Determining The Optimal Product-Mix Using Integer Programming: An Application in Audio Speaker Production

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Abstract. In manufacturing sector, production planning or scheduling is the most important managerial task in order to achieve profit maximization and cost minimization. With limited resources, the management has to satisfy customer demand and at the same time fulfill company's objective, which is to maximize profit or minimize cost. Hence, planning becomes a significant task for production site in order to determine optimal number of units for each product to be produced. In this study, integer programming technique is used to develop an appropriate product-mix planning to obtain the optimal number of audio speaker products that should be produced in order to maximize profit. Branch-and-bound method is applied to obtain exact integer solutions when non-integer solutions occurred. Three major resource constraints are considered in this problem: raw materials constraint, demand constraint and standard production time constraint. It is found that, the developed integer programming model gives significant increase in profit compared to the existing method used by the company. At the end of the study, sensitivity analysis was performed to evaluate the effects of changes in objective function coefficient and available resources on the developed model. This will enable the management to foresee the effects on the results when some changes happen to the profit of its products or available resources.

Keywords: Product-mix problem, integer programming, Branch-and-bound technique, audio speaker production.
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INTRODUCTION

Efficient planning, scheduling and coordination of all production activities are an important aspect of a manufacturing process. During planning phase, the forecasted product demand will be translated into a production plan before it can be finalized in a production master schedule. Proper planning will ensure an effective and efficient operation of manufacturing processes. As such, planning and control problems are inherently optimization problems, where the objective is to develop a plan that meets demand at minimum cost or that fills the demand that maximizes profit [1].

The product-mix problem of a production planning is the process of determining the optimal number of each type of product that should be produced subject to a given demand and resource constraints in order to maximize total profits or minimize total cost. The management of a company is responsible in making decision of the right product-mix. As emphasized on a research by Donato and Simas [2], decision of product-mix to be produced is linked closely to medium-term planning. Besides, product-mix strategy can bridge the gap between company's objectives and the customer demand for a wide range of products [3].

LITERATURE REVIEW

Many decision faced by an operation manager is focused on finding the best way to achieve the objectives of the company subject to a number of constraints related to raw materials, available production time, available capacity etc. [4].

The common constraints considered in manufacturing optimization problems are relates to limited labor hours, limited machine hours and limited production capacity. For example Donato and Simas [2] in their study to select optimal product-mix of a Brazilian mechanical manufacturer considers assembly line capacity at production floor as one of the major constraint. Graves [1] include available labor hour constraint in the study on optimization models for production planning of discrete parts in batch manufacturing environment. While Sakar and Prasanta [5] mentioned that production capacity needs information such as factory capacity in hours, product standard time and line efficiency. In a study on a sport wear manufacturer in Hong Kong and China, Zhao and Katehakis [6]

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emphasizes that Minimum Order Quantity (MOQ) become a big challenge in managing supply chain efficiently. The reason is MOQ reduces manufacturer's flexibility of its production operation. Donato and Simas [2] conducted a study to select optimal product-mix subject to the component production constraint to ensure that production of a given product is limited to the available capacity to produce each component.

Integer and Linear programming methods (IP and LP) are two of the most widely used methods to solve production planning problems. LP is a model that consist of linear relationship representing a firm's decision(s), given an objective and resource constraints [7]. It can be implemented to a wide range of different operational problem such as manufacturing, banking and human resources and marketing [4]. Donato and Simas [2] conducted a study on efficiency of Sales and Operation Planning (S&OP) through a linear programming model that uses cost and profit data to select optimal product-mix of mechanical manufacturer. The objective of the study is to improve the medium-term operation strategies for manufacturer. Blomer and Gunther [8] use LP model for a multi-product in the chemical industry. The model is aimed for sequencing and scheduling of allocation chemical products. The minimization of the make span is deemed to be the objectives function. Bhat [3] conducted a study on various range of products offered by a soap and detergent manufacturer in India by implementing IP. 21 products with different brand are used in his study for the purpose of product-mix adjustment. Javanmard and Habibollah [9] conducted a study by implementing IP for various products of dairy milk industry with different flavors and fat content in Sala, Iran. The objective of his study is to reduce major sources of variable cost which associated with production schedule. The model takes into account relevant constraint in production such as raw materials, machine capacity, and manpower restrictions. As a result, complete production schedule was obtained in terms of products that should be produced, quantities and inventory level.

In this study, the researcher used IP technique to find the optimal production mix in a speaker manufacturing company, Sound Vision Corporation. Sound Vision is one of the global manufacturers making high quality audio systems with almost 60 percent of its products are supplied to automotive sector. The company manufactures more than 100 types of speaker models used in various end products. In this study, only the models with significant contribution towards the company's profit will be considered. The models are deemed significant as they turn out to be the top selling products and normally running at production line every month. This study develops a product-mix planning to obtain the optimal number of speakers that should be produced in order to maximize profit. The constraints considered are raw materials, production capacity and customer demand.

PROBLEM FORMULATION

The decision variable, number of units that have to be manufactured for each type of speaker, is denoted by x_i. The rest of the variables used in the IP formulation is defined below:

x_i = the number of units of product i to be manufactured,
i = types of product where i = 1, 2,…, n
r_i = raw material requirement for per unit of product i,
R = total amount of MOQ in pieces,
L_i = the lower demand limit for product i,
U_i = the upper demand limit for product i,
n = total products types,
p_i = profit per unit of product i,
s_i = standard time of product i,
S = the available of overall standard time for selected line in hours.
The complete IP formulation for this problem is as follows:

Maximize profit, $$Z = \sum_{i=1}^{n} p_i x_i \quad \text{, where } i = 1, 2, \ldots, n. \qquad (1)$$
Subject to,

$$\sum_{i=1}^{n} r_i x_i \geq R, \quad \text{where } i = 1, 2, \ldots, n \qquad (2)$$

$$\sum_{i=0}^{n} s_i x_i \leq S, \quad \text{where } i = 1, 2, \ldots, n. \qquad (3)$$

$$L_i \leq x_i \leq U_i, \quad \text{where } i = 1, 2, \ldots, n. \qquad (4)$$

$$x_1, x_2, \ldots, x_n \geq 0 \text{ and integer}$$

In the IP formulation, (1) represents the objective function, (2) represents raw material constraint, (3) represents production line capacity constraint and (4) represents demand constraints. This IP formulation is solved using LINDO software package to determine the optimal number of each type of speakers that should be produced in order to maximize profit. Next, sensitivity analysis is performed to evaluate the effects on maximal profit caused by changes in either profit contribution of each product or changes in available raw materials, production time or demand.

DATA ANALYSIS AND RESULTS

For the purpose of this study, only one production line is selected from 4 main lines in Sound Vision's production site. Twenty-eight products with significant contribution towards the company's profit are considered in this study. The products are deemed significant as they are the top selling products and normally running at the production line every month. The purpose of objective function is to maximize the profit of 28 selected active products. Profit per unit of each product is the coefficient of the respective variables in the objectives function. Selling price and profit per unit for the selected products are shown in Table 1.

TABLE (1). Name of product, selling price and profit per unit of each product

Product Code	Decision Variable Name	Selling Price / Unit (RM)	Profit / Unit (RM)
SP10B-2	x_1	2.7717	0.83
SP10A-3	x_2	14.3667	4.31
SP12A-3	x_3	3.1074	0.93
SP13C-3	x_4	15.0590	4.52
SP13G-3	x_5	15.0590	4.52
SP16A-7	x_6	15.7281	4.72
SP16A-3	x_7	9.4105	2.82
SP16B-3	x_8	9.4105	2.82
SP16A-3	x_9	8.5680	2.57
SP78A-3	x_{10}	5.4520	1.64
SP79A-4	x_{11}	6.6138	1.98
SP81A-5	x_{12}	3.2000	0.96
SP82A-3	x_{13}	5.9518	1.79
SP82C-3	x_{14}	5.9518	1.79
SP82D-5	x_{15}	6.0827	1.82
SP82A-4	x_{16}	4.6496	1.39
SP82A-5	x_{17}	4.8743	1.46
SP82A-7	x_{18}	5.9856	1.80
SP82B-5	x_{19}	6.1325	1.84
SP82C-5	x_{20}	6.0932	1.83
SP1035E-7	x_{21}	13.3032	3.99
SP1035F-7	x_{22}	12.4901	3.75
SP1064C-5	x23	10.6026	3.18
SP1278A-7	x24	10.7330	3.22
SP1278D-7	x25	9.3100	2.79
SP1280A-7	x26	8.9806	2.69
SP16179A-3	x27	8.5680	2.57
SP902B-7	x28	13.6370	4.09

Information such as raw material with MOQ, standard time in hours as well as minimum and maximum demand in pieces was used to build the constraint functions of the proposed model. Table 2 shows the related data of each product which are implemented in the constraint functions formulation.

TABLE (2). Products MOQ, production time and demand

Decision Variable (Product)	Raw material with Minimum Order Quantity (MOQ)	Standard time in hours	Minimum production quantity in pieces	Maximum demand in pieces
x_1	Damper	0.1334	1000	12000
x_2	Damper	0.0900	100	5400
x_3	Pole piece	0.0716	100	2000
x_4	Pole piece	0.0806	100	6100
x_5	Pole piece	0.0806	100	6100
x_6	Magnet	0.0927	1000	4000
x_7	Magnet	0.0968	1000	6650
x_8	Magnet	0.0862	100	4900
x_9	Magnet	0.0840	1000	9000
x_{10}	Dust cap	0.0655	100	2000
x_{11}	Dust cap	0.0546	1000	9000
x_{12}	Cone paper	0.0624	100	7500
x_{13}	Cone paper	0.0608	100	9000
x_{14}	Cone paper	0.0600	100	2300
x_{15}	Cone paper	0.0634	300	1050
x_{16}	Cone paper	0.0567	400	3000
x_{17}	Cone paper	0.0574	1000	9000
x_{18}	Cone paper	0.0442	100	1000
x_{19}	Cone paper	0.0476	100	1000
x_{20}	Cone paper	0.0476	100	1000
x_{21}	Not related	0.1284	300	3000
x_{22}	Not related	0.1450	100	1700
x_{23}	Not related	0.0550	100	1000
x_{24}	Not related	0.1327	100	6000
x_{25}	Not related	0.1327	100	2000
x_{26}	Not related	0.1228	100	4200
x_{27}	Magnet	0.0840	1000	12000
x_{28}	Dust cap	0.1658	100	22000

Several materials must be purchased with MOQ which imposed by supplier. The MOQ of raw material in a month was given by the purchasing department.

TABLE (3). MOQ of raw material for related products

Raw material type	Description	Related product(s)	MOQ in pieces
PO	Pole piece	SP12A-3, SP13C-3, SP13G-3	3000
DA	Damper	SP10A-2,SP10A-3	1000
DC	Dust cap	SP78A-3, SP79A-4, SP902B-7	1200
MG	Magnet	SP16A-7, SP16A-3, SP16B-3, SP16A-3, SP16179A-3	15,000
CP	Cone paper	SP81A-5,SP82A-3, SP82C-3, SP82D-5, SP82A-4, SP82A-5,SP82A-7,SP82B-5, SP82C-5	25,000

All the 28 products with their optimal production are summarized in Table 4. From the obtained solution it is found that the optimal profit that can be achieved is RM 249639.20.

TABLE (4). Results of optimal product-mix

Decision Variable	Product Code	Number of units
x_1	SP10B-2	1000
x_2	SP10A-3	5400
x_3	SP12A-3	100
x_4	SP13C-3	6100
x_5	SP13G-3	6100
x_6	SP16A-7	4000
x_7	SP16A-3	6650
x_8	SP16B-3	4900
x_9	SP16A-3	9000
x_{10}	SP78A-3	2000
x_{11}	SP79A-4	9000
x_{12}	SP81A-5	100
x_{13}	SP82A-3	9000
x_{14}	SP82C-3	2300
x_{15}	SP82D-5	1050
x_{16}	SP82A-4	552
x_{17}	SP82A-5	9000
x_{18}	SP82A-7	1000
x_{19}	SP82B-5	1000
x_{20}	SP82C-5	1000
x_{21}	SP1035E-7	3000
x_{22}	SP1035F-7	1700
x_{23}	SP1064C-5	1000
x_{24}	SP1278A-7	100
x_{25}	SP1278D-7	100
x_{26}	SP1280A-7	100
x_{27}	SP16179A-3	1000
x_{28}	SP902B-7	1966

Next, model evaluation is performed by comparing the model output with actual data from the company which is depicted in Table 5.

TABLE (5). Comparison of model results and company data

Item	Model result	Company data	Difference
Profit (RM)	249,639.22	233,961.30	16,677.92
Raw material used-Damper (in pieces)	6400	12,540	6140
Raw material used-Pole Piece (in pieces)	12,300	1420	10,880
Raw material used-Dust cap (in pieces)	12,000	16,100	4100
Raw material used-Magnet (in pieces)	25,550	39,550	14,000
Raw material used-Cone paper (in pieces)	25,000	20,150	4850
Standard time (in hours)	7005.97	8904.72	1142.75

In terms of profit, the proposed IP model shows a higher optimal profit compared to profit obtained by the company using the current product mix. The difference is RM16677.92 which is a 7.13% increase in profit. The increase shows that the proposed model is helpful in maximizing profit for various ranges of audio speaker products at Sound Vision Company compared to the existing practice.

In terms of raw materials consumed, it is found that all raw materials have been fully utilized for the production of these speakers. The obvious difference in raw material usage is magnet (a difference of 14,000 pieces) which the current production had consumed more than the proposed model. On the other hand, for pole piece usage, the model had consumed 10,880 pieces more than the company data. This is due to products which use pole piece such as x_3 (SP12A-3), x_4 (SP13C-3) and x_5 (SP13G-3) are produced more than the company's current production level. Nevertheless, both company data and the proposed model satisfy the MOQ requirement by raw material supplier.

It is also can be observed that, in the proposed models solution the standard time is fully utilized and no extra working days or overtime (OT) is required to produce all the products. However, it is found that, the current production schedule used by the company needs additional standard time (overtime) to satisfy all the production level. The company has spent 8904.72 hours, out of available amount of 7761.98 hours (an additional 1142.82 hours of overtime). The excess 1142.82 hours is equivalent to almost 4 days of overtime (standard time for normal working day is 352.8173 hours per day).

Next, sensitivity analysis on the proposed model is performed to determine the effects on optimal solution value due to changes in either objective function coefficient or right-hand side values of constraints. Model evaluation is necessary to observe the fluctuations of the model output when several amendments are done into the model input. The goodness of model evaluation is it facilitates the management to make decision whether to increase or decrease in profit (due to competitive market place with various competitors) with current basis remains optimal. Secondly, it can be a guide to the management in terms of available resource (right hand side amount for each constraint) if they would like to cut off the excess raw material or additional standard time while optimal product-mix composition remains unchanged.

For ranges of each objective function coefficient, it is given in the objective coefficient ranges portion of the model output as shown in Table 6. The allowable increase column shows that the amount by which an objective function coefficient can be increased with the current basis remaining optimal. In the same way, the allowable decrease column shows the amount by which an objective function coefficient can be decreased with current basis remaining optimal.

If we take product x_1, allowable increase is infinity and allowable decrease is 0.83 as shown in Table 6. Hence, unit profit of x_1 can be increased up to infinity and can be decreased by RM0.83 without affecting the current optimal solution mix. For example, if the profit per unit of product x_1 becomes RM2, which is an increase of RM1.17, then the profit will increase by RM1.17 per unit without affecting the optimal product-mix. As a result, the new profit is calculated as RM1.17 (1000 pieces) + RM249, 639.20 = RM250, 809.20. From this analysis, the management has a clear picture whether the decision to decrease profit of related products is possible (if the price offered by competitors is more attractive to customers in business market). On the other hand, the management can boost profit of associated products if demand flexibility is understandable to increase more revenue.

TABLE (6). Objective function ranges in which the basis is unchanged

Product item	Allowable increase	Allowable decrease
x_1	infinity	0.83
x_2	infinity	4.31
x_3	infinity	0.93
x_4	infinity	4.52
x_5	infinity	4.52
x_6	infinity	4.72
x_7	infinity	2.82
x_8	infinity	2.82
x_9	infinity	2.57
x_{10}	infinity	1.64
x_{11}	infinity	1.98
x_{12}	infinity	0.96
x_{13}	infinity	1.79
x_{14}	infinity	1.79
x_{15}	infinity	1.82
x_{16}	infinity	1.39
x_{17}	infinity	1.46

TABLE (6). Objective function ranges in which the basis is unchanged [continued]

Product item	Allowable increase	Allowable decrease
x_{18}	infinity	1.80
x_{19}	infinity	1.84
x_{20}	infinity	1.83
x_{21}	infinity	3.99
x_{22}	infinity	3.75
x_{23}	infinity	3.18
x_{24}	infinity	3.22
x_{25}	infinity	2.79
x_{26}	infinity	2.69
x_{27}	infinity	2.57
x_{28}	infinity	4.09

Next we will determine the range of right-hand side values that can be adjusted without affecting the current basis. This information can help the producer to determine the effects of changes in available resources on current solution. For example, if we consider constraint R_1 (maximum demand for product SP10A-2), currently the right-hand side of this maximum demand constraint is 12,000 pieces. The current basis remains optimal if R_1 is decreased until 11,000 pieces (the allowable decrease for R_1) or could be increased infinitely (the allowable increase for R_1).

This result can be used as a guide to decrease the available monthly production resources such as raw material and standard time. In addition, a large amount of extra profit can be achieved if production manager makes the decision to cut off the excess raw materials or standard time. Thus, this beneficial information can assist the management to make better decision.

Table (7). Right-hand side (RHS) ranges for maximum demand in which the basis is unchanged

Constraint	Current RHS	Allowable increase	Allowable decrease
R_1	12000	infinity	11000
R_2	5400	0.320284	5300
R_3	2000	infinity	1900
R_4	6100	0.357637	6000
R_5	6100	0.357637	6000
R_6	4000	0.31096	3000
R_7	6650	0.297785	5650
R_8	4900	0.334403	4800
R_9	9000	0.343162	8000
R_{10}	2000	0.440085	1900
R_{11}	9000	0.527941	8000
R_{12}	7500	infinity	7400
R_{13}	9000	0.474105	2
R_{14}	2300	0.480426	2
R_{15}	1050	0.454662	2
R_{16}	3000	infinity	2448
R_{17}	9000	0.502188	2
R_{18}	1000	0.652162	2
R_{19}	1000	0.605579	2
R_{20}	1000	0.605579	2
R_{21}	3000	0.224498	2700
R_{22}	1700	0.198797	1600
R_{23}	1000	0.524101	900
R_{24}	6000	infinity	5900
R_{25}	2000	infinity	1900
R_{26}	4200	infinity	4100
R_{27}	12000	infinity	11000
R_{28}	22000	infinity	20034

CONCLUSION

The result from model output shows that the proposed model gives a solution with better profit compared to the current practice. Therefore, we can conclude that the proposed IP model is more beneficial and helpful to apply for the speaker production operation at Sound Vision Company. Furthermore from the sensitivity analysis report, the management of the company can determine the effect of changes in per unit profit of each product or the available resources on the current solution. This will enable them to make more robust decision in terms of pricing and resource allocation.

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