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Analyst Reputation, Communication, and Information Acquisition

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ABSTRACT

Earlier studies have shown that reputational concerns tend to reduce agents' opportunistic behavior. However, a recent study by Morris argued that analysts' (experts') reputational concerns may discourage truthful communication when they try to avoid being perceived as being misaligned with investors. In this paper, I examine the effect of reputational concerns on communication in a setting where analysts can choose their precision endogenously. Because both misaligned and aligned analysts want investors to trust their reports in the future, both will aim to build a reputation for being aligned. In equilibrium, aligned analysts will acquire more information than misaligned analysts. As a result, investors may favorably update their beliefs about the analysts' type when the report is proven to be accurate. Therefore, both types of analysts will have reputational incentives to communicate truthfully. The paper also derives conditions under which the analysts' reputational

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concerns have a nonmonotonic impact on aligned analysts' equilibrium precision choices and investors' welfare.

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1. Introduction

In many situations, decision makers turn to experts for information and advice. However, experts' preferences or incentives are often not perfectly aligned with those of decision makers; hence experts may engage in opportunistic reporting. Conventional wisdom and prior studies (e.g., Benabou and Laroque [1992]) suggest that experts' future (reputational) concerns are an effective way to discipline opportunistic reporting and encourage truthful communication. The reason is that, if the aligned experts are committed to communicate truthfully, then to build reputation for being aligned, the misaligned experts will also have reputational incentives to communicate truthfully. One important implicit assumption here is that the aligned experts are nonstrategic and always communicate truthfully.

A recent study by Morris (2001) has challenged the conventional wisdom by endogenizing the aligned experts' strategic communication behavior. Specifically, Morris shows that, if the misaligned experts have a strong tendency to issue a certain message, then, to build reputation, the aligned experts will have an incentive to avoid sending this particular message, which leads to information loss when the aligned experts are sufficiently concerned about the future. Morris refers to this effect as "political correctness."

Morris's finding is disturbing in that experts' future concerns *hurt* communication exactly in those cases where experts care about the future. One of the important assumptions in Morris's argument is that all types of experts are endowed with the same amount of information and don't engage in information gathering afterwards. Consequently, there is only one way to build reputation for being aligned, which is to avoid sending the particular message favored by the misaligned type.

In the real world, experts may engage in all kinds of information acquisition activities. Therefore, the question left open by Morris (2001) is, in a setting where experts can endogenously choose the precision of their information, how do experts' future concerns affect their communication with decision makers? In this paper, I aim to answer this question, especially in the case where experts' future concerns are important, that is, when experts care sufficiently about the future.

To model the reputation formation process, I build on Morris (2001) and consider a repeated cheap talk game with two communication periods, preceded by an information acquisition stage. In each period, the decision maker makes a decision based on information strategically communicated by the expert. The decision maker is uncertain about the expert's type. An *aligned* expert internalizes the decision maker's preference in each period and always wants the decision maker to make the correct decision. A *misaligned* expert, in contrast, always prefers the decision maker to choose a higher action. The expert is endowed with some noisy private information about the true state of the world. At the outset, the expert may engage in (unobservable) costly information acquisition to increase the precision of her signal for both periods. At the end of the first period, the decision maker updates his belief about the expert's type based on the expert's first-period report and the realized state; this updated belief about the expert's type is labeled "expert reputation." The second period then unfolds similarly, with a new state of the world. How much the expert values the second period is interpreted as the expert's "future (reputational) concerns."

As a generic expert and decision-maker model, this paper can be applied to various settings in which there are repeated interactions between an expert and a decision maker. In particular, the analyst setting is a natural application. In practice, analysts may have investment banking incentives or trading commission incentives, which lead analysts' preferences to be misaligned with those of investors. Unfortunately, investors usually don't have an exact idea about analysts' preferences and can only rely on analysts' track records to infer their type. Furthermore, an especially salient feature of financial analysts is that they actively engage in various forms of information acquisition. Therefore, this model is particularly descriptive of the analyst setting and can shed some light on how analysts' future concerns affect their repeated strategic communication with investors in the presence of information acquisition.

To demonstrate the main result, it is useful to first examine the communication game in the second period. Because this is the last period in the model, the analyst does not care about maintaining her reputation. Consequently, the aligned analyst will report truthfully, and the misaligned analyst will issue a high report independent of her signal. Hence, if the investor receives a low report, he learns with certainty that the analyst is aligned. If the investor receives a high report, analyst reputation (formed in the first period) matters in that the greater the assessed likelihood that the analyst is aligned, the more seriously the investor will take the analyst's report and invest accordingly. As a result, both types of analysts benefit from a high reputation (along the equilibrium path). In addition, because the misaligned analyst is more likely to exploit her reputation (she always issues a high report in the second period), she benefits more from a high reputation than does the aligned analyst.

Now consider the analyst's incentives to acquire information. Note that, in general, the analyst benefits from better information through two channels. First, better information increases the analyst's ability to build reputation. I label this the "reputation effect." Recall that the misaligned analyst benefits from a high reputation even more than does the aligned one. Second, better information enables the analyst to guide investors toward more profitable decisions in each period, holding reputation constant. I label this the "precision effect." Because the aligned analyst internalizes the investors' preferences, this increases the aligned analyst's payoff. In contrast, precision per se does not matter to the misaligned analyst because her payoff is independent of the state. Combining these two arguments, it is not clear, a priori, which type of analyst benefits more from greater precision. To evaluate whose benefit is larger, note that the investor's second-period action responds more to the analyst's report than to the analyst's reputation. Given that the aligned type has a bigger impact on the investor's second-period action (through improving the accuracy of her report) than the misaligned type (through improving her reputation), it is the aligned type who benefits more and will acquire higher precision than the misaligned type in equilibrium.

The preceding *endogenous* link between the analyst's degree of incentive alignment and her precision choice differentiates this study from Morris (2001). In the latter, the analyst's precision is exogenously given and identical across different types. Therefore, there is only one way for the analyst to build reputation for being aligned, that is, by issuing a low report, because the misaligned type is known to have an upward bias. As a result, the aligned analyst with important future concerns will tend to issue low reports independent of her signals, which makes communication uninformative. However, if, in equilibrium, the aligned type acquires more precise signals than the misaligned type, then truth-telling (first period) communication may resurface. The reason is that now there are two alternative ways to build reputation: (1) by issuing a report as accurately as possible (the "accuracy" motive), and (2) by issuing a low report (the "contrarian" motive). Which motive prevails in equilibrium depends on how much the analyst values the future. When the analyst cares sufficiently about the future, the accuracy motive is more important, so that both types of analysts have reputational incentives to report truthfully. That is, future concerns serve as an effective way to encourage truthful communication exactly in those cases of important future concerns, where, in Morris's (2001) world, informative communication breaks down.

If the analyst cares little about the future, then the analyst's current period payoff will be the predominant consideration when deciding on the first-period communication strategy. As a result, in the first period, the aligned type will truthfully communicate, while the misaligned type will always issue a high report. Now the analyst's reputation will improve upon issuing a low report for either realized state; that is, the contrarian motive prevails in equilibrium. In summary, my analysis suggests that how the analyst values the future affects the reputation updating process: If the analyst is sufficiently concerned about the future, her reputation will improve upon issuing more accurate reports; conversely, if the analyst does not care about the future, her reputation will improve upon issuing low reports.

This paper also examines how the analyst's future concerns affect her information acquisition. Specifically, the analyst's future concerns may have a nonmonotonic impact on the aligned type's information acquisition effort. When future concerns are unimportant, the equilibrium is such that, in each period, the aligned type truthfully communicates, while the misaligned type always reports high. Given that the first-period communication is (imperfectly) informative, in expectation, the aligned analyst improves her reputation for the second period, which leads to a more responsive investor action (to the analyst's report) in the second period compared with the first period. That is, the benefit of the analyst having high precision is more prominent for the second period (compared with the first period). Therefore, an increase in future concerns (i.e., the weight on the second-period payoff, which also implies a corresponding decrease in the weight on the first-period payoff) will increase the aligned analyst's marginal benefit of acquiring information, which will lead her to choose higher equilibrium precision. Conversely, when the analyst is sufficiently concerned about the future, the equilibrium is such that, in the first period, both types truthfully communicate, whereas in the second period, only the aligned type tells the truth. Therefore the investor's action responds more to the analyst's report in the first period than in the second, which in turn suggests that the benefit of the analyst having high precision is stronger for the first period. Hence an increase in the weight on the second-period payoff will decrease the aligned analyst's marginal benefit of acquiring information and lead to a lower equilibrium precision choice.

Additionally, the analyst's future concerns also have a nonmonotonic impact on the investor's welfare. In the case of unimportant future concerns, an increase in future concerns leads to a higher precision choice by the aligned analyst and therefore makes the investor better off; conversely, when the analyst is sufficiently concerned about her future, an increase in future concerns induces the aligned analyst to choose a lower precision level and eventually makes the investor worse off.

1.1 RELATED LITERATURE

Pioneered by Kreps and Wilson (1982) and Milgrom and Roberts (1982), and extended by Fudenberg and Levine (1989), the earlier reputation literature argues that reputational concerns enhance the players' commitment power and reduce opportunistic behavior.[1] Although the main message of this paper is related, there are some fundamental differences. Specifically, the earlier literature introduces a "commitment type" who always plays a specific strategy, and, consequently, a strategic player aims to build reputation by mimicking the commitment type's behavior. In contrast, in this

[1] Some recent follow-up works (e.g., Ely and Valimaki [2003], Ely, Fudenberg, and Levine [2008]) actually demonstrate that reputation may be bad. The key to bad reputation is that participation is optional for the short-run players, and that every action of the long-run player that guarantees the short-run players' participation prompts the risk of being interpreted as a signal that the long-run player is "bad."

paper all types are strategic, and the aligned type builds reputation by distinguishing herself from the misaligned type.[2]

Reputation is also studied in the cheap talk literature initiated by Crawford and Sobel (1982). Focusing on reputation dynamics, Sobel (1985), Benabou and Laroque (1992), Kim (1996), Stocken (2000), Morris (2001), and Wang (2009) study repeated cheap talk games and examine how future concerns affect communication. In these earlier papers, the expert's precision is exogenously given. The innovation of this paper is that I endogenize the expert's precision choice and study then how future concerns affect communication.[3]

The effect of reputation on experts' communication behavior in a static model is the focus of Trueman (1994), Jackson (2005), and Ottaviani and Sorensen (2006). In all three papers, analysts' reputation (type) is with regard to their precision (ability), which is exogenously given; whereas in this paper, I allow analysts to choose the precision of their information, and analysts' reputation (type) is with regard to their exogenous degrees of incentive alignment.

Prior research has studied analysts' communication and information acquisition without future concerns. Morgan and Stocken (2003) study communication between analysts and investors when investors are uncertain about analysts' incentives but keep analysts' precision exogenous. Hayes (1998) examines how incentives to generate commissions affect analysts' information acquisition but assumes that analysts report truthfully. Fischer and Stocken (2010) endogenize both analysts' information acquisition and communication. They investigate how public information affects analysts' information acquisition and their communication with investors.

There is also a large body of empirical literature on analysts' behavior. Representative studies include Stickel (1992), Dugar and Nathan (1995), Womack (1996), Lin and McNichols (1998), and Michaely and Womack (1999). These articles show that, on average, analysts' reports are informative but optimistic, which is in line with the prediction of Beyer and Guttman (2011).

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 studies the communication game in each period for exogenous and commonly known analysts' precision. Section 4 fully characterizes the equilibrium of the model where analysts' precision choices are

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[2] In addition, in the earlier reputation literature, there is only one type of uncertainty: the long-run player could be the strategic type or the commitment type. However, in this paper, a decision maker experiences two types of uncertainty: (1) uncertainty about the expert's preferences, and (2) uncertainty about the expert's precision (which is endogenously derived).

[3] Closely related to my paper, Xu (2011) also extends Morris (2001) and studies reputational concerns with endogenous information acquisition. However, Xu (2011) focuses on the effect of future concerns on information acquisition instead of communication. Specifically, Xu (2011) assumes a nonstrategic misaligned type who always reports high, and, as a result, being contrarian is the only way for the aligned expert to build reputation. Therefore, the aligned type's communication behavior is the same as in Morris (2001).

endogenous and unobservable. Section 5 discusses the impact of analysts' future concerns on their precision choices and the investor's welfare, and section 6 concludes. All proofs are contained in the appendix.

2. *Model Setup*

In this section, I describe the basic setup of the model, which follows Morris (2001). I consider an investor ("he") who is uninformed about the state of the world and makes decisions based on the advice provided by an analyst ("she"). The analyst could be aligned (A) or misaligned (M). An aligned analyst wants the investor to make correct investment decisions in each period. A misaligned analyst, in contrast, always wants the investor to make "buy" decisions (independent of the state of the world). The investor is uncertain about the analyst's type, $J \in \{A, M\}$, and only knows that the prior probability of the analyst being aligned is 1/2.

The game has one information-acquisition stage and two communication periods. At stage 0, the analyst engages in an *unobservable* information acquisition effort to increase the precision of her signal for the following two periods. In period 1, the state of the world w_1 can take the value of 0 or 1; each state occurs with equal probability. The analyst observes an informative signal $s_1 \in \{0, 1\}$ about the state of the world, and the precision of the signal $Pr(s_1 = w_1 | w_1) = \gamma \in [\bar{\gamma}, 1]$ is determined by the analyst's stage 0 information-acquisition effort. The effort cost $c(\gamma)$ is a twice differentiable function that satisfies $c'(\gamma) \geq 0$ and $c''(\gamma) > 0$; that is, the analyst's cost of gathering information increases in precision and at an increasing rate. If the analyst does not acquire information at stage 0, she keeps her default precision $\bar{\gamma} \geq 3/4$;[4] that is, $c(\bar{\gamma}) = 0$.

After observing the signal, the analyst issues a report $m_1 \in \{0, 1\}$. The report does not need to be truthful. The investor then makes an investment decision $a_1 \in R$ according to his inference about the state based on the analyst's report m_1. After the action a_1 is taken, the state w_1 is publicly observed. Then the investor updates his belief about the analyst's type based on the realized state w_1 and the received report m_1. As a result, the analyst now has reputation $\lambda_2 = \Lambda(m_1, w_1)$ entering period 2. Period 2 then unfolds similarly to period 1, with a new and independent state w_2 (again equally likely to be 0 or 1), a new signal s_2 observed by the analyst with precision γ, a new report m_2 sent by the analyst, and a new action a_2 taken by the investor. The sequence of events is as depicted in figure 1.

Because the investor's action in each period is payoff-irrelevant for other periods, how the investor values the future is insignificant for the game. In each period, the investor aims to adjust his investment decision a_t to

[4] The assumption that the default precision $\bar{\gamma} \geq 3/4$ is made to simplify the analysis of the equilibrium in section 4 and the comparative statics analysis in section 5. Assuming instead that the default precision is 1/2 and adding additional (rather clunky) constraints on the cost function $c(\gamma)$ would work, too.

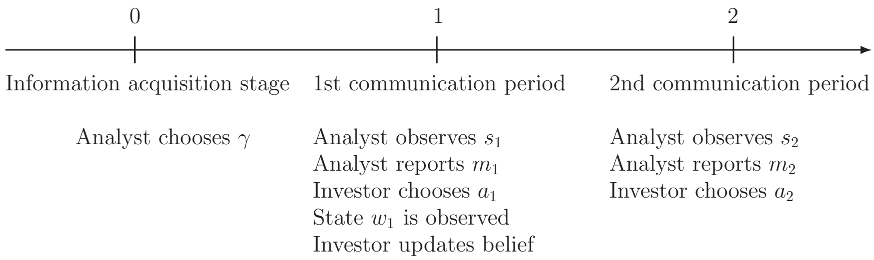

FIG. 1.—Timeline.

the state of the world w_t. His utility in period t is given by a quadratic loss function[5]:

$$-(a_t - w_t)^2.$$

The aligned analyst has identical preferences over a_t as the investor in each period. The utility of the aligned analyst is given by

$$-(1 - x^A)(a_1 - w_1)^2 - x^A(a_2 - w_2)^2 - c(\gamma^A).$$

The misaligned analyst, in contrast, always wants the high action to be chosen, independent of the state.[6] Her utility is given by

$$-(1 - x^M)(a_1 - 1)^2 - x^M(a_2 - 1)^2 - c(\gamma^M),$$

where $0 < x^J < 1$ captures the weight type J ($J = A, M$) analyst puts on period 2 payoff, and $1 - x^J$ is the weight on period 1 payoff. I refer to x^J as the analyst's exogenous *future (reputational) concerns*. Note that the cost of acquiring information $c(\cdot)$ is assumed to be independent of the analyst's type. To ensure a unique, interior optimal choice of precision, I assume that $c(\gamma)$ satisfies the following additional conditions: (i) $\lim_{\gamma \to \bar{\gamma}} c'(\gamma) = 0$, (ii) $\lim_{\gamma \to 1} c'(\gamma) \geq 3/2$, and (iii) $c''(\gamma) \geq 1$.[7]

An equilibrium in this game is characterized by the analyst's information acquisition strategy at stage 0, the analyst's communication strategy in each period, the decision rule for the investor in each period, and the belief function of the investor. The type J analyst's information acquisition strategy specifies the precision she will choose at stage 0 when her future

[5] The investor could be short-lived or long-lived. If the investor is short-lived and there are different generations of investors, the later generation has access to the analyst's report history at no cost and thus can update his belief about the analyst's type.

[6] This assumption is made without loss of generality. Assuming instead that the misaligned analyst has a downward bias will lead to qualitatively similar results. Put differently, the direction of the misaligned analyst's bias is insignificant. The key here is that the misaligned analyst is biased in a certain direction.

[7] As is shown in the appendix, either analyst's marginal benefit of acquiring precision is less than 3/2, hence $\lim_{\gamma \to 1} c'(\gamma) \geq 3/2$ is sufficient to guarantee an interior solution. An example of the cost function satisfying all these conditions is the quadratic function $c(\gamma) = a(\gamma - \bar{\gamma})^2$.

concerns are x^J; I denote this by $\gamma^J(x^J)$. The type J analyst's communication strategy in period t is a function $\sigma_t^J : \{0, 1\} \times [\bar{\gamma}, 1] \to [0, 1]$, where $\sigma_t^J(s_t, \gamma^J)$ is the probability of the type J analyst reporting 1 in period t when her signal is s_t and precision is γ^J. The investor's decision rule in period t is a function $a_t : \{0, 1\} \times [0, 1] \to R$, where $a_t(m_t, \lambda_t)$ is the investor's action in period t when he receives message m_t and his belief of the analyst being aligned is λ_t. To save on notation, I suppress the argument λ_1 in $a_1(\cdot)$, because $\lambda_1 = \lambda = 1/2$, a constant. As is implied by the notation, I allow the analyst to play mixed communication strategies.

The solution concept I employ in the paper is *Perfect Bayesian Nash Equilibrium*, which requires that players' actions maximize their expected utilities and their beliefs satisfy Bayes' rule whenever possible. The investor's action in period t, $a_t(m_t, \lambda_t)$, maximizes his expected utility in period t, given the message received m_t and analyst reputation λ_t. The type J analyst's period t communication strategy $\sigma_t^J(s_t, \gamma^J)$ maximizes her expected utility in period t given signal s_t and precision level γ^J. Anticipating how the information is communicated in future periods, the type J analyst's precision choice $\gamma^{J*}(x^J)$ maximizes her expected utility at stage 0. The formal definition of the equilibrium is relegated to appendix A.

Extending standard arguments from the cheap talk literature in which babbling equilibria always exist, in my model, there always exists an equilibrium in which neither analyst acquires information and communication in each period is babbling. Suppose that in each period the analyst issues a report randomly, independent of her type and signal. Then the investor will rationally make his investment decision solely based on his prior knowledge of the state. Given such a response from the investor, the analyst has an incentive neither to deviate from her uninformative report nor to become better informed. The interesting question then is whether and when there exist informative equilibria, and which, if any, type of analyst chooses to acquire information. In line with earlier studies, I select as the focal equilibrium the most informative one when there exist multiple equilibria. In the following analysis, without loss of generality, I assume $a_t(1, \lambda_t) \geq a_t(0, \lambda_t)$.

Before proceeding, I discuss some key features of the model. First, the model captures the information asymmetry between the investor and the analyst about the latter's preference. The prior literature has agreed that little is known about analysts' preferences. Different analysts may have different preferences because of their respective compensation contracts, the effectiveness of the "Chinese wall" between investment banking and research groups of their respective employers, or their levels of integrity. Investors have little knowledge of those attributes and can only try to infer analysts' types through their track records. Second, in the model, there are two dimensions along which analysts can differ: their preferences and their precision. I treat preferences as the primitive difference and precision as the endogenously derived difference. The motivation for this specific model choice is that, in practice, we do observe that analysts actively engage in information acquisition through various channels such as developing industry knowledge and analyzing financial reports. Hence it appears more descriptive to allow analysts to choose their precision levels.

3. The Repeated Communication Game: Exogenous and Commonly Known Precision

For now, to illustrate the key features of the communication game, I take the analyst's precision γ^J as exogenously given and commonly known. I will relax this assumption in section 4. The communication game can be solved by backward induction.

3.1 THE SECOND-PERIOD COMMUNICATION GAME

At the end of period 1, the investor updates his belief about the analyst's type based on the analyst's report and the realized state, and the analyst now has commonly known reputation λ_2 entering period 2. Because period 2 is the last period, neither type of analyst has an incentive to protect her reputation and instead both simply seek to maximize their payoffs in that period.

In line with the cheap talk literature, I assume that informative communication, if it can be supported in equilibrium, is played in each period. The following argument demonstrates that pure strategy informative communication always obtains in the second period. Suppose this is the case, then $a_2(1, \lambda_2) > a_2(0, \lambda_2)$. Therefore, the misaligned analyst has a strict incentive to report 1, and the aligned analyst must have a strict incentive to report her signal truthfully.[8] If the investor receives message 0, he learns with certainty that the analyst is aligned and truthfully reporting her signal. Given the aligned analyst's precision γ^A, the investor will assign probability $1 - \gamma^A$ to state 1 and choose action $a_2(0, \lambda_2) = 1 - \gamma^A$. If the investor receives message 1, he will be uncertain about the analyst's type and choose his action based on the updated belief:[9]

$$\begin{aligned} a_2(1, \lambda_2) &= \Pr[w_2 = 1 | m_2 = 1] \\ &= \frac{\frac{1}{2}[\lambda_2 \gamma^A + (1 - \lambda_2)]}{\frac{1}{2}[\lambda_2 \gamma^A + (1 - \lambda_2)] + \frac{1}{2}[\lambda_2(1 - \gamma^A) + (1 - \lambda_2)]} \\ &= \frac{1 - \lambda_2 + \lambda_2 \gamma^A}{2 - \lambda_2}. \end{aligned}$$

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[8] The argument is as follows: given that the misaligned analyst reports 1 all the time, for $a_2(1, \lambda_2) > a_2(0, \lambda_2)$ to hold, the aligned analyst must report 1 more often when she observes signal 1 than when she observes signal 0. Because I focus here on pure strategy, this means that the aligned analyst must report her signal truthfully.

[9] This confirms Morgan and Stocken's (2003) finding that the investor's uncertainty about the analyst's incentive makes it impossible for the aligned analyst to credibly reveal good news.

Clearly $a_2(1, \lambda_2) \in [1/2, \gamma^A] > a_2(0, \lambda_2)$. Therefore, the misaligned analyst will indeed always report 1. It is also true that the aligned analyst will indeed truthfully report her signal.[10] Hence, pure strategy informative communication does obtain in the second period. In fact, it can be shown that this pure strategy informative equilibrium is the unique informative equilibrium in the second period.

All else equal, the action induced by a high second-period report, $a_2(1, \lambda_2)$, is increasing in analyst reputation λ_2. The higher the probability with which an analyst is believed to be aligned, the more credible her report is perceived to be, and hence the investor will choose a higher action accordingly.

Given analyst reputation λ_2, write $V^J(\lambda_2)$ for the type J analyst's second-period expected utility (calculated at the beginning of the second period, before s_2 is realized), which is also the analyst's value function for reputation.[11] The aligned analyst's value function for reputation is therefore

$$\begin{aligned} V^A(\lambda_2) &= -\frac{1}{2}\gamma^A[a_2(1,\lambda_2) - 1]^2 - \frac{1}{2}(1-\gamma^A)[a_2(0,\lambda_2) - 1]^2 \\ &\quad -\frac{1}{2}(1-\gamma^A)[a_2(1,\lambda_2) - 0]^2 - \frac{1}{2}\gamma^A[a_2(0,\lambda_2) - 0]^2 \\ &= -\frac{1}{2}(a_2(0,\lambda_2) - 1)^2 - \frac{1}{2}(a_2(1,\lambda_2))^2 + \gamma^A(a_2(1,\lambda_2) - a_2(0,\lambda_2)). \end{aligned} \tag{1}$$

The misaligned analyst's value function for reputation equals

$$V^M(\lambda_2) = -[a_2(1,\lambda_2) - 1]^2. \tag{2}$$

Note that $a_2(0, \lambda_2)$ is independent of λ_2. Hence it is straightforward to show that

$$\begin{aligned} \frac{dV^A(\lambda_2)}{d\lambda_2} &= (\gamma^A - a_2(1,\lambda_2))\frac{da_2(1,\lambda_2)}{d\lambda_2} \geq 0, \\ \frac{dV^M(\lambda_2)}{d\lambda_2} &= 2(1 - a_2(1,\lambda_2))\frac{da_2(1,\lambda_2)}{d\lambda_2} > 0. \end{aligned}$$

[10] If the aligned analyst observes signal 0, she will compare her utility conditional on sending message 0, $U_2^A(m_2 = 0|s_2 = 0) = -\gamma^A(a_2(0,\lambda_2) - 0)^2 - (1-\gamma^A)(a_2(0,\lambda_2) - 1)^2$, with her utility conditional on sending message 1, $U_2^A(m_2 = 1|s_2 = 0) = -\gamma^A(a_2(1,\lambda_2) - 0)^2 - (1 - \gamma^A)(a_2(1,\lambda_2) - 1)^2$. It is straightforward to show that $U_2^A(m_2 = 0|s_2 = 0) - U_2^A(m_2 = 1|s_2 = 0) = (a_2(1,\lambda_2) - a_2(0,\lambda_2))[a_2(1,\lambda_2) + a_2(0,\lambda_2) - 2(1-\gamma^A)] > 0$. Hence the aligned analyst will indeed report 0 when she observes signal 0. Similarly, it can be shown that the aligned analyst will truthfully report 1 when she observes signal 1.

[11] Note that, in this section, γ^J is an exogenous parameter. To save on notation, I suppress the dependency of $V^J(\cdot)$ on γ^J.

Also,

$$\frac{dV^A(\lambda_2)}{d\lambda_2} - \frac{dV^M(\lambda_2)}{d\lambda_2} = (\gamma^A + a_2(1, \lambda_2) - 2)\frac{da_2(1, \lambda_2)}{d\lambda_2} < 0.$$

Both types of analysts benefit from a high reputation, *with the misaligned type benefiting even more.* To generate intuition for this result, notice that the investor's action upon receiving message 1 is increasing in analyst reputation λ_2, while the action induced by message 0 is independent of λ_2. The misaligned analyst always reports 1, and hence her reputation pays off in all scenarios. In contrast, with (ex ante) probability 1/2, the aligned analyst reports 0, in which case her payoff is independent of her reputation λ_2. Therefore, the misaligned analyst is more likely to exploit her reputation and hence benefits more from a high reputation than her aligned peer.

To conclude, in the second period, the aligned analyst reports truthfully, and the misaligned analyst always reports 1. Both types of analysts benefit from a high reputation, with the misaligned analyst benefiting from it even more.

3.2 THE FIRST-PERIOD COMMUNICATION GAME

In the first period, the analyst anticipates the reputational consequences in the second period as she chooses her first-period communication strategy. Specifically, the aligned analyst's objective in the first communication period includes both her first-period payoff and her second-period expected utility and is given by

$$-(1 - x^A)(a_1 - w_1)^2 + x^A V^A(\Lambda(m_1, w_1)),$$

where $\Lambda(m_1, w_1)$ is the posterior analyst reputation updated based on the analyst's first-period report m_1 and the realized state w_1. Analogously, the misaligned analyst's objective in the first communication period is given by

$$-(1 - x^M)(a_1 - 1)^2 + x^M V^M(\Lambda(m_1, w_1)).$$

Note that first-period communication can be informative in terms of either the analyst's type or the underlying state. I argue that in equilibrium, however, it has to convey information about *both* dimensions. Suppose it only conveyed information about the underlying state, while being uninformative about the analyst's type. In this case, there would be no reputational reporting consequences, and the analyst would act only on her current reporting incentive in that the aligned type would tell the truth, while the misaligned type would always report 1. However, these optimal reporting strategies themselves are informative about the analyst's type, indicating a contradiction. Conversely, suppose communication in the first period were uninformative about the underlying state, yet informative about the analyst's type. In that case, because there would be no current reporting consequences, both types of analysts would end up choosing communication strategies that will give them the highest reputation, which makes communication completely uninformative, indicating another contradiction.

Thus, in equilibrium, informative communication in the first period must convey information about both dimensions.

The striking finding of Morris (2001) is that, when the aligned analyst cares sufficiently about the future, that is, x^A becomes large, then no information can be conveyed in the first period. I replicate Morris's result in my setting in the following Lemma:

LEMMA 1 (Morris 2001). *Suppose both types of analysts have the same precision, that is,* $\gamma^A = \gamma^M = \gamma^o$. *For any* x^M, *there exist cutoff values* $\underline{x}^A(\gamma^o, x^M) \leq \bar{x}^A(\gamma^o, x^M)$ *such that, if the aligned analyst's future concerns are sufficiently important, that is,* $x^A > \bar{x}^A(\gamma^o, x^M)$, *communication in the first period is babbling. Conversely, if and only if the aligned analyst's future concerns are sufficiently unimportant, that is,* $x^A < \underline{x}^A(\gamma^o, x^M)$, *there exists an informative equilibrium in which the aligned analyst truthfully reports her signal in the first period.*

To understand the intuition behind this result, first I argue that, when both types of analysts have the same precision, there is only one way to build reputation for being aligned, which is to issue a low report. Therefore, both types of analysts will have reputational incentives to report 0. If the aligned analyst's future concerns are sufficiently important, then her reputational reporting incentive dominates, and she will report 0 all the time, which makes communication uninformative.[12] Conversely, if the aligned analyst cares sufficiently about her current payoff, then her current reporting incentive dominates, and she will report her signal truthfully.

The preceding result is valid for any x^M. Note that the cutoff values $\underline{x}^A(\gamma^o, x^M)$ and $\bar{x}^A(\gamma^o, x^M)$ are functions of x^M. Roughly speaking, in the proposed first-period informative equilibrium, the misaligned analyst has reputational incentive to report 0, while her current incentive is to report 1. Therefore her future concerns x^M play an important role in determining her equilibrium reporting strategy, which, in turn, affects how the investor updates his beliefs and thereby the aligned analyst's reporting behavior.

[12] One would think that such communication can still be informative since the misaligned analyst's communication strategy may be signal-dependent. However, the following argument demonstrates that this cannot be the case. Suppose the aligned analyst always reports 0, independent of her signal. Then for communication to be informative, the misaligned analyst must report 1 more often when she observes signal 1 compared with signal 0. Given that the aligned analyst reports 0 all the time, then the investor will rationally anticipate that (1) the possibility of a report 0 coming from the aligned analyst is higher if the state is 1 than if the state is 0, which implies that $\Lambda(0, 1) > \Lambda(0, 0)$; (2) the analyst must be misaligned if the report is 1, that is, $\Lambda(1, 1) = \Lambda(1, 0) = 0$. Therefore, the reputation enhancement by reporting 0 when the state is 1 is greater than that when the state is 0, that is, $\Lambda(0, 1) - \Lambda(1, 1) > \Lambda(0, 0) - \Lambda(1, 0)$. This in turn implies that the misaligned analyst has stronger reputational incentive to report 0 when she observes signal 1 rather than signal 0. Now notice that the misaligned analyst's current reporting incentive is to always report 1 and that incentive is independent of her signal. Therefore, the misaligned analyst will report 1 more often when her signal is 0, which contradicts $\sigma_1^M(1) > \sigma_1^M(0)$. Hence, for first-period communication to be informative, the aligned analyst cannot report 0 all the time.

I now allow for the analysts to have different precision for exogenous reasons. If the misaligned analyst has more precise information than does the aligned analyst, then, again, there is only one way to build reputation for being aligned, which is to issue a low report. The preceding arguments therefore apply, and the same result prevails, in that first-period communication takes the form of babbling when the aligned analyst's future concerns are sufficiently important. I now ask the central question for the remainder of this section: if the aligned analyst has greater precision, can informative (first-period) communication obtain in those cases where it fails to exist before?

PROPOSITION 1. *Suppose it is common knowledge that the aligned analyst is better informed than the misaligned analyst, that is, $\gamma^A > \gamma^M$; then there exists a value $\bar{x}^M(\gamma^A, \gamma^M)$ such that, if the misaligned analyst's future concerns are important, that is, $x^M > \bar{x}^M(\gamma^A, \gamma^M)$, then, for any x^A, there exists a truth-telling equilibrium in which both types of analysts truthfully report their signals in the first period.*

The value of $\bar{x}^M(\gamma^A, \gamma^M)$ is derived in the appendix.[13] If the aligned analyst is better informed than the misaligned analyst, then informative communication will resurface in the first period for any level of the aligned analyst's future concerns, given that the misaligned analyst cares sufficiently about the future.

If the aligned type has an informational advantage, then there are two alternative ways for the analyst to build reputation for being aligned: (1) by issuing a report as *accurately* as possible (I label this the "accuracy" motive), and (2) by issuing a *low* report, because it is commonly known that the misaligned analyst is upwardly biased. (I label this the "contrarian" motive.) The following argument shows that, when the misaligned analyst cares sufficiently about the future, accuracy is the dominant way to build reputation. Suppose the accuracy motive dominates; then both types of analysts will have reputational incentives to truthfully report their signals. For the aligned analyst, she now has both current and reputational incentives to tell the truth and hence will report her signal truthfully. For the misaligned analyst, when her future concerns are sufficiently important, her reputational reporting incentive dominates, and hence she will also truthfully report her signal. Given that both types of analysts tell the truth and the aligned type has higher precision, it is indeed rational for the investor to update favorably regarding the analyst's type when the report is consistent with the realized state. Conversely, suppose the contrarian motive dominates and, for any realized state, the investor updates favorably when he receives a low

[13] Note that the cutoff $\bar{x}^M(\cdot)$ is independent of x^A. If the aligned analyst is better informed than the misaligned analyst, then, in the postulated informative communication, both the aligned analyst's reputational and current reporting incentives are to report truthfully. Therefore the aligned analyst's future concerns x^A will not affect her equilibrium reporting strategy and hence will have no effect on the investor's beliefs.

report. Then, when the misaligned analyst's future concerns are sufficiently important, her reputational reporting incentive dominates, and she will always issue a low report, which actually makes the investor's updating rule irrational. Therefore, when the misaligned analyst is sufficiently concerned about the future, the accuracy motive prevails in equilibrium; that is, the analyst builds reputation for being aligned by issuing reports as accurately as possible.

To conclude, for exogenous and commonly known precision, if the misaligned analyst has equal or higher precision than the aligned analyst, first-period communication takes the form of babbling when the aligned analyst's future concerns are sufficiently important. Conversely, if the aligned analyst is better informed, informative communication can obtain in the first period regardless of the aligned analyst's future concerns, given that the misaligned analyst values the future sufficiently. Furthermore, both types of analysts now have reputational incentives to truthfully report; that is, the possible detrimental role of future concerns on communication, as documented in Morris (2001), disappears.

4. *Unobservable Choice of Precision*

In this section, I return to the full-fledged model in which the analyst's precision choice is endogenous and unobservable.[14] I consider pure reporting strategies here to simplify the analysis.[15] To ensure that the main results are not driven by differences in time preferences across different types of analysts, going forward I assume that both types of analysts have the same future concerns. That is,

ASSUMPTION 1. $x^A = x^M = x$.

The striking finding of Morris (2001) is that, when the expert's precision is exogenous and identical, no information can be conveyed in the first period if the aligned expert is sufficiently concerned about the future. Proposition 1 shows that informative communication may resurface when the aligned analyst has access to a more precise signal for exogenous reasons. Now the important question is whether indeed the aligned type will acquire an informational advantage once we endogenize the analyst's precision choice and assume it is unobservable to the investor. Without getting into too many details, it is obvious that the aligned analyst must always acquire *weakly* more information than the misaligned one, because the

[14] Note that the communication game is not a proper subgame because the analyst's precision choice is unobservable, that is, the communication game starts from a nonsingleton information set.

[15] Furthermore, restricting attention to pure reporting strategies makes it harder to find informative first-period communication, which biased against my finding that the informative first-period communication resurfaces for important future concerns once the analyst's precision choice is endogeneized.

misaligned analyst would have an incentive to throw away information to avoid losing reputation if she were the better informed one.[16] However, the following argument shows that it is not clear, a priori, whether the aligned analyst has a *strictly* stronger incentive to acquire information.

Loosely speaking, the analyst benefits from acquiring information through two channels: First, more precise information increases the analyst's ability to build reputation. Recall that, while both types of analysts benefit from a high reputation, the misaligned analyst benefits even more. Second, more precise information enables the analyst to guide the investor toward more profitable decisions, holding reputation constant. Because the aligned analyst internalizes the investor's preference, better information in effect increases her payoff. In contrast, precision per se does not matter to the misaligned analyst because her payoff is independent of the state. For later reference, I label the first source of incentive the "reputation effect" and the second source of incentive the "precision effect." Combining these two effects, it is not clear which type of analyst has a stronger incentive to acquire information. The next result demonstrates that, overall, the aligned type has a *strictly* stronger incentive to acquire information, and, consequently, when the analyst's future concerns are sufficiently important, both types of analysts truthfully report in the first period.

PROPOSITION 2. *There exists some* $x^o \in (0, 1)$ *such that, if the analyst's future concerns are important, that is,* $x > x^o$, *then the informative equilibrium is that:*

(i) Both types of analysts acquire information, and the aligned type acquires more information than the misaligned type, that is, $\gamma^{A^*}(x) > \gamma^{M^*}(x) > \bar{\gamma}$.

(ii) Communication in each period is informative. Specifically, both types of analysts report truthfully in the first period. In the second period, the aligned analyst reports truthfully and the misaligned analyst reports 1 all the time.

(iii) The analyst's reputation is improved if the analyst's report is consistent with the realized state; more specifically,

$$\Lambda(0,0) = \Lambda(1,1) > \frac{1}{2} > \Lambda(1,0) = \Lambda(0,1).$$

Following the rationale established in section 3.1 and Proposition 1, it is straightforward to demonstrate the following: If the investor holds the (proposed equilibrium) conjectures that (a) the aligned analyst acquires higher precision than the misaligned analyst, (b) in the first period, both types of analysts truthfully report their signals, and (c) in the second period, the aligned analyst reports truthfully and the misaligned analyst always reports 1, then for the analyst's future concerns being important, the optimal communication strategy of the analyst with the conjectured precision (as in (a)) is indeed consistent with the investor's conjectures (as in (b) and

[16] I thank the referee for providing this explanation.

(c)). Now it remains to show that, for $x > x^o$, the analyst's precision choice is consistent with the investor's conjecture (as in (a)). To that end, we need also to examine the analyst's communication behaviors for off-equilibrium precision choices.

Given the proposed conjectures about the analyst's precision choice and communication behaviors, the investor will favorably update his belief about the analyst's type when the report is consistent with the realized state. As I show in the appendix, the aligned analyst will report her signal truthfully in each period even if her true precision differs from the conjectured precision $\hat{\gamma}^A$. Write $V^J(\lambda_2, \gamma^J)$ for the type J analyst's second-period expected utility (calculated at the beginning of the second period) when her reputation is λ_2 and her actual precision is γ^J. Hence, the aligned analyst's utility at the information acquisition stage is as follows:

$$\begin{aligned} U_0^A(\gamma^A, x) = {} & \frac{1}{2}\gamma^A[-(1-x)(a_1(1)-1)^2 + xV^A(\Lambda(1,1),\gamma^A)] \\ & + \frac{1}{2}(1-\gamma^A)[-(1-x)(a_1(0)-1)^2 + xV^A(\Lambda(0,1),\gamma^A)] \\ & + \frac{1}{2}\gamma^A[-(1-x)(a_1(0)-0)^2 + xV^A(\Lambda(0,0),\gamma^A)] \\ & + \frac{1}{2}(1-\gamma^A)[-(1-x)(a_1(1)-0)^2 + xV^A(\Lambda(1,0),\gamma^A)] \\ & - c(\gamma^A). \end{aligned}$$

Note that both $\Lambda(m_1, w_1)$ and $a_1(m_1)$ depend on the investor's conjectures about the analyst's precision choice $\hat{\gamma}^J$ and communication behaviors, which vary with x. Collecting terms and adding back the omitted variables in $\Lambda(m_1, w_1)$ and $a_1(m_1)$ for completeness, we get[17]

$$\begin{aligned} U_0^A(\gamma^A, x) = {} & \gamma^A[-(1-x)(a_1(0|\hat{\gamma}^A, \hat{\gamma}^M, x))^2 + xV^A(\Lambda(0,0|\hat{\gamma}^A, \hat{\gamma}^M, x), \gamma^A)] \\ & + (1-\gamma^A)\big[-(1-x)(a_1(1|\hat{\gamma}^A, \hat{\gamma}^M, x))^2 \\ & + xV^A(\Lambda(1,0|\hat{\gamma}^A, \hat{\gamma}^M, x), \gamma^A)\big] - c(\gamma^A). \end{aligned}$$

To avoid clutter, going forward, I again suppress the arguments x, $\hat{\gamma}^A$, and $\hat{\gamma}^M$ in $\Lambda(m_1, w_1)$ and $a_1(m_1)$.

The aligned analyst's optimal precision choice thus has to satisfy the following first-order condition (imposing the equilibrium condition $\gamma^J = \hat{\gamma}^J = \gamma^{J^*}$ after taking derivative with respect to γ^A):

[17] Given that the investor holds the conjectures that both types of analysts truthfully report their signals in the first period, it is immediate that $a_1(0) + a_1(1) = 1$, $\Lambda(0, 0) = \Lambda(1, 1)$, and $\Lambda(1, 0) = \Lambda(0, 1)$.

$$c'(\gamma^{A*}) = (1-x)\underbrace{(a_1(1)-a_1(0))}_{PE_1^A(x>x^o,\gamma^{A*},\gamma^{M*})} + x\underbrace{[V^A(\Lambda(0,0),\gamma^{A*}) - V^A(\Lambda(1,0),\gamma^{A*})]}_{RE^A(x>x^o,\gamma^{A*},\gamma^{M*})}$$
$$+ x\underbrace{\left[\gamma^{A*}\frac{\partial V^A(\Lambda(0,0),\gamma^A)}{\partial\gamma^A} + (1-\gamma^{A*})\frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial\gamma^A}\right]}_{PE_2^A(x>x^o,\gamma^{A*},\gamma^{M*})}, \qquad (3)$$

where $RE^J(\cdot)$ and $PE_t^J(\cdot)$ represent, respectively, the type J analyst's "reputation effect" and her period t "precision effect." The left-hand side of equation (3) is the aligned type's marginal cost of acquiring precision γ^{A*}. The right-hand side of equation (3) represents her marginal benefit of acquiring precision γ^{A*}, which I denote by $MB^A(x,\gamma^{A*},\gamma^{M*}) = (1-x)PE_1^A(\cdot) + xRE^A(\cdot) + xPE_2^A(\cdot)$.

Given that the aligned analyst truthfully reports in each period, better information will enable her not only to provide more accurate guidance to the investor in each period, but also to build reputation. Hence the aligned analyst's total benefit from acquiring information comprises her reputation effect and two periods' precision effects.

For the misaligned analyst, one needs to identify conditions on x to ensure that there are no deviations not only regarding her reporting strategies but also regarding her precision choice. By Proposition 1, for x sufficiently large, the misaligned analyst with equilibrium precision γ^{M*} will report her signal truthfully. Therefore, for there to be any deviation from truth-telling, it must be a joint deviation in terms of both precision choice *and* reporting strategies. The analysis in the appendix shows, however, that, when $x > x^o$, any such joint deviation is not profitable. Therefore, when $x > x^o$, the misaligned analyst truthfully reports even with off-equilibrium precision. Hence, the misaligned analyst's utility at the information acquisition stage is (again adding back the omitted variables in $\Lambda(m_1, w_1)$ and $a_1(m_1)$ for completeness)

$$\begin{aligned} U_0^M(\gamma^M, x) = &-\frac{1}{2}(1-x)[(a_1(1|\hat{\gamma}^A,\hat{\gamma}^M,x)-1)^2 + (a_1(0|\hat{\gamma}^A,\hat{\gamma}^M,x)-1)^2] \\ &+ x\big[\gamma^M V^M(\Lambda(0,0|\hat{\gamma}^A,\hat{\gamma}^M,x)) \\ &+ (1-\gamma^M)V^M(\Lambda(1,0|\hat{\gamma}^A,\hat{\gamma}^M,x))\big] - c(\gamma^M). \end{aligned}$$

Note that γ^M does not affect $V^M(\cdot)$ because the misaligned analyst always reports 1 in the second period, independent of her precision. Hence, I suppress the argument γ^M in $V^M(\cdot)$. Also, the ex ante probability for the analyst to observe 0 or 1 is always 1/2, irrespective of her precision. Therefore, the probability of the misaligned analyst reporting 0 or 1 (in the first period) is 1/2, given that she reports truthfully in the first period. Taking derivative with respect to γ^M, the misaligned analyst's optimal precision choice has to satisfy the following first-order condition (imposing the equilibrium condition $\gamma^J = \hat{\gamma}^J = \gamma^{J^*}$ here):

$$c'(\gamma^{M*}) = x \underbrace{[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]}_{RE^M(x>x^o,\gamma^{A*},\gamma^{M*})}. \tag{4}$$

That is, by acquiring information, the misaligned analyst stands only to benefit by boosting her reputation. I denote the right-hand side of equation (4) by $MB^M(x, \gamma^{A*}, \gamma^{M*})$, which represents the misaligned analyst's marginal benefit of acquiring information.

Note that the analyst's equilibrium precision choice γ^{A*} and γ^{M*} are jointly determined by the system of equations (3) and (4). Now it remains to show that *the aligned analyst benefits strictly more from acquiring information than does the misaligned analyst.* This comparison may seem trivial at first glance, because the aligned analyst benefits from both reputation and precision effects, whereas the misaligned analyst benefits only from the reputation effect. However, recall that the misaligned analyst reaps greater benefit from a high reputation. Thus, further analysis is needed to evaluate the comparison.

A sufficient condition for the aligned analyst to benefit strictly more from high precision is that $PE_2^A(\cdot) > RE^M(\cdot)$. As I will show now, this is indeed the case. To illustrate this observation, first note that the aligned analyst's second-period precision effect stems from her impact on the investor's decision in the second period by making her report more informative, while holding reputation λ_2 constant:

$$\frac{\partial V^A(\lambda_2, \gamma^A)}{\partial \gamma^A} = a_2(1, \lambda_2) - a_2(0, \lambda_2). \tag{5}$$

In contrast, the misaligned analyst's reputation effect stems from her impact on the investor's decision in the second period by improving her reputation (recall that she always reports 1 in the second period):

$$\begin{aligned} RE^M(\cdot) &= -(a_2(1, \Lambda(0,0)) - 1)^2 + (a_2(1, \Lambda(1,0)) - 1)^2 \\ &< a_2(1, \Lambda(0,0)) - a_2(1, \Lambda(1,0)). \end{aligned} \tag{6}$$

The key to understanding Proposition 2 now is that the aligned analyst's impact on the investor's decision in the second period through *differential reporting* (as in equation (5)) is greater than the misaligned analyst's impact on the investor's second-period decision through *differential reputation* (as in equation (6)). In the second period, the aligned analyst truthfully reports her signal, while the misaligned analyst always reports 1. Hence, $a_2(m_2 = 0, \lambda_2) = 1 - \hat{\gamma}^A$, and $a_2(m_2 = 1, \lambda_2)$ depends on analyst reputation λ_2. Clearly, $a_2(m_2 = 1, \lambda_2 = 0) = \frac{1}{2}$ and $a_2(m_2 = 1, \lambda_2 = 1) = \hat{\gamma}^A$. Therefore, the *maximum* effect of *differential reputation* on the investor's second-period decision is $a_2(m_2 = 1, \lambda_2 = 1) - a_2(m_2 = 1, \lambda_2 = 0) = \hat{\gamma}^A -$

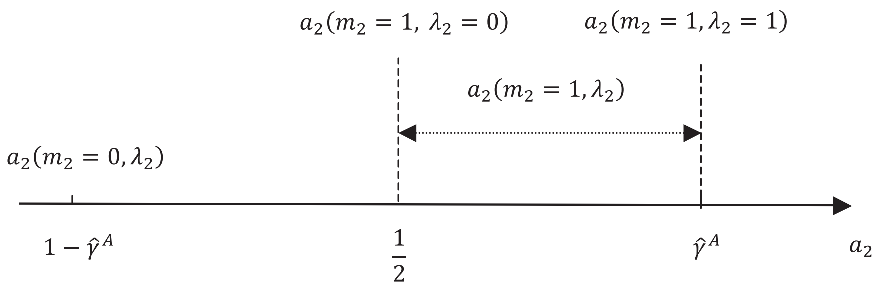

FIG. 2.—The impact of m_2 and λ_2 on $a_2(m_2, \lambda_2)$.

$\frac{1}{2}$, while the *minimum* effect of *differential reporting* on the investor's second-period decision is $a_2(m_2 = 1, \lambda_2 = 0) - a_2(m_2 = 0, \lambda_2) = \hat{\gamma}^A - \frac{1}{2}$. That is, the aligned analyst's *differential reporting* has a bigger impact on the investor's decision than does the misaligned analyst's *differential reputation.* (See figure 2 for an illustration.) Hence, the aligned analyst's second-period precision effect is greater than the misaligned analyst's reputation effect. As a result, in equilibrium, the aligned analyst will acquire higher precision than the misaligned analyst.

I now address the remaining case of the analyst's future concerns being unimportant:

PROPOSITION 3. *There exists some $x^{oo} \in (0, 1) \leq x^o$ such that, if the analyst's future concerns are unimportant, that is, $x < x^{oo}$, then the informative equilibrium is that:*

(i) Only the aligned analyst acquires information, that is, $\gamma^{A^*}(x) > \gamma^{M^*}(x) = \bar{\gamma}$.
(ii) Communication in each period is informative. Specifically, in both periods, the aligned analyst reports truthfully and the misaligned analyst reports 1 all the time.
(iii) The analyst's reputation is improved if report 0 is issued in the first period:

$$\Lambda(0, 0) = \Lambda(0, 1) = 1 > \frac{1}{2} > \Lambda(1, 1) > \Lambda(1, 0).$$

The argument for the case of unimportant future concerns is straightforward. If $x < x^{oo}$, then the current-period payoff will be the predominant consideration when the analyst determines her reporting strategies in the first period. Therefore, in the first period, the aligned analyst reports truthfully, while the misaligned type always reports 1. Note that the main difference between the equilibrium under $x > x^o$ (Proposition 2) and the equilibrium under $x < x^{oo}$ (Proposition 3) is due to the misaligned type's different reporting behaviors in the first period. If $x < x^{oo}$, the misaligned type always reports 1 in the first period, therefore (a) the misaligned analyst has no incentives to acquire additional information; (b) if the aligned type reports 0 in the first period, she perfectly reveals her type (no matter what the realized state is); and (c) anticipating that the misaligned analyst always reports 1 in the first period, the investor's inferences about the state $a_1(m_1)$ and the analyst's type $\Lambda(m_1, w_1)$ are independent of $\hat{\gamma}^M$.

The aligned analyst's optimal precision choice is determined by the following first-order condition (imposing the equilibrium condition $\gamma^J = \hat{\gamma}^J = \gamma^{J^*}$):

$$c'(\gamma^{A*}) = (1-x)\underbrace{(a_1(1) - a_1(0))}_{PE_1^A(x<x^{oo},\gamma^{A*})} + x\underbrace{\frac{1}{2}[V^A(\Lambda(1,1),\gamma^{A*}) - V^A(\Lambda(1,0),\gamma^{A*})]}_{RE^A(x<x^{oo},\gamma^{A*})}$$
$$+\underbrace{x\frac{1}{2}\left[\gamma^{A*}\frac{\partial V^A(\Lambda(1,1),\gamma^A)}{\partial\gamma^A} + (1-\gamma^{A*})\frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial\gamma^A} + \frac{\partial V^A(\lambda_2 = 1,\gamma^A)}{\partial\gamma^A}\right]}_{PE_2^A(x<x^{oo},\gamma^{A*})}. \tag{7}$$

The right-hand side of equation (7) represents the aligned analyst's marginal benefit of acquiring precision γ^{A*}, which I denote by $MB^A(x, \gamma^{A*})$. Given that $\hat{\gamma}^M$ doesn't affect the investor's inferences about the state $a_1(m_1)$ and the analyst's type $\Lambda(m_1, w_1)$, $MB^A(\cdot)$ is independent of $\hat{\gamma}^M$ (hence γ^{M*}). In addition, because the aligned analyst reporting 0 perfectly reveals her type, only when she reports 1, which happens with ex ante probability 1/2 (because she truthfully reports and the ex ante probability of observing 1 is 1/2), does the aligned type enjoy the benefit of reputation enhancement.

To conclude, Propositions 2 and 3 demonstrate that, once we endogenize the analyst's precision choice, the aligned analyst will acquire an informational advantage in equilibrium (in case of sufficiently important or unimportant future concerns). As a result, Morris's "political correctness" effect disappears. Specifically, when the analyst cares sufficiently about the future, both types of analysts will truthfully communicate in the first period. In contrast, if the analyst cares little about the future, her current reporting incentive dominates, and hence only the aligned type will truthfully communicate in the first period, while the misaligned type will always submit a high report.[18]

5. *Comparative Statics and Investor's Welfare*

5.1 COMPARATIVE STATICS

Propositions 2 and 3 describe the analyst's equilibrium precision choice for $x > x^o$ and $x < x^{oo}$. The next natural question is how the analyst's equilibrium precision choice varies with x. Owing to some technical

[18] A limitation of the analysis is that, for complexity reasons, the characterization of the equilibrium for the intermediate region of future concerns, $[x^{oo}, x^o]$, is not feasible. Instead, I include a numerical example characterizing the missing part of the equilibrium in the appendix.

difficulties, for $x > x^o$, I cannot unambiguously determine the effect of x on the misaligned type's equilibrium precision γ^{M*}. Instead, I focus on how the aligned analyst's equilibrium precision γ^{A*} varies with x.

PROPOSITION 4. *If the analyst's information acquisition cost function is sufficiently convex, the aligned analyst's equilibrium precision γ^{A*} is nonmonotonic in x:*

(1) *If* $x < x^{oo}$, $\frac{d\gamma^{A*}(x)}{dx} > 0$.

(2) *If* $x > x^o$, $\frac{d\gamma^{A*}(x)}{dx} < 0$.

To make sense of this result, first note that, by equations (3) and (7), the aligned analyst's marginal benefit of acquiring information is affected by x both directly (through the weight on first- and second-period payoffs) and indirectly (through the equilibrium precision $\gamma^{J*}(x)$). As shown in the appendix, the dominant effect is the direct effect—an increase in x *directly* increases the weight on the aligned analyst's reputation effect and second-period precision effect but decreases the weight on her first-period precision effect. Therefore, to examine how an increase in x affects the aligned analyst's marginal benefit of acquiring information, we should focus on the comparison of $PE_1^A(\cdot)$ and $PE_2^A(\cdot) + RE^A(\cdot)$.

Recall that the analyst's precision effect in each period derives from her impact on the investor's decision in that period by changing her report. If $x < x^{oo}$, the equilibrium is such that each type's reporting strategy in the first period is the same as in the second period. Furthermore, given that first-period communication is informative, in expectation, the aligned type improves her reputation for the second period, which leads the investor to respond more to the analyst's report in the second period compared with the first; that is,

$$PE_2^A(\cdot) > PE_1^A(\cdot).$$

It then follows that an increase in x increases the aligned type's marginal benefit of acquiring information, which leads to a higher γ^{A*}.

Conversely, when $x > x^o$, the equilibrium is such that both types of analysts truthfully report in the first period and only the aligned type truthfully reports in the second period. Therefore, the investor responds more to the analyst's report in the first period than in the second period. That is, the aligned analyst's first-period precision effect is greater than her second-period precision effect. At the same time, when the information acquisition cost function is sufficiently convex, the equilibrium precision differential $\gamma^{A*} - \gamma^{M*}$ cannot be too large, and hence the aligned type's reputation effect $RE^A(\cdot)$ is bounded from above. Overall, it is shown in the appendix that, when the information acquisition cost function is sufficiently convex,

$$PE_2^A(\cdot) + RE^A(\cdot) < PE_1^A(\cdot).$$

Therefore, an increase in x decreases the aligned type's marginal benefit of acquiring information, which leads to a lower γ^{A*}.

5.2 INVESTOR'S WELFARE

In this section, I examine how the analyst's future concerns affect the investor's welfare, defined as the investor's ex ante expected utility. Clearly, the investor's welfare is affected by how the investor discounts the future. One natural candidate is to assume that the investor has the same time preference as the analyst. However, such an assumption would introduce a mechanical link between the investor's welfare and the analyst's future concerns (time preference). To avoid such a mechanical link, instead I assume that the investor puts equal weights on the first-period and second-period payoffs; that is, the investor treats the two periods equally. An alternative explanation is that there are two different generations of investors, and the regulators treat the two generations of investors equally.

The investor's ex ante expected utility measures how much information is reflected in the investor's decisions. The more information that is incorporated into the investor's decisions, the higher the investor's ex ante expected utility will be. The total amount of information incorporated into the investor's decisions is determined both by the communication between the analyst and the investor, and also by the analyst's equilibrium precision choice.

If $x < x^{oo}$, in both periods, only the aligned type truthfully communicates, while the misaligned type always reports 1. Therefore, the investor's welfare is independent of the misaligned analyst's precision. In contrast, higher precision of the aligned analyst not only directly translates into better decision making (by the investor) in each period through her truthful communication, but also facilitates the investor's learning about the analyst's type, which also leads to better decision making in the second period.

Now consider the case in which $x > x^{o}$. Then, again, the aligned analyst truthfully communicates in both periods, whereas the misaligned analyst only truthfully communicates in the first period. Therefore, higher precision of the aligned analyst implies that not only more information is directly incorporated into the investor's decision making in each period, but also the investor's learning about the analyst's type is more effective. Both factors contribute to a higher investor's welfare. For the effect of the misaligned analyst's precision, a different logic applies: On one hand, higher precision of the misaligned analyst leads to better decision making (by the investor) in the first period, given that she truthfully communicates. On the other hand, keeping the aligned analyst's precision constant, the higher the precision of the misaligned analyst, the smaller the precision differential between the two types, and thus the less effective the investor's learning about the analyst's type, which affects second-period decision making negatively. As shown in the appendix, the first-period effect is stronger, and therefore the overall effect of the misaligned type's precision on the investor's welfare is positive. Furthermore, note that the investor's first-period decision making is equally (positively) affected by either type's precision, whereas the second-period decision making is positively affected by the aligned type's precision and negatively affected by the misaligned type's precision. Therefore, the overall impact of the aligned type's precision on the investor's welfare is stronger (more positive) than that of the misaligned type's precision.

I now analyze how the investor's welfare is affected by the analyst's future concerns x. Denote the investor's welfare by $U_0^I(\cdot)$.

PROPOSITION 5. *If the analyst's information acquisition cost function is sufficiently convex and the third derivative of the cost function is not too large,*[19] *then the investor's welfare is nonmonotonic in the analyst's future concerns x:*

(1) *If $x < x^{oo}$, the investor's welfare is increasing in x. That is,*

$$\frac{dU_0^I(x < x^{oo})}{dx} > 0.$$

(2) *If $x > x^o$, the investor's welfare is decreasing in x. That is,*

$$\frac{dU_0^I(x > x^o)}{dx} < 0.$$

If $x < x^{oo}$, the investor's welfare is increasing in the aligned analyst's precision and is independent of the misaligned type's precision. At the same time, Proposition 4 shows that, when the cost function is sufficiently convex, the aligned analyst's equilibrium precision is increasing in x. Therefore, the effect of x (through γ^{A*}) on the investor's welfare is positive.

Conversely, when $x > x^o$, the investor's welfare is increasing in both analysts' precision. As Proposition 4 shows, for important future concerns, the aligned analyst acquires less information as future concerns become more important. Therefore, the effect of future concerns on the investor's welfare through γ^{A*} is negative. Of course, future concerns also affect the investor's welfare through the impact on the misaligned analyst's equilibrium precision γ^{M*}. Note that (1) as the appendix shows, if the analyst's information acquisition cost function is sufficiently convex and the third derivative of the cost function is not too large, then future concerns x affect the aligned analyst's precision choice γ^{A*} more than that of the misaligned analyst γ^{M*} (if the two effects are in different directions)[20] and (2) the aligned analyst's equilibrium precision γ^{A*} has a stronger impact on the investor's welfare than that of the misaligned analyst γ^{M*}. Both forces suggest that future concerns affect the investor's welfare more through γ^{A*}

[19] An example of such a cost function is quadratic cost function.

[20] Technically, $\frac{d\gamma^{A*}}{dx} + \frac{d\gamma^{M*}}{dx} < 0$. That is, if $\frac{d\gamma^{M*}}{dx}$ is positive, then it must hold that $\frac{d\gamma^{M*}}{dx} < |\frac{d\gamma^{A*}}{dx}|$.

than through γ^{M*}. Therefore, the overall impact of future concerns on the investor's welfare is negative.

To conclude, Propositions 4 and 5 show that, under plausible constraints on the analyst's information acquisition cost function, the analyst's future concerns have a nonmonotonic impact on the aligned analyst's equilibrium precision choice and the investor's welfare.

6. *Conclusion*

This paper investigates how future concerns affect analysts' incentives to invest in information acquisition and their subsequent reporting to investors in the form of "repeated cheap talk." In equilibrium, aligned analysts will acquire more information than misaligned analysts. Hence there are two ways to build reputation for being aligned: (1) by issuing reports as accurately as possible, and (2) by issuing reports contrary to the misaligned type's bias. If analysts' future concerns are sufficiently important, the first way of building reputation prevails. Then truth-telling communication can be sustained in equilibrium, and analysts' communication in the reputation-formation stage conveys information both about their types (the exogenous degrees of incentive alignment) and their precision (given the endogenous association between analysts' types and precision). If, instead, analysts' future concerns are small, then analysts' current reporting incentives dominate: aligned analysts communicate truthfully, while misaligned analysts always issue high reports. As a result, analysts' reputation will improve after low reports have been issued.

Certain key features of the model warrant further discussion. For example, I assume that analysts' precision is perfectly correlated over time. In the extreme opposite case of zero serial correlation (i.e., acquiring information only increases analysts' precision in the current period), neither type will acquire information in the first period if both types of analysts care sufficiently about future transactions.[21] In practice, however, some elements affecting analysts' precision are clearly persistent (e.g., industry knowledge, the ability to analyze financial statements). My findings will continue to hold qualitatively when the correlation is sufficiently large.

Finally, in this paper, I assume that both types of analysts face the same information acquisition cost, which rules out misaligned analysts' potential privileged access to management inside information. Regulation FD aims to

[21] The key to understanding this result is that misaligned analysts benefit more from a high reputation than do their aligned peers. When both types of analysts care sufficiently about the future, they acquire information (in the first period) mainly for reputation-building purposes. Then misaligned analysts will have stronger incentives to acquire information than aligned analysts. Hence, no equilibria can exist in which aligned analysts acquire strictly more information. As a result, Morris's "political correctness" effect prevails, and the first-period communication has to be babbling, which in turn eliminates both types' incentives to acquire information in the first period.

prohibit selective disclosures with the goal of creating a more even playing field among analysts. I therefore expect that my model speaks more to the post-Reg FD regime. Furthermore, my results imply that, if misaligned analysts have a significant informational advantage over aligned analysts, then communication may become uninformative when analysts care sufficiently about the future. Hence Reg FD has the additional benefit of fostering communication by limiting misaligned analysts' informational advantage.

APPENDIX A

Formal Definition of the Equilibrium

I begin by introducing the analyst's utility at each decision node. To that end, denote by $U_t^J(m_t|s_t, \gamma^J, \lambda_t, x^J)$ the type J analyst's utility at period t when she reports m_t while her signal is s_t, precision is γ^J, reputation is λ_t, and future concerns are x^J. Note that $\lambda_1 = \frac{1}{2}$, therefore, to save on notation, I suppress the argument λ_1 in $U_1^J(\cdot)$. At the same time, since the second period is the last period, the analyst's future concerns x^J have no effect on her expected utility upon observing s_2, and hence is omitted in $U_2^J(\cdot)$. Let $U_0^J(\gamma^J, x^J)$ denote type J analyst's utility at the information acquisition stage when she chooses precision γ^J and future concerns are x^J.[22]

Given the investor's second-period optimal decision rule $a_2(m_2, \lambda_2)$, where $\lambda_2 = \Lambda(m_1, w_1)$ (to be specified below), the type J analyst's second-period utility upon observing s_2 is:

$$\begin{aligned} U_2^A\left(m_2|s_2, \gamma^A, \lambda_2\right) &= -\gamma^A(a_2(m_2, \lambda_2) - s_2)^2 \\ &\quad -(1-\gamma^A)(a_2(m_2, \lambda_2) - (1-s_2))^2, \\ U_2^M\left(m_2|s_2, \gamma^M, \lambda_2\right) &= -(a_2(m_2, \lambda_2) - 1)^2. \end{aligned}$$

Denote by $\sigma_t^J(s_t, \gamma^J, \lambda_t, x^J)$ the type J analyst's optimal reporting strategy at period t upon observing signal s_t when her precision is γ^J, reputation is λ_t, and future concerns are x^J. For the same reason as in the case of $U_t^J(\cdot)$, when there is no scope for confusion, I omit the argument λ_1 in $\sigma_1^J(\cdot)$ and x^J in $\sigma_2^J(\cdot)$. Then,

$$\sigma_2^J\left(s_2, \gamma^J, \lambda_2\right) \in \underset{\sigma_2^J\in[0,1]}{\operatorname{argmax}}\ \sigma_2^J U_2^J\left(1|s_2, \gamma^J, \lambda_2\right) + \left(1-\sigma_2^J\right) U_2^J\left(0|s_2, \gamma^J, \lambda_2\right).$$

Hence, the type J analyst's second-period expected utility (calculated at the beginning of the second period, before s_2 is realized) is:

$$\begin{aligned} V^J\left(\lambda_2, \gamma^J\right) = \frac{1}{2}\sum_{s_2=0,1} \Big\{&\sigma_2^J\left(s_2, \gamma^J, \lambda_2\right) U_2^J\left(1|s_2, \gamma^J, \lambda_2\right) \\ &+ \left(1-\sigma_2^J\left(s_2, \gamma^J, \lambda_2\right)\right) U_2^J\left(0|s_2, \gamma^J, \lambda_2\right)\Big\}. \end{aligned}$$

[22] To save on notation, I suppress the functional dependence of the players' utilities and strategies on their conjectures about their counterparties' actions.

Given the investor's first-period optimal decision rule $a_1(m_1)$, the type J analyst's utility upon observing s_1 is then given by:

$$\begin{aligned}
U_1^A(m_1|s_1,\gamma^A,x^A) &= (1-x^A)\left\{-\gamma^A(a_1(m_1)-s_1)^2-(1-\gamma^A)(a_1(m_1)-(1-s_1))^2\right\}\\
&\quad +x^A\left\{\gamma^A V^A(\Lambda(m_1,s_1),\gamma^A)+(1-\gamma^A)V^A(\Lambda(m_1,1-s_1),\gamma^A)\right\},\\
U_1^M(m_1|s_1,\gamma^M,x^M) &= -(1-x^M)(a_1(m_1)-1)^2\\
&\quad +x^M\left\{\gamma^M V^M(\Lambda(m_1,s_1),\gamma^M)+(1-\gamma^M)V^M(\Lambda(m_1,1-s_1),\gamma^M)\right\}.
\end{aligned}$$

Then,

$$\sigma_1^J(s_1,\gamma^J,x^J)\in \underset{\sigma_1^J\in[0,1]}{\operatorname{argmax}}\ \sigma_1^J U_1^J(1|s_1,\gamma^J,x^J)+(1-\sigma_1^J)U_1^J(0|s_1,\gamma^J,x^J).$$

Thus, the type J analyst's utility at the information acquisition stage is given by

$$\begin{aligned}
U_0^J(\gamma^J,x^J) &= \frac{1}{2}\sum_{s_1=0,1}\left\{\sigma_1^J\left(s_1,\gamma^J,x^J\right)U_1^J(1|s_1,\gamma^J,x^J)\right.\\
&\quad \left.+\left(1-\sigma_1^J\left(s_1,\gamma^J,x^J\right)\right)U_1^J(0|s_1,\gamma^J,x^J)\right\}-c(\gamma^J).
\end{aligned}$$

Next, I introduce the players' belief functions. Let $\phi_t^J(m_t|w_t)$ denote the investor's conjecture about the probability of the type J analyst sending message m_t given state w_t in period t. Then,[23]

$$\phi_t^J(1|w_t)=\hat{\gamma}^J\hat{\sigma}_t^J\left(w_t,\hat{\gamma}^J,\lambda_t,x^J\right)+(1-\hat{\gamma}^J)\hat{\sigma}_t^J\left(1-w_t,\hat{\gamma}^J,\lambda_t,x^J\right),$$

and $\phi_t^J(0|w_t)=1-\phi_t^J(1|w_t)$. The belief function $\Gamma_t(m_t,\lambda_t)$ states the investor's inference of the actual state being 1 in period t. By Bayes rule, it is given by

$$\Gamma_t(m_t,\lambda_t)=\frac{\lambda_t\phi_t^A(m_t|1)+(1-\lambda_t)\phi_t^M(m_t|1)}{\lambda_t\phi_t^A(m_t|1)+(1-\lambda_t)\phi_t^M(m_t|1)+\lambda_t\phi_t^A(m_t|0)+(1-\lambda_t)\phi_t^M(m_t|0)}. \tag{A1}$$

$\Gamma_t(m_t,\lambda_t)$ is well defined when the denominator is nonzero. I adopt the convention that $\Gamma_t(m_t,\lambda_t)=1/2$, the prior, if the denominator is zero. That is, when the posterior belief of the state is undefined according to Bayes's rule, the investor keeps his prior belief about the state. In particular, $\lambda_1=\lambda=1/2$ is the prior reputation, and $\lambda_2=\Lambda(m_1,w_1)$ denotes the posterior

[23] I adopt the standard notation in the literature where $\hat{}$ represents the conjecture.

reputation, defined as the investor's belief of the analyst being aligned if report m_1 is received and state w_1 is realized:

$$\Lambda(m_1, w_1) = \frac{\phi_1^A(m_1|w_1)}{\phi_1^A(m_1|w_1) + \phi_1^M(m_1|w_1)}. \tag{A2}$$

Again, I adopt the convention that $\Lambda(m_1, w_1) = \lambda = 1/2$, the prior, if the denominator is zero.

Now I am in a position to define the equilibrium of the game formally.

DEFINITION A1. *A Perfect Bayesian Nash equilibrium of the game is a strategy-belief profile* $(\gamma^{A^*}(\cdot), \gamma^{M^*}(\cdot), \sigma_t^A(\cdot), \sigma_t^M(\cdot), a_t(\cdot), \Gamma_t(\cdot), \Lambda(\cdot))$ *satisfying the following properties:*

(1)

$$\gamma^{J^*}(x^J) \in \underset{\gamma^J \in [\bar{\gamma}, 1]}{\operatorname{argmax}} \quad U_0^J(\gamma^J, x^J).$$

(2)

$$a_t(m_t, \lambda_t) \in \underset{a_t \in R}{\operatorname{argmax}} \quad -\Gamma_t(m_t, \lambda_t)(a_t - 1)^2 - (1 - \Gamma_t(m_t, \lambda_t))\, a_t^2.$$

(3)

$$\sigma_t^J\left(s_t, \gamma^J, \lambda_t, x^J\right) \in \underset{\sigma_t^J \in [0,1]}{\operatorname{argmax}} \ \sigma_t^J U_t^J\left(1|s_t, \gamma^J, \lambda_t, x^J\right) + (1 - \sigma_t^J)\, U_t^J(0|s_t, \gamma^J, \lambda_t, x^J).$$

(4) *The state and type inference functions,* $\Gamma_t(m_t, \lambda_t)$ *and* $\lambda_2 \equiv \Lambda(m_1, w_1)$*, are derived from the analyst's equilibrium strategy according to inference rules* (A1) *and* (A2)*. Specifically,*

$$\phi_t^J(1|w_t) = \gamma^{J^*}\sigma_t^J(w_t, \gamma^{J^*}, \lambda_t, x^J) + (1 - \gamma^{J^*})\sigma_t^J(1 - w_t, \gamma^{J^*}, \lambda_t, x^J).$$

When the posteriors are undefined according to Bayes rule, I adopt the convention that the investor sticks to his priors.

APPENDIX B

Proofs of the Main Results

Proof of Lemma 1. This proof is similar to the proof of Proposition 2 in Morris (2001) and thus is omitted. ■

Proof of Proposition 1. Suppose the investor holds conjectures that both types of analysts truthfully report in the first period. Then

$$\Lambda(1,1) = \Lambda(0,0) = \frac{1}{1 + \frac{\gamma^M}{\gamma^A}}, \qquad \Lambda(1,0) = \Lambda(0,1) = \frac{1}{1 + \frac{1-\gamma^M}{1-\gamma^A}},$$

$$a_1(1) = \frac{1}{2}(\gamma^A + \gamma^M), \qquad a_1(0) = 1 - \frac{1}{2}(\gamma^A + \gamma^M).$$

Given that $\gamma^A > \gamma^M$, it is immediate that $\Lambda(1,1) = \Lambda(0,0) > \Lambda(1,0) = \Lambda(0,1)$. Now I examine whether indeed the analysts' best responses are consistent with the investor's conjectures.

In the first period, if the type J analyst with precision γ^J observes signal s_1, $s_1 = 0, 1$, she will compare her payoff conditional on reporting 1 versus reporting 0:

$$\begin{aligned} & U_1^A(m_1 = 1|s_1 = 1, \gamma^A, x^A) - U_1^A(m_1 = 0|s_1 = 1, \gamma^A, x^A) \\ &= (1 - x^A)(a_1(1) - a_1(0))(2\gamma^A - 1) \\ &\quad + x^A(2\gamma^A - 1)[V^A(\Lambda(0,0)) - V^A(\Lambda(1,0))] \\ &> 0, \end{aligned}$$

$$\begin{aligned} & U_1^A(m_1 = 1|s_1 = 0, \gamma^A, x^A) - U_1^A(m_1 = 0|s_1 = 0, \gamma^A, x^A) \\ &= -(1 - x^A)(a_1(1) - a_1(0))(2\gamma^A - 1) \\ &\quad - x^A(2\gamma^A - 1)[V^A(\Lambda(0,0)) - V^A(\Lambda(1,0))] \\ &< 0, \end{aligned}$$

$$\begin{aligned} & U_1^M(m_1 = 1|s_1 = 1, \gamma^M, x^M) - U_1^M(m_1 = 0|s_1 = 1, \gamma^M, x^M) \\ &= (1 - x^M)(a_1(1) - a_1(0)) + x^M(2\gamma^M - 1)[V^M(\Lambda(0,0)) \\ &\quad - V^M(\Lambda(1,0))] \\ &> 0, \end{aligned}$$

$$\begin{aligned} & U_1^M(m_1 = 1|s_1 = 0, \gamma^M, x^M) - U_1^M(m_1 = 0|s_1 = 0, \gamma^M, x^M) \\ &= (1 - x^M)(a_1(1) - a_1(0)) - x^M(2\gamma^M - 1)[V^M(\Lambda(0,0)) \\ &\quad - V^M(\Lambda(1,0))]. \end{aligned}$$

Therefore, if $x^M > \bar{x}^M(\gamma^A, \gamma^M) \equiv \frac{1}{x_c^M(\gamma^A,\gamma^M)+1}$, where

$$\begin{aligned} x_c^M(\gamma^A, \gamma^M) &\equiv \frac{(2\gamma^M - 1)\left[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))\right]}{a_1(1) - a_1(0)} \\ &= \frac{(2\gamma^M - 1)\left[V^M\left(\frac{1}{1+\frac{\gamma^M}{\gamma^A}}\right) - V^M\left(\frac{1}{1+\frac{1-\gamma^M}{1-\gamma^A}}\right)\right]}{\gamma^M + \gamma^A - 1}, \end{aligned}$$

then $U_1^M(m_1 = 1|s_1 = 0, \gamma^M, x^M) - U_1^M(m_1 = 0|s_1 = 0, \gamma^M, x^M) < 0$. That is, both types of analysts will truthfully report, consistent with the investor's conjectures.

To summarize, if $\gamma^A > \gamma^M$, the truth-telling equilibrium in which both types of analysts truthfully report their signals obtains in the first period for $x^M > \bar{x}^M(\gamma^A, \gamma^M)$. ■

Proof of Proposition 2. Assume that the investor holds conjectures that (a) the aligned analyst chooses higher precision than the misaligned analyst, $\hat{\gamma}^A > \hat{\gamma}^M$; (b) both types of analysts truthfully report in the first period; and (c) in the second period the aligned analyst reports truthfully and the misaligned analyst always reports 1. Then the investor will update favorably the analyst's reputation when the analyst's report is consistent with the realized state, that is,

$$\Lambda(1,1) = \Lambda(0,0) = \frac{1}{1 + \dfrac{\hat{\gamma}^M}{\hat{\gamma}^A}} > \Lambda(1,0) = \Lambda(0,1) = \frac{1}{1 + \dfrac{1-\hat{\gamma}^M}{1-\hat{\gamma}^A}}.$$

And the investor's actions in each period upon receiving the analyst's report are:

$$a_1(1) = \frac{1}{2}(\hat{\gamma}^A + \hat{\gamma}^M) > a_1(0) = 1 - \frac{1}{2}(\hat{\gamma}^A + \hat{\gamma}^M),$$

$$a_2(1, \lambda_2) = \frac{1 - \lambda_2 + \lambda_2\hat{\gamma}^A}{2 - \lambda_2} > a_2(0, \lambda_2) = 1 - \hat{\gamma}^A.$$

Now we need to examine the analyst's best responses with regard to both the precision choice and the communication strategies.

(1) The aligned analyst:

(i) In the *second* period, if the aligned analyst with precision γ^A observes signal s_2, $s_2 = 0, 1$, she will compare her utility conditional on reporting 1 versus reporting 0:

$$\begin{aligned}
&U_2^A(m_2 = 1|s_2 = 1, \gamma^A, \lambda_2) - U_2^A(m_2 = 0|s_2 = 1, \gamma^A, \lambda_2)\\
&= [a_2(1,\lambda_2) - a_2(0,\lambda_2)][2\gamma^A - a_2(1,\lambda_2) - a_2(0,\lambda_2)]\\
&> 0,\\
&U_2^A(m_2 = 1|s_2 = 0, \gamma^A, \lambda_2) - U_2^A(m_2 = 0|s_2 = 0, \gamma^A, \lambda_2)\\
&= [a_2(1,\lambda_2) - a_2(0,\lambda_2)][2(1-\gamma^A) - a_2(1,\lambda_2) - a_2(0,\lambda_2)]\\
&= [a_2(1,\lambda_2) - a_2(0,\lambda_2)]\frac{2(1-\lambda_2)(\hat{\gamma}^A - \gamma^A) - (2\gamma^A - 1)}{2 - \lambda_2}\\
&< 0.
\end{aligned}$$

The last inequality derives from the fact that $2(1-\lambda_2)(\hat{\gamma}^A-\gamma^A) \leq 2(\hat{\gamma}^A-\gamma^A) \leq 1/2$ and $2\gamma^A-1 \geq 1/2$, given $\hat{\gamma}^A \in [\bar{\gamma}, 1] \in [3/4, 1]$ and $\gamma^A \in [\bar{\gamma}, 1] \in [3/4, 1]$. Therefore, in the second period the aligned analyst with precision γ^A will truthfully report.
Then the aligned analyst's second-period expected utility is:

$$\begin{aligned} V^A(\lambda_2, \gamma^A) &= -\frac{1}{2}\gamma^A(a_2(1,\lambda_2)-1)^2 - \frac{1}{2}(1-\gamma^A)(a_2(0,\lambda_2)-1)^2 \\ &\quad -\frac{1}{2}\gamma^A(a_2(0,\lambda_2))^2 - \frac{1}{2}(1-\gamma^A)(a_2(1,\lambda_2))^2 \\ &= -\frac{1}{2}(a_2(0,\lambda_2)-1)^2 - \frac{1}{2}(a_2(1,\lambda_2))^2 \\ &\quad + \gamma^A[a_2(1,\lambda_2)-a_2(0,\lambda_2)]. \end{aligned}$$

Higher precision will increase $V^A(\cdot)$, whereas the effect of reputation on $V^A(\cdot)$ is ambiguous:

$$\begin{aligned} \frac{\partial V^A(\lambda_2,\gamma^A)}{\partial \gamma^A} &= a_2(1,\lambda_2) - a_2(0,\lambda_2) = \frac{2\hat{\gamma}^A-1}{2-\lambda_2} > 0, \\ \frac{\partial V^A(\lambda_2,\gamma^A)}{\partial \lambda_2} &= (\gamma^A - a_2(1,\lambda_2))\frac{da_2(1,\lambda_2)}{d\lambda_2} \\ &= \left(\gamma^A - \frac{1-\lambda_2+\lambda_2\hat{\gamma}^A}{2-\lambda_2}\right)\frac{da_2(1,\lambda_2)}{d\lambda_2}. \end{aligned}$$

Clearly, if $\gamma^A \geq \hat{\gamma}^A$, $\frac{\partial V^A(\lambda_2,\gamma^A)}{\partial \lambda_2} \geq 0$. Otherwise, given that $\gamma^A \in [\bar{\gamma}, 1] \in [3/4, 1]$ and $\hat{\gamma}^A \in [\bar{\gamma}, 1] \in [3/4, 1]$, we need $\lambda_2 \leq 2/3$ to guarantee that $\frac{\partial V^A(\lambda_2,\gamma^A)}{\partial \lambda_2} \geq 0$.

(ii) In the *first* period, if the aligned analyst with precision γ^A observes signal s_1, $s_1 = 0, 1$, she will compare her utility conditional on reporting 1 versus reporting 0:

$$\begin{aligned} &U_1^A(m_1 = 1|s_1 = 1, \gamma^A, x) - U_1^A(m_1 = 0|s_1 = 1, \gamma^A, x) \\ &= (1-x)(a_1(1)-a_1(0))(2\gamma^A-1) \\ &\quad + x(2\gamma^A-1)[V^A(\Lambda(0,0),\gamma^A) - V^A(\Lambda(1,0),\gamma^A)], \end{aligned}$$

$$\begin{aligned} &U_1^A(m_1 = 1|s_1 = 0, \gamma^A, x) - U_1^A(m_1 = 0|s_1 = 0, \gamma^A, x) \\ &= -(1-x)(a_1(1)-a_1(0))(2\gamma^A-1) \\ &\quad -x(2\gamma^A-1)[V^A(\Lambda(0,0),\gamma^A) - V^A(\Lambda(1,0),\gamma^A)]. \end{aligned}$$

Given that $3/4 \leq \bar{\gamma} \leq \hat{\gamma}^M < \hat{\gamma}^A \leq 1$, it can be shown that $\Lambda(1,0) < \Lambda(0,0) < 2/3$, therefore $V^A(\Lambda(0,0),\gamma^A) - V^A(\Lambda(1,0),\gamma^A) > 0$. As a result, $U_1^A(m_1 = 1|s_1 = 1, \gamma^A, x) - U_1^A(m_1 = 0|s_1 = 1, \gamma^A, x) > 0$ and $U_1^A(m_1 = 1|s_1 = 0, \gamma^A, x) - U_1^A(m_1 = 0|s_1 = 0, \gamma^A, x) < 0$. That is, the aligned analyst with precision γ^A will truthfully report in the first period.

(iii) The aligned analyst's choice of precision:
Given that the aligned analyst with precision γ^A truthfully reports in the first period, the aligned analyst's utility at the information acquisition stage is as follows:

$$\begin{aligned}
U_0^A(\gamma^A, x) &= \frac{1}{2}\gamma^A[-(1-x)(a_1(1)-1)^2 + xV^A(\Lambda(1,1),\gamma^A)] \\
&\quad + \frac{1}{2}(1-\gamma^A)[-(1-x)(a_1(0)-1)^2 + xV^A(\Lambda(0,1),\gamma^A)] \\
&\quad + \frac{1}{2}\gamma^A[-(1-x)(a_1(0)-0)^2 + xV^A(\Lambda(0,0),\gamma^A)] \\
&\quad + \frac{1}{2}(1-\gamma^A)[-(1-x)(a_1(1)-0)^2 + xV^A(\Lambda(1,0),\gamma^A)] - c(\gamma^A) \\
&= \gamma^A[-(1-x)(a_1(0))^2 + xV^A(\Lambda(0,0),\gamma^A)] \\
&\quad + (1-\gamma^A)[-(1-x)(a_1(1))^2 + xV^A(\Lambda(1,0),\gamma^A)] - c(\gamma^A).
\end{aligned}$$

Taking derivative with respect to γ^A, the aligned analyst's optimal precision choice has to satisfy the following first-order condition (after imposing the equilibrium condition $\gamma^J = \hat{\gamma}^J = \gamma^{J^*}$):

$$\begin{aligned}
c'(\gamma^{A*}) = &\ (1-x)\underbrace{(a_1(1)-a_1(0))}_{PE_1^A(x>x^o,\gamma^{A*},\gamma^{M*})} + x\underbrace{[V^A(\Lambda(0,0),\gamma^{A*}) - V^A(\Lambda(1,0),\gamma^{A*})]}_{RE^A(x>x^o,\gamma^{A*},\gamma^{M*})} \\
&+ x\underbrace{[\gamma^{A*}\frac{\partial V^A(\Lambda(0,0),\gamma^A)}{\partial \gamma^A} + (1-\gamma^{A*})\frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial \gamma^A}}_{PE_2^A(x>x^o,\gamma^{A*},\gamma^{M*})},
\end{aligned}$$

where $PE_t^A(\cdot)$ stands for "precision effect" in period t for the aligned analyst and $RE^A(\cdot)$ stands for "reputation effect" for the aligned analyst. It is straightforward to show that:

$$\begin{aligned}
PE_2^A(\cdot) &= \gamma^{A*}\frac{\partial V^A(\Lambda(0,0),\gamma^A)}{\partial \gamma^A} + (1-\gamma^{A*})\frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial \gamma^A} \\
&= \gamma^{A*}[a_2(1,\Lambda(0,0)) - a_2(0,\Lambda(0,0))] \\
&\quad + (1-\gamma^{A*})[a_2(1,\Lambda(1,0)) - a_2(0,\Lambda(1,0))] \\
&> a_2(1,\lambda_2 = 0) - a_2(0,\lambda_2 = 0) \\
&= \gamma^{A*} - 1/2.
\end{aligned}$$

Hence,

$$\begin{aligned} c'(\gamma^{A*}) &= (1-x)PE_1^A(\cdot) + xRE^A(\cdot) + xPE_2^A(\cdot) \\ &> xPE_2^A(\cdot) \\ &> x(\gamma^{A*} - 1/2). \end{aligned} \tag{B1}$$

Finally, let's check the second-order condition:

$$\begin{aligned} \frac{\partial^2 U_0^A(\cdot)}{\partial \gamma^{A^2}} &= 2x\left[\frac{\partial V^A(\Lambda(0,0),\gamma^A)}{\partial \gamma^A} - \frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial \gamma^A}\right] \\ &\quad - c''(\gamma^A) \\ &= 2x[a_2(1,\Lambda(0,0)) - a_2(1,\Lambda(1,0))] - c''(\gamma^A) \\ &\leq 1 - c''(\gamma^A) \\ &\leq 0. \end{aligned}$$

(2) The misaligned analyst:

(i) In the *second* period, since $a_2(1,\lambda_2) > a_2(0,\lambda_2)$, the misaligned analyst's best response is to always report 1 no matter what her true precision is.
The misaligned analyst's second-period expected utility equals:[24]

$$V^M(\lambda_2) = -[a_2(1,\lambda_2) - 1]^2.$$

Clearly, $\frac{\partial V^M(\lambda_2)}{\partial \lambda_2} > 0$.

(ii) In the *first* period, if the misaligned analyst with precision γ^M observes signal s_1, $s_1 = 0, 1$, she will compare her utility conditional on reporting 1 versus reporting 0:

$$\begin{aligned} & U_1^M(m_1 = 1|s_1 = 1, \gamma^M, x) - U_1^M(m_1 = 0|s_1 = 1, \gamma^M, x) \\ &= (1-x)(a_1(1) - a_1(0)) + x(2\gamma^M - 1)[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))] \\ &> 0, \\ & U_1^M(m_1 = 1|s_1 = 0, \gamma^M, x) - U_1^M(m_1 = 0|s_1 = 0, \gamma^M, x) \\ &= (1-x)(a_1(1) - a_1(0)) - x(2\gamma^M - 1)[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]. \end{aligned}$$

Define γ_c^M such that $U_1^M(m_1 = 1|s_1 = 0, \gamma_c^M, x) - U_1^M(m_1 = 0|s_1 = 0, \gamma_c^M, x) = 0$.[25] Then if $\gamma^M \geq \gamma_c^M$, $U_1^M(m_1 = 1|s_1 = 0, \gamma^M, x) - U_1^M(m_1 = 0|s_1 = 0, \gamma^M, x) \leq 0$. That is, the misaligned analyst with precision $\gamma^M \geq \gamma_c^M$ will truthfully report. On the other hand, if $\gamma^M < \gamma_c^M$, $U_1^M(m_1 = 1|s_1 = 0, \gamma^M, x) - U_1^M(m_1 = 0|s_1 = 0, \gamma^M, x) >$

[24] Note that γ^M does not affect $V^M(\cdot)$ because the misaligned analyst always reports 1 in the second period, independent of her precision. Hence, I suppress the argument γ^M in $V^M(\cdot)$.

[25] Note that $\gamma_c^M(\cdot)$ is a function of x and $\hat{\gamma}^J$, $J = A, M$.

0. That is, the misaligned analyst with precision $\gamma^M < \gamma_c^M$ will always report 1.

(iii) The misaligned analyst's choice of precision:
If the misaligned analyst chooses $\gamma^M \geq \gamma_c^M$, she will truthfully report in the first period. This is the equilibrium we are seeking here. Then her utility at the information acquisition stage is:

$$\begin{aligned} U_0^M(\gamma^M \geq \gamma_c^M, x) = & -\frac{1}{2}(1-x)[(a_1(1)-1)^2 + (a_1(0)-1)^2] \\ & + x[\gamma^M V^M(\Lambda(0,0)) + (1-\gamma^M) V^M(\Lambda(1,0))] \\ & - c(\gamma^M). \end{aligned}$$

Taking derivative with respect to γ^M, the misaligned analyst's optimal precision choice has to satisfy the following first-order condition (after imposing the equilibrium condition $\gamma^J = \hat{\gamma}^J = \gamma^{J^*}$):

$$c'(\gamma^{M*}) = x \underbrace{[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]}_{RE^M(x>x^o, \gamma^{A*}, \gamma^{M*})}.$$

The second-order condition is also satisfied:

$$\frac{\partial^2 U_0^M(\cdot)}{\partial \gamma^{M2}} = -c''(\gamma^M) < 0.$$

Note that

$$\begin{aligned} RE^M(x > x^o, \gamma^{A*}, \gamma^{M*}) &= V^M(\Lambda(0,0)) - V^M(\Lambda(1,0)) \\ &= [a_2(1, \Lambda(0,0)) - a_2(1, \Lambda(1,0))] \\ &\quad [2 - a_2(1, \Lambda(0,0)) - a_2(1, \Lambda(1,0))] \\ &< a_2(1, \Lambda(0,0)) - a_2(1, \Lambda(1,0)) \\ &< a_2(1, \lambda_2 = 1) - a_2(1, \lambda_2 = 0) \\ &= \gamma^{A*} - 1/2. \end{aligned}$$

Therefore, by (B1), $c'(\gamma^{M*}) = xRE^M(\cdot) < xPE_2^A(\cdot) < c'(\gamma^{A*})$. Given that $c''(\gamma) > 0$, it follows that $\gamma^{A*} > \gamma^{M*}$. In addition, the following shows that either type's marginal benefit of acquiring information is less than 3/2, which, combined with the assumption that $\lim_{\gamma \to 1} c'(\gamma) \geq 3/2$, implies that $\gamma^{M*} < \gamma^{A*} < 1$.

$$\begin{aligned} c'(\gamma^{M*}) &= xRE^M(\cdot) < \gamma^{A*} - 1/2 \leq 1/2; \\ c'(\gamma^{A*}) &= (1-x)PE_1^A(\cdot) + xPE_2^A(\cdot) + xRE^A(\cdot) \\ &< (1-x)PE_1^A(\cdot) + xPE_2^A(\cdot) + xRE^M(\cdot) \\ &< 1 + 1/2 = 3/2. \end{aligned} \tag{B2}$$

The first inequality derives from $RE^A(\cdot) < RE^M(\cdot)$. The second inequality is due to the facts that (1) both $PE_1^A(\cdot)$ and $PE_2^A(\cdot)$ stem from the impact on the investor's decision in each period by changing report. Given that $a_t(1) - a_t(0) \leq 1$, such impact is smaller than 1; and (2) $xRE^M(\cdot) \leq 1/2$.
Note that $a_1(m_1)$ and $\Lambda(m_1, w_1)$ are functions of γ^{A*} and γ^{M*}, hence the values of γ^{A*} and γ^{M*} are determined by the following system of equations (B.3a) and (B.3b):

$$c'(\gamma^{A*}) = (1-x)\underbrace{(a_1(1) - a_1(0))}_{PE_1^A(x>x^o,\gamma^{A*},\gamma^{M*})} + x\underbrace{[V^A(\Lambda(0,0),\gamma^{A*}) - V^A(\Lambda(1,0),\gamma^{A*})]}_{RE^A(x>x^o,\gamma^{A*},\gamma^{M*})}$$

$$+\ x\underbrace{\left[\gamma^{A*}\frac{\partial V^A(\Lambda(0,0),\gamma^A)}{\partial\gamma^A} + (1-\gamma^{A*})\frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial\gamma^A}\right]}_{PE_2^A(x>x^o,\gamma^{A*},\gamma^{M*})}, \qquad \text{(B3a)}$$

$$c'(\gamma^{M*}) = x\underbrace{[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]}_{RE^M(x>x^o,\gamma^{A*},\gamma^{M*})}. \qquad \text{(B3b)}$$

(iv) Finally, we need to find conditions on x to ensure that there are no deviations regarding the misaligned analyst's reporting strategies and precision choice.

Part (a): To ensure that there is no deviation regarding the misaligned analyst's reporting strategies along the equilibrium path, we need the misaligned analyst's net benefit of deviating from truthfully reporting to be negative.

Define

$$\begin{aligned} D_1(x) &\equiv U_1^M(m_1 = 1|s_1 = 0, \gamma^{M*}, x) - U_1^M(m_1 = 0|s_1 = 0, \gamma^{M*}, x) \\ &= (1-x)(a_1(1) - a_1(0)) \\ &\quad - x(2\gamma^{M*} - 1)[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]. \end{aligned} \qquad \text{(B4)}$$

Note that $a_1(m_1)$ and $\Lambda(m_1, w_1)$ are functions of γ^{A*} and γ^{M*}, and γ^{A*} and γ^{M*} are functions of x. Clearly, $D_1(x=0) > 0$, and $D_1(x = 1) < 0$. Then by continuity, there must exist a $x^+ \in (0,1)$ such that $D_1(x = x^+) = 0$ and $D_1(x > x^+) < 0$. That is, $x > x^+$ guarantees that there is no deviation regarding the misaligned analyst's reporting strategies along the equilibrium path.

For later use, let's try to examine the value of x^+. x^+ is determined by setting (B4) equal to 0. Therefore,

$$\frac{1-x^+}{x^+} = \frac{(2\gamma^{M*} - 1)[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]}{a_1(1) - a_1(0)}$$

$$
\begin{aligned}
&= \frac{(2\gamma^{M*}-1)[V^M(\Lambda(0,0)) - V^M(\Lambda(1,0))]}{\gamma^{A*}+\gamma^{M*}-1} \\
&< V^M(\Lambda(0,0)) - V^M(\Lambda(1,0)) \\
&\equiv RE^M(\gamma^{A*}, \gamma^{M*}). \qquad \text{(B5)}
\end{aligned}
$$

Tedious algebra shows that

$$
\begin{aligned}
\frac{\partial RE^M(\cdot)}{\partial \gamma^{A*}} &= \frac{2[\gamma^{M*}+\gamma^{A*}(1-\gamma^{A*})][(\gamma^{A*})^2+\gamma^{M*}(4\gamma^{A*}-1)]}{(2\gamma^{M*}+\gamma^{A*})^3} \\
&\quad + \frac{2[(1-\gamma^{M*})+(1-\gamma^{A*})^2][(1-\gamma^{M*})(1-4(1-\gamma^{A*}))-(1-\gamma^{A*})^2]}{(2(1-\gamma^{M*})+1-\gamma^{A*})^3} \\
&> 0; \\
\frac{\partial RE^M(\cdot)}{\partial \gamma^{M*}} &= -2(2\gamma^{A*}-1)\left[\frac{\gamma^{A*}(\gamma^{M*}+\gamma^{A*}(1-\gamma^{A*}))}{(2\gamma^{M*}+\gamma^{A*})^3} + \frac{(1-\gamma^{A*})(1-\gamma^{M*}+(1-\gamma^{A*})^2)}{(2(1-\gamma^{M*})+1-\gamma^{A*})^3}\right] \\
&< 0.
\end{aligned}
$$

Note that $3/4 < \gamma^{M*} < \gamma^{A*} < 1$, therefore

$$
\frac{1-x^+}{x^+} < RE^M(\gamma^{A*}, \gamma^{M*}) < RE^M(1, 3/4) = \frac{4}{25}. \qquad \text{(B6)}
$$

Part (b): To make sure that there is no deviation regarding the misaligned analyst's precision choice, note that the misaligned analyst has two options when choosing precision:

(1) Choose $\gamma^M \geq \gamma_c^M$ so that she rationally truthfully reports in the first period. Along this path, her utility reaches maximum when she chooses γ^{M*};

(2) Choose $\gamma^M < \gamma_c^M$ so that she rationally always reports 1 in the first period. Along this path, her utility reaches maximum when she keeps the default precision $\bar{\gamma}$.
Define $D_2(x)$ as the misaligned analyst's net benefit of deviating from the equilibrium precision choice.

$$
\begin{aligned}
D_2(x) &\equiv U_0^M(\bar{\gamma}, x) - U_0^M(\gamma^{M*}, x) \\
&= \frac{1}{2}(1-x)[a_1(1)-a_1(0)] - x\left(\gamma^{M*}-\frac{1}{2}\right)[V^M(\Lambda(0,0))-V^M(\Lambda(1,0))] \\
&\quad + c(\gamma^{M*}) \\
&= \frac{1}{2}D_1(x) + c(\gamma^{M*}).
\end{aligned}
$$

Clearly $D_2(x = x^+) = c(\gamma^{M*}) > 0$, and

$$
D_2(x=0) = \frac{1}{2}[a_1(1) - a_1(0)] + c(\gamma^{M*}) > 0,
$$

$$
\begin{aligned}
D_2(x=1) &= -\left(\gamma^{M*}-\frac{1}{2}\right)[V^M(\Lambda(1,1)) - V^M(\Lambda(1,0))] + c(\gamma^{M*}) \\
&\leq c'(\gamma^{M*})(\gamma^{M*}-\bar{\gamma}) - \left(\gamma^{M*}-\frac{1}{2}\right)[V^M(\Lambda(1,1)) - V^M(\Lambda(1,0))] \\
&= -\left(\bar{\gamma}-\frac{1}{2}\right)c'(\gamma^{M*}) \\
&< 0.
\end{aligned}
$$

The last equality follows from the first-order condition (B.3b). Therefore, by continuity, there must exist an $x^o \in (x^+, 1)$ such that for all $x > x^o$, $D_2(x) < 0$. That is, if $x > x^o$, there is no deviation regarding the misaligned analyst's precision choice. At the same time, there also must exist an $x^{++} \in (0, x^o]$ such that for all $x < x^{++}$, $D_2(x) > 0$. Consequently, the misaligned analyst will deviate from the equilibrium precision choice and thus disturb the proposed equilibrium. Note that in the case that $D_2(x) = 0$ has a unique solution, $x^{++} = x^o$.

In sum, if $x > x^o$, the best responses of the analyst are consistent with the investor's conjectures. Hence there exists an informative equilibrium in which $\gamma^{A*} > \gamma^{M*} > \bar{\gamma}$, both types truthfully report in the first period, and in the second period, the aligned type truthfully reports and the misaligned type always reports 1. In contrast, for $x < x^{++}$, the proposed equilibrium does not exist. ■

Proof of Proposition 3. The proof of Proposition 3 follows three steps: (1) First, following the similar arguments as in Morris (2001), it could be shown that any first-period (pure strategy) informative communication must be that the aligned type truthfully reports her signal and the misaligned type truthfully reports 1 when observing 1; (2) As the proof of Proposition 2 shows, for $x < x^{++}$, both types truthfully report in the first period cannot be sustained in equilibrium. Consequently, the only possible first-period (pure strategy) informative communication is that the aligned type truthfully reports and the misaligned type always reports 1; (3) Finally, we show that, for $x < x^{oo} \equiv min\{x^{++}, x^*, x^{**}\}$, the equilibrium indeed exists such that in the first period the aligned type truthfully reports and the misaligned type always reports 1. The following argument is to prove the existence of such an equilibrium.

Assume that the investor holds conjectures that the type J analyst chooses precision $\hat{\gamma}^J$, and in each period the aligned analyst reports truthfully and the misaligned analyst always reports 1. Then the investor will update favorably the analyst's reputation when the analyst's report is 0, that is,

$$
\Lambda(0,1) = \Lambda(0,0) = 1 > \Lambda(1,1) = \frac{1}{1+\frac{1}{\hat{\gamma}^A}} > \Lambda(1,0) = \frac{1}{1+\frac{1}{1-\hat{\gamma}^A}}.
$$

And, the investor's actions in each period upon receiving the analyst's report are:

$$a_1(1) = \frac{1+\hat{\gamma}^A}{3} > a_1(0) = 1-\hat{\gamma}^A,$$
$$a_2(1,\lambda_2) = \frac{1-\lambda_2+\lambda_2\hat{\gamma}^A}{2-\lambda_2} > a_2(0,\lambda_2) = 1-\hat{\gamma}^A.$$

Now we need to examine the analyst's best responses with regard to both the precision choice and the reporting strategies.

(i) In the *second* period, the same analysis as in the proof of Proposition 2 shows that regardless of their true precision, the aligned analyst will truthfully report, and the misaligned analyst will always report 1.

(ii) In the *first* period, if the aligned analyst with precision γ^A observes signal s_1, $s_1 = 0, 1$, she will compare her utility conditional on reporting 1 versus reporting 0. Accordingly, define

$$\begin{aligned} D^A_{s_1=1}(x,\gamma^A) &\equiv U^A_1(m_1=1|s_1=1,\gamma^A,x) - U^A_1(m_1=0|s_1=1,\gamma^A,x) \\ &= (1-x)[a_1(1)-a_1(0)][2\gamma^A - a_1(1) - a_1(0)] \\ &\quad - x[V^A(\lambda_2=1,\gamma^A) - \gamma^A V^A(\Lambda(1,1),\gamma^A) - (1-\gamma^A)V^A(\Lambda(1,0),\gamma^A)]. \end{aligned}$$

And,

$$\begin{aligned} D^A_{s_1=0}(x,\gamma^A) &\equiv U^A_1(m_1=1|s_1=0,\gamma^A,x) - U^A_1(m_1=0|s_1=0,\gamma^A,x) \\ &= (1-x)[a_1(1)-a_1(0)][2(1-\gamma^A) - a_1(1) - a_1(0)] \\ &\quad - x[V^A(\lambda_2=1,\gamma^A) - (1-\gamma^A)V^A(\Lambda(1,1),\gamma^A) - \gamma^A V^A(\Lambda(1,0),\gamma^A)] \\ &< 0. \end{aligned}$$

Note that

$$\begin{aligned} D^A_{s_1=1}(x=0,\gamma^A) &= [a_1(1)-a_1(0)][2\gamma^A - a_1(1) - a_1(0)] \\ &= [a_1(1)-a_1(0)]\left[2\gamma^A - \frac{4}{3} + \frac{2}{3}\hat{\gamma}^A\right] \\ &> 0. \end{aligned}$$

Therefore, by continuity, there must exist a $x^* > 0$ such that $D^A_{s_1=1}(x < x^*, \gamma^A) > 0$ for any $\gamma^A \in [\bar{\gamma}, 1]$. That is, for $x < x^*$, the aligned analyst will truthfully report, regardless of her true precision.

Similarly, for the misaligned analyst with precision γ^M, she will compare her utility conditional on reporting 1 versus reporting 0 after observing signal s_1. Accordingly, define

$$
\begin{aligned}
&D^M_{s_1=1}(x, \gamma^M) \\
&\equiv U_1^M(m_1 = 1|s_1 = 1, \gamma^M, x) - U_1^M(m_1 = 0|s_1 = 1, \gamma^M, x) \\
&= (1-x)[(a_1(0)-1)^2 - (a_1(1)-1)^2] - x[V^M(\lambda_2 = 1) \\
&\quad -\gamma^M V^M(\Lambda(1,1)) - (1-\gamma^M)V^M(\Lambda(1,0))].
\end{aligned}
$$

And,

$$
\begin{aligned}
&D^M_{s_1=0}(x, \gamma^M) \\
&\equiv U_1^M(m_1 = 1|s_1 = 0, \gamma^M, x) - U_1^M(m_1 = 0|s_1 = 0, \gamma^M, x) \\
&= (1-x)[(a_1(0)-1)^2 - (a_1(1)-1)^2] - x[V^M(\lambda_2 = 1) \\
&\quad -(1-\gamma^M)V^M(\Lambda(1,1)) - \gamma^M V^M(\Lambda(1,0))].
\end{aligned}
$$

Note that $\Lambda(1,1) > \Lambda(1,0)$ and $\frac{dV^M(\cdot)}{d\lambda_2} > 0$; therefore, for given x and γ^M,

$$
\begin{aligned}
D^M_{s_1=1}(x, \gamma^M) &> D^M_{s_1=0}(x, \gamma^M) \\
&> D^M(x) \\
&\equiv (1-x)\left[(a_1(0)-1)^2 - (a_1(1)-1)^2\right] \\
&\quad - x[V^M(\lambda_2 = 1) - V^M(\Lambda(1,0))]. \qquad \text{(B7)}
\end{aligned}
$$

The fact that $D^M(x=0) > 0$ and $D^M(x=1) < 0$ implies that, by continuity, there must exist an $x^{**} \in (0,1)$ such that $D^M(x \leq x^{**}) \geq 0$. It then follows that $D^M_{s_1=1}(x \leq x^{**}, \gamma^M) > D^M_{s_1=0}(x \leq x^{**}, \gamma^M) > D^M(x \leq x^{**}) \geq 0$ for any γ^M. That is, for $x < x^{**}$, the misaligned type always reports 1, regardless of her true precision.

In sum, for $x < \min\{x^*, x^{**}\}$, in the **first** period, the aligned analyst will truthfully report and the misaligned type will always report 1, regardless of their true precision.

(iii) The analyst's precision choice:
Given that, for $x < \min\{x^*, x^{**}\}$, the misaligned analyst always reports 1 in both periods, she thus has no incentive to acquire any additional information.

In contrast, for the aligned analyst, she truthfully reports in the first period for $x < \min\{x^*, x^{**}\}$; hence her utility at the information acquisition stage is as follows:

$$
\begin{aligned}
U_0^A(\gamma^A, x) = &\frac{1}{2}\gamma^A[-(1-x)(a_1(1)-1)^2 + xV^A(\Lambda(1,1), \gamma^A)] \\
&+ \frac{1}{2}(1-\gamma^A)[-(1-x)(a_1(0)-1)^2 + xV^A(\Lambda(0,1), \gamma^A)]
\end{aligned}
$$

$$+\frac{1}{2}\gamma^A[-(1-x)(a_1(0)-0)^2 + xV^A(\Lambda(0,0),\gamma^A)]$$
$$+\frac{1}{2}(1-\gamma^A)[-(1-x)(a_1(1)-0)^2 + xV^A(\Lambda(1,0),\gamma^A)]$$
$$-c(\gamma^A).$$

Taking derivative with respect to γ^A, the aligned analyst's optimal precision choice has to satisfy the following first-order condition (after imposing the equilibrium condition $\gamma^J = \hat{\gamma}^J = \gamma^{J^*}$. Here $\Lambda(m_1, w_1)$ and $a_1(m_1)$ are specified in the beginning of the proof of Proposition 3.):

$$c'(\gamma^{A*}) = (1-x)\underbrace{(a_1(1)-a_1(0))}_{PE_1^A(x<x^{oo},\gamma^{A*})} + \underbrace{x\frac{1}{2}[V^A(\Lambda(1,1),\gamma^{A*}) - V^A(\Lambda(1,0),\gamma^{A*})]}_{RE^A(x<x^{oo},\gamma^{A*})}$$
$$+\underbrace{x\frac{1}{2}\left[\gamma^{A*}\frac{\partial V^A(\Lambda(1,1),\gamma^A)}{\partial\gamma^A} + (1-\gamma^{A*})\frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial\gamma^A} + \frac{\partial V^A(\lambda_2=1,\gamma^A)}{\partial\gamma^A}\right]}_{PE_2^A(x<x^{oo},\gamma^{A*})}. \tag{B8}$$

Finally, let's check the second-order condition:

$$\frac{\partial^2 U_0^A(\cdot)}{\partial{\gamma^A}^2} = x\left[\frac{\partial V^A(\Lambda(1,1),\gamma^A)}{\partial\gamma^A} - \frac{\partial V^A(\Lambda(1,0),\gamma^A)}{\partial\gamma^A}\right] - c''(\gamma^A)$$
$$= x[a_2(1,\Lambda(1,1)) - a_2(1,\Lambda(1,0))] - c''(\gamma^A)$$
$$\le \frac{1}{2} - c''(\gamma^A)$$
$$<0.$$

In sum, if $x < \min\{x^*, x^{**}\}$, the best responses of the analyst are consistent with the investor's conjectures. Hence there exists an informative equilibrium in which $\gamma^{A*} > \gamma^{M*} = \bar{\gamma}$, the misaligned analyst always reports 1 in each period, and the aligned analyst truthfully reports in each period.

(iv) Define $x^{oo} \equiv min\{x^*, x^{**}, x^{++}\}$. Then, for $x < x^{oo}$, the most informative (pure strategy) equilibrium is such that $\gamma^{A*} > \gamma^{M*} = \bar{\gamma}$, the misaligned analyst always reports 1 in each period, and the aligned analyst truthfully reports in each period.

Finally, note that $x^{oo} \equiv min\{x^*, x^{**}, x^{++}\} \le x^{++} \le x^o$. ■

PROPOSITION 4. *There exists a positive bounded number a such that, if $c''(\gamma) \ge a$, that is, the analyst's information acquisition cost function is sufficiently convex, the aligned analyst's equilibrium precision γ^{A*} is nonmonotonic in x.*

(1) If $x < x^{oo}$, $\frac{d\gamma^{A*}(x)}{dx} > 0$.

(2) If $x > x^o$, $\frac{d\gamma^{A*}(x)}{dx} < 0$.

Proof.

(1) If $x < x^{oo}$, the first-order condition determining the analyst's equilibrium precision choice is specified in (B8). Let $MB^A(x < x^{oo}, \cdot)$ denote the right-hand side of equation (B8). Taking derivative with respect to x on equations (B8), we get:

$$\underbrace{\left(c''(\gamma^{A*}) - \frac{\partial MB^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}}\right)}_{+} \frac{d\gamma^{A*}}{dx}$$
$$= \underbrace{-PE_1^A(x < x^{oo}, \cdot) + PE_2^A(x < x^{oo}, \cdot) + RE^A(x < x^{oo}, \cdot)}_{+}.$$

The following arguments show how to determine the signs of the above terms, respectively. Tedious algebra shows that (given that $\gamma^{A*} \in (\bar{\gamma}, 1) \in (\frac{3}{4}, 1)$):

$$\frac{\partial PE_1^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} = \frac{4}{3},$$
$$\frac{\partial PE_2^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} = 5\left(\frac{1}{(3-\gamma^{A*})^2} + \frac{1}{(2+\gamma^{A*})^2}\right)$$
$$> 5\left(\frac{1}{(3-3/4)^2} + \frac{1}{(2+1)^2}\right) > \frac{4}{3},$$
$$\frac{\partial RE^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} = \frac{5(1-2\gamma^{A*})^2(19-\gamma^{A*}(1-\gamma^{A*}))}{2(3-\gamma^{A*})^3(2+\gamma^{A*})^3} > 0.$$

Therefore,

$$\frac{\partial^2 MB^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*} \partial x}$$
$$= -\frac{\partial PE_1^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} + \frac{\partial PE_2^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} + \frac{\partial RE^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}}$$
$$> 0.$$

That is, $\frac{\partial MB^A(x<x^{oo},\cdot)}{\partial \gamma^{A*}}$ is increasing in x. Given that $x \leq 1$,

$$\frac{\partial MB^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} = (1-x)\frac{\partial PE_1^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} + x\frac{\partial PE_2^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}}$$
$$+ x\frac{\partial RE^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}}$$
$$\leq \frac{\partial PE_2^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}} + \frac{\partial RE^A(x < x^{oo}, \cdot)}{\partial \gamma^{A*}}$$
$$= \frac{25}{2}\left(\frac{1}{(3-\gamma^{A*})^3} + \frac{1}{(2+\gamma^{A*})^3}\right)$$
$$< 3.$$

Therefore, for $c''(\gamma) \geq a$, where $a \geq 3$, $c''(\gamma^{A*}) - \frac{\partial MB^A(x<x^{oo},\cdot)}{\partial \gamma^{A*}} > 0$. At the same time, note that:

$$PE_1^A(x < x^{oo}, \cdot) = a_1(1) - a_1(0) = a_2\left(1, \lambda_2 = \frac{1}{2}\right) - a_2(0),$$

$$\begin{aligned} & PE_2^A(x < x^{oo}, \cdot) \\ = & \frac{1}{2}\gamma^{A*}[a_2(1, \Lambda(1,1)) - a_2(0, \Lambda(1,1))] \\ & + \frac{1}{2}(1-\gamma^{A*})[a_2(1, \Lambda(1,0)) - a_2(0, \Lambda(1,0))] \\ & + \frac{1}{2}[a_2(1, \lambda_2 = 1) - a_2(0, \lambda_2 = 1)] \\ \geq & \, a_2\left(1, \lambda_2 = \frac{1}{2}\left[\gamma^{A*}\Lambda(1,1) + (1-\gamma^{A*})\Lambda(1,0) + 1\right]\right) - a_2(0) \\ > & \, a_2\left(1, \lambda_2 = \frac{1}{2}\right) - a_2(0). \end{aligned}$$

The first inequality is due to the convexity of $a_2(1, \lambda_2)$ in λ_2 and the independence of $a_2(0, \lambda_2)$ on λ_2. The second inequality stems from the fact that the expectation of the aligned analyst's posterior reputation, $\frac{1}{2}[\gamma^{A*}\Lambda(1,1) + (1-\gamma^{A*})\Lambda(1,0) + 1]$, is greater than the ex ante reputation, $\frac{1}{2}$. Therefore,

$$-PE_1^A(x < x^{oo}, \cdot) + PE_2^A(x < x^{oo}, \cdot) + RE^A(x < x^{oo}, \cdot) > 0.$$

Then it follows immediately that $\frac{\partial \gamma^{A*}}{\partial x} > 0$ for $x < x^{oo}$.

(2) If $x > x^o$, the analyst's precision choices, γ^{A*} and γ^{M*}, are jointly determined by the system of equations (B3a) and (B3b). Let $MB^A(x > x^o, \cdot)$ and $MB^M(x > x^o, \cdot)$ denote, respectively, the right-hand side of equation (B3a) and (B3b). Taking derivative with respect to x on equations (B3a) and (B3b), and after some algebra manipulation, we get

$$\begin{aligned} & \underbrace{\left(c''(\gamma^{A*}) - \frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{A*}}\right)}_{+} \frac{d\gamma^{A*}}{dx} \\ & = \underbrace{\frac{\partial MB^A(x > x^o, \cdot)}{\partial x}}_{-} + \underbrace{\frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{M*}}}_{-\text{ for } x > x^o} \frac{d\gamma^{M*}}{dx}, \end{aligned} \tag{B9}$$

$$\underbrace{\left(c''(\gamma^{M*}) - \frac{\partial MB^M(x > x^o, \cdot)}{\partial \gamma^{M*}}\right)}_{+} \frac{d\gamma^{M*}}{dx}$$

$$= \underbrace{\frac{\partial MB^M(x > x^o, \cdot)}{\partial x}}_{+} + \underbrace{\frac{\partial MB^M(x > x^o, \cdot)}{\partial \gamma^{A*}}}_{+} \frac{d\gamma^{A*}}{dx}. \tag{B10}$$

Given the signs of the above terms (which I will show in the next step), it is clear that $\frac{d\gamma^{A*}}{dx} < 0$: Suppose not; then by (B10), it follows that $\frac{d\gamma^{M*}}{dx} > 0$, which, by (B9), implies that $\frac{d\gamma^{A*}}{dx} < 0$; a contradiction. Therefore, $\frac{d\gamma^{A*}}{dx} < 0$.

The following arguments show how to determine the signs of the above terms, respectively. First, given that $3/4 \le \bar{\gamma} < \gamma^{M*} < \gamma^{A*} < 1$, tedious algebra shows that (where $\propto$ indicates that both sides have the same sign):

$$\frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{A*}} = \frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{M*}} = 1;$$

$$\begin{aligned}\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{A*}} &= \frac{2(\gamma^{M*})^2(1+4\gamma^{M*})}{(2\gamma^{M*}+\gamma^{A*})^2} + \frac{2(1-\gamma^{M*})^2(1+4(1-\gamma^{M*}))}{[2(1-\gamma^{M*})+(1-\gamma^{A*})]^2}\\ &> 0;\end{aligned}$$

$$\begin{aligned}\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} &= (2\gamma^{A*}-1)\left[-\frac{(\gamma^{A*})^2}{(2\gamma^{M*}+\gamma^{A*})^2} + \frac{(1-\gamma^{A*})^2}{[2(1-\gamma^{M*})+(1-\gamma^{A*})]^2}\right]\\ &\propto \gamma^{M*} - \gamma^{A*} < 0;\end{aligned}$$

$$\begin{aligned}\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{A*}} &= (2\gamma^{A*}-1)^2\left[\frac{(\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{A*})^3} + \frac{(1-\gamma^{M*})^2}{[2(1-\gamma^{M*})+(1-\gamma^{A*})]^3}\right]\\ &\quad + 2(2\gamma^{A*}-1)\left[\frac{(1-\gamma^{M*})^2}{[2(1-\gamma^{M*})+(1-\gamma^{A*})]^2} - \frac{(\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{A*})^2}\right]\\ &> 2(2\gamma^{A*}-1)\left[\frac{(1-\gamma^{M*})^2}{[2(1-\gamma^{M*})+(1-\gamma^{A*})]^2} - \frac{(\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{A*})^2}\right]\\ &\propto \gamma^{A*} - \gamma^{M*}\\ &> 0;\end{aligned}$$

$$\begin{aligned}\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}} &= (2\gamma^{A*}-1)^2\left[-\frac{\gamma^{A*}\gamma^{M*}}{(2\gamma^{M*}+\gamma^{A*})^3} - \frac{(1-\gamma^{A*})(1-\gamma^{M*})}{[2(1-\gamma^{M*})+(1-\gamma^{A*})]^3}\right]\\ &< 0;\end{aligned}$$

$$\begin{aligned}\frac{\partial RE^M(x > x^o, \cdot)}{\partial \gamma^{A*}} &= \frac{2[\gamma^{M*}+\gamma^{A*}(1-\gamma^{A*})][(\gamma^{A*})^2+\gamma^{M*}(4\gamma^{A*}-1)]}{(2\gamma^{M*}+\gamma^{A*})^3}\\ &\quad + \frac{2[(1-\gamma^{M*})+(1-\gamma^{A*})^2][(1-\gamma^{M*})(1-4(1-\gamma^{A*}))-(1-\gamma^{A*})^2]}{(2(1-\gamma^{M*})+1-\gamma^{A*})^3}\\ &> 0;\end{aligned}$$

$$\begin{aligned}\frac{\partial RE^M(x > x^o, \cdot)}{\partial \gamma^{M*}} &= -2(2\gamma^{A*}-1)\left[\frac{\gamma^{A*}(\gamma^{M*}+\gamma^{A*}(1-\gamma^{A*}))}{(2\gamma^{M*}+\gamma^{A*})^3} + \frac{(1-\gamma^{A*})(1-\gamma^{M*}+(1-\gamma^{A*})^2)}{(2(1-\gamma^{M*})+1-\gamma^{A*})^3}\right]\\ &< 0.\end{aligned}$$

The rough economic intuition for $\frac{\partial RE^J(\cdot)}{\partial \gamma^{A*}} > 0$ and $\frac{\partial RE^J(\cdot)}{\partial \gamma^{M*}} < 0$ is that, keeping the other type's equilibrium precision constant, the higher (lower) the γ^{A*} (γ^{M*}), the larger the precision differential between the two types, then the more effective the investor's updating about the analyst's type, and hence the higher the type J analyst's reputation effect. Furthermore, the more effective the investor's learning about the analyst's type, the higher the expectation of the aligned analyst's posterior reputation (because the expected reputation improvement of the aligned analyst is higher), which leads to a bigger second-period precision effect $PE_2^A(\cdot)$. That is, $\frac{\partial PE_2^A(\cdot)}{\partial \gamma^{A*}} > 0$ and $\frac{\partial PE_2^A(\cdot)}{\partial \gamma^{M*}} < 0$. In contrast, the investor's response to the analyst's differential report in the first period, $a_1(1) - a_1(0)$, is increasing in either type's precision $\gamma^{J*}, J = A, M$, given that both types of analysts truthfully communicate in the first period.

Note that $MB^M(x > x^o, \cdot) = xRE^M(x > x^o, \cdot)$. Therefore,

$$\frac{\partial MB^M(x > x^o, \cdot)}{\partial \gamma^{A*}} = x\frac{\partial RE^M(x > x^o, \cdot)}{\partial \gamma^{A*}} > 0,$$

$$\frac{\partial MB^M(x > x^o, \cdot)}{\partial \gamma^{M*}} = x\frac{\partial RE^M(x > x^o, \cdot)}{\partial \gamma^{M*}} < 0,$$

$$\frac{\partial MB^M(x > x^o, \cdot)}{\partial x} = RE^M(x > x^o, \cdot) > 0.$$

Hence the signs of the terms in (B10) are verified.

Given that $MB^A(x > x^o, \cdot) = (1 - x)PE_1^A(x > x^o, \cdot) + xPE_2^A(x > x^o, \cdot) + xRE^A(x > x^o, \cdot)$, then

$$\frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{A*}} > 0,$$

$$\frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{M*}} = (1 - x)\frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + x\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + x\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}},$$

$$\frac{\partial MB^A(x > x^o, \cdot)}{\partial x} = -PE_1^A(x > x^o, \cdot) + PE_2^A(x > x^o, \cdot) + RE^A(x > x^o, \cdot).$$

To determine the signs of $c''(\gamma^{A*}) - \frac{\partial MB^A(x>x^o,\cdot)}{\partial \gamma^{A*}}$, $\frac{\partial MB^A(x>x^o,\cdot)}{\partial x}$, and $\frac{\partial MB^A(x>x^o,\cdot)}{\partial \gamma^{M*}}$, the proof is much more involved. So I break down the proof into small steps respectively.

(I) To show that, for $c''(\gamma) \geq a$, where a is a positive bounded number,

$$\frac{\partial MB^A(x > x^o, \cdot)}{\partial x} < 0.$$

Proof.

(i) First I argue that $-PE_1^A(x > x^o, \cdot) + PE_2^A(x > x^o, \cdot) < 0$.

$$\begin{aligned} PE_1^A(x > x^o, \cdot) &= a_1(1) - a_1(0) = \gamma^{A*} + \gamma^{M*} - 1, \\ PE_2^A(x > x^o, \cdot) &= \gamma^{A*}[a_2(1, \Lambda(0,0)) - a_2(0, \Lambda(0,0))] \\ &\quad + (1-\gamma^{A*})[a_2(1, \Lambda(1,0)) - a_2(0, \Lambda(1,0))] \\ &< a_2\left(1, \lambda_2 = \frac{4}{7}\right) - a_2(0) \\ &= \gamma^{A*} + \frac{4\gamma^{A*}+3}{10} - 1 \\ &< \gamma^{A*} + \gamma^{M*} - 1. \end{aligned}$$

The first inequality derives from the fact that $\Lambda(1,0) < \Lambda(0,0) < \frac{4}{7}$ and $a_2(1, \lambda_2)$ is increasing in λ_2. The second inequality is due to $\frac{3}{4} \leq \bar{\gamma} < \gamma^{M*} < \gamma^{A*} < 1$.

(ii) Second, I argue that, if the equilibrium precision differential $\Delta \equiv \gamma^{A*} - \gamma^{M*}$ is smaller than some positive threshold $\bar{\Delta}_1$, $\frac{\partial MB^A(x>x^o,\cdot)}{\partial x} = -PE_1^A(x > x^o, \cdot) + PE_2^A(x > x^o, \cdot) + RE^A(x > x^o, \cdot) < 0$.

Clearly if $\Delta = 0$, $RE^A(\cdot) = 0$, and $\frac{\partial MB^A(x>x^o,\cdot)}{\partial x} = -PE_1^A(\cdot) + PE_2^A(\cdot) + RE^A(\cdot) < 0$. Then, by continuity, there must exist a threshold $\bar{\Delta}_1 > 0$ such that for $\Delta < \bar{\Delta}_1$, $\frac{\partial MB^A(x>x^o,\cdot)}{\partial x} < 0$.

(iii) Third, I show that, if $c''(\gamma) > 1/\bar{\Delta}_1$, the equilibrium precision differential $\Delta < \bar{\Delta}_1$.

By equations (B3a) and (B3b), we get:

$$\begin{aligned} c'(\gamma^{A*}) - c'(\gamma^{M*}) &= (1-x)PE_1^A(\cdot) + xPE_2^A(\cdot) + xRE^A(\cdot) - xRE^M(\cdot) \\ &< (1-x)PE_1^A(\cdot) + xPE_2^A(\cdot) \\ &\leq 1. \end{aligned}$$

The first inequality derives from $RE^A(\cdot) < RE^M(\cdot)$. The second inequality is due to the fact that both $PE_1^A(\cdot)$ and $PE_2^A(\cdot)$ stem from the impact on the investor's decision in each period by changing report. Given that $a_t(1) - a_t(0) \leq 1$, such impact is smaller than 1.

By mean value theorem, $c'(\gamma^{A*}) - c'(\gamma^{M*}) = c''(\delta)(\gamma^{A*} - \gamma^{M*})$ for some $\delta \in (\gamma^{M*}, \gamma^{A*})$. Therefore, if $c''(\gamma) \geq 1/\bar{\Delta}_1$ for any γ, $\Delta \equiv \gamma^{A*} - \gamma^{M*} < 1/c''(\delta) \leq \bar{\Delta}_1$. Since $\bar{\Delta}_1$ is positive and bounded away from 0, $1/\bar{\Delta}_1$ is a positive bounded number.

Combining the above three steps, I show that, if $c''(\gamma) \geq a$, where $a \geq 1/\bar{\Delta}_1$, the equilibrium precision differential $\Delta < \bar{\Delta}_1$, which implies that

$$\frac{\partial MB^A(x > x^o, \cdot)}{\partial x} = -PE_1^A(x > x^o, \cdot) + PE_2^A(x > x^o, \cdot) + RE^A(x > x^o, \cdot) < 0.$$

(II) To show that, for $c''(\gamma) \geq a$, where a is a positive bounded number,

$$\frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{M*}} < 0.$$

Proof. First, note that

$$\begin{aligned}
&\frac{\partial^2 MB^A(x > x^o, \cdot)}{\partial \gamma^{M*} \partial x} \\
&= -\frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + \frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + \frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}} \\
&< 0.
\end{aligned}$$

That is, $\frac{\partial MB^A(x>x^o,\cdot)}{\partial \gamma^{M*}}$ is decreasing in x. Recall that $x^o > x^+$ (the proof of Proposition 2), therefore, here $x > x^o > x^+$, and

$$\begin{aligned}
\frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{M*}} &= (1-x)\frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + (x)\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} \\
&\quad + (x)\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}} \\
&< (1-x^+)\frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + (x^+)\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} \\
&\quad + (x^+)\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}} \\
&= (1-x^+) + (x^+)\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + (x^+)\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}} \\
&< x^+\left[RE^M(\cdot) + \underbrace{\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{M*}} + \frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{M*}}}_{-}\right].
\end{aligned}$$

Where the last inequality stems from (B5).

If $\Delta \equiv \gamma^{A*} - \gamma^{M*}$ goes to 0, $RE^M(\cdot) \to 0$. Then, by continuity, there must exist a positive number $\bar{\Delta}_2$ (bounded away from 0) such that, for $\Delta < \bar{\Delta}_2$, $\frac{\partial MB^A(x>x^o,\cdot)}{\partial \gamma^{M*}} < 0$. Therefore, following the same argument as in part (I), there must exist a bounded number $1/\bar{\Delta}_2$ such that, if $c''(\gamma) \geq 1/\bar{\Delta}_2$, Δ is small enough to ensure $\frac{\partial MB^A(x>x^o,\cdot)}{\partial \gamma^{M*}} < 0$.

(III) To show that, for $c''(\gamma) \geq a$, where a is a positive bounded number,

$$c''(\gamma^{A*}) - \frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{A*}} > 0.$$

Proof.

(i) First, I show that

$$\begin{aligned}
\frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{A*}} &= (1-x)\frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{A*}} + x\frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{A*}} \\
&\quad + x\frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{A*}} \\
&< \frac{\partial PE_1^A(x > x^o, \cdot)}{\partial \gamma^{A*}} + \frac{\partial PE_2^A(x > x^o, \cdot)}{\partial \gamma^{A*}} \\
&\quad + \frac{\partial RE^A(x > x^o, \cdot)}{\partial \gamma^{A*}} \\
&= 1 + \underbrace{\frac{(\gamma^{M*})^2(1+4\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{A*})^3}}_{f(\gamma^{A*},\gamma^{M*})} \\
&\quad + \underbrace{\frac{(1-\gamma^{M*})^2(1+4(1-\gamma^{M*}))^2}{(2(1-\gamma^{M*})+1-\gamma^{A*})^3}}_{g(\gamma^{A*},\gamma^{M*})}.
\end{aligned}$$

Note that, due to $3/4 < \gamma^{M*} < \gamma^{A*} < 1$,

$$\begin{aligned}
f(\gamma^{A*}, \gamma^{M*}) &= \frac{(\gamma^{M*})^2(1+4\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{A*})^3} \\
&< \frac{(\gamma^{M*})^2(1+4\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{M*})^3} \\
&= \frac{(1+4\gamma^{M*})^2}{27\gamma^{M*}} \\
&< \frac{(1+4*1)^2}{27*3/4} \\
&= \frac{100}{81}.
\end{aligned}$$

$$\begin{aligned}
g(\gamma^{A*}, \gamma^{M*}) &= \frac{(1-\gamma^{M*})^2(1+4(1-\gamma^{M*}))^2}{(2(1-\gamma^{M*})+1-\gamma^{A*})^3} \\
&< \frac{(1-\gamma^{M*})^2(1+4(1-\gamma^{M*}))^2}{(2(1-\gamma^{A*})+1-\gamma^{A*})^3} \\
&< \frac{(1-3/4)^2(1+4(1-3/4))^2}{(2(1-\gamma^{A*})+1-\gamma^{A*})^3} \\
&= \frac{1}{108(1-\gamma^{A*})^3}.
\end{aligned}$$

Hence

$$c''(\gamma^{A*}) - \frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{A*}} > c''(\gamma^{A*}) - \frac{181}{81} - \frac{1}{108(1-\gamma^{A*})^3}.$$

(ii) Second, I aim to show that, for $c''(\gamma) \geq a$, where a is a positive bounded number,

$$c''(\gamma^{A*}) - \frac{181}{81} - \frac{1}{108(1-\gamma^{A*})^3} > 0.$$

Note that, by (B2) and the assumption that $c'(\bar{\gamma}) = 0$,

$$c'(\gamma^{A*}) - c'(\bar{\gamma}) < 3/2.$$

By mean value theorem, $c'(\gamma^{A*}) - c'(\bar{\gamma}) = c''(\delta)(\gamma^{A*} - \bar{\gamma})$ for some $\delta \in (\bar{\gamma}, \gamma^{A*})$. Therefore, if $c''(\gamma) > a$ for any γ,

$$\gamma^{A*} - \bar{\gamma} < \frac{3}{2c''(\delta)} < \frac{3}{2a}$$

$$\Leftrightarrow \; \gamma^{A*} < \bar{\gamma} + \frac{3}{2a}.$$

Hence, if $c''(\gamma) > a$,

$$c''(\gamma^{A*}) - \frac{181}{81} - \frac{1}{108(1-\gamma^{A*})^3} > \underbrace{a - \frac{181}{81} - \frac{1}{108(1-\bar{\gamma}-\frac{3}{2a})^3}}_{h(a)}.$$

Given that $h(a)$ is increasing in a and $\lim_{a\to\infty} h(a) \to \infty$, there must exist a bounded number a^o such that $h(a^o) = 0$. That is, if $c''(\gamma) > a^o$,

$$c''(\gamma^{A*}) - \frac{\partial MB^A(x > x^o, \cdot)}{\partial \gamma^{A*}} > c''(\gamma^{A*}) - \frac{181}{81} - \frac{1}{108(1-\gamma^{A*})^3} > 0.$$

Finally, pick $a = \max\{3, 1/\bar{\Delta}_1, 1/\bar{\Delta}_2, a^o\}$. ■

PROPOSITION 5. *There exists a positive bounded number a such that, if $c''(\gamma) \geq a$ and $c'''(\gamma) \leq \frac{25}{58}a$, that is, the analyst's information acquisition cost function is sufficiently convex and the third derivative of the cost function is not too large, then the investor's welfare is nonmonotonic in the analyst's future concerns x:*

(1) If $x < x^{oo}$, the investor's welfare is increasing in x. That is,

$$\frac{dU_0^I(x < x^{oo})}{dx} > 0.$$

(2) If $x > x^o$, the investor's welfare is decreasing in x. That is,

$$\frac{dU_0^I(x > x^o)}{dx} < 0.$$

Proof. Define $V^I(\lambda_2, \gamma^{A*})$ as the investor's second-period expected utility (calculated at the beginning of the second period) with posterior analyst reputation λ_2 and aligned analyst's equilibrium precision γ^{A*}. Note that γ^{M*} does not affect $V^I(\cdot)$ because the misaligned analyst always reports 1 in the second period, independent of her precision. Hence:

$$\begin{aligned} V^I(\lambda_2, \gamma^{A*}) = & -\frac{1}{2}(\gamma^{A*}\lambda_2 + 1 - \lambda_2)[a_2(1, \lambda_2) - 1]^2 \\ & -\frac{1}{2}(1 - \gamma^{A*})\lambda_2[a_2(0, \lambda_2) - 1]^2 \\ & -\frac{1}{2}((1 - \gamma^{A*})\lambda_2 + 1 - \lambda_2)[a_2(1, \lambda_2) - 0]^2 \\ & -\frac{1}{2}\gamma^{A*}\lambda_2[a_2(0, \lambda_2) - 0]^2 \\ = & \frac{[1 + 2(\gamma^{A*} - 1)\gamma^{A*}]\lambda_2 - 1}{2(2 - \lambda_2)}. \end{aligned}$$

It's easy to show that $V^I(\cdot)$ is increasing and convex in λ_2. Also $V^I(\cdot)$ is increasing in γ^{A*}.

(1) If $x < x^{oo}$, then, by Proposition 3, the equilibrium is such that, in the first period, the aligned analyst truthfully communicates and the misaligned analyst always reports 1. Thus the investor's ex ante expected utility is as follows:

$$\begin{aligned} U_0^I(x < x^{oo}) = & \frac{1}{4}(\gamma^{A*} + 1)[-(a_1(1) - 1)^2 + V^I(\Lambda(1, 1), \gamma^{A*})] \\ & +\frac{1}{4}(1 - \gamma^{A*})[-(a_1(0) - 1)^2 + V^I(\Lambda(0, 1), \gamma^{A*})] \\ & +\frac{1}{4}(1 - \gamma^{A*} + 1)[-(a_1(1) - 0)^2 + V^I(\Lambda(1, 0), \gamma^{A*})] \\ & +\frac{1}{4}\gamma^{A*}[-(a_1(0) - 0)^2 + V^I(\Lambda(0, 0), \gamma^{A*})] \\ = & \underbrace{[-\frac{1}{6}(1 + 2(1 - \gamma^{A*})\gamma^{A*})]}_{\equiv W_1(\gamma^{A*}),\ \text{First Period}} \\ & \underbrace{+\frac{1}{4}\left[V^I(\lambda_2 = 1, \gamma^{A*}) + (\gamma^{A*} + 1)V^I\left(\frac{1}{1 + \frac{1}{\gamma^{A*}}}, \gamma^{A*}\right) + (2 - \gamma^{A*})V^I\left(\frac{1}{1 + \frac{1}{1 - \gamma^{A*}}}, \gamma^{A*}\right)\right]}_{\equiv W_2(\gamma^{A*}),\ \text{Second Period}}. \end{aligned}$$

Apparently, for $x < x^{oo}$, x affects $U_0^I(\cdot)$ indirectly through γ^{A*} alone.

$$\frac{dU_0^I(x < x^{oo})}{d\gamma^{A*}} = \frac{dW_1(\gamma^{A*})}{d\gamma^{A*}} + \frac{dW_2(\gamma^{A*})}{d\gamma^{A*}}$$

$$= \frac{2\gamma^{A*} - 1}{3} + \frac{125(2\gamma^{A*} - 1)}{8(\gamma^{A*} - 3)^2(\gamma^{A*} + 2)^2}$$
$$> 0.$$

Recall that, by Proposition 4, when $c''(\gamma) > a$, $\frac{d\gamma^{A*}}{dx} > 0$ for $x < x^{oo}$. Therefore,

$$\frac{dU_0^I(x < x^{oo})}{dx} = \frac{dU_0^I(x < x^{oo})}{d\gamma^{A*}} \frac{d\gamma^{A*}}{dx} > 0.$$

(2) If $x > x^o$, then, in equilibrium, both types of analysts truthfully communicate in the first period, thus the investor's ex ante expected utility is as follows:

$$\begin{aligned}
U_0^I(x > x^o) = &\frac{1}{2}\left[\frac{1}{2}(\gamma^{A*} + \gamma^{M*})\right][-(a_1(1) - 1)^2 + V^I(\Lambda(1,1), \gamma^{A*})] \\
&+\frac{1}{2}\left[1 - \frac{1}{2}(\gamma^{A*} + \gamma^{M*})\right][-(a_1(0) - 1)^2 + V^I(\Lambda(0,1), \gamma^{A*})] \\
&+\frac{1}{2}\left[1 - \frac{1}{2}(\gamma^{A*} + \gamma^{M*})\right][-(a_1(1) - 0)^2 + V^I(\Lambda(1,0), \gamma^{A*})] \\
&+\frac{1}{2}\left[\frac{1}{2}(\gamma^{A*} + \gamma^{M*})\right][-(a_1(0) - 0)^2 + V^I(\Lambda(0,0), \gamma^{A*})] \\
= &\underbrace{\left[-\frac{1}{2}(\gamma^{A*} + \gamma^{M*})(1 - \frac{1}{2}(\gamma^{A*} + \gamma^{M*}))\right]}_{\equiv Z_1(\gamma^{A*}, \gamma^{M*}),\ \text{First Period}}
\end{aligned}$$

$$+\underbrace{\left[\frac{1}{2}(\gamma^{A*} + \gamma^{M*})V^I\left(\frac{1}{1 + \frac{\gamma^{M*}}{\gamma^{A*}}}, \gamma^{A*}\right) + \left(1 - \frac{1}{2}(\gamma^{A*} + \gamma^{M*})\right)V^I\left(\frac{1}{1 + \frac{1-\gamma^{M*}}{1-\gamma^{A*}}}, \gamma^{A*}\right)\right]}_{\equiv Z_2(\gamma^{A*}, \gamma^{M*}),\ \text{Second Period}}.$$

If $x > x^o$, then x affects $U_0^I(\cdot)$ indirectly through both γ^{A*} and γ^{M*}. The proof then includes the following three steps: Step (i) shows how $U_0^I(\cdot)$ is affected by γ^{A*} and γ^{M*}; Step (ii) shows how γ^{A*} and γ^{M*} are affected by x; Step (iii) combines the first two steps and shows how x affects $U_0^I(\cdot)$.

Step (i): To show that

$$\frac{\partial U_0^I(x > x^o)}{\partial \gamma^{A*}} > \frac{\partial U_0^I(x > x^o)}{\partial \gamma^{M*}} > 0.$$

Proof. It is straightforward that

$$\frac{\partial Z_1(\cdot)}{\partial \gamma^{A*}} = \frac{\partial Z_1(\cdot)}{\partial \gamma^{M*}} = \frac{\gamma^{A*} + \gamma^{M*} - 1}{2} > 0.$$

Also,

$$\begin{aligned}
\frac{\partial Z_2(\cdot)}{\partial \gamma^{A*}} &= \frac{1}{4}\left[8\gamma^{M*} - 4 + \frac{(1-\gamma^{M*})^2(1+4(1-\gamma^{M*}))^2}{(2(1-\gamma^{M*})+(1-\gamma^{A*}))^2} - \frac{(\gamma^{M*})^2(1+4\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{A*})^2}\right] \\
&> \frac{1}{4}\left[8\gamma^{M*} - 4 + \frac{(1-\gamma^{M*})^2(1+4(1-\gamma^{M*}))^2}{(2(1-\gamma^{M*})+(1-\gamma^{M*}))^2} - \frac{(\gamma^{M*})^2(1+4\gamma^{M*})^2}{(2\gamma^{M*}+\gamma^{M*})^2}\right] \\
&= \frac{1}{3}(2\gamma^{M*}-1) \\
&> 0.
\end{aligned}$$

The first inequality arises from the facts that $\gamma^{A*} > \gamma^{M*}$ and $\frac{\partial Z_2(\cdot)}{\partial \gamma^{A*}}$ is increasing in γ^{A*}.

At the same time,

$$\begin{aligned}
\frac{\partial Z_2(\cdot)}{\partial \gamma^{M*}} &= \frac{(2\gamma^{A*}-1)^2(\gamma^{A*}-\gamma^{M*})((\gamma^{A*}-2)\gamma^{A*}+(2\gamma^{A*}-1)\gamma^{M*})}{2(2\gamma^{M*}+\gamma^{A*}-3)^2(2\gamma^{M*}+\gamma^{A*})^2} \\
&< \frac{(2\gamma^{A*}-1)^2(\gamma^{A*}-\gamma^{M*})(3\gamma^{A*}-3)\gamma^{A*}}{2(2\gamma^{M*}+\gamma^{A*}-3)^2(2\gamma^{M*}+\gamma^{A*})^2} \\
&< 0.
\end{aligned}$$

Therefore,

$$\frac{\partial U_0^I(x > x^o)}{\partial \gamma^{A*}} - \frac{\partial U_0^I(x > x^o)}{\partial \gamma^{M*}} = \frac{\partial Z_2(\cdot)}{\partial \gamma^{A*}} - \frac{\partial Z_2(\cdot)}{\partial \gamma^{M*}} > 0.$$

And tedious algebra shows that

$$\frac{\partial Z_2(\cdot)}{\partial \gamma^{M*}}(\gamma^{A*}, \gamma^{M*}) \geq \frac{\partial Z_2(\cdot)}{\partial \gamma^{M*}}(1, 3/4) = -\frac{1}{50}.$$

Therefore

$$\frac{\partial U_0^I(x > x^o)}{\partial \gamma^{M*}} = \frac{\partial Z_1(\cdot)}{\partial \gamma^{M*}} + \frac{\partial Z_2(\cdot)}{\partial \gamma^{M*}} \geq \frac{1}{4} - \frac{1}{50} > 0.$$

Step (ii): To show that, if $c''(\gamma) \geq a$ and $c'''(\gamma) \leq \frac{25}{58}a$, then, for $x > x^o$,

$$\frac{d\gamma^{A*}}{dx} + \frac{d\gamma^{M*}}{dx} < 0.$$

Proof. If $x > x^o$, adding (B9) and (B10), we get

$$\underbrace{\left[c''(\gamma^{A*}) - \frac{\partial MB^A(\cdot)}{\partial \gamma^{A*}} - \frac{\partial MB^M(\cdot)}{\partial \gamma^{A*}}\right]}_{\equiv C_1} \frac{d\gamma^{A*}}{dx}$$

$$+\underbrace{\left[c''(\gamma^{M*}) - \frac{\partial MB^A(\cdot)}{\partial \gamma^{M*}} - \frac{\partial MB^M(\cdot)}{\partial \gamma^{M*}}\right]}_{\equiv C_2, >0} \frac{d\gamma^{M*}}{dx}$$

$$= -PE_1^A(x > x^o, \cdot) + PE_2^A(x > x^o, \cdot)$$

$$+ RE^A(x > x^o, \cdot) + RE^M(x > x^o, \cdot).$$

Following the similar argument as the proof of Proposition 4, I argue that there exists a bounded number $1/\bar{\Delta}_3$ such that, if $c''(\gamma) \geq 1/\bar{\Delta}_3$, the equilibrium precision differential $\Delta \equiv \gamma^{A*} - \gamma^{M*}$ is smaller than some positive threshold $\bar{\Delta}_3$, and as a result $-PE_1^A(\cdot) + PE_2^A(\cdot) + RE^A(\cdot) + RE^M(\cdot) < 0$. That is, $C_1 \frac{d\gamma^{A*}}{dx} + C_2 \frac{d\gamma^{M*}}{dx} < 0$.

By Proposition 4, $\frac{d\gamma^{A*}}{dx} < 0$ for $x > x^o$. Therefore, if $C_1 \leq C_2$,

$$C_2 \frac{d\gamma^{A*}}{dx} + C_2 \frac{d\gamma^{M*}}{dx} \leq C_1 \frac{d\gamma^{A*}}{dx} + C_2 \frac{d\gamma^{M*}}{dx} < 0.$$

Put differently, a sufficient condition for $\frac{d\gamma^{A*}}{dx} + \frac{d\gamma^{M*}}{dx} < 0$ is that $C_1 \leq C_2$.

$$\begin{aligned}
& C_1 \leq C_2 \\
\Leftrightarrow\ & c''(\gamma^{A^*}) - c''(\gamma^{M^*}) \leq \frac{\partial MB^A(\cdot)}{\partial \gamma^{A*}} + \frac{\partial MB^M(\cdot)}{\partial \gamma^{A*}} - \frac{\partial MB^A(\cdot)}{\partial \gamma^{M*}} - \frac{\partial MB^M(\cdot)}{\partial \gamma^{M*}} \\
\Leftrightarrow\ & c'''(\delta)(\gamma^{A*} - \gamma^{M*}) \leq \frac{\partial MB^A(\cdot)}{\partial \gamma^{A*}} + \frac{\partial MB^M(\cdot)}{\partial \gamma^{A*}} - \frac{\partial MB^A(\cdot)}{\partial \gamma^{M*}} - \frac{\partial MB^M(\cdot)}{\partial \gamma^{M*}} \\
\Leftrightarrow\ & c'''(\delta)(\gamma^{A*} - \gamma^{M*}) \qquad\qquad \text{(B11)} \\
\leq\ & \underbrace{x\left[\frac{\partial PE_2^A(\cdot)}{\partial \gamma^{A*}} + \frac{\partial RE^A(\cdot)}{\partial \gamma^{A*}} + \frac{\partial RE^M(\cdot)}{\partial \gamma^{A*}} - \frac{\partial PE_2^A(\cdot)}{\partial \gamma^{M*}} - \frac{\partial RE^A(\cdot)}{\partial \gamma^{M*}} - \frac{\partial RE^M(\cdot)}{\partial \gamma^{M*}}\right]}_{Q(\cdot)}
\end{aligned}$$

Note that $x > x^o > x^+ > \frac{25}{29}$ (by (B6)), and due to $\frac{3}{4} \leq \bar{\gamma} < \gamma^{M*} < \gamma^{A*} < 1$,

$$\begin{aligned}
Q(\cdot) &> \frac{\partial PE_2^A(\cdot)}{\partial \gamma^{A*}} - \frac{\partial PE_2^A(\cdot)}{\partial \gamma^{M*}} \\
&> \frac{2(\gamma^{M*})^2(1 + 4\gamma^{M*})}{(2\gamma^{M*} + \gamma^{A*})^2} + \frac{2(1 - \gamma^{M*})^2(1 + 4(1 - \gamma^{M*})) - (2\gamma^{A*} - 1)(1 - \gamma^{A*})^2}{[2(1 - \gamma^{M*}) + (1 - \gamma^{A*})]^2} \\
&> \frac{2(\gamma^{M*})^2(1 + 4\gamma^{M*})}{(2\gamma^{M*} + \gamma^{A*})^2} + \frac{(1 - \gamma^{A*})^2[2 + 8(1 - \gamma^{M*}) - (2\gamma^{A*} - 1)]}{[2(1 - \gamma^{M*}) + (1 - \gamma^{A*})]^2} \\
&> \frac{2(\gamma^{M*})^2(1 + 4\gamma^{M*})}{(2\gamma^{M*} + \gamma^{A*})^2} \\
&> \frac{2(3/4)^2(1 + 4 * 3/4)}{(2 * 1 + 1)^2} = \frac{1}{2}.
\end{aligned}$$

Therefore, a sufficient condition for (B11) to hold is that

$$c'''(\delta)(\gamma^{A*} - \gamma^{M*}) \leq \frac{25}{58}. \tag{B12}$$

Note that, if $c''(\gamma) \geq a$, $\gamma^{A*} - \gamma^{M*} \leq \frac{1}{a}$. Hence a sufficient condition for (B12) to hold is that

$$c'''(\gamma) \leq \frac{25}{58}a, \quad \text{for} \quad \gamma \in (\gamma^{M*}, \gamma^{A*}).$$

Put everything together, if $c''(\gamma) \geq a$ and $c'''(\gamma) \leq \frac{25}{58}a$, then $\frac{d\gamma^{A*}}{dx} + \frac{d\gamma^{M*}}{dx} < 0$.

Now $a = \max\{3, 1/\bar{\Delta}_1, 1/\bar{\Delta}_2, a^o, 1/\bar{\Delta}_3\}$.

Step (iii): Finally, to complete the argument, if $\frac{d\gamma^{A*}}{dx} + \frac{d\gamma^{M*}}{dx} < 0$, then

$$\begin{aligned}\frac{dU_0^I(x > x^o)}{dx} &= \frac{\partial U_0^I(x > x^o)}{\partial \gamma^{A*}}\frac{d\gamma^{A*}}{dx} + \frac{\partial U_0^I(x > x^o)}{\partial \gamma^{M*}}\frac{d\gamma^{M*}}{dx} \\ &< \frac{\partial U_0^I(x > x^o)}{\partial \gamma^{M*}}\left(\frac{d\gamma^{A*}}{dx} + \frac{d\gamma^{M*}}{dx}\right) \\ &< 0.\end{aligned}$$

The first inequality stems from $\frac{\partial U_0^I(x>x^o)}{\partial \gamma^{A*}} > \frac{\partial U_0^I(x>x^o)}{\partial \gamma^{M*}} > 0$ (Step (i)) and $\frac{d\gamma^{A*}}{dx} < 0$ (Proposition 4). ■

APPENDIX C

A Numerical Characterization of the Equilibrium

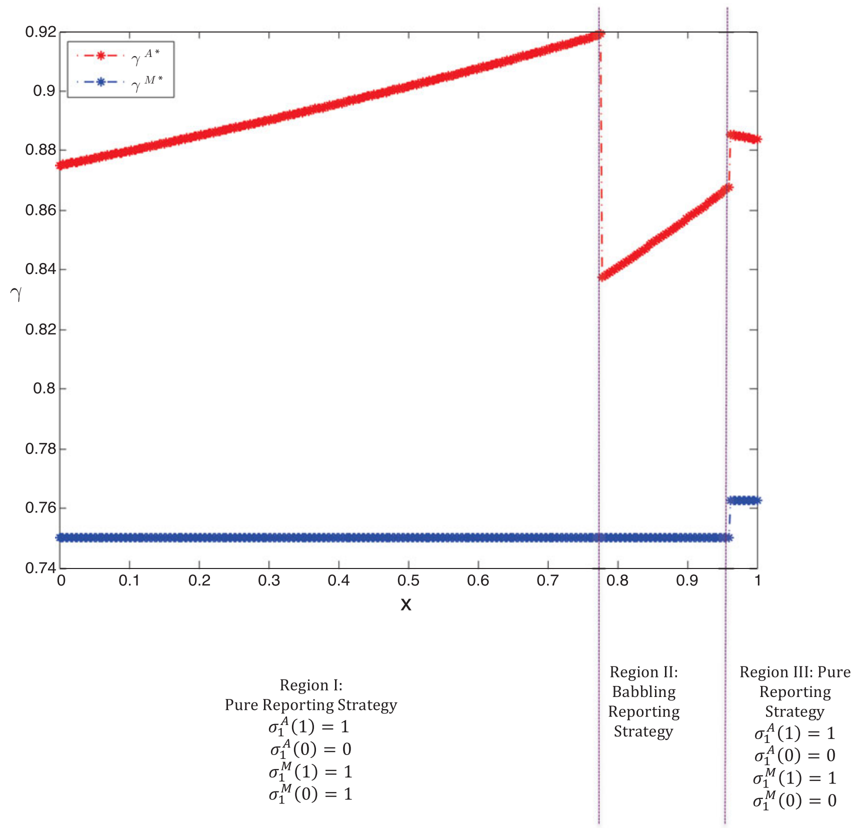

FIG. C1.—Numerical characterization of the equilibrium when $\lim_{\gamma\to\bar{\gamma}} c'(\gamma) = 0$. For this figure, the default precision $\bar{\gamma} = 3/4$, and the information acquisition cost function is $c(\gamma) = 2(\gamma - \bar{\gamma})^2$. The reporting strategy listed in this figure refers specifically to the first period. The analyst's second-period reporting strategy is the same for all values of x: the aligned type reports truthfully and the misaligned type always reports 1. If I assume instead that $\lim_{\gamma\to\bar{\gamma}} c'(\gamma) \in (0, \bar{c})$, then in Region II there may exist informative equilibrium in which the analyst plays a mixed reporting strategy in the first period.

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