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Contract-auction based distributed resource allocation for cooperative communications

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Abstract: Cooperative communication significantly improves the performance of wireless systems. Transmission of signal in such a system can be accomplished by the help of intermediate relay nodes. However, due to the selfish nature of network nodes, an incentive mechanism is required to stimulate relay nodes to cooperate. On the other hand, the authors assume that the source nodes are ill-informed about channel conditions of the relay nodes, which may result in asymmetry of information. In this study, the authors propose a distributed power allocation and price assignment algorithm over cooperative wireless networks. The proposed solution aims to achieve optimum power allocation to the source nodes and best price of power at the relay nodes, in the presence of asymmetric channel state information. To this end, the authors combine contract theory and auction mechanism in order to provide the highest possible utility for both the source and the relay nodes. The proposed distributed approach benefits the source nodes by preventing the relay nodes from cheating behaviour. Additionally, it favours the relay nodes by letting them assign the final price of power. Finally, the authors present simulation results in order to demonstrate efficiency of the proposed distributed multi-user algorithm.

1 Introduction

Cooperative communication, in which a source node transmits information to a destination via selected relay node(s) has attracted considerable attention recently. The main idea of cooperative relaying is to utilise neighbouring nodes as virtual antennas and provide the benefits of multi-input multi-output communications. Several cooperative strategies have been proposed and developed so far, including amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward. Related publications show how cooperative communication improves the overall performance of wireless systems [1, 2]. Nevertheless, in order to fully exploit the potential advantages of relay-based cooperative communication, it is crucial to design efficient resource allocation, such as relay selection [3–5] or power control [6, 7].

In cooperative wireless networks, nodes may not serve a common goal. Therefore, a mechanism is required to stimulate nodes to cooperate. On the other hand, due to unavailability of global channel state information (CSI), there is an asymmetry of information among nodes. According to such characteristics, in this work we employ contract theory and auction mechanism to optimally allocate power and assign its price. We assume that a source node designs contract and proposes it to the relay nodes. On the other hand, the relay nodes have the right to accept or reject the proposed contract. We categorise relay nodes based on their channel conditions to the destination and we refer to it as *type* of relay. The designed contract in this work consists of one contract pair for each type. Each contract pair is in the form of (D, ζ), where D is demand of source node for signal-to-noise ratio (SNR) at the destination and ζ is unit price of power at the helping relay. If a relay node accepts the proposed contract, then it may play an auction game. The main reason to do this is that the initial price proposed by the contract, only maximises utility of the source node. However, other factors may affect the price of power and need to be taken into account as well, including utility of the relay nodes and the competition among the candidate relays. In such a system and with dynamic changes in the price of power, the allocated power to the source node is required to change dynamically.

This work is a follow-up of our recent conference contribution [8], where the proposed solution considers one source node and multiple relay nodes. However, in the existence of multiple source nodes, a competition occurs among nodes for the available resources. This competition finally affects the price of power at helping relays. Considering multiple source nodes, this work discusses how the price of power is set at the relay nodes, taking utility of nodes into account. Moreover, a relay node may not be always able to provide the whole required SNR demand by a source node. Therefore, this work considers a scenario when power budget of a relay node is less than a source node's demand. Furthermore, in order to have less computational complexity in each step, a source node excludes those relay nodes who are bidding for infeasible prices. Finally, the whole process continues until a source node finds the best relay node and a participant relay node sets the best price of power. The proposed model in this work is simple to implement and does not require availability of global CSI. Additionally, the simulation results show that the proposed algorithm achieves comparable performance to the centralised scheme.

The organisation of this paper is outlined as follows. Section 2 reviews the related works. Section 3 presents the system model and defines the utility functions of source node and relay node. The proposed distributed power allocation and price assignment algorithm is described in Section 4. Section 5 presents performance evaluation, followed by conclusions in Section 6.

2 Related works

Recently, optimising resource allocation in cooperative wireless systems has obtained researchers attention [9]. For instance, Alam *et al.* [10] propose a joint relay selection, power allocation and sub-carrier assignment approach. Sadek *et al.* [11] address the relay assignment problem, based on the global availability of channel statistics. Specifically, they analyse the performance of two relay assignment protocols: distributed nearest neighbour relay assignment and infrastructure-based relay assignment. As a result, significant gains can be obtained when applying these algorithms over direct transmission in terms of coverage area, transmit power and spectral efficiency. Zhao *et al.* [12] also consider availability of CSI and channel statistics at all the nodes. An optimal power allocation is derived in order to minimise the outage probability, considering total and individual power constraints. Additionally, a selection AF scheme is proposed to select only one relay node. This scheme maintains full diversity order and achieves better outage behaviour, compared with all participate AF (AP-AF) and optimum power allocation schemes.

The above-mentioned works and most proposed relay selection and resource allocation algorithms require complete CSI as a global knowledge. However, this is an unrealistic assumption for real-world scenarios. Thus, distributed resource allocation techniques that only work with local or partial channel information observe more attention. In [13], a distributed relay selection scheme is proposed, considering finite-state Markov chain channels. A stochastic control modelling framework is utilised in order to provide a solution for the relay selection problem. Krikidis *et al.* [14] propose a solution for partial relay selection, when only neighbouring channel information is available to the source. In [15], the authors propose a distributed power allocation with partial CSI. First, each relay node individually decides whether to cooperate or not. Then that relay applies one of the proposed power allocation strategies, which works with limited CSI.

Game theory, as a powerful and flexible tool, has been widely utilised in cooperative wireless networks. Song *et al.* [16] provide game theoretical based distributed solutions to the resource allocation problems for device-to-device communication. Chen *et al.* [17] apply Stakelberg game in a distributed market-based pricing framework. Since a multi-hop wireless relay communication is considered, the payment by the source node should be shared among all the participant relays in a delivery process. However, game theory-based solutions require further investigation for existence, uniqueness and computation of Nash equilibrium, which pose extra challenges for these methods. In [18], an ascending-clock auction algorithm is proposed, in order to allocate relay power to the source nodes. The proposed distributed algorithm enforces truthful power demands by the source nodes and converges in a finite number of time steps to the unique Walrasian equilibrium allocation. The announced price vector in this work is set only by relay nodes, assuming that they are honest at all times. Although the authors discuss about truth telling of source nodes, they simply assume that relay nodes behave in a trustworthy manner. Moreover, there is no discussion on feasibility of the proposed prices by relay nodes.

Recently, contract theory-based communication is applied in cognitive radio networks [19] and cooperative cellular networks [20–22]. For instance, Gao *et al.* [19] study the problem of designing proper economic incentive for the success of dynamic spectrum sharing. The proposed ContrAuction mechanism focuses on maximising profit of a primary owner in a hybrid market. In [20], a relay selection scheme is proposed in the presence of incomplete knowledge of channels. The proposed contract theory-based scheme selects the best relay nodes, in order to minimise cost of communication for the source node, when it tries to transmit its signal to a destination via relays. The authors also prove that a relay node with maximum harmonic mean value of the source–relay and relay–destination channel conditions has the least communication cost. Finally, the relay selection approach proposed in [21] applies contract theory in order to find the best relay nodes under budget constraints. However, this mechanism leaves the bargaining power completely to the source node that eventually leads to a static contract and reduces flexibility of the proposed solution. Considering this issue, the proposed method in this work does not follow accept/reject mechanism and it provides the opportunity for nodes to negotiate for the price of power.

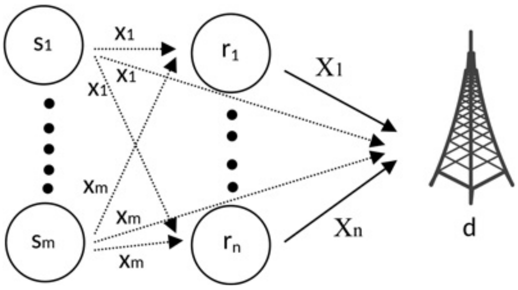

Fig. 1 *Considered system with m source nodes and n relay nodes. Dotted lines represent the broadcasting phase, while the solid lines represent the cooperation phase*

3 System model and utility functions

3.1 System model

Consider a wireless network with m source nodes, denoted as $s_1, s_2, \ldots, s_m$. The source nodes assumed to have data symbols $x_1, x_2, \ldots, x_m$, respectively. They aim at communicating their data symbols to a common destination node D via a set of n relay nodes $r_1, r_2, \ldots, r_n$. We assume that transmissions use orthogonal channels through time division multiplexing (Fig. 1).

Relay nodes are ready to devote their transmission power to the source node. We divide the signal transmission into two phases. During the first phase, called *broadcasting phase* a source node s_j broadcasts its data. The received signals at the relay node r_i and at the destination node d are

$$\begin{aligned} y_{s_j,i} &= \sqrt{p_{s_j}} h_{s_j,i} x_j + n_{s_j,i}, \quad i = 1, \ldots, n \\ y_{s_j,d} &= \sqrt{p_{s_j}} h_{s_j,d} x_j + n_{s_j,d}, \end{aligned} \tag{1}$$

where p_{s_j} is the broadcast transmit power of the source node s_j. x_j, y_{s_j}, i and y_{s_j}, d denote the transmitted signal by the source s_j, the signals received at relay node r_i and the destination node, respectively. $h_{s_{j,i}}$ and $h_{s_{j,d}}$ represent the channel coefficients of s_j–r_i and s_j–destination links. $n_{s_{j,i}}$ and $n_{s_{j,d}}$ are the zero-mean additive white Gaussian noise (AWGN) samples with variance N_0 at the relay node r_i and the destination, respectively.

During the second phase, called *cooperation phase*, relay node r_i uses the AF protocol, i.e. normalises the received signal and transmits it to the destination in its assigned time slot. AF relaying is chosen for the simplicity and transparency of the relaying process compared with DF relaying. However, other cooperation methods are also applicable in this work. The received signal at the destination node by the relay node r_i is

$$y_{i,d} = \sqrt{p_{r_i}} h_{i,d} X_i + n_{i,d}, \tag{2}$$

where $h_{i,d}$ is the channel coefficient from relay node r_i to the destination node and p_{r_i} is the power used by the relay node r_i for transmission in its time slot. $n_{i,d}$ denotes the AWGN of the r_i-destination channel. Without loss of generality, we assume that the noise variance is the same for all the links. X_i is the normalised transmitted signal from the relay node r_i to the destination node d and is given by

$$X_i = \frac{y_{s_j,i}}{|y_{s_j,i}|}. \tag{3}$$

The achieved SNR at the destination node that results from the direct transmission can be expressed as

$$\gamma_{s_j,d} = \frac{p_{s_j} |h_{s_j,d}|^2}{N_0}, \tag{4}$$

and the achieved SNR by the help of relay node r_i is equal to

$$\gamma_{s_j,i,d} = \frac{p_{s_j}|h_{s_j,i}|^2 p_{r_i}|h_{i,d}|^2}{N_0(p_{s_j}|h_{s_j,i}|^2 + p_{r_i}|h_{i,d}|^2 + N_0)}. \tag{5}$$

Using (5), the s_j–destination channel capacity when relay node r_i participates could be written as

$$C_{s_j,r_i,d} = \frac{1}{2}\log_2(1 + \gamma_{s_j,d} + \gamma_{s_j,i,d}), \tag{6}$$

Accordingly, if n relay nodes participate in cooperation, then the capacity formula can be written as

$$C_{s_j,r,d} = \frac{1}{n+1}\log_2\left[1 + \gamma_{s_j,d} + \sum_{i=1}^{n}\gamma_{s_j,i,d}\right]. \tag{7}$$

Hence, the overall communication between the source nodes and the destination node could be performed over $m + n$ time slots.

3.2 Utility functions

As we stated above, we consider a typical wireless cooperative network, in which a particular mobile node intends to transmit its data to a destination node. We assume that the source node asks for neighbouring nodes assistance, if it suffers from poor channel conditions to the destination. This work tries to answer two fundamental questions. First, what is the optimum amount of power that a source node should buy from each relay node to optimise its utility. Second, what is the best price of power at each relay node that optimises utility of the relay. Here, we formulate utility functions for source and relay nodes. These utility functions are later needed for proper power allocation and price assignment.

(i) *Source node:* The main objective of a source node is to transmit its signal to a destination node. Therefore, it may ask other nodes to forward its signal, if it suffers from poor channel conditions to the destination. Moreover, the source node does not have free access to the relay nodes' resources. Consequently, a reimbursement mechanism is required to stimulate relay nodes to devote their power to the source. Assume that the relay node r_i is helping the source node and in return, the source node pays credit for the service it receives. The received payment at the relay node r_i is equal to

$$\beta_i = p_{r_i} * \zeta_i, \tag{8}$$

where ζ_i is the price per unit of power at the relay node r_i. Accordingly, if n relay nodes cooperate during signal transmission, the total reimbursement paid by the source node s_j to the relay nodes is equal to

$$\beta_j = \sum_{i=1}^{n}\beta_i. \tag{9}$$

The utility of the source node s_j, denoted as U_{s_j}, is equal to the service it receives from the helping relay nodes minus cost of that service. U_{s_j} can be formulated as

$$U_{s_j} = aC_{s_j,r,d} - \beta_j, \tag{10}$$

where a is gain per unit of capacity.
(ii) *Relay node:* The main objective of a relay node is to gain as much profit as possible. The utility of a relay node is equal to the profit it receives by a source node minus cost of forwarding that source node's signal to the destination. For instance, utility of relay node r_i can be written as

$$U_{r_i} = (\zeta_i - c_i)p_{r_i}, \tag{11}$$

where c_i is cost of spending one unit of power.
The objective of both the source node and the relay node is to maximise their utility. From (10) and (11), it is clear that the price of power plays an important role in the achieved utility by the nodes.

4 Contract-auction-based solution

This section explains the proposed algorithm in detail Fig. 2. The algorithm utilises contract theory and auction mechanism, in order to optimise power allocation and price assignment. We first explain how a contract is designed. Then, auction game is explained, following by the proposed algorithm.

4.1 Contract theory

As stated before, the principle of contract theory has been applied in this work. We first categorise relay nodes to different types based on their channel quality to the destination. We denote a vector of types $\boldsymbol{\Theta} = \{\theta_1, \theta_2, \ldots, \theta_n\}$ and without loss of generality, we assume that $\theta_1 < \theta_2 < \cdots < \theta_n$. More specifically, for a relay node r_i, if $\theta_j \le |h_{i,d}|^2 < \theta_{j+1}$, then this relay node is of type θ_j. We consider a basic contract form without punishment, consisting of (i) a source node's demand D_i for SNR at the destination node and (ii) the proposed initial price of power ζ_i by the source node at that relay node. The contract could be formulated as

$$\mathcal{C} = \{(D_i, \zeta_i), \quad \forall\theta_i \in \boldsymbol{\Theta}\}. \tag{12}$$

Two main constraints should be considered, while designing a contract. First, the designed contract should follow the *individual rationality (IR)* constraint. Since nodes are rational and selfish, they all follow the IR constraint. A node is rational, only if it receives non-negative payoff by helping others. This happens if the price per unit of power sets equal or higher than its actual cost, such that

$$\zeta_i - c_i \ge 0, \quad \forall\theta_i \in \boldsymbol{\Theta}. \tag{13}$$

The second constraint is the *incentive compatibility (IC)*, which means that each individual relay will receive the highest utility by accepting the contract pair designed for its type only. In other words, the IC constraint stops relay nodes from cheating behaviour and forces them to reveal their types to the source node truthfully [23], such that

$$\frac{D_i}{\theta_i}(\zeta_i - c_i) \ge \frac{D_j}{\theta_i}(\zeta_j - c_i), \quad \forall\theta_i \in \boldsymbol{\Theta}. \tag{14}$$

Additionally, there is a constraint for the price of power that needs to be taken into account. The price of power should not be too low that results in negative utility for the relay nodes. Moreover, it should not be too high that results in negative utility for the source nodes. Based on the utility of the source node (10) and the *IR* constraint (13), the feasible price range could be formulated as

$$c_i \le \zeta_i \le \frac{aC_{s_j,r_i,d}}{p_{r_i}}, \quad \forall i. \tag{15}$$

A source node designs contract as follows. For each relay of type $\theta_i \in \boldsymbol{\Theta}$, a contract pair is designed in the form of a two-tuple consisting of optimal power demand and price of one unit of power. Price of one unit of power in the initial contract is selected in a way that maximises utility of the source node. Therefore, according to (15) the initial price per unit of power for relay of type θ_i is set to c_i. Then according to [24] the initial optimum

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Algorithm 1 Distributed Contract-Auction Algorithm
1. A source node, who suffers from poor channel conditions to its destination, designs and broadcasts a contract, $\mathcal{C} = \{(D_1, \zeta_1), (D_2, \zeta_2), ..., (D_n, \zeta_n)\}$.
2. do
    • Each interested relay accepts the proper contract pair.
    • Each candidate relay may be interested to bid for the price of power.
    • If (the the number of $r_i$'s neighbor $< 3$) or (required power for relay $r_i$ to provide $D_i$ at the destination $> p_{r_i}^{max}$) Then $r_i$ increases its price by the step size $\zeta_i = \zeta_i + \mathcal{S}$.
    • Else If (the the number of $r_i$'s neighbor $> 3$) Then $r_i$ decreases its price by the step size and sets it as the new price, if it is in feasible price range $\zeta_i = \zeta_i - \mathcal{S}$.
    • Interested relay nodes announce their bid to the source node.
    • ($\forall$ received $\zeta_i$) If ($\zeta_i$ is not a feasible price) then source node excludes $r_i$ from list of candidate relays.
    • The source node designs and broadcasts a new contract based on the updated prices.
while (the proposed contract remains unchanged).
3. Source node accepts the lowest bid, calculates the the optimal power demand from that relay node and communication starts.
4. If the selected relay is only able to provide part of the required SNR, then the source node selects the relay with the second lowest bid and this process continues until the source node could accomplish its task.
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Fig. 2 *Distributed contract-auction algorithm*

power demand from relay node r_i that maximises utility of the source node is calculated as follows

$$p_{r_i}^{*} = \max\left[0,\ \min\left(\sqrt{\frac{A_i B_i}{\zeta_i}}\frac{Y + \sqrt{Y^2 + 4X\tilde{w}}}{2X} - B_i, p_{r_i}^{\max}\right)\right], \quad (16)$$

where

$$A_i = \frac{p_{s_j}|h_{s_j,i}|^2}{(N_0 + p_{s_j}|h_{s_j,d}|^2)}, \quad B_i = \frac{p_{s_j}|h_{s_j,i}|^2 + N_0}{|h_{i,d}|^2}$$

$$\tilde{w} = \frac{a}{\ln 2}, \quad X = 1 + \sum_{i=1}^{n} A_i$$

and $Y = \sum_{i=1}^{n} \sqrt{\zeta_i A_i B_i} \cdot p_{r_i}^{\max}$ is the power constraint at the relay node r_i. Finally, the source node calculates the optimal SNR demand from the relay node r_i as

$$D_i = \theta_i p_{r_i}^{*}. \quad (17)$$

The question arises here, how a source node calculates optimal power demand while it does not have access to the relay node's private information like $h_{i,d}$. When a source node utilises contract theory, it categorises relay nodes based on their types and proposes a contract pair for each relay type θ. Consequently, the source node does not have access to the value of $h_{i,d}$, but it substitutes $h_{i,d}$ by θ_i for all $\theta_i \in \boldsymbol{\Theta}$.

4.2 Auction theory

An auction is a decentralised market for allocating resources and has recently been introduced to several areas of wireless communications, including power allocation [25] and spectrum sharing [26, 27]. Auction theory has been applied in this work, as part of a solution for the mentioned problems. We assume that a relay node acts as *seller* and sells power to the source node. On the other hand, a source node is a *buyer* and buys power from the relay nodes. Relay nodes may play an auction game, if they are interested to cooperate. Therefore, a candidate relay node may bid for the price of power, after accepting the proposed contract by the source. The announced price of power by the relay nodes could be less or more than the announced price of power by the contract.

4.3 Proposed distributed contract-auction algorithm

In this section, we propose a distributed algorithm for power allocation and price assignment problems, when the source node does not have access to the relays' CSI. Some available works in the literature try to overcome this problem by assuming that CSI is broadcast by the nodes. However, due to the selfish and greedy nature of the nodes, relay nodes may lie about their private CSI. This is very likely to happen if the relays are supposed to acquire some benefits based on this private CSI. Considering the fact that nodes may not be honest at all times, persuades us to apply contract theory that prevents relay nodes from cheating behaviour.

We divide the operation of the proposed algorithm into two main phases. During the first phase, named *contract phase*, the source node designs a contract following the above-mentioned constraints and broadcasts it in the network. Then those neighbouring relay nodes, who receive a copy of the contract, respond to the source node if they are interested to cooperate. When a relay node accepts a contract pair, it reveals its type to the source node. Relay nodes also prefer not to cheat or pretend that they are from other types, due to the IC of the proposed contract.

During the second phase, named *auction phase*, nodes play an auction game. After a helping node accepts the proper contract pair, it becomes aware of the initial price of power that the source is interested to pay. In this phase, candidate relay nodes may bid for the price of power. Again, due to the greedy nature of nodes, the relay nodes are interested to maximise their utility as much as possible. Therefore, a relay node may bid for a higher price of power, if there is limited number of nodes in its neighbourhood (<3). Accordingly, the new price would be increased by $\mathcal{S}$, where $\mathcal{S}$ is the step size. On the other hand, higher number of nodes decreases chance of a relay to be selected by a source node. Hence, a relay node may be interested to bid for a lower price of power in order to obtain the source node's attention. A relay node may bid for a lower price of power, when the number of neighbour nodes is large (>3). Therefore, the new price would be decreased by $\mathcal{S}$. Additionally, the source node's demand for SNR can affect the price of power as well. For instance, if the required power from relay node r_i is greater than its power budget $((D_i/\theta_i) > p_{r_i}^{\max})$, then r_i increases its price in order to force the source node to reduce its SNR demand. When the source node receives the new bids announced by the relays, it excludes those relays which offer infeasible prices. The reason is that the *IR* constraint emphasises that each node should receive non-negative profit. However, an infeasible price may result in a negative utility for the source or for the relay node. Applying the new prices, the source node designs a new contract and calculates optimal SNR demand based on (16) and (17). If the newly designed contract is identical to the contract designed in previous step, then the source node starts buying power from the relay nodes and communication starts (steps 3 and 4). Otherwise the source node broadcasts a new contract and the algorithm jumps back to step 2.

Having more than one source node in the system may result in receiving multiple requests of forwarding signal by a relay node. This relay node can take advantage of such a scenario and increase the price of power in order to earn more revenue. Moreover, the relay node r_i may announce different price of power for different source nodes since the announced price is a function of r_i's distance to both the source node and its destination node. Consequently, in scenarios with less number of helping relay nodes compared with the source nodes, the utility of PUs decreases due to the competition among them.

5 Performance evaluation

To evaluate the performance of the proposed algorithm, the simulation results are presented in this section. First for the purpose of comparison, we investigate the centralised power allocation in Section 5.1, followed by outage probability formulation in Section 5.2. In Section 5.3, we compare the result with equal power allocation (EPA) scheme. EPA is the conventional AP-AF scheme where a source node s_j and all its helping relays p_{r_i} use the same power $(p_{r_i} = p_{s_j} = p_{\text{total}}/(n+1))$ [12].

5.1 Centralised optimal power allocation scheme

For the purpose of comparison, in this subsection we investigate a centralised optimal power allocation problem with its solution. Let us assume that the system resources are shared by all available n relay nodes. The optimal power allocation problem that maximises C_{s_j}, r, d and thereby minimises outage probability can be formulated as follows

$$\begin{aligned}&\max \quad \frac{1}{n+1}\log_2\left(1+\gamma_{s_j,d}+\sum_{i=1}^{n}\gamma_{s_j,i,d}\right)\\ &\text{s.t.} \quad p_{s_j}+\sum_{i=1}^{n}p_{r_i} \le p_{\text{total}}, \quad 0 \le p_{r_i} \le p_{r_i}^{\max}, \ 0 \le p_{s_j} \le p_{s_j}^{\max} \ \forall i,\end{aligned} \tag{18}$$

where $\gamma_{s_{j,d}}$ and $\gamma_{s_{j,i,d}}$ are defined in (4) and (5). This optimisation problem is modelled with both sum and individual power constraints. $p_{s_j}^{\max}$ is the power constraint at the source node s_j and the total available power is limited to p_{total}.

Since $\log_2(1+x)$ is monotonically increasing function of x, we can get an equivalent optimisation problem as in [12]

$$\begin{aligned}&\min \quad \sum_{i=1}^{n}\frac{p_{s_j}^2 a_i^2 + p_{s_j}a_i}{p_{s_j}a_i + p_{r_i}b_i + 1}\\ &\text{s.t.} \quad p_{s_j}+\sum_{i=1}^{n}p_{r_i} \le p_{\text{total}}, \quad 0 \le p_{r_i} \le p_{r_i}^{\max}, \ 0 \le p_{s_j} \le p_{s_j}^{\max} \ \forall i,\end{aligned} \tag{19}$$

where $a_i = (|h_{s_j,i}|^2/N_0)$ and $b_i = (|h_{i,d}|^2/N_0)$.

The solution of the optimal power allocation (19) among the relay nodes to maximise the capacity of the system, given a fixed transmit power p_{s_j} for the source node s_j, with total and individual power constraint is

$$p_{r_i} = \left(\sqrt{\frac{p_{s_j}^2 a_i^2 + p_{s_j}a_i}{b_i}}\lambda - \frac{p_{s_j}a_i + 1}{b_i}\right)_0^{p_{r_i}^{\max}}, \tag{20}$$

where λ is a constant chosen to meet the total power constraint and $(x)_l^u$ is defined as

$$(x)_l^u = \begin{cases} l, & x < l, \\ x, & l \le x \le u, \\ u, & u < x, \end{cases} \tag{21}$$

which can be considered as an extended water-filing process. According to [12], in the high SNR regime the optimal power allocation reduces to

$$p_{r_i} = \left(\frac{p_{s_j}a_i}{\sqrt{b_i}}\lambda - \frac{p_{s_j}a_i}{b_i}\right)_0^{p_{r_i}^{\max}}. \tag{22}$$

5.2 Outage probability

An outage occurs when the capacity C falls below a certain threshold R_0, with outage probability $P_{\text{out}} = P[C < R_0]$. The threshold is determined according to the application and the transmitter/receiver structure. Based on [12], in the high SNR regime, the upper bound for the outage probability is

$$P_{\text{out}}^{\text{AP}} \le \frac{\lambda_0 \prod_{i=1}^{n}(\lambda_i + \delta_i)}{n+1}\left(\frac{2^{(n+1)R_0} - 1}{\gamma}\right)^{(n+1)}, \tag{23}$$

where $\gamma = (1/N_0)$. For a given set of channels and noise coefficients, the transmitted SNR at each node and the received SNR at the destination node are proportional to $(1/N_0)$. Therefore, γ is served as the measure of system SNR. λ_0 is the exponential parameter of $p_{s_j}|h_{s_j,d}|^2$ and can be written as

$$\lambda_0 = \frac{1}{E\{p_{s_j}|h_{s_j,d}|^2\}} = \frac{1}{p_{s_j}\nu_{s_j,d}}, \tag{24}$$

where $E\{.\}$ denotes the expectation operator. $\nu_{s_{j,d}}$ is the variance of the Rayleigh fading channel $h_{s_{j,d}}$. Similarly, λ_i and δ_i could be written as

$$\begin{aligned}\lambda_i &= \frac{1}{p_{s_j}\nu_{s_j,i}},\\ \delta_i &= \frac{1}{p_{r_i}\nu_{i,d}},\end{aligned} \tag{25}$$

where $h_{s_j,i} \sim \mathcal{CN}(0, \nu_{s,i})$ and $h_{i,d} \sim \mathcal{CN}(0, \nu_{i,d})$.

To minimise the outage probability in (23), the following part should be minimised [12]

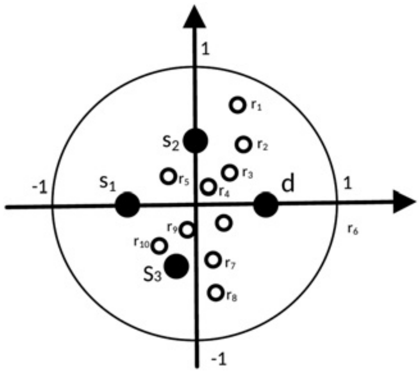


Fig. 3 *Simulation scenario: a cooperative wireless network with three source nodes, one destination node and ten relay nodes*

$$\begin{aligned} \min \quad & \lambda_0 \prod_{i=1}^{n} (\lambda_i + \delta_i) \\ \text{s.t.} \quad & p_{s_j} + \sum_{i=1}^{n} p_{r_i} \le p_{\text{total}}, \quad 0 \le p_{s_j} \le p_{s_j}^{\max}, \;\; 0 \le p_{r_i} \\ & \le p_{r_i}^{\max} \; \forall i. \end{aligned} \tag{26}$$

5.3 Simulation and results

For illustration purposes, here we provide some simulation results. As shown in Fig. 3, this example places all the nodes in a circle, centred at the origin of the x–y plane with radius $r = 1$. The locations of the destination node d are fixed at (0.5, 0). The x coordinate of a relay node varies within the range [–0.5, 0.5] and the y coordinate varies within the range [–1, 1].

The relay types are normalised, independent and uniformly distributed between 50 and 300. We quantise the range of types with a quantisation factor K to be 10. The channel between two nodes is $h_{i,j} \sim \mathcal{CN}(0, 1/d^{\nu})$, where d is the distance between the two nodes, and $\nu = 2.5$ is the path-loss exponent. We assume that all the noise variances are equal ($N_0 = 1$). The power constraints are set as $p_{s_j} = 1$ and $p_{r_i}^{\max} = 2$. The gain per unit of capacity is $a = 1$ and the cost per unit of power is the same for all the relay nodes and taken to be 0.2. The step size $\mathcal{S}$ is equal to 10^{-2} and the numerical results are obtained by averaging over 500 independent runs with randomly generated relay locations for each run.

Fig. 4 depicts the average capacity against system SNR (γ) in the presence of one source–destination pair ($s_1 - d$). We assume that there are ten candidate relay nodes available to forward the source node's signal to the destination node. This figure illustrates that our proposed algorithm outperforms the EPA scheme. Moreover, the performance under the proposed algorithm is comparable to that of the centralised scheme.

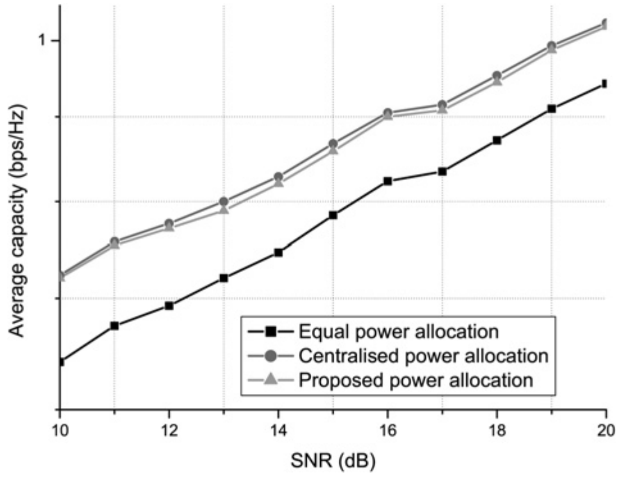


Fig. 4 *Average capacity against SNR for the three schemes (m = 1, n = 10)*

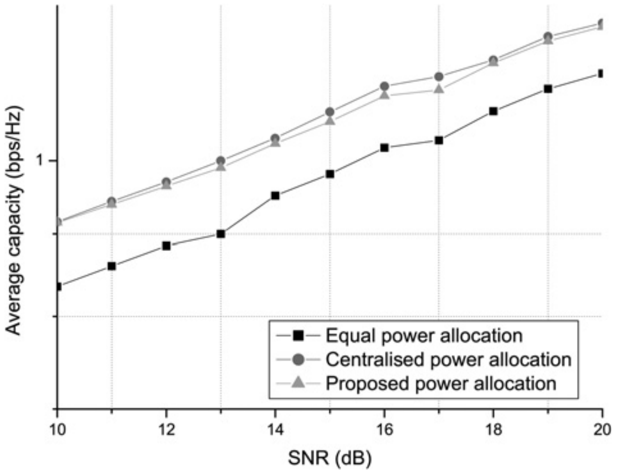


Fig. 5 *Average capacity against SNR for the three schemes (m = 3, n = 10)*

Similar to Fig. 4, Fig. 5 also compares the average capacity against system SNR (γ). However, in this figure, the number of available source nodes is three. In the presence of more than one source node, a relay node may receive multiple contract offers and based on its greedy nature, it responds to a contract pair that provides more benefits for it. The achieved capacity in this figure slightly improves compared with Fig. 4, due to the location of s_2 and s_3. Since s_2 and s_3 are closer to the destination node, they achieve higher SNR and thus higher capacity compared with the s_1. This will affect the total average capacity and hence the achieved performance in Fig. 5 is higher compared with Fig. 4.

In the above two mentioned scenarios, the number of source nodes is less than the number of candidate relay nodes. However, if the number of source nodes is more than the number of relay nodes, the achieved average capacity reduces, due to inadequate number of helping relays in the system.

Fig. 6 shows the outage probability for the three mentioned schemes. Centralised power allocation and our proposed algorithm improve outage probability, significantly. It is also clear from the figure that there is no considerable difference between these two schemes. However, outage probability for EPA is higher, since allocating equal power to all the nodes in not an optimum way of managing resources.

We also analyse utility of the nodes in Figs. 7 and 8. The plotted graphs illustrate utility of the source node and average utility of the

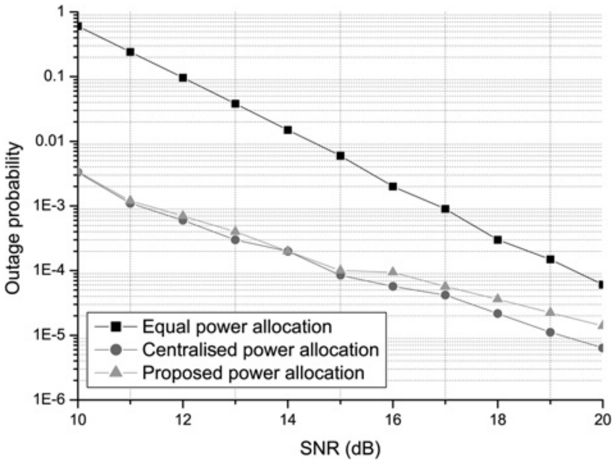


Fig. 6 *Outage probability of the three schemes (m = 3, n = 10)*

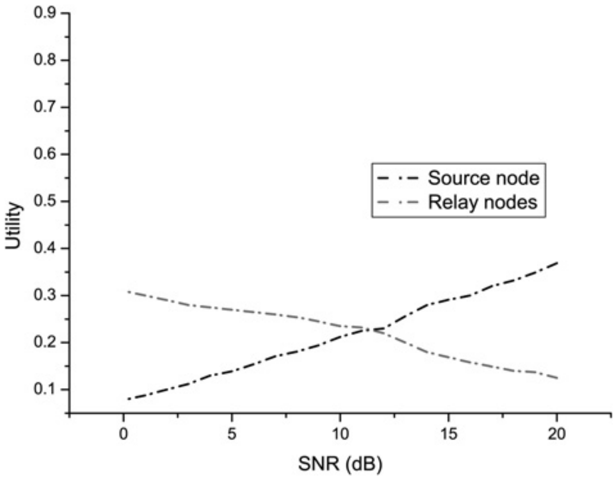


Fig. 7 *Utility of the source node and average utility of the relay nodes (m = 1, n = 3)*

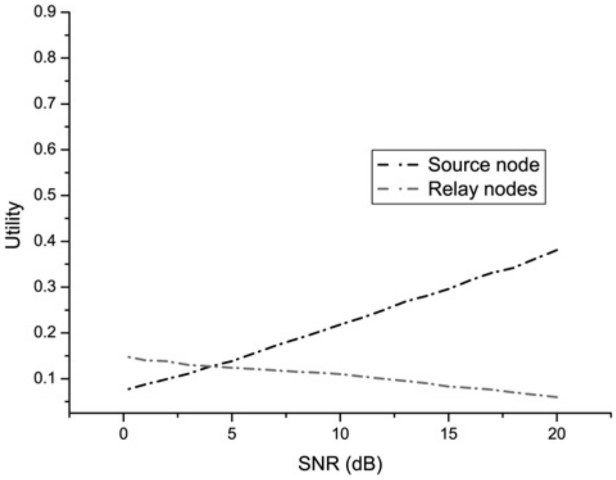


Fig. 8 *Utility of the source node and average utility of the relay nodes (m = 1, n = 10)*

relay nodes versus system SNR. Fig. 7 considers one source node and three relay nodes, while Fig. 8 considers one source node and ten relay nodes. Considering these two figures, it is clear that utility of the source node is linearly increasing, while system SNR is improving. This is due to the fact that improved system SNR results in better channel quality between the source node and the destination node. Better channel quality helps the source node to be less dependent on the relays. This will finally result in less payment to the relay nodes and increases utility of the source node. On the other hand, average utility of the relay nodes increases when system SNR decreases. Considering low system SNR, the source node then needs relay nodes' help to forward its data. This will result in more payment to the relay nodes and consequently increases their utility.

Comparing these two figures, we notice that by increasing number of candidate relay nodes from 3 to 10, their average utility decreases. This is due to the fact that when relay nodes are in excess, they bid for lower prices of power and therefore their average utility degrades.

5.4 Convergence

Each relay node r_i updates its price ζ_i so that its utility U_{r_i} satisfies the following equality

$$\frac{\partial U_{r_i}}{\partial \zeta_i} = \frac{\partial}{\partial \zeta_i}\left[(\zeta_i - c_i)p^*_{r_i}\right] = p^*_{r_i} + (\zeta_i - c_i)\frac{\partial p^*_{r_i}}{\partial \zeta_i} = 0, \qquad (27)$$

with the equality holding if and only if ζ_i reaches the optimum.

Rearranging the above equation, we have

$$\zeta_i = I(\zeta_i) = c_i - \frac{p^*_{r_i}}{\partial p^*_{r_i}/\partial p_i}. \qquad (28)$$

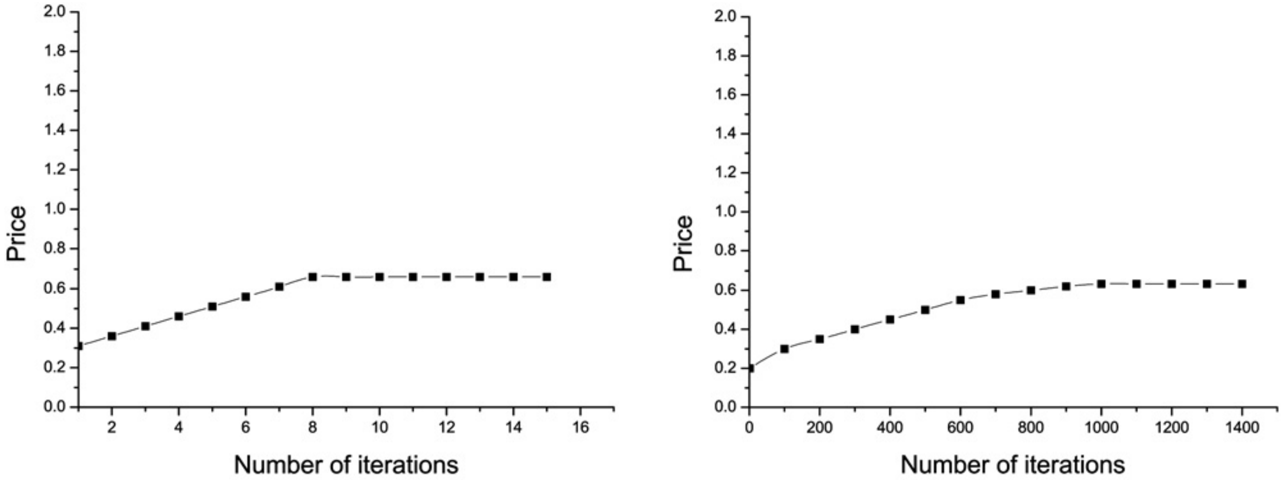
We demonstrate the convergence of the proposed algorithm by proving that $I(\zeta_i)$ is a standard function [28].

Definition: A function $I(\zeta)$ is standard if $\forall \zeta \geq 0$, the following properties satisfies [28]

- Positivity: $I(\zeta) > 0$.
- Monotonicity: If $\zeta \geq \zeta'$, then $I(\zeta) \geq I(\zeta')$.
- Scalability: For all $\alpha > 1$, $\alpha I(\zeta) > I(\alpha\zeta)$.

It is straightforward to prove that $I(\zeta_i)$ is a standard function.

The convergence of the proposed algorithm is illustrated in Fig. 9. As it is clear from the figure, the convergence speed of the proposed algorithm is highly dependent on the assigned value to the step size $\mathcal{S}$. We first equate the step size to 10^{-2}. Fig. 9*a* shows that it takes eight iterations until the price converges to an optimum price. Then by equating the step size to 10^{-4}, the convergence takes an



a *b*

Fig. 9 *Price of power with different step sizes*
a $\mathcal{S} = 10^{-2}$
b $\mathcal{S} = 10^{-4}$

average 900 iterations and the algorithm converges very slowly to an optimum price.

5.5 Complexity analysis and overhead

During each iteration, each source node broadcasts a contract, which is received by each of the n relay nodes. Additionally, each relay node negotiates about the price of power with source node via appropriate control channels. Therefore, $\mathcal{O}(m+n)$ messages are exchanged in each iteration. The number of iterations $\mathcal{I}(\mathcal{S})$ is dependent on the step size $\mathcal{S}$. Therefore, the total number of exchanged messages is $\mathcal{O}((m+n)\mathcal{I}(\mathcal{S}))$.

As explained in previous sections, in centralised power allocation, a centralised controller must obtain all the required information including perfect CSI. After collecting the required information, the centralised controller solves the optimisation problem given in Section 5.1. However, this method is not feasible to implement due to the substantial feedback requirements and the latency in collecting/exchanging CSI. Also in a network with large number of nodes, the computational complexity becomes practically prohibitive. In the proposed distributed mechanism, the power demand of each source node depends on the channel coefficient of the relay–destination link. However, this channel coefficient is relay's private information. Therefore, by applying contract theory, a relay node may accept a contract pair and reveals its type to the source node due to the IC constraint. Clearly, the overhead of this method is much less than having a centralised controller.

6 Conclusion

In this paper, we studied the problem of power allocation and price assignment in wireless systems under asymmetric knowledge of channel. The asymmetry of information happened when a party hides whole or parts of its information from others. In the proposed system, we assumed that relay nodes hide their CSI from source nodes, which made it difficult for a source node to determine its optimal power demand from the relays. To this end, we applied contract theory in order to stimulate candidate relay nodes to truthfully reveal their channel conditions to the source node at the time of cooperation. We propose an algorithm that distributively coordinates the relay power allocation and price assignment. The proposed mechanism combined contract theory and auction game and aimed to optimise the utility of both source and relay nodes. In the first phase, we applied contract theory to propose an offer for each type of relays. The proposed contract by a source node consisted of the source node SNR demand at the destination node and also the proposed price of power for that relay. Due to the IC constraint, each interested relay node chose a contract pair designed for its type only. By accepting the right contract pair, the interested relay node revealed its type to the source node. However, the amount of proposed price of power in the contract was selected in a way that maximises utility of the source node. In the second place and after accepting the contract pair, each relay node might participate in an auction game and bid for the price of power. The proposed contract-auction mechanism can be extended as a resource allocation technique in cognitive radio networks to offer a fair resource allocation among all participants nodes.

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8 Appendix

8.1 Analytical comparison between the centralised scheme and the proposed solution

Here, we sketch the analytical comparison between the centralised scheme in Section 5.1 and the proposed contract-auction-based solution. Initially, according to (18), the Lagrangian of the centralised optimal scheme can be presented as follows

$$\begin{aligned} L_{\text{cen}}(p_r, \lambda, \nu) = C_{s_j,r,d} + \sum_{i=1}^{n} \nu_i(-p_{r_i}) + \sum_{i=1}^{n} \lambda_i\left(p_{r_i} - p_{r_i}^{\max}\right) \\ + \lambda_{n+1}\left(\sum_{i=1}^{n} p_{r_i} - p_{\text{total}}\right), \end{aligned} \qquad (29)$$

where the Lagrangian multipliers are $\lambda = (\lambda_1, \ldots, \lambda_{n+1})$ and $\nu = (\nu_1, \ldots, \nu_n)$, with $\lambda_i, \nu_i \geq 0$. Since each node aims to maximise its own utility in the proposed solution defined in (10) and (11), therefore the objective can be viewed equivalently as a vector optimisation, and the scalarisation can be presented as follows

$$\begin{aligned} \max \quad & U_{s_j} + \sum_{i=1}^{n} w_i U_{r_i} \\ \text{s.t.} \quad & 0 \leq p_{r_i} \leq p_{r_i}^{\max}, \quad i = 1, \ldots, n, \\ & \zeta_i \geq 0, \quad i = 1, \ldots, n, \end{aligned} \tag{30}$$

where $w = (w_1, \ldots, w_n)$ is any weight vector, and $w_i > 0$, $\forall i$. Similarly, we can express the Lagrangian for the scalarised optimisation as

$$\begin{aligned} \tilde{L}_{\text{pro}}(p_r, \zeta, \tilde{\lambda}, \tilde{\nu}, \tilde{\mu}) = {} & U_s + \sum_{i=1}^{n} w_i U_{r_i} + \sum_{i=1}^{n} \tilde{\mu}_i(-\zeta_i) \\ & + \sum_{i=1}^{n} \tilde{\nu}_i(-p_{r_i}) + \sum_{i=1}^{n} \tilde{\lambda}_i\left(p_{r_i} - p_{r_i}^{\max}\right), \end{aligned} \tag{31}$$

where the Lagrangian multipliers are $\tilde{\lambda} = (\tilde{\lambda}_1, \ldots, \tilde{\lambda}_n)$, $\tilde{\mu} = (\tilde{\mu}_1, \ldots, \tilde{\mu}_n)$ and $\tilde{\nu} = (\tilde{\nu}_1, \ldots, \tilde{\nu}_n)$, with $\tilde{\lambda}_i, \tilde{\mu}_i, \tilde{\nu}_i \geq 0$, $\forall i$.

Substituting (10) and (11) into (31) and after some manipulation, $\tilde{L}_{\text{pro}}(p_r, \zeta, \tilde{\lambda}, \tilde{\nu}, \tilde{\mu})$ becomes

$$\begin{aligned} \tilde{L}_{\text{pro}}(p_r, \zeta, \tilde{\lambda}, \tilde{\nu}, \tilde{\mu}) = {} & aC_{s_j,r,d} + \sum_{i=1}^{n} \left[w_i(\zeta_i - c_i) - \zeta_i\right] p_{r_i} \\ & - \sum_{i=1}^{n} \tilde{\mu}_i \zeta_i + \sum_{i=1}^{n} \tilde{\nu}_i(-p_{r_i}) \\ & + \sum_{i=1}^{n} \tilde{\lambda}_i(p_{r_i} - p_{r_i}^{\max}). \end{aligned} \tag{32}$$

The above Lagrangian can be written as

$$\begin{aligned} \tilde{L}'_{\text{pro}}(p_r, \zeta, \lambda, \nu, \mu) = {} & R_{s_j,r,d} + \sum_{i=1}^{n} \frac{\left[w_i(\zeta_i - c_i) - \zeta_i\right]}{a} p_{r_i} \\ & - \sum_{i=1}^{n} \frac{\tilde{\mu}_i}{a} \zeta_i + \sum_{i=1}^{n} \frac{\tilde{\nu}_i}{a}(-p_{r_i}) \\ & + \sum_{i=1}^{n} \frac{\tilde{\lambda}_i}{a}(p_{r_i} - p_{r_i}^{\max}). \end{aligned} \tag{33}$$

After comparing (29) and (33), we can find that they have similar terms, which can be viewed as one-to-one mappings, i.e. $\lambda_i \leftrightarrow (\tilde{\lambda}_i/a)$, $\nu_i \leftrightarrow (\tilde{\nu}_i/a)$ and

$$\lambda_{n+1}\left(\sum_{i=1}^{n} p_{r_i} - p_{\text{total}}\right) \leftrightarrow \frac{\sum_{i=1}^{n} [w_i(\zeta_i - c_i) - \zeta_i] p_{r_i} - \sum_{i=1}^{n} \tilde{\mu}_i \zeta_i}{a}.$$

From both the simulation and the above analysis, due to the equivalence of Lagrangian in the centralised and proposed approaches, the proposed solution achieves comparable performance to that in the centralised optimal scheme.