried to derive the relationship from levels 1 to 3, and had found the rules between the numbers of cubes required. He then used the rule to infer the required numbers for 7 levels.

To answer the area question, this student used the 3 level shapes to calculate the separate surface area from six different angles. Then, he again used the relation found in 3 levels to infer the total surface area for 7 levels. This solution is excellent, as once the rule has been derived, it can apply to solve any level of n.

Figure 9. Excellent solutions made by students using the multimedia whiteboard system

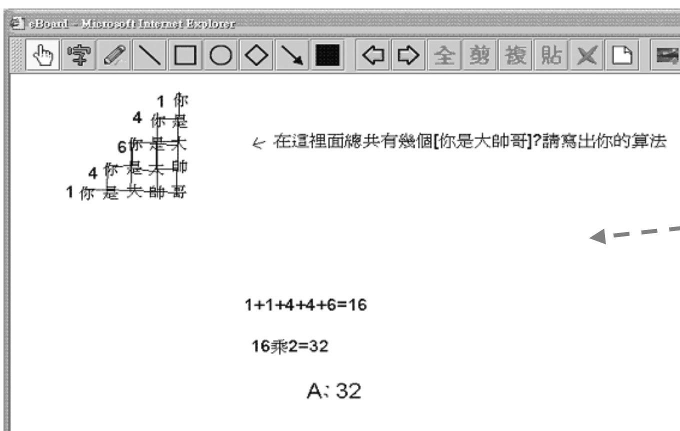
When using the multimedia whiteboard system, the students can be stimulated to try their best to solve problems actively, so that several innovative and excellent solutions could be generated. Many students do not use just the known formula to solve a problem but also derive fantastic solutions using their reasoning and creative thinking abilities. For example, the teachers found several excellent works in the geometric problems as shown in Figure 9. Sung (2005) also pointed out that the peer evaluation 7mechanism should be employed for helping students to understand the problem's properties and to reexamine their own solutions, which then strengthen students' critical thinking and creativity.

Numeral Problem 5: Students were asked to write 5 serial numbers or a 5-words Chinese sentence in the form of a triangle, please calculate how many possible paths can be linked together such that the numbers, or words are connected as a sequence or a sentence.



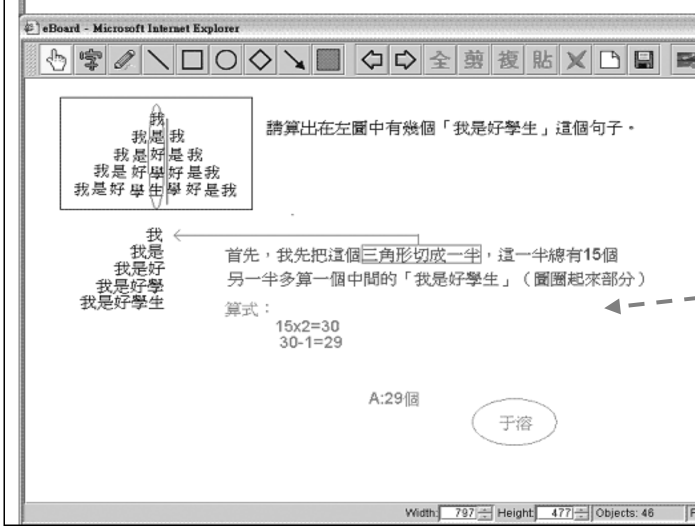
Teacher's comments on a correct solution

This is a correct solution. The student cut the numbers from the middle and then he calculated the possible paths from each number 1 in the left part. Finally, the left sum multiplied by 2 and subtracted 1, because the middle paths had been counted twice. Only 15 students (60%) got correct answers for this problem.



Teacher's comments on a wrong solution 1

This student's solution was similar to the above one, but the mistake was that he forgot to subtract 1.
Only 2 students (8%) had this kind of mistake in this problem.

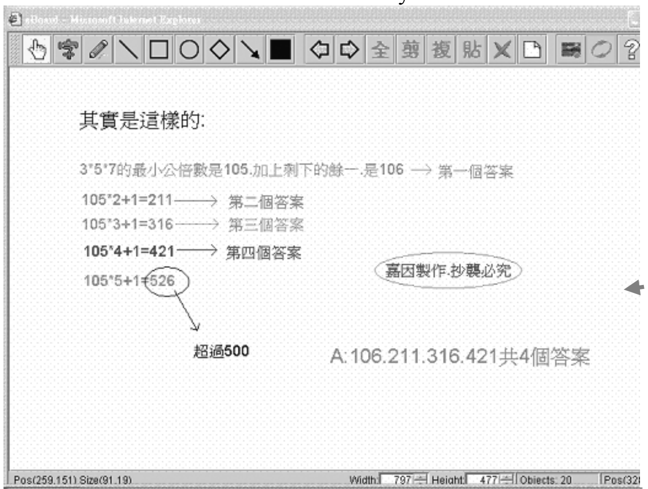


Teacher's comments on a wrong solution 2

This student had the correct reasoning toward the problem, but the mistake occurred on the miscounting one for level 3, after multiplied by 2, the answer became less 2 comparing to the correct answer. There were 6 students (24%) had this kind of mistake in this problem.

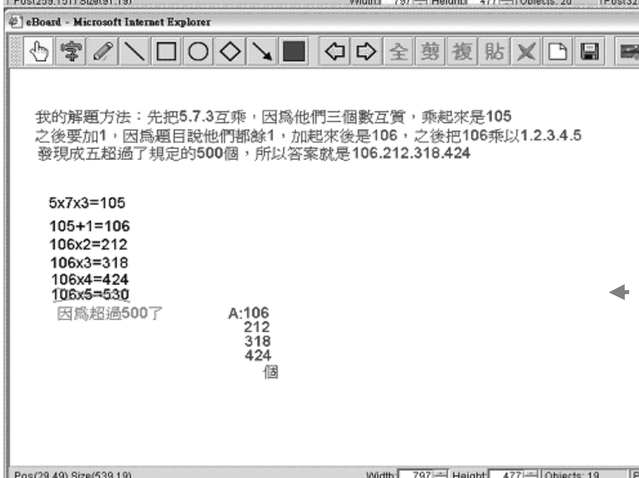
Figure 10. Types of mistakes made in Numeric Problem 5

Numeral Problem 7: There is a stack of cubes less than 500, of which if the cubes are grouped by 3, 5 or 7, then only one cube is left. Please calculate how many cubes are in this stack.



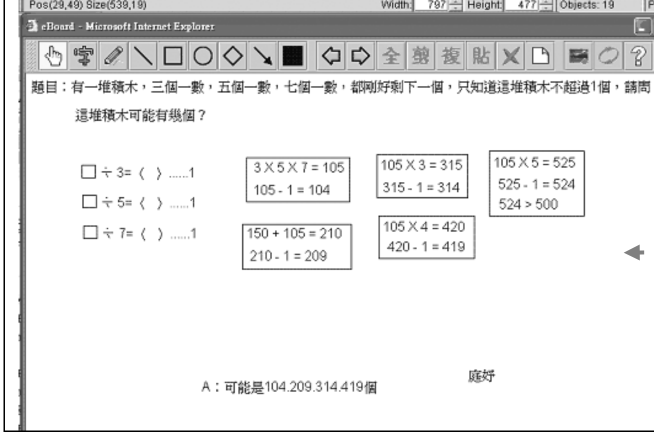
Teacher's comments on a correct solution

Most students know that they need to calculate the least common multiplier (l. c. m) of 3, 5, and 7, which is 105 first. Then by adding 1 equals to 106. As this number is still far smaller than 500, so continue to multiple 105 by 2, 3, 4 and 5, the last time will get a value 525 larger than 500. Therefore, there are 4 possible answers for this problem. Up to 20 students (80%) got correct answers for this problem.



Teacher's comment on a wrong solution 1

This student got the first correct answer 106 by adding 1 to 105(l.c.m), He, however, made a mistake by multiplying the first answer 106 instead of 105 with 2, 3, 4 to get the other three wrong answers.. There were 3 students (12%) having this kind of mistake for this problem.



Teacher's comment on a wrong solution 2

This student did not really understand the meaning of remaining 1 in the problem. So, he subtracted 105 by 1 instead of added 105 by 1. Only 2 students (8%) had this kind of mistake for this problem.

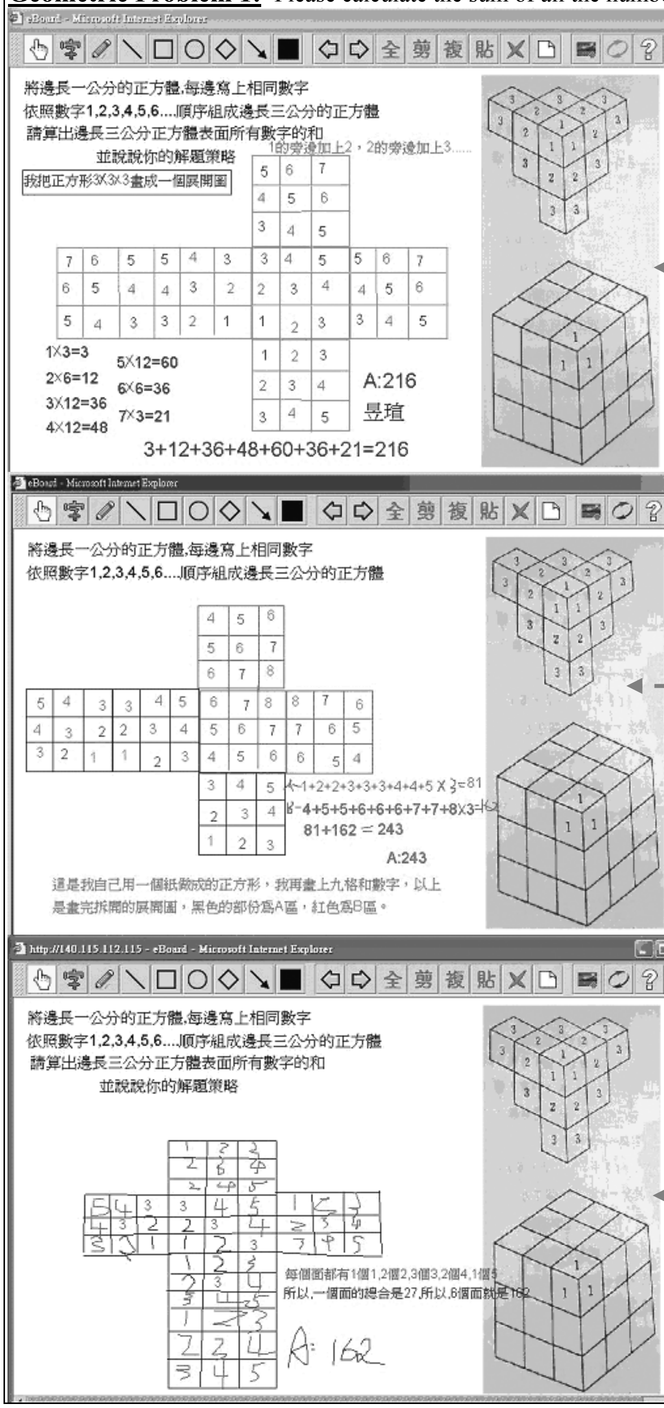
Figure 11. Types of mistakes made in Numeric Problem 7

4.6 Detecting Students' Misunderstanding

Using the Multimedia Whiteboard System to facilitate students learning mathematical problem solving can stimulate students to generate more creative solutions and also help teachers detect what kind of mistakes students might make. The teachers have found several types of mistakes that students often made in numerical and geometric problem solving. Some of the critical mistakes are shown in Figures 10, 11, 12 and 13. By analyzing the mistaken solutions, the teacher can know exactly where the students made mistakes and what caused the misconception. This allows the teacher to give better comments and suggestions accordingly. The teacher also evaluated and improved their class instructions according to the lessons learned in this mathematical problem solving experiment.

Students were better facilitated in applying their solution methods using the multimedia whiteboard system. For example, in Numeric Problem 5, a student used different colors to draw the sequences such that he would not recount the middle column numbers as those numbers have been counted already in the left part. This would reduce the chance of making a mistake.

Geometric Problem 1: Please calculate the sum of all the numbers shown on the 3x3 cube.



Teacher's comments on a correct solution

This student expanded the cube into a two dimensional shape such that all six surfaces could be clearly seen with corresponding numbers on it. She counted the numbers of 1, 2, 3, 4, 5, 6, and 7 respectively, then all the calculated numbers were summed up together to get the final answer. Up to 22 students (88%) got the correct answers for this problem.

Teacher's comments on a wrong solution 1

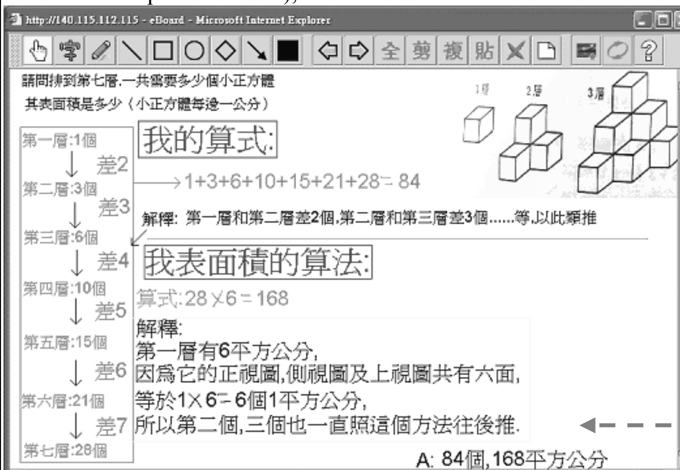
This student also expanded the cube, but he did not begin from 1 to fill the cells so all the sequences were wrong. There were 2 students (8%) had this kind of mistake for this problem.

Teacher's comments on a wrong solution 2

The student misunderstood the sequence of the numbers on the cube. When he filled the cells on the expanded shape, the numbers were wrong. Only this student made in this type of mistake.

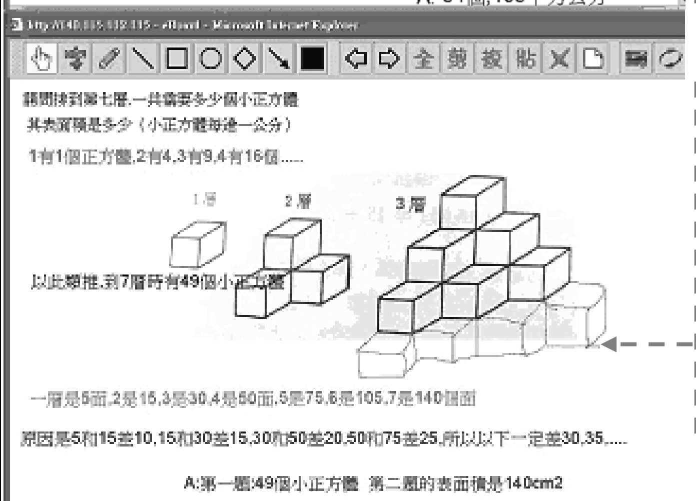
Figure 12. Types of mistakes made in Geometric Problem 1

Geometric Problem 3: Please calculate how many cubes are needed to form a similar shape with 7 levels (only 1, 2 and 3 level shapes are shown), and what is the total area of the surfaces in this shape.



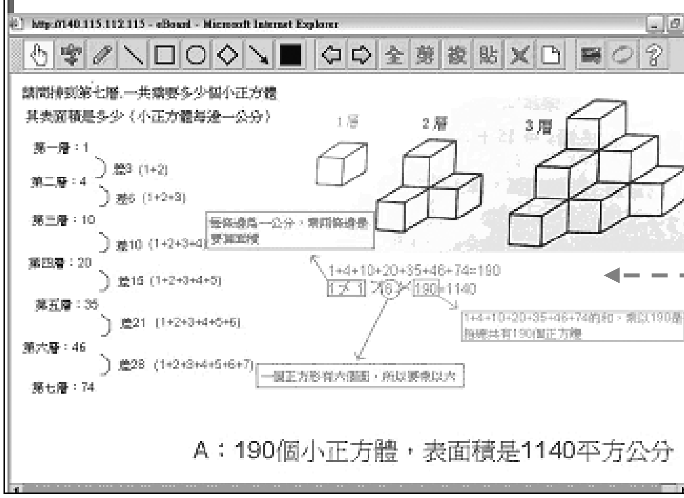
Teacher's comments on a correct solution

This student found the differences for number of cubes between levels 1, 2, and 3, then he inferred the differences for other levels, and the numbers cubes needed at each level. Finally, all the numbers at each level were added up to get the final answer. For the surface area of the shape, the student found that the area of all six dimensions was the same as the number multiplying with 1 cm^2. Therefore, the area is equal to the cube numbers multiplying with 6 cm^2. Up to 23 students (92%) got the correct answers for this problem



Teacher's comments on a wrong solution 1

This student got the right number of cubes for level 1 and level 2, but was incorrect for the third level. The number he got was 9 (10 is the correct answer). Therefore he inferred the number of cubes needed is equal to the square of level (7 X 7 = 49). He also missed the bottom side so the answer was also wrong. Only this student made this type of mistake.



Teacher's comment on a wrong solution 2

This student made a mistake by putting the total number of cubes for the third level as the number of cubes for only level 3. For example, he put 10 (1 + 3 + 6) as the number of cubes for level 3, but 6 is the right answer. Consequently, the area of surface was also wrong. Only this student made this type of mistake

Figure 13. Types of mistakes made in Geometric Problem 3

5. Conclusions & Suggestions

This study explored how multiple representation skills and creativity affect mathematical problem solving using a multimedia whiteboard system. We summarize the main findings according to the proposed three research questions.

5.1 Representation Skills of T and G Are the Keys to Mathematical Problem Solving

Most students could easily apply formulas to get their first solution without any detailed explanation. However, many students obtained good solutions with enhanced T and G representation skills after participating in the criticism and response activities. Only a few students could not do it after this. We classified students into 'Balanced Group' and 'Unbalanced Group' according to their representation skills. The student performance in using representation skill R between the two groups was not different. However, the Balanced Group students performed significantly better than the Unbalanced Group students on representation skills T and G. This finding matches the revised Bloom cognitive taxonomy for the six levels of cognition process, that is 'remembering', 'understanding', 'applying', 'analyzing', 'evaluation', and 'creating'. In this study, the students not only carried with 'remembering', and 'applying' but also 'understanding' the problems and the formulas by elaboration to their solution. Once the students used multiple representations like text, voice, symbol and graph to explain their solutions, the teacher could further investigate whether students really 'understood and applied' or merely 'remembered' the formulas. Therefore, the T and G representation skills play the most important roles in linking the learning process among 'remembering', 'understanding' and 'applying'. We conclude that T and G representation skills are the keys to successful mathematical problem solving for students.

5.2 Profound Effect of Elaboration Ability in Creativity on Multiple Representation Skills

The elaboration ability of students is very essential for them to be able to elaborate their own solutions or to criticize others' solutions using various methods and perspectives. In the Williams' creativity test analysis, elaboration ability is the key factor that stimulates students to create their own knowledge during a problem solving process. The students with high elaboration ability could manipulate T and G representation skills well in problem solving. Students with high elaboration ability could take better advantages of peer interactions and teacher guidance to generate more diversified ideas and solutions in problem solving. In contrast, students with low elaboration ability had great difficulty in manipulating their representation skills well. We conclude that elaboration ability in creativity is one of the critical factors that affects student multiple T and G representation skills in mathematical problem solving.

5.3 Advantages and Disadvantages of Using Multimedia Whiteboard in Mathematical Problem Solving☐

Applying multiple representation skills to solve mathematical problems using the designed multimedia whiteboard system with mutual criticism was helpful in stimulating students with prosperous perspectives on problem solving and criticizing. In the face-to-face classroom, most students follow only the approach that the teacher teaches in class. They seldom try to follow their own ideas and approaches to solve math problems. It is not easy to stimulate students to use multiple representations in math problem solving and explaining. However, in this study, it was shown from teachers' observations that the students enjoyed using the multimedia whiteboard and felt it was very interesting and useful for them to solve mathematical problems. Therefore, they were highly engaged in problem solving in the computer classroom. Even in the criticism activity, they paid good attention to giving comments to others. When the students explained their solutions, criticized others' solutions and responded to others' comments using text, voice, graph, or symbol, they had the chance to reflect on whether they really understood the problem. The teachers were able to identify students that misunderstood points in each component of problem solving, and provide immediate assistance and suggestions.

One drawback of the designed multimedia whiteboard system is that it only provides mouse, keyboard, or microphone as input devices. Students might get frustrated using a mouse to draw mathematical symbols or graphics. Adopting a tablet PC or digital ink technology might solve this problem.

5.4 Suggestions

To cultivate students' critical thinking and reasoning ability through mathematical problem solving is essential. Students multiple representation skills must be stimulated and applied for explanation and criticism in the problem solving process. Due to the time and tool limitations in the physical classroom, ICT tools should be adopted to support students while applying multiple representation skills to mathematical problem solving. The designed web-based multimedia whiteboard system can better facilitate students in giving multiple representations during mutual criticism. It is also recommended that teachers design mathematical problem solving activities supported by multimedia whiteboard systems to improve students' multiple representation skills. Regarding future study related to creativity, the affective factors in Williams' Creativity test, like curiosity, imagination, challenge, and risk-taking that were not addressed in this paper, are worth further study.

Acknowledgment

This study was supported by the National Science Council under the grant number: NSC94 2520-S-110-001.

References

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33* (2), 131-152.

Anderson, L.W., & Krathwohl, D. R. (2001). *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*, New York: Longman.

Arthur, W. (1997). *AIMS plans pattern-based Math curriculum*, Retrieved May 25, 2007, from http://www.aimsedu.org/Documents/Pattern/Pattern.html.

Baer, J. (1993). *Creativity and Divergent Thinking: A Task-Specific Approach*, Hillsdale, N.J.: Lawrence Erlbaum.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *Mathematical Behavior, 21* (4), 401-421.

Canady, J. E. (1982). CPS for the Educational Administrator. *The Journal of Creative Behavior*, *16* (2), 132-149.

Center for Creative Learning (2002). *Creativity assessment*, Retrieved May 23, 2007, from http://www.creativelearning.com/Assess/test21.htm.

Chen, M. F. (1995). *The Research of "Student Factor & Question Factor's" Impact on Multiply and Division Application Problem Solution for Elementary School Studen*t, Unpublished Doctoral Dissertation of Education Psychology & Guidance School, National Normal University, Taiwan (In Chinese).

David, N. (2001). *A comparison of the National Assessment of Educational Progress (NAEP), the Third international Mathematics and Science Study Repeat (TIMSS-R),and the Program for International Student Assessment (PISA), NCES 2001-7*, Washington: U.S. Department of Education, National Center for Education Statistics.

Firestien, R. L., & Treffinger, D. J. (1983). Creative problem solving: guidelines and resources for effective facilitation. *Gifted Child Toda*y, January/February, 2-10.

Forbes, R. (1996). Creative problem solving. *The Journal of Product Innovation Management, 13* (5), 463.

Ford Foundation. (2005). *QUASAR: The revolution of the possible*, Retrieved May 23, 2007, from http://www.fordfound.org/elibrary/documents/5024/022.cfm.

Gagne, E.D. (1985). *The Cognitive Psychology of School Learning*, Boston: Little, Brown and company.

Gerard, J. P. (1999). *Two dimensions of creativity: level and style*, Retrieved May 10, 2007, from http://www.buffalostate.edu/orgs/cbir/readingroom/html/Puccio-99a.html.

Guidance In Jiouhu (2005). *Aptitude test – Williams Creativity Assessment Packet*, Retrieved May 10, 2007, from http://www.psychtest.com/curr01/CATLG031.HTM.

Howe, R.(1996). Instruction and Experience for abilities related to creative processes and products. *Journal of Creative Behavior, 30*, 156-178.

Hwang W. Y., Chen N. S., & Hsu J. L. (2006). Development and Evaluation of Multimedia Whiteboard System for Improving Mathematical Problem Solving. *Computers & Education, 46* (2), 105-121.

Isaksen, S.G., Dorval, K.B., & Treffinger, D.J. (2000). *Creative Approaches to Problem Solving: A Framework for Change*, Buffalo, New York: Creative Problem Solving - Group Buffalo.

Jeffery, P. (1993). *Multi-Modal Learning Newsletter,* 4, July, Swinburne Press.

Jennifer, A. R. (2003). *Creative problem solving in the classroom*, Retrieved May 21, 2007, from http://web.indstate.edu/soe/blumberg/cpsstu3.html.

Johanna, E. D. (2003). *A brief review of creativity*, Retrieved Retrieved May 21, 2007, from http://www.personalityresearch.org/papers/dickhut.html.

Jonassen, D. H (2000). *Computers as mind tools for schools: Engaging critical thinking*, Columbus. OH: Merrill/Prentice Hall.

Johnson, S. (1998). What's in a representation, why do we care, and what does it mean? Examining evidence from psychology. *Automation in Construction, 8.*

Kaput, J. J. (1987a). Toward a theory of symbol use in mathematics. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (159-195). Hillsdale, NJ: Erlbaum.

Kaput, J. J. (1987b). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (19-26). Hillsdale, NJ: Erlbaum.

Kuang, G. H.(1993). *The Influence of Ceramics' Creative Thinking Teaching Class on creativity developments in the fifth and sixth grades' kids*, Unpublished Master Dissertation of Compulsory Education School of National Hsin-Chu Teachers' College (In Chinese).

Lane, S., Stone, C. A., Ankenmann, R. D., & Liu, M. (1995).Examination of the assumptions and properties of the graded: Item Response Model: An example of using a Mathematics performance assessment. *Applied Measurement In Education, 8* (4), 313-340.

Lane, S., & Wang, N.(1996). Detection of gender-related differential item functioning in a Mathematics performance assessment. *Applied Measurement In Education, 9* (2), 175-199.

Lane, S. (1999). *Validity evidence for assessment*, Reidy Interactive Lecture Series, October 14-15, Providence, R.I.

Lange, J.D.(1999). *Framework for Classroom Assessment in Mathematics*, National Center for Improving Student Learning and Achievement in Mathematics and Science / Freudenthal Institute. Madison: WCER.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (33-40). Hillsdale, NJ: Erlbaum.

Lin, W. C. (2002).*The Research of Creativity and Creative Thinking in Education*, Unpublished Master Dissertation of Education School, National Sun Yat-Sen University, Taiwan (In Chinese).

Lubart, T. I., & Sternberg, R. J. (1996). Investing in Creativity. *American Psychologist, 51* (7), 677-688.

Mayer, R. E. (1992).*Thinking , Problem Solving, Cognition*, New York: W. H. Freeman and Company.

Milrad, M. (2002). Using Construction Kits, Modeling Tools and System Dynamics Simulations to Support Collaborative Discovery Learning. *Educational Technology & Society 5* (4), 76-87.

Moskal, B. M., & Leydens, J.A. (2000). Scoring rubric development: validity and reliability. practical assessment, *Research & Evaluation, 7* (10), Retrieved May 20, 2007 from http://PAREonline.net/getvn.asp?v=7&n=10.

NAGB(2001). *NAEP framework project: Mathematics framework for 2005*, US Department of Education.

NAGB(2002). *Mathematics framework project: Mathematics framework for the 2003*, US Department of Education.

National Council of Teacher of Mathematics (2000). Principles and Standards for School Mathematics, Retrieved May 20, 2007 from http://www.nctm.org.

Osborn, A.F.(1953). *Applied Imagination*, New York: Scribner's.

Parke, C. S. (2002). Mathematics performance assessment: discovering why some Items or rubrics don't measure up. *RMLE Online*, 25 (1), Retrieved May 21, 2007, from http://www.nmsa.org/research/rmle/rmle/rmle_jan_2002_c.html.

Parker, J. P. (1978). We all have Problems...who doesn't? But can they all be solved. *Gifted Child Today*, March/April, 61-63.

Pilgrim, C. (1996). Multi-modal learning: a case study. *Proceedings of the Third International Interactive Multimedia Symposium*, 316-322. Perth, Australia, Retrieved May 21, 2007, from http://www.aset.org.au/confs/iims/1996/lp/pilgrim.html.

Psychology Press.(2005).*Aptitude Test – Williams Creativity Assessment Packet*, Retrieved May 21, 2007, from http://www.psy.com.tw/ (In Chinese).

Silver, E. A. (1989). QUASAR. *The Ford Foundation Letter*, 20, 1–3.

Sternberg, R. J. (1999). *Handbook of creativity*, New York: Cambridge University Press.

Sung, Y.-T., Chang, K.-E., Chiou, S.-K., & Hou, H.-T. (2005). The design and application of a web-based self- and peer-assessment system. *Computers & Education, 45* (2), 187-202.

Tennyson, R. D., & Breuer, K. (2002). Improving problem solving and creativity through use of complex-dynamic simulations. *Computers in Human Behavior, 18* (6), 650-668.

Truman, S., & Mulholland, P. (2003). *Creative-collaborative music learning: A computer supported approach*, Retrieved May 21, 2007, from http://kmi.open.ac.uk/people/sylvia/papers%20pdf/TrumanMulholland%202003.pdf.

Wallas, G. (1926) *The Art of Thought*, New York: Harcourt-Brace.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science, 21* (2), 179-217.