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Opportunistic usage of television white space with respect to the long term evolution-advanced parameters

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Abstract: Change of transmission from analogue to digital in television (TV) systems has resulted in the formation of considerable spectrum holes in various geographical locations. Cognitive radio (CR) is one of the best structures that can be opportunistically used for optimal exploitation of these spectral sources. In this study, the authors intend to investigate the use of multiple-input multiple-output orthogonal frequency division multiple access-based structure in the TV white spaces bands in the downlink path with respect to long term evolution advanced (LTE-A) parameters. For this purpose, while different spectrum holes in the TV frequency range have been introduced, their limitations in practical applications have fully been discussed. Besides, since the previous power allocation algorithms are not effective enough to be applied in the present scenario, a proper algorithm for maximising the total capacity of the CR system has been introduced. The performance efficiency of the proposed power allocation algorithm has been evaluated and justified through the consideration of the system parameters based on the LTE-A standard.

1 Introduction

Recently, a global movement has been started to convert television (TV) transmissions from analogue to digital, which is called the digital switch-over (DSO) [1]. Owing to the spectrum efficiency of digital television (DTV), however, a lot of the spectrum bands which are used by analogue TV will be free for other wireless communication applications. Moreover, a number of TV frequency bands are left unused within each geographical location to avoid interference into the adjacent DTV systems. Therefore after DSO a large part of the spectrum, including the interleaved bands and cleared bands remains unused in a given geographical location which creates an exceptional opportunity for secondary users' (SU) applications.

The cognitive radio (CR) is a key technology for adopting the unused spectrum opportunistically in much efficient manner. In fact, the CR has an intelligent structure that can acquire the vacant bands in any location and any time and use these bands dynamically for SUs' transmission in a non-interfering manner [2, 3].

Owing to the fragment nature of the TV white space, selecting of a good physical layer technology for the CR system has an important role in efficient usage of these sources. In fact, this technology should be able to use the spectrum holes as efficient as possible while avoiding interfering the licensed TV users.

One technique that seems to meet both these requirements is the multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM). MIMO-OFDM is capable of deactivating subcarriers across its transmission bandwidth that could potentially interfere with the transmission of TV users according to using OFDM structure for its transmission. Moreover, it can support a high aggregate data rate with the remaining subcarriers, because of the advantages offered by its MIMO nature [4].

Among different standards which are currently considering MIMO-OFDM structure for their transmission, long term evolution-advance (LTE-A) has a good feature to apply in TV white spaces (TVWS) channels. Some of the key characteristics of the LTE-A which make it suitable for this application are [5, 6]:

- *Carrier aggregation:* Having the capability of combining different chunks of the spectrum which is known as carrier aggregation, leads to the scalable bandwidth nature of the LTE-A with ranging from 1 to 100 MHz. This capability makes it suitable to cover a lot of DTV channels' bandwidth.
- *Supporting MIMO:* LTE-A support advance antenna systems with different aspects, including spatial multiplexing configuration which can be used for efficient utilisation of the TVWS.
- *Scheduling capability:* LTE-A base-station predicted to have the ability of proper radio resource management. This nature makes it possible to allocate proper power to different CR users with controlling the caused interference to the TV systems.

In this paper, we intend to investigate using MIMO-OFDMA based structure in TVWS bands in downlink path with respect to LTE-A parameters. To this aim by exploring the existing unused resources in TV bands deeply, different kinds of free spaces with related limitations are introduced. Since, the conventional resource allocation schemes are not hence efficient for using in the considered scenario, the main purpose of this paper is to extract a proper resource allocation algorithm to use the TVWS channels in the best manner. We formulate the main problem as an optimisation problem with objective function that maximises the total capacity of the CR system with respect to different interference constraints. By converting the main optimisation problem into a tractable convex one, a proper resource allocation scheme is proposed by using the Karush–Kuhn–Tucker (KKT) and ellipsoid methods.

1.1 Related work and paper organisation

A lot of works have been done on the context of co-existence of SU and the primary ones (PU) most of which consider OFDM as physical layer of the CR system. Generally, these works can be classified into two different groups according to the modelling of the interference limitation. The first group such as [7, 8] considers a scenario in which the SU and PU are located in the same geographical location with side-by-side bands. Since, the sub-carriers of two systems are located close to each other, there are some leakages from the SU sub-carriers into PU bands related to the sub-carriers' side-lobes leakages. The main purpose of this group is to allocate proper power to each CR sub-carrier to maximise the capacity of CR network while the caused interferences into the primary bands keep below a specific level.

In the second one, it is assumed that the SU and PU users use the same sub-carriers for their transmission but in different geographical location with a specific distance [9, 10]. In this scenario, the SU sub-carriers' power managed in such a way that optimal utilisation of them obtained while the total power of them keeps below enough to avoid causing interference to the PU systems which are located in the specific distance from SU.

It will be shown in this paper that for optimal utilisation of TVWS by CR networks, both of these modes should be considered for resource allocation. Specially, we introduce two different kinds of white spaces in TV bands with different power usage limitation, which make the proposed optimisation problem different from previous works, which have been done in this context.

Efficient power allocation has been investigated for single user MIMO-OFDM-based CR systems in [4, 11]. Shahraki *et al.*, [12] extended the work of Shahraki and Mohamed-Pour [4] for multi-user case and extracted a proper resource allocation using dual Lagrange method. Integer linear programming method was used for the proper resource allocation in MIMO-OFDMA-based CR networks in [13]. However, all of these works are based on the interference consideration according to the first group scenario discussed above.

The rest of the paper is organised as follows. In Section 2, the regulation regimes for DTV spectrum usage which leads to the white space appearance is explored briefly. After that, different kinds of white space in the TV bands and their related power limitation for CR networks' application, is introduced.

The CR system structure is presented in Section 3 and maximisation of the CR system's capacity with respect to the power usage limitations is formulated as an optimisation problem. Proper resource allocation scheme is extracted in Section 4 by solving the problem via combined KKT and ellipsoid methods. The complexity of the proposed algorithm for practical application is explained in Section 5. In Section 6, the performances of the new algorithm is investigated by computer simulation. Results are interpreted and finally, the conclusions are drawn in Section 7.

2 TV white spaces

2.1 TVWS definition

Up to now, both UK and US have allowed the usage of TVWS by the license-exempt CR systems and it is predicted that it becomes as a general rule throughout Europe in near future [1]. TVWS spectrum includes a large part of the ultrahigh frequency (UHF) and very high frequency bands that become available for the secondary usage after the digital switch-over. Fig. 1, illustrates the amount of available TVWS after DSO [14].

As Fig. 1 shows, there are three different types of channel in the considered frequency band. The part which is marked in green is the cleared spectrum which can be licensed for the new wireless services. The large part which is shown with purple colour refers to the interleaved spectrum and the channel which is shown by pink colour is the section which is licensed for exclusive access for wireless microphones.

The main concept of the opportunistic access of the TVWS returns to the interleaved part of the spectrum which is about 256 MHz. Interleaved part arises because DTV transmitters in a given geographical location do not use all DTV channels to avoid causing interference to the other DTV transmitters. Hence, in each geographical location a large part of the spectrum remains unused which can be used by the CR systems. Therefore the following discussions and assumptions in this article will be based on the efficient usage of this opportunity in TVWS bands.

2.2 TVWS detection

The first issue in using the cognitive systems is that how we can successfully detect TVWS in the certain location to avoid harmful interference to licensed users. To do so, two different methods proposed which are known as 'geolocation combined with access to a database' and 'spectrum sensing' [15].

In the geolocation combined with data based method, the CR base-station determines its location and accesses a database to determine the unused TV channels at that location. For instance, GPS can be used to support the CR base-station to determine its location and an entity in CR base-station can be used for sustaining the database.

In the sensing method, CR users autonomously detect the presence of TV signals and only use the channels that are not used by TV broadcaster. The main inability of this method is the hidden node problem. This problem arises when the CR device cannot detect the active TV signals through blockage and consequently causing harmful interference to the TV transmission.

According to the last report of the FCC, it has been decided that cognitive devices use both spectrum sensing and geolocation combined with data-based method [16]. Therefore, it can be assumed that the CR base-station can determine the spectrum holes in any geographical location with very good accuracy.

2.3 LTE-A applied in TVWS

Generally, LTE-A was premeditated to support three different kinds of carrier aggregation. These are contiguous aggregation in a single band, non-contiguous aggregation in a single band and, non-contiguous aggregation in different bands [6]. Current work is based on the first case in which side-by-side bands are aggregated together. Each of these bands can be up to 20 MHz according to the LTE structure and the number of aggregated bands can be up to 5 bands. Therefore, LTE-A based structure can have the bandwidth up to 100 MHz and cover a lot of DTV bands in a given geographical location.

However, the assumptions and definitions in the paper are in such a way that the obtained results can be generalised for the other types of aggregation.

2.4 Different kinds of white space

To illustrate different kinds of free spaces of the spectrum in a particular geographical location, consider Fig. 2. As this Fig. 2 shows, each DTV station covers a certain geographical area shown by a circle. The circles have to be non-overlapping to avoid interference between DTV coverage areas. The CR network has an

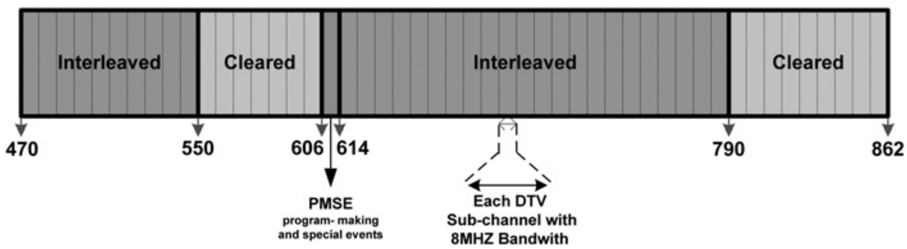

Fig. 1 *UHF spectrum allocation in the UK after DSO*

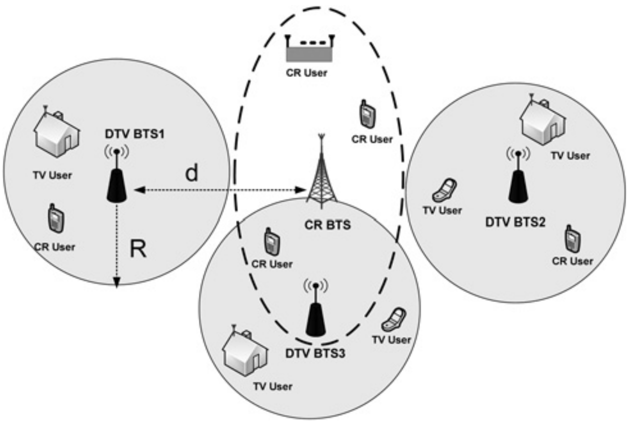

Fig. 2 *Coexistence of the CR network with the TV cells*

individual base-station which is located among the DTVs' covering area.

According to Fig. 2 and the facts about spectrum sensing which are discussed above, the CR base station can determine three kinds of spectrum in any location.

Channel type A: Using the database and the feedbacked spectrum sensing information from CR users, the CR base-station concludes that these parts of the spectrum are currently used by primary systems and cannot be used by the CR network.

Channel type B: The CR base-station determines that these parts of the spectrum are not used in the considered geographical location of the CR base-station with sufficiently large radius. For instance, according to Fig. 2, using the pre-established database, CR base-station knows that none of the DTV's stations use the channel NO.40 (622–630 MHZ) for a specific period of time.

Channel type C: It can be seen in Fig. 2 that the gap among the non-overlapping circles creates a spectrum hole which can be used by CR devices. For instance, according to Fig. 2, the CR base-station can use the DTV channels which are used by DTV1 and DTV2 stations but not used by DTV3's station. However, it should be noted that cognitive devices should be carefully designed to use these parts of the spectrum with proper power applications so as to be compatible with the DTV networks and avoid interfering them.

2.5 Different kinds of limitation

Since active primary users have priority than secondary ones, SUs' transmission must not produce any disturbance on primary users' transmission. In this sub-section different kinds of limitation on CR power usage will be introduced.

Limitation on channel type A: Since these parts of the spectrum are used by the primary systems, the CR systems do not allow transmitting any data in these parts and all the sub-carriers in this sub-channel should be modulated by zero data during the transmission. However, according to the leakage of power through the sub-carriers' sidelobes, it is not proper to expect that the transmit power in these sub-channels to be zero, since the sidelobes of the sub-carriers in other sub-channels certainly induce non-zero transmit power in these sub-channels [9]. Therefore, a transmit power threshold should be set for these sub-channels (γ).

Limitation on channel type B: These parts of the spectrum determined as a free parts which are not used in the considered geographical location with the DTV systems. Therefore at the first glance, it seems that CR systems can use these parts with no power limitation. However, as discussed above, there are some power leakages from the CR transmission into the primary users' bands through the sidelobes of the OFDM structure. Therefore, it is necessary that the CR system control the allocated power in these bands in such a way that the introduced interference into the channels type A does not exceed the pre-determined threshold level, γ.

For a MIMO-OFDM system, The interference introduced by the jth antenna of ith sub-carrier to the lth DTV sub-channel, $I_{ij}^{l}(d_{il}, P_{ij})$, can be obtained as follows [4, 17]

$$I_{ij}^{l}(d_{il}, P_{ij}) = P_{ij}T_{\rm S}\int_{d_{il}-B_l/2}^{d_{il}+B_l/2}\left(\frac{\sin \pi f T_{\rm S}}{\pi f T_{\rm S}}\right)^2 {\rm d}f \quad (1)$$

where P_{ij} is the transmit power of the CR base-station on the jth antenna of ith sub-carrier, d_{il} represents the frequency distance between the ith sub-carrier of CR users' band and the lth sub-channels which is used by a DTV network with B_l bandwidth and $T_{\rm S}$ is the OFDM symbol duration.

Limitation on channel type C: As depicted in Fig. 2, in these parts of the spectrum each DTV transmitter defines a protection area whose radius is R. The CR base-station should planned its transmission power in these bands in such a way that the interfering CR power at any potential receiver in a DTV area be lower than a certain value, say η. Zhao and Sadler [10] has been shown that CR transmitter can use these parts of the spectrum subject to the following power constraint

$$P_{tl} \leq \eta(d - R)^{\beta} \quad (2)$$

where d is the distance between the CR base station and the related DTV transmitter, β is the path attenuation factor and P_{tl} is the total transmit power on lth type C sub-channel. It should be noted that in the above limitation criteria the simple distance path loss model considered for the attenuation of the CR signals.

In addition to the above limitation, we should also consider the same limitation as channel type B, related to the power leakage into the channel type A.

3 System model and problem formulation

Consider a CR base-station which was located in a specific geographical location. The proposed CR system is based on the LTE-A parameters with sufficiently large system's bandwidth, which can cover several DTV sub-channels.

As Fig. 3 shows, the system's total bandwidth is divided into Q sub-channels with the bandwidth equal to each DTV channel. The CR BTS determines the different types of the channel on its bandwidth domain according to the assumptions given in previous section and deactivate the type A channels' sub-carriers.

Let us classify the determined sub-channels into different sets. Set $\mathcal{A}$, containing the A type sub-channels' number with L elements and Set $\mathcal{B}$ and set $\mathcal{C}$ which are containing the B type and C type sub-channels' number, respectively. It is assumed that each sub-channel contains X sub-carriers indicated by q_1 to q_X for qth sub-channel (see Fig. 3). Furthermore, the following assumptions are considered consequently:

$\mathcal{A} \cup \mathcal{C}$ with total G elements' number, indicates the set of sub-channels with the interference power constraint.

$\mathcal{B} \cup \mathcal{C}$ indicates the set of sub-channels which can be used by CR network including total N sub-carriers.

Now the main challenge is how to allocate the existing resource to the CR users, to obtain maximum capacity for the CR networks subject to the constraints were introduced in the previous section.

Suppose that the CR base station is equipped with $N_{\rm T}$ transmit antennas and K CR users have $M_1, M_2, \ldots, M_K$ antennas, respectively. The kth SU's MIMO channel for ith sub-carrier is denoted by an $M_k \times N_{\rm T}$ matrix $\boldsymbol{H}_k(i)$, where its element $h_{i,n,m}^{k}$ denotes the channel gain between mth transmit antenna and nth receive antenna. The MIMO channel matrix $\boldsymbol{H}_k(i)$ is assumed to be perfectly known to the transmitter and kth user's receiver by channel estimation.

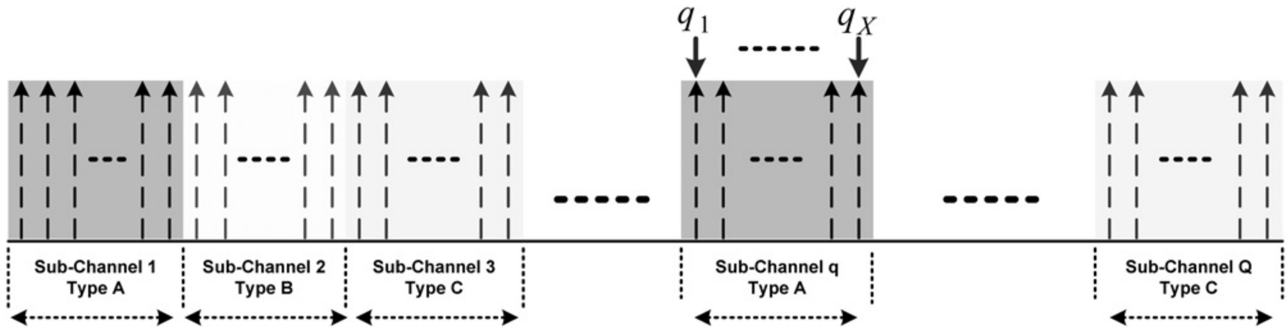

Fig. 3 *Different kinds of DTV channels in a given geographical location*

It has been shown that, when the channel state information is known perfectly at the transmitter, the maximisation of the MIMO mutual information can be achieved by using singular value decomposition (SVD) [18]. By decomposing the MIMO channel into $n_{\min} = \operatorname{rank}(\boldsymbol{H}_k(i))$ independent sub-channels with SVD and multiplexing independent data onto these independent channels, the capacity of kth user on the ith sub-carrier can be obtained by

$$C_k(i) = \sum_{j=1}^{n_{\min}} \log_2\left(1 + \frac{P_{kj}(i)\lambda_{kj}^2(i)}{N_0}\right) \quad \text{bit/s/Hz} \tag{3}$$

where $P_{kj}(i)$ denotes the kth user's transmit power at the jth antenna of ith sub-carrier, $\lambda_{kj}(i)$ is the jth diagonal element of a diagonal matrix containing the singular values of $\boldsymbol{H}_k(i)$ after SVD and N_0 is the additive white Gaussian noise variance.

Therefore, the main objective function with the related constraints can be summarised as the following optimisation problem

$$\begin{aligned}
&\text{maximise} \quad C = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} \boldsymbol{\alpha}_{ki} \cdot C_{kij}(P_{kj}(i))\\
&\text{Subject to} \quad \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} \boldsymbol{\alpha}_{ki} \cdot P_{kj}(i) \cdot I_{ij}^{g} \le I_{\mathrm{TH}}^{g} \quad g = 1, 2, \ldots, G\\
&\qquad \sum_{k=1}^{K} \boldsymbol{\alpha}_{ki} \le 1 \quad \forall i, \quad \boldsymbol{\alpha}_{ki} \in \{0, 1\} \quad \forall i, k\\
&\qquad \boldsymbol{\alpha}_{ki} \cdot P_{kj}(i) \ge 0 \quad \forall \quad i, j \text{ and } k
\end{aligned} \tag{4}$$

where, N is the number of sub-carriers which can be used by CR network and the other parameters has the following definition

$$C_{kij}(P_{kj}(i)) = \log_2\left(1 + \frac{P_{kj}(i)\lambda_{kj}^2(i)}{N_0}\right) \tag{4-1}$$

$$I_{ij}^{g} = \begin{cases} T_{\mathrm{S}} \int_{d_{im}-B_m/2}^{d_{im}+B_m/2} \left(\dfrac{\sin \pi f T_{\mathrm{S}}}{\pi f T_{\mathrm{S}}}\right)^2 \mathrm{d}f & \text{if } g \in \{A\} \\ 1 & \text{if } g \in \{C\} \text{ and } i \in \left[C_{g1}, C_{gX}\right] \\ 0 & \text{if } g \in \{C\} \text{ and } i \notin \left[C_{g1}, C_{gX}\right] \end{cases} \tag{4-2}$$

and

$$I_{\mathrm{TH}}^{g} = \begin{cases} \boldsymbol{\gamma}_g & \text{if } g \in \{A\} \\ \boldsymbol{\eta}_g (d_g - R_g)^{\beta_g} & \text{if } g \in \{C\} \end{cases} \tag{4-3}$$

The second constraint in the optimisation problem (4) is related to the OFDMA nature of the LTE standard in downlink path which indicates that each sub-carrier is only allocated to one user. In other words, $\boldsymbol{\alpha}_{ki}$ which is the assignment indicator defined in such a way that $\boldsymbol{\alpha}_{ki} = 1$ means the ith sub-carrier is allocated to the kth user and $\boldsymbol{\alpha}_{ki} = 0$ has the inverse meaning.

The simplest interpretation of the (4) is how we can assign the active sub-carriers to the CR users ($\boldsymbol{\alpha}_{ki}$), and how much power should be allocated to each branch to maximise the total capacity of the system with respect to the interference constraints ($P_{kj}(i)$).

4 Proper resource allocation

The problem (4) is not a concave or convex problem with respect to the ($\boldsymbol{\alpha}_{ki}$, $P_{kj}(i)$) and it cannot be solved by the standard convex optimisation methods. However, using a little change, the problem can be converted to a convex problem without loss of generality. We use a similar technique which was used in [19] and introduce a new variable as $S_{kj}(i) = \boldsymbol{\alpha}_{ki} \cdot P_{kj}(i)$ and allow α_{ki} being a real number in [0, 1]. Thus, the problem (4) can be rewritten as follows

$$\begin{aligned}
&\text{maximise} \quad C = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} \boldsymbol{\alpha}_{ki} \cdot C_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right)\\
&\text{Subject to} \quad \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} S_{kj}(i) \cdot I_{ij}^{g} \le I_{\mathrm{TH}}^{g} \quad g = 1, 2, \ldots, G\\
&\qquad \sum_{k=1}^{K} \boldsymbol{\alpha}_{ki} \le 1 \quad \forall i, \quad 0 \le \boldsymbol{\alpha}_{ki} \le 1 \quad \forall i, k\\
&\qquad S_{kj}(i) \ge 0 \quad \forall i, j \text{ and } k
\end{aligned} \tag{5}$$

In the above problem, $\boldsymbol{\alpha}_{ki} \cdot C_{kij}((S_{kj}(i))/(\boldsymbol{\alpha}_{ki}))$ known as the perspective function of $C_{kij}(P_{kj}(i))$ with $\boldsymbol{\alpha}_{ki} > 0$. Thus, since the $C_{kij}(P_{kj}(i))$ is a concave function of $P_{kj}(i)$, then so is its perspective function with respect to ($\boldsymbol{\alpha}_{ki}$, $P_{kj}(i)$) [20].

Now, note that according to the definition $S_{kj}(i) = \boldsymbol{\alpha}_{ki} \cdot P_{kj}(i)$, if $\boldsymbol{\alpha}_{ki} = 0$ then $S_{kj}(i) = 0$ and for $\boldsymbol{\alpha}_{ki} \ne 0$ the problem can be solved by the (KKT) method as follows.

The Lagrangian of (4) is

$$\begin{aligned}
&\mathcal{L}\Big(\boldsymbol{\alpha}_{ki}, S_{kj}(i), \boldsymbol{\mu}_{kj}(i), \boldsymbol{\nu}_i, \boldsymbol{\rho}_{ki}, \boldsymbol{\xi}_{ki}, \boldsymbol{\psi}_g\Big) = \\
&-\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} \boldsymbol{\alpha}_{ki} \cdot C_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right) + \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} \boldsymbol{\mu}_{kj}(i)\Big(0 - S_{kj}(i)\Big)\\
&+\sum_{i=1}^{N} \boldsymbol{\nu}_i \left(\sum_{k=1}^{K} \boldsymbol{\alpha}_{ki} - 1\right) + \sum_{k=1}^{K}\sum_{i=1}^{N} \boldsymbol{\rho}_{ki}\big(\boldsymbol{\alpha}_{ki} - 1\big)\\
&+\sum_{k=1}^{K}\sum_{i=1}^{N} \boldsymbol{\xi}_{ki}\big(0 - \boldsymbol{\alpha}_{ki}\big) + \sum_{g=1}^{M} \boldsymbol{\psi}_g \left(\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} S_{kj}(i) \cdot I_{ij}^{g} - I_{\mathrm{TH}}^{g}\right)
\end{aligned} \tag{6}$$

where $\boldsymbol{\mu}_{kj}(i)$, ν_i, $\boldsymbol{\rho}_{ki}$, $\boldsymbol{\xi}_{ki}$ and $\boldsymbol{\psi}_g$ are the nonnegative Lagrangian multipliers.

The KKT conditions can be expressed as

$$\partial_{S_{kj}(i)}L = -C'_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right) - \boldsymbol{\mu}_{kj}(i) + \sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g = 0 \tag{7-1}$$

$$\partial_{\boldsymbol{\alpha}_{ki}}L = -\sum_{j=1}^{n_{\min}}\left[C_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right) - \frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}} \cdot C'_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right)\right] + \boldsymbol{\nu}_i + \boldsymbol{\rho}_{ki} - \boldsymbol{\xi}_{ki} = 0 \tag{7-2}$$

$$\boldsymbol{\mu}_{kj}(i) \cdot \left(0 - S_{kj}(i)\right) = 0 \tag{7-3}$$

$$\boldsymbol{\nu}_i \cdot \left(\sum_{k=1}^{K}\boldsymbol{\alpha}_{ki} - 1\right) = 0 \tag{7-4}$$

$$\boldsymbol{\rho}_{ki} \cdot (\boldsymbol{\alpha}_{ki} - 1) = 0 \tag{7-5}$$

$$\boldsymbol{\xi}_{ki} \cdot (0 - \boldsymbol{\alpha}_{ki}) = 0 \tag{7-6}$$

$$\boldsymbol{\psi}_g \cdot \left(\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} S_{kj}(i) \cdot I_{ij}^g - I_{\mathrm{TH}}^g\right) = 0 \tag{7-7}$$

From (7-1) and (4-1)

$$S_{kj}^*(i) = \boldsymbol{\alpha}_{ki} \cdot \left(\frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g - \boldsymbol{\mu}_{kj}(i)\right)} - \frac{N_0}{\lambda_{kj}^2(i)}\right) \tag{8}$$

Since $S_{kj}^*(i) \geq 0$ and $\boldsymbol{\alpha}_{ki} > 0$ hence

$$\frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g - \boldsymbol{\mu}_{kj}(i)\right)} \geq \frac{N_0}{\lambda_{kj}^2(i)} \tag{9}$$

If

$$\frac{N_0}{\lambda_{kj}^2(i)} \leq \frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g\right)}$$

then $\boldsymbol{\mu}_{kj}(i) = 0$ and therefore from (8) we have

$$S_{kj}^*(i) = \boldsymbol{\alpha}_{ki} \cdot \left(\frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g\right)} - \frac{N_0}{\lambda_{kj}^2(i)}\right) \tag{10}$$

If

$$\frac{N_0}{\lambda_{kj}^2(i)} > \frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g\right)}$$

Then from (9)

$$\frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g\right)} < \frac{N_0}{\lambda_{kj}^2(i)} \leq \frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g - \boldsymbol{\mu}_{kj}(i)\right)}$$

Which, implies that

$$\boldsymbol{\mu}_{kj}(i) > 0$$

and it can be conclude from (7-3) that

$$S_{kj}^*(i) = 0$$

Therefore from the considered case we can conclude that

$$S_{kj}^*(i) = \boldsymbol{\alpha}_{ki} \cdot \left[\frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g\right)} - \frac{N_0}{\lambda_{kj}^2(i)}\right]^+ \tag{11}$$

and consequently

$$P_{kj}^*(i) = \left[\frac{1}{Ln2 \cdot \left(\sum_{g=1}^{G}\boldsymbol{\psi}_g \cdot I_{ij}^g\right)} - \frac{N_0}{\lambda_{kj}^2(i)}\right]^+ \tag{12}$$

Now, let us define G_{ijk} according to (7-2) as follow

$$G_{ijk} = \sum_{j=1}^{n_{\min}}\left[C_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right) - \frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}} \cdot C'_{kij}\left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}}\right)\right] = \boldsymbol{\nu}_i + \boldsymbol{\rho}_{ki} - \boldsymbol{\xi}_{ki} \tag{13}$$

It can be concluded from (7-5) and (7-6) that

If $\boldsymbol{\alpha}_{ki} = 1$ then

$\boldsymbol{\xi}_{ki} = 0$ and $\boldsymbol{\rho}_{ki} \geq 0$

If $0 < \boldsymbol{\alpha}_{ki} < 1$ then

$\boldsymbol{\rho}_{ki} = 0$ and $\boldsymbol{\xi}_{ki} = 0$

Hence it can be conclude from (13) that

$$G_{ijk} = \begin{cases} \boldsymbol{\nu}_i & \text{if } 0 < \boldsymbol{\alpha}_{ki} < 1 \\ \geq \boldsymbol{\nu}_i & \text{if } \boldsymbol{\alpha}_{ki} = 1 \end{cases} \tag{14}$$

In view of the fact that only one user can be assigned to each sub-carrier, a proper sub-carrier assignment method is inspired by considering (14), as

$$\boldsymbol{\alpha}_{ki}^* = \begin{cases} 1 & \text{if } k^* = \arg\max_k (G_{ijk}) \\ 0 & \text{else} \end{cases} \tag{15}$$

Therefore, the main interpretation of (11) and (14) can be summarised as follows.

With the fixed $\boldsymbol{\psi} = [\psi_1, \psi_2, \ldots, \psi_g, \ldots, \psi_G]$ substituting (11) into (13) and searching among all users, joint sub-carrier assignment and power allocation can be done simultaneously, according to (14).

The remaining task is to find optimum value for $\boldsymbol{\psi}$. To do so, iterative methods can be used and the final optimum value of $\boldsymbol{\psi}^*$ can be obtained by updating the value of $\boldsymbol{\psi}$ at any step. By the present study, the ellipsoid method is used for finding the optimal value of $\boldsymbol{\psi}$ according the following discussions [21].

4.1 Ellipsoid method

Ellipsoid method is, in fact, the multi-dimensional extension of the bi-section method. In this method, an arbitrary ellipsoid is given in such a way that encloses the optimal value of the problem. After that by evaluating the sub-gradient at the centre of the ellipsoid, roughly half of the ellipsoid is eliminated in each iteration. The iterations continue until the desired accuracy is reached. Different steps of the ellipsoid method are summarised in Algorithm 1 with the following descriptions.

At the first step the initial ellipsoid is selected with the parameters $\boldsymbol{E}_0$ and $\boldsymbol{\psi}_0$ where $\boldsymbol{\psi}_0$ is a G dimensional matrix which indicates the centre of the ellipsoid and $\boldsymbol{E}_0$ is a $G \times G$ positive-definite matrix. Parameter t indicated different steps number. After selecting the initial ellipsoid, sub-gradient of the optimisation problem is evaluated at the centre of the given ellipsoid as $\boldsymbol{d}_t = [d_1, d_2, \ldots, d_g, \ldots, d_G]$ where

$$d_g = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{n_{\min}} S_{kj}(i) \cdot I_{ij}^g - I_{\mathrm{TH}}^g \tag{16}$$

After that by updating the ellipsoid parameters' value in each iteration, the loop is continued until the considered accuracy, $\boldsymbol{\varepsilon}$, is reached.

Therefore, by using Algorithm 1 and finding the optimal value for $\boldsymbol{\psi}$, joint sub-carrier and power assignment can be done such that the total capacity of the system is maximum while all the interference constraints are fulfilled. Different steps of the extracted resource allocation method can be summarised as Algorithm 2.

Algorithm 1: Different steps of the ellipsoid algorithm
Initialise $\boldsymbol{E}_0$, $\boldsymbol{\psi}_0$, $t = 1$
Repeating Loop

Evaluate $\boldsymbol{d}_t$ according to the (16)

$$\tilde{\boldsymbol{d}}_t = \frac{\boldsymbol{d}_t}{\sqrt{\boldsymbol{d}_t^{\mathrm{T}} \boldsymbol{E}_t^{-1} \boldsymbol{d}_t}}$$

$$\boldsymbol{\psi}_{t+1} = \boldsymbol{\psi}_t - \frac{1}{G+1} \boldsymbol{E}_t^{-1} \tilde{\boldsymbol{d}}_t$$

$$\boldsymbol{E}_{t+1}^{-1} = \frac{G^2}{G^2 - 1} \times \left(\boldsymbol{E}_t^{-1} - \frac{2}{G+1} \boldsymbol{E}_t^{-1} \tilde{\boldsymbol{d}}_t \tilde{\boldsymbol{d}}_t^{\mathrm{T}} \boldsymbol{E}_t^{-1} \right)$$

Stopping criteria

$$\sqrt{\boldsymbol{d}_t^{\mathrm{T}} \boldsymbol{E}_t^{-1} \boldsymbol{d}_t} < \boldsymbol{\varepsilon}$$

Algorithm 2: Ellipsoid method-based resource allocation steps
Initialise $\boldsymbol{\psi}_0$
Repeat

Find $\{P_{kj}(i)\}$ and $\boldsymbol{\alpha}_{ki}^{*}$ by solving (12) and (14).
Update $[\psi_1, \psi_2, \ldots, \psi_g, \ldots, \psi_G]$ using ellipsoid method

Until the convergence of $\boldsymbol{\psi}$
Return $\{P_{kj}(i)\}$ and $\boldsymbol{\alpha}_{ki}^{*}$

5 Additional aspects

Generally, in the CR networks the main limiting factor is the caused interference to the primary systems' bands, however in some cases the total power of the CR base-station's transmitter may be added to the existing limitations, say P_{total}. It means that, we should add another constraint to the (5) by which control the total power which is allocated to the CR sub-carriers. To do so, the optimisation problem can be rewritten as follow

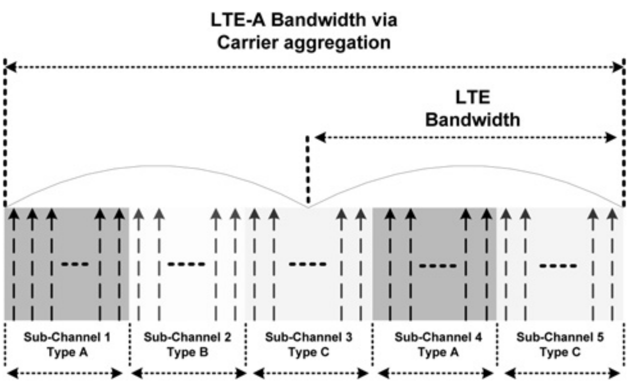

Fig. 4 *Distribution of the different kinds of DTV sub-channels in the LTE-A based aggregated spectrum domain*

$$\begin{aligned} &\text{maximise} \quad C = \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} \boldsymbol{\alpha}_{ki} \cdot C_{kij} \left(\frac{S_{kj}(i)}{\boldsymbol{\alpha}_{ki}} \right) \\ &\text{Subject to} \quad \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{n_{\min}} S_{kj}(i) \cdot I_{ij}^{g} \le I_{\text{TH}}^{g} \quad g = 0, 1, 2, \ldots, G \\ &\qquad \sum_{k=1}^{K} \boldsymbol{\alpha}_{ki} \le 1 \quad \forall \quad i, 0 \le \boldsymbol{\alpha}_{ki} \le 1 \quad \forall i, k \\ &\qquad S_{kj}(i) \ge 0 \quad \forall \quad i, j \text{ and } k \end{aligned} \tag{17}$$

where, $I_{ij}^{0} = 1 \quad \forall i, j$ and $I_{\text{TH}}^{0} = P_{\text{total}}$.

It can be seen that the optimisation problem (17) has no conflicts with the problem (5) and can be solved with the same approach.

5.1 Complexity analysis

Let us assume that the mean number of $n_{\min}$ is equal to the M, the number of secondary sub-carriers is N and the number of CR users is K. Finding the optimal solution for the considered problem with the fixed $\boldsymbol{\psi}$, using the (12) and (14) has the complexity of $\mathcal{O}$(KNM). In the updating procedure of $\boldsymbol{\psi}$ according to the Algorithm 1, $\mathcal{O}(G^2)$ operations are needed [21]. Therefore the total complexity of the introduced algorithm in this paper is $\mathcal{O}(\text{KNMG}^2)$. It can be seen that the proposed approach has polynomial times complexity which is acceptable for the practical implementations.

6 Simulation results and discussions

The performance of the proposed resource allocation scheme is evaluated in this section using computer simulation. The simulations are carried out for a MIMO-OFDMA CR with 3GPP-LTE-A downlink parameters.

Let us assume a scenario similar to Fig. 4, in which the CR base-station is designed based on the LTE-A structure with total 40 MHZ bandwidth, which can cover five neighbouring DTV channels. The CR base-station determines various white space using the method which was explained in the paper as indicated by Fig. 4. As this Fig. shows, in the considered spectrum domain, there are two active DTV sub-channels, which cannot be used by CR system and three sub-channels, including one type B sub-channel and two type C sub-channels. In the LTE standard, the distance between sub-carriers is fixed for all system's bandwidth equal to 15 KHz; therefore each DTV sub-channel with 8 MHz bandwidth contains around 533 sub-carriers.

The number of sub-carriers that can be used by the CR system in the considered scenario is $N = 1599$ and the number of sub-channels with the power limitation is $G = 4$.

Determining the exact value of DTV tolerable interference thresholds needs more studies and measurements. However, since the present study aims at evaluating the proposed algorithm, the values provided in [22] is used in the simulations with the following details.

It has been shown that the interfering signal level should be less than −92 dbm to prevent causing harmful interference into the DVT transmission. With regard to this value the tolerable interference threshold for the channels type A can be determined by considering the minimum distance between the DTV users and CR base-station. If this minimum distance is considered equal to 30 m, then by using the simple path loss model the path loss between the CR transmitter and DTV receiver becomes around 55 dbm. Therefore the maximum value for γ would be around −37 dbm.

The maximum transmittable power by the channel type C depends on the distance between the CR base station and the nearest DTV cell edge [(d-R) in Fig. 2]. Simulations have been carried out for different values of this distance.

Spatial channel model developed by 3GPP SCM which is highly recommended for evaluating the LTE, has been employed to simulate the channel gains(h) between CR base-station and CR users. Urban Macrocell environment has been selected. A linear array of N_{T} equispaced antennas at the CR base-station, an array

Table 1 Spatial channel model base on 3gpp-scm

Urban macro	
number of paths (U)	6
number of sub-path per–path (R)	20
antenna spacing at the transmitter	4λ
antenna spacing at the receiver	0.5λ
mean AS at BS	$E(\sigma_{AS}) = 15°$
AS at BS as a lognormal RV	$\mu_{AS} = 1.18$
$\sigma_{AS} = 10\wedge(\varepsilon_{AS}x + \mu_{AS}),\ x \sim \eta(0, 1)$	$\varepsilon_{AS} = 0.210$
$r_{AS} = \sigma_{AoD}/\sigma_{AS}$	1.3
per path AS at BS	2 deg
BS per-path AoD distribution	$\eta\left(0, \sigma^2_{AOD}\right)$ where $\sigma_{AoD} = r_{AS}\sigma_{AS}$
mean AS at user receiver	$E(\sigma_{AS,\ mobile}) = 68°$
per-path AS at user receiver	35°
AoA distribution at user receiver	$\eta\left(0, \sigma^2_{AOA}(Pr)\right)$
delay spread as a lognormal RV	$\mu_{DS} = -6.18$
$\sigma_{DS} = 10\wedge(\varepsilon_{DS}x + \mu_{DS}),\ x \sim \eta(0, 1)$	$\varepsilon_{DS} = 0.18$
mean total RMS delay spread	$E(\sigma_{DS}) = 0.65$ µs
$r_{DS} = \sigma_{delays}/\sigma_{DS}$	1.7
lognormal shadowing standard deviation	8 dB

of M antennas at each CR user's receiver and single antenna at the primary user's receiver have been considered.

Table 1 shows the main parameters of the channel used in our simulation. More details about the channel specification are given in [23].

First, effect of the amount of white space caused by the existing gap between DTV cells has been investigated. Therefore, in the given scenario, for the channel type A, the maximum tolerable interference is assumed to be constant and equal to −37 dBm. Then by changing the d-R value, the power allocation procedure for each of TV channels has been illustrated.

Fig. 5 shows the power allocation procedure for each TV channel with respect to different spaces. As it can be observed, the amount of allocated power to channels 1 and 4 for any given space is zero.

For d-$R = 500$ m, since the only limitation on the subcarriers of channel 2 is the interference constraint caused by the power leakage on the channels 1 and 4, thus, it can be observed that considerable amount of power has been allocated to the subcarriers of this channel. However, for channels 3 and 5, since, in addition to the said constraint, there is also a constraint associated with the TV channels, the amount of power allocated to their corresponding subcarriers is low because of their quite close distance to the TV channels.

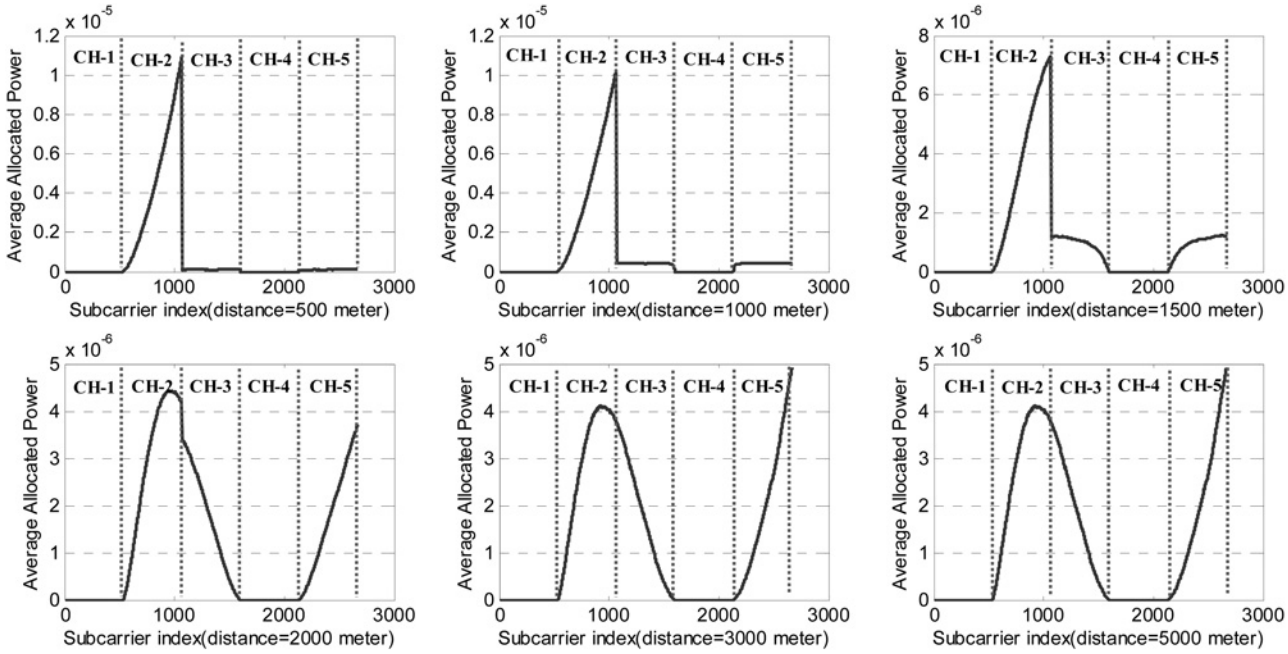

Fig. 5 *Power allocation procedure for each TV channel with respect to different space between DTV cells' edge and CR base-station*

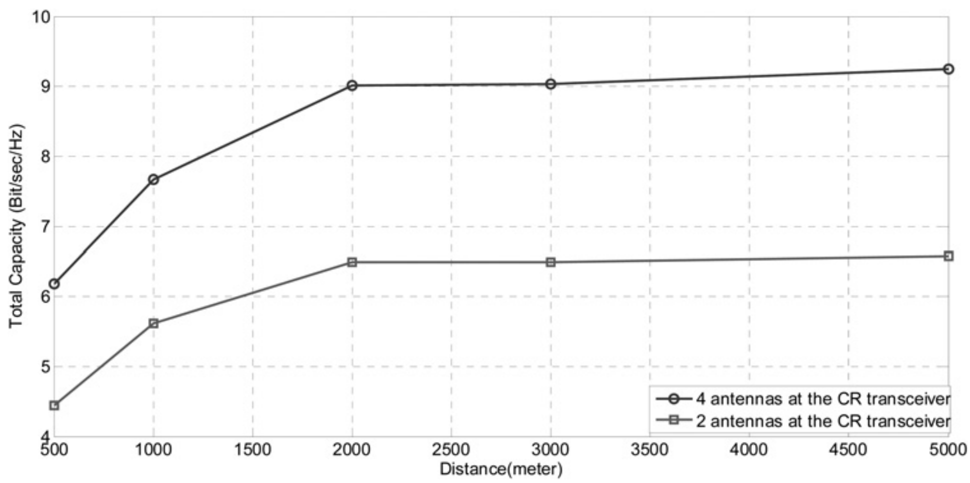

Fig. 6 *Effect of the distance from DTV cells on the total capacity of the CR system with the different number of antennas*

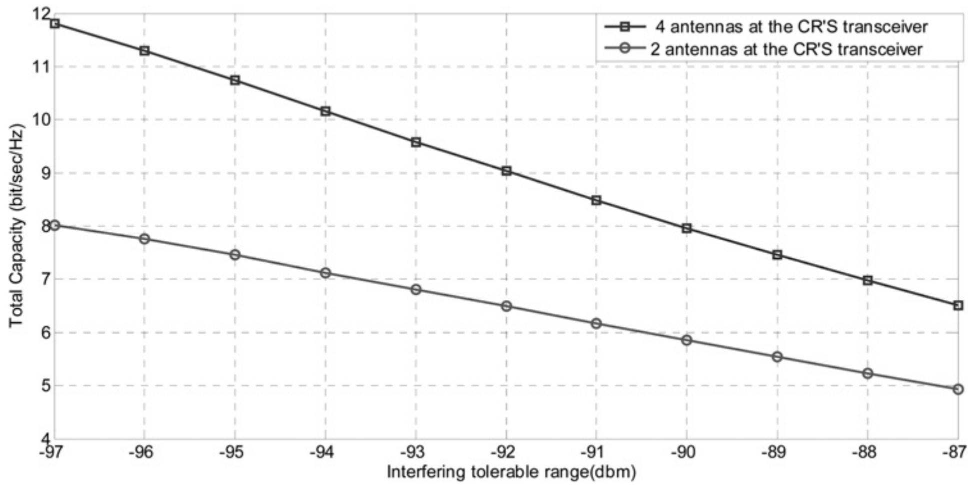

Fig. 7 *Effect of the TVS' interfering tolerable threshold on the total capacity of the CR system*

As the distance from the geographical boundary of TV cells increases to 1000, 1500 and 3000 m, it can be observed that the amount of power allocated to bands 3 and 5 gradually increases and is finally maximised at the distance of 3000 m.

As illustrated, for the distances above 3000 m, no significant variations in the amount of power allocated to CR subcarriers can be observed. It can be accounted for by the fact that after surpassing the said threshold, the major constraint is the constraint because of the interference imposed on bands 1 and 4. In other words, in spite of the fact that the restriction on the power allocation has been reduced in view of the geographical distance, the allocated power cannot exceed a given threshold because of the power leakage through the side lobes of the subcarriers of bands 1 and 4. This can be verified by the similarity of the powers allocated at the distances of 3000 and 5000 m.

Fig. 6 shows the procedure for changing the total system capacity through the variations in the distances from the geographical boundary of TV cells for 2×2 and 4×4 structures. Regarding the power allocation procedure explained in Fig. 5, it is observed that by increasing in the distance from the geographical boundary of TV cells up to around 3000 m, the total system capacity in both cases increases and then remains constant. Besides, through the investigation of the related diagrams for this Fig, it can be shown that by the increase in the number of antennas of the system, the total system capacity increases as expected.

As mentioned before, determination of the exact value of threshold for tolerable interference on the DTV receivers requires more studies and measurements. In the previous scenario, this value was considered to be −92 dBm. In the present section, to show the effect of this threshold limit on the total system capacity, the resultant capacity has been investigated with respect to the different values of the said threshold. Fig. 7 shows the trend for variations in the total system capacity in a range from −87 dBm to −97 dBm. Simulation has been performed for both 2×2 and 4×4 structures. It is clear that by the reduction in the amount of the considered threshold, which can also be interpreted as more interferences on the DTV receivers, the total capacity of the CR network increases.

7 Conclusions

In the present study, the possibility of using spectrum holes created by 'digital switch over' in the CR networks has been considered. Through investigation of different kinds of created white spaces, the limitations on the opportunistic exploitation of these sources have been examined. It has been shown that for optimal use of these sources by MIMO-OFDMA structure, an appropriate power allocation algorithm is required. In this context, the problem has been formulated as an optimisation problem, and then the proper algorithm for this structure has been extracted. Through simulation of the CR network based on the LET-A standard parameters and by consideration of real values for TV networks, the extracted algorithm has been evaluated. Simulation results verify the proper performance of our proposed algorithm.

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