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ORIGINAL ARTICLE

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The efficiency range of economical cutting conditions for a multistage transfer machine under a failure replacement strategy

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Abstract This paper presents models for calculating the optimal cutting feed rate and spindle speed at each stage in a multistage transfer machine. The optimal cutting conditions are determined by taking into account the cutting constraints for three objective functions, which are: minimum expected cycle time, minimum expected cost per unit, and maximum expected profit rate, using a one-dimensional search procedure. The efficiency range in which the optimal solutions for the three objective functions can be found is also analyzed. In addition, the optimal cutting conditions at each stage are compared to those of a stand-alone cutting machine.

Keywords Multistage machining system · Machining economics · Tool life · Replacement · Transfer machines

1 Introduction

The machining economics problem has been studied extensively for a single machine with a single tool. In practice, however, the conversion of raw materials into a product is rarely performed in a single process. Rather, it is processed through a series of multiple-stage operations, as are performed on a flow-type transfer machine, such as *Gnutti FMF*. In a transfer machine, the cycle time is the same for all the stages and there are no buffers between any two consecutive stages. In the present paper, a transfer machine is analyzed in which each part is processed through M production stages sequenced in a production-technological order, where each production stage maintains a single machine tool. The optimal cutting conditions for the multistage machining system is determined under three different objective functions—minimum expected cycle time, minimum expected cost per component, and maximum expected contribution to profit per unit of time (profit rate), taking into account cutting constraints. The optimization is done under the failure replacement strategy (FRS), where a tool is replaced only when a failure occurs. A proof that the number of stages does not affect the computational complexity of each problem is given, and a one-dimensional search procedure for optimizing each of the objective functions is suggested. In addition, the efficiency range of economical cutting conditions in which the optimal solution for the three objective functions can be found is presented and analyzed.

Hitomi [8] developed algorithms to calculate the optimal machining conditions for a multistage machining system. He assumed an unlimited buffer size between the machines. This paper, on the other hand, deals with a flow-type transfer machine with no buffers between the machine stages. Agapiou [1] described a mathematical algorithm for the optimization of the multistage machining system problem. Agapiou's goal was to minimize the unit cost under the physical limitations of the machines (maximum available cutting force, speed, and feed) for a given cycle time.

Our models are based on the economical trade-off between a short and a long tool life. A short tool life is uneconomical because of high tool replacement costs, while the use of low speed and low feed which increase the tool lives is uneconomical also because of a low production rate. The trade-off between tool life, feed, and speed is presented by Taylor's well known tool-life equation, $V\overline{T}^n f^m = C$, where $\overline{T}$ is the tool life; V is the cutting speed; f is the feed; and n, m, and C are constants. Recommended values for the parameters n, m, and C can be found in standard machining handbooks (e.g., Rembold et al. [12]). In most cases, $n<m<1$.

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The following assumptions are made for all of the models described in this paper:

1. The first machine is never "starved" because the demand is greater than the capacity of the transfer machine.
2. The revenue per component is not dependent on the production rate; that is, a linear revenue function is assumed. This assumption is commonly used when the profit rate criterion is under consideration (e.g., Hitomi [6–8]), and is very realistic for nonmonopolistic organizations. Cowton and Wirth [5] suggested a nonlinear revenue function to maximize profits, such as would be faced by a monopolist.
3. The number of passes that each tool performs is predetermined, and, without loss of generality, a single pass at each stage is performed.

The reminder of the paper is organized as follows. Section 2 presents the basic model, the cutting constraints, and the minimization of the expected cycle time function. In Section 3, an algorithm for minimizing the expected cost per unit function is presented. The optimization of the expected profit rate function is introduced in Section 4, and, in Section 5, the efficiency range of the feed rate at each stage is presented and analyzed. In Section 6, a numerical example is presented and analyzed. A summary concludes the paper.

2 Production rate criterion

2.1 Expected cycle time function

The expected cycle time of a multistage machining system under the FRS is the sum of two terms: the maximal production time (i.e., the production time of the bottleneck stage) and the expected tool changing time. It is given by the following equation:

$$E(Tp) = \max_{i=1,\ldots,M}(Tm_i + Te_i) + \sum_{i=1}^{M} Td_i \times \frac{Tm_i}{\overline{T}_i} \tag{1}$$

where M is the number of stages and $E(Tp)$ is the expected production time per component (min). At stage i, $\overline{T}_i$ is the expected tool life (min); Tm_i is the machining time per component (min); Te_i is the sum of the tool retract time, load, unload, and set-up time per component (min); and Td_i is the tool changing time (min).

The machining time for the turning, boring, and drilling operations is expressed as (Agapiou [1]):

$$Tm_i = \frac{\pi D_i \tilde{L}_i}{12 V_i f_i} \tag{2}$$

while for milling operations it is:

$$Tm_i = \frac{\pi D_i(\tilde{L}_i + \varepsilon_i)}{12 Nt_i V_i f_i} \tag{3}$$

where, at stage i, $\tilde{L}_i$ is the workpiece length to be machined (in); f_i is the feed (in/rev); D_i is the diameter of a generated surface or cutter (in); Nt_i is the number of inserts per milling cutter body; and ε_i is the overtravel of the milling cutter on the workpiece (in). We denote $L_i = \tilde{L}_i$ for turning, boring, and drilling operations, and $L_i = \frac{\tilde{L}_i + \varepsilon_i}{Nt_i}$ for milling operations.

The expected cycle time (Eq. 1) may be expressed in terms of the cutting parameters through the following relationships (see Agapiou [1] and Cakir and Gürarda [4]):

$$Tm_i = \frac{L_i}{Hf_i} \tag{4}$$

$$V_i = \frac{\pi D_i N_i}{12_i} \tag{5}$$

$$f_i = \frac{Hf_i}{N_i} \tag{6}$$

$$\overline{T}_i = C_i^{1/n_i} \times \left(\frac{\pi D_i N_i}{12}\right)^{-1/n_i} \times \left(\frac{Hf_i}{N_i}\right)^{-m_i/n_i} \tag{7}$$

where, at stage i, Hf_i is the feed rate (in/min) and N_i is the spindle speed (rpm).

By using Eqs. 2, 4, 5, 6 and 7, the expected cycle time function is rewritten as:

$$E(Tp) = \max_{i=1,\ldots,M}\left(\frac{L_i}{Hf_i} + Te_i\right) + \sum_{i=1}^{M} K_i \times Td_i \times Hf_i^{\frac{m_i - n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} \tag{8}$$

where:

$$K_i = L_i \times C_i^{-1/n_i} \times \left(\frac{\pi D_i}{12}\right)^{1/n_i} \tag{9}$$

The expected cycle time function has to be minimized under the various cutting constraints as described in the following section.

2.2 Cutting constraints

The maximum feed is restricted by its maximum permissible shear stress. For example, for a drilling operation, the maximum permissible feed, $f_{\max}$, can be given by the relationship (Bhattacherya and Ham [3]):

$$f_{\max} = C_s \times K_l \times D^{0.6} \tag{10}$$

where C_s depends upon the work material being drilled and K_l is a coefficient that takes the drilling hole length into account (for values of C_s as a function of the work material and K_l as a function of the drill length, see Bhattacherya and Ham [3]). Therefore (see Cakir and Gürarda [4]):

$$N_i \geq \max\left(N_{\min_i}, \frac{Hf_i}{f_{\max_i}}\right) \tag{11}$$

where $N_{\min_i}$ is the minimum value of the available spindle speed at stage i.

Since the selected feed rate of the drill head, Hf_i, has to be within the feasible feed rate range at each stage, Hf_i should satisfy the two following constraints:

$$Hf_i \leq Hf_{\max_i} \tag{12}$$

and:

$$Hf_i \geq Hf_{\min_i} \tag{13}$$

2.3 Minimum expected cycle time under the failure replacement strategy

Our objective is to determine the feed rate and spindle speed at each stage in order to minimize the expected cycle time function (Eq. 8) under the relevant cutting constraints (Eqs. 11, 12, 13). The problem can be formulated as:

$$\min E(Tp) = U + \sum_{i=1}^{M} K_i \times Td_i \times Hf_i^{\frac{m_i - n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} \tag{P1}$$

subject to:

$$\text{Constraints } [1,\ M]:\ U \geq \frac{L_i}{Hf_i} + Te_i \quad \forall i = 1, \ldots, M$$

$$\text{Constraints } [M+1,\ 2M]:\ N_i \geq N_{\min_i} \quad \forall i = 1, \ldots, M$$

$$\text{Constraints } [2M+1,\ 3M]:\ N_i \geq \frac{Hf_i}{f_{\max}} \quad \forall i = 1, \ldots, M$$

$$\text{Constraints } [3M+1,\ 4M]:\ Hf_i \geq Hf_{\min_i} \forall i = 1, \ldots, M$$

$$\text{Constraints } [4M+1,\ 5M]:\ Hf_i \leq Hf_{\max_i} \forall i = 1, \ldots, M$$

where $U_i = \frac{L_i}{Hf_i} + Te_i$ is the sum of the processing time, tool retract time, load, unload, and set-up time per component at stage i, i.e., U_i includes the total operation times at stage i per cycle in cases where no breakages occur. Therefore, $U = \max\limits_{i=1,\ldots,M}(U_i) = \max\limits_{i=1,\ldots,M}\left(\frac{L_i}{Hf_i} + Te_i\right)$ presents the production time of the bottleneck machine. From the constraints above, it is easy to observe that the U value is upper bounded by $U_{\max} = \max\limits_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\}$ and lower bounded by $U_{\min} = \max\limits_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\max_i}} + Te_i\right\}$.

Since the maximum between convex functions and the sum of the convex functions is a convex function itself, the expected cycle time is a convex function. In addition, all of the constraints are also convex functions. Thus, any feasible solution that satisfies the Karush Kuhn-Tucker (KKT) conditions is optimal for P1.

P1 is a convex programming problem that includes $2M$+1 variables (U, Hf_i, and N_i for i=1,..., M) and $5M$ constraints. In the following, we prove two different properties of the optimal solution that will help us to reduce both the number of variables and the number of constraints in P1.

Property 1 The minimal available spindle speed at each stage minimizes the expected cycle time.

Proof The derivative of the expected cycle time with respect to the spindle speed at stage i is positive for any chosen feed rate at the different stages. Thus, as shown in Eq. 14, the minimal available spindle speed is selected at each stage:

$$N_i = \max\left(N_{\min_i}, \frac{Hf_i}{f_{\max_i}}\right) \quad \forall i = 1,\ 2, \ldots,\ M \tag{14}$$

Property 2 Under the optimal solution of P1, either $U_i = U = \max\limits_{j=1,\ldots,M}\left(\frac{L_j}{Hf_j} + Te_j\right) \forall i = 1,\ 2, \ldots,\ M,$ or at least one of the feed rate boundary constraints (constraints [$3M$+1, $4M$]) is satisfied as an equality such that, in at least one stage, the processing is being done at the minimal feasible feed rate.

The proof of property 2 is straightforward from the KKT conditions.

From property 2, we get that the value of Hf_i is determined directly from the U value, as shown in Eq. 15:

$$Hf_i = \max\left(\frac{L_i}{U - Te_i},\ Hf_{\min_i}\right) \quad \forall i = 1,\ 2, \ldots,\ M, \tag{15}$$

By substituting Eq. 15 into Eq. 14 we get:

$$N_i = \max\left(N_{\min_i},\ \max\left(\frac{L_i}{U - Te_i}\right),\ \frac{Hf_{\min_i}}{f_{\max_i}}\right) \quad \forall i = 1,\ 2, \ldots,\ M, \tag{16}$$

i.e., the value of N_i is also directly determined from the U value, as shown in Eq. 16.

As a result of the above analysis, P1 can be reduced to a single-variable problem (U) with a single feasibility constraint as given below:

$$\min E(Tp(U)) = U + \sum_{i=1}^{M} K_i \times Td_i \times \max\left\{ \max\left(Hf_{\min_i},\ \frac{L_i}{U-Te_i}\right)^{\frac{m_i-n_i}{n_i}} \times N_{\min_i}^{\frac{1-m_i}{n_i}},\ \max\left(Hf_{\min_i},\ \frac{L_i}{U-Te_i}\right)^{\frac{1-n_i}{n_i}} \times f_{\max_i}^{\frac{m_i-1}{n_i}} \right\} \tag{P1.1}$$

subject to:

$$U_{\min} = \max_{i=1,\ldots,\ n}\left\{\frac{L_i}{Hf_{\max_i}} + Te_i\right\} \le U \le \max_{i=1,\ldots,\ n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\} = U_{\max}$$

Since the objective function of P1.1 is a convex function, the optimal solution for this problem is obtained by using any one-dimensional search procedure over all of the possible values of U. For example, according to the bisection method (Bazaraa and Shetty [2]), the optimal value of U can be found in $\log\left(\frac{U_{\max}-U_{\min}}{\varepsilon}\right)$ iterations to within an accuracy of ε.

In order to reduce the search interval, in the following, we prove that the optimal feed rate for the single-stage problem is an upper bound to the optimal feed rate at any stage in a multistage machining system, i.e., the optimal value of U for the single-stage problem is a lower bound to the optimal value of U in a multistage machining system.

Property 3 The optimal feed rate of the single-stage problem is an upper bound to the optimal feed rate at each stage of the multistage machining system.

Proof The expected cycle time function of the multistage machining system (the objective function of P1) can be rewritten as:

$$E(Tp) = \max_{k=1,\ldots,\ M}(E(Tp_k)) \tag{17}$$

where:

$$E(Tp_k) = U_k + \sum_{i=1}^{M} K_i \times Td_i \times Hf_i^{\frac{m_i-n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} \tag{18}$$

For the single-stage problem, with the same parameters as those of stage k in the multistage machining system, the expected cycle time is:

$$E\left(\tilde{T}p_k\right) = U_k + K_k \times Td_k \times Hf_k^{\frac{m_k-n_k}{n_k}} \times N_k^{\frac{1-m_k}{n_k}} \tag{19}$$

By inserting Eq. 19 into Eq. 18, we obtain:

$$E(Tp_k) = E\left(\tilde{T}p_k\right) + \sum_{i \ne k} K_i \times Td_i \times Hf_i^{\frac{m_i-n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} \tag{20}$$

For constant feed rates in stage $i{\neq}k$, if $Hf_k > Hf_{p_k}^{**}$, where $Hf_{p_k}^{**}$ is the optimal feed rate for the single-stage problem under the minimum expected cycle time criterion, $\frac{\partial E(Tp_k)}{\partial Hf_k} = \frac{\partial E\left(\tilde{T}p_k\right)}{\partial Hf_k} > 0$, $\frac{\partial E(Tp_k)}{\partial Hf_k} = \frac{\partial E\left(\tilde{T}p_k\right)}{\partial Hf_k} > 0$, and $\frac{\partial E(Tp_i)}{\partial Hf_k} = \left(\frac{m_i-n_i}{n_i}\right) \times K_i \times Td_i \times Hf_i^{\frac{m_i-2n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} > 0\ \forall i \ne k$. Thus, in order to minimize $E(Tp)$, Hf_k has to be reduced to at least $Hf_{p_k}^{**}$ (for obtaining the optimal cutting conditions for the single-stage problem, see Shabtay and Kaspi [13]).

Using property 3, the search interval for the optimal solution of P1.1 can be reduced, since the lower bound of this range becomes $U_{\min} = \max_{i=1,\ldots,\ n}\left\{\frac{L_i}{Hf_{p_k}^{**}} + Te_i\right\}$.

The analysis of this section is summarized by the following optimization algorithm for minimizing the expected cycle time.

Algorithm 1: minimizing the expected cycle time

Step 1

Determine the search interval $U_{\min} \le U \le U_{\max}$, where $U_{\min} = \max_{i=1,\ldots,\ n}\left\{\frac{L_i}{Hf_{p_k}^{**}} + Te_i\right\}$ and $U_{\max} = \max_{i=1,\ldots,\ n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\}$.

Step 2

(Search procedure) Determine the optimal U value that minimizes the objective in P1.1 by applying the bisection method (Bazaraa and Shetty [2]) within the search interval.

Step 3

Determine the optimal feed rate and spindle speed at each stage by using Eqs. 14 and 15.

3 Minimizing the expected cost per unit criterion under the failure replacement strategy

In most studies, the machining economics problem has to be optimized for the expected cost per unit criterion, since reduced production costs tend to increase profitability in the long run (e.g., La Commare et al. [11], Iakovou et al. [9], Jianqiang and Keow [10]). The expected cost per unit function is the sum of two terms: the labor cost per unit, which is proportional to the expected cycle time, and the tool failure cost. The expected cost per unit function for a multistage system is:

$$E(\text{cost}) = Co \times E(Tp) + \sum_{i=1}^{M} Ct_i \times \frac{Tc_i}{\overline{T}_i} \tag{21}$$

where Ct_i is the tool failure cost at stage i, which is the sum of the edge and the defective part costs ($), and Co is the operating cost ($/min). By inserting Eqs. 4, 5, 6, 7, 8 into Eq. 21, the expected cost as a function of the feed rate and spindle speed at each stage can be expressed as:

$$\begin{aligned} E(\text{cost}) = {} & Co \times \max_{i=1,\ldots,M}\left(\frac{L_i}{Hf_i} + Te_i\right) + \sum_{i=1}^{M} K_i \\ & \times Hf_i^{\frac{m_i-n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} \times (Co \times Td_i \times Ct_i) \end{aligned} \tag{22}$$

Our objective is to determine the feed rate and spindle speed at each stage in order to minimize the expected cost per unit (Eq. 22) under the relevant cutting constraints (Eqs. 11, 12, 13). The problem can be formulated as:

$$\begin{aligned} \min E(\text{cost}) & \\ = {} & Co \times U \\ & + \sum_{i=1}^{M} K_i \times Hf_i^{\frac{m_i-n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}} \times (Co \times Td_i \times Ct_i) \end{aligned} \tag{P2}$$

subject to:

$$\begin{aligned} &\text{Constraints } [1,\ M]:\ U \ge \frac{L_i}{Hf_i} + Te_i \quad \forall i = 1, \ldots, M \\ &\text{Constraints } [M+1,\ 2M]:\ N_i \ge N_{\min_i} \quad \forall i = 1, \ldots, M \\ &\text{Constraints } [2M+1,\ 3M]:\ N_i \ge \frac{Hf_i}{f_{\max}} \quad \forall i = 1, \ldots, M \\ &\text{Constraints } [3M+1,\ 4M]:\ Hf_i \ge Hf_{\min_i}\ \forall i = 1, \ldots, M \\ &\text{Constraints } [4M+1,\ 5M]:\ Hf_i \le Hf_{\max_i} \forall i = 1, \ldots, M \end{aligned}$$

Since P1 and P2 have the same formulation structure, it is easy to show that, P2, like P1, also obeys properties 1 and 2. As a result, P2 can be reformulated as:

$$\begin{aligned} &\min E(\text{cost}(U)) \\ &= Co \times U + \sum_{i=1}^{M} K_i \times (Co \times Td_i \times Ct_i) \times \max\left\{ \max\left(Hf_{\min_i}, \frac{L_i}{U - Te_i}\right)^{\frac{m_i-n_i}{n_i}} \times N_{\min_i}^{\frac{1-m_i}{n_i}},\ \max\left(Hf_{\min_i}, \frac{L_i}{U - Te_i}\right)^{\frac{1-n_i}{n_i}} \times f_{\max_i}^{\frac{m_i-1}{n_i}} \right\} \end{aligned} \tag{P2.1}$$

subject to:

$$\begin{aligned} U_{\min} = \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\max_i}} + Te_i\right\} \le U \\ \le \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\} = U_{\max} \end{aligned}$$

Since the objective function of P2.1 is a convex function, the optimal solution is obtained by using any one-dimensional search procedure. Actually, the search interval for the optimal solution for P2.1 can be reduced by using a tighter lower bound, $U_{\min} = \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{c_i}^{**}} + Te_i\right\}$, where $Hf_{c_k}^{**}$ is the optimal feed rate for minimizing the expected cost per unit in a single-stage cutting machine with the same parameters as those of stage k in the multistage machining system.

The analysis of this section is summarized by the following optimization algorithm for minimizing the expected cost per unit.

Algorithm 2: minimizing the expected cost per unit

Step 1

Determine the search interval $U_{\min} \le U \le U_{\max}$, where $U_{\min} = \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{c_i}^{**}} + Te_i\right\}$ and $U_{\max} = \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\}$.

Step 2

(Search procedure) Determine the optimal U value that minimizes the objective in P2.1 by applying the bisection method (Bazaraa and Shetty [2]) within the search interval.

Step 3

Determine the optimal feed rate and spindle speed at each stage by using Eqs. 14 and 15.

4 Maximization of the expected profit rate under the failure replacement strategy

The maximum expected profit rate has been found to be the best criterion for optimization of the machining economics problem (Hitomi [7] and Cowton and Wirth [5]). It is defined as:

$$E(\Omega) = \frac{r - E(\text{cost})}{E(Tp)} \tag{23}$$

where r is the revenue per component. Inserting Eqs. 4, 5, 6, 7, 8 and 22 into Eq. 23, the expected profit rate as a function of the feed rate and spindle speed at each stage can be expressed as:

$$E(\Omega) = \frac{r - \sum_{i=1}^{M} Ct_i \times K_i \times Hf_i^{\frac{m_i - n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}}}{\max_i\left(\frac{L_i}{Hf_i} + Te_i\right) + \sum_{i=1}^{M} K_i \times Td_i \times Hf_i^{\frac{m_i - n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}}} - Co \tag{24}$$

Our goal is to determine the feed rate and the spindle speed at each stage in order to maximize Eq. 24 under the constraints for P1 and P2. The problem is formulated as:

$$\max E(\Omega) = \frac{r - \sum_{i=1}^{M} Ct_i \times K_i \times Hf_i^{\frac{m_i - n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}}}{U + \sum_{i=1}^{M} K_i \times Td_i \times Hf_i^{\frac{m_i - n_i}{n_i}} \times N_i^{\frac{1-m_i}{n_i}}} \tag{P3}$$

and is subject to the same constraints as those of P1 and P2.

It is easy to prove that P3 also obeys properties 1 and 2. Thus, P3 is reformulated as a one-decision variable problem (U):

$$\max E(\Omega) = \frac{r - \sum_{i=1}^{M} Ct_i \times K_i \times \max\left\{\max\left(Hf_{\min_i}, \frac{L_i}{U-Te_i}\right)^{\frac{m_i-n_i}{n_i}} \times N_{\min_i}^{\frac{1-m_i}{n_i}},\ \max\left(Hf_{\min_i}, \frac{L_i}{U-Te_i}\right)^{\frac{1-n_i}{n_i}} \times f_{\max_i}^{\frac{m_i-1}{n_i}}\right\}}{U + \sum_{i=1}^{M} K_i \times Td_i \times \max\left\{\max\left(Hf_{\min_i}, \frac{L_i}{U-Te_i}\right)^{\frac{m_i-n_i}{n_i}} \times N_{\min_i}^{\frac{1-m_i}{n_i}},\ \max\left(Hf_{\min_i}, \frac{L_i}{U-Te_i}\right)^{\frac{1-n_i}{n_i}} \times f_{\max_i}^{\frac{m_i-1}{n_i}}\right\}} \tag{P3.1}$$

subject to:

$$U_{\min} = \max_{i=1,\dots,n}\left\{\frac{L_i}{Hf_{\max_i}} + Te_i\right\} \le U \le \max_{i=1,\dots,n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\} = U_{\max}$$

Under the assumption that there is at least one point for which $r > E(\text{cost})$, we can conclude that the expected profit rate is a unimodal function, since the numerator of the derivative of the expected profit rate function with respect to U is a decreasing non-continuous monotonic function. Thus, any one-dimensional search procedure can be used to maximize the expected profit rate.

The analysis of this section is summarized by the following optimization algorithm for maximizing the expected profit rate.

Algorithm 3: maximizing the expected profit rate

Step 1

Determine the search interval $U_{\min} \le U \le U_{\max}$, where $U_{\min} = \max_{i=1,\dots,n}\left\{\frac{L_i}{Hf_{\max_i}} + Te_i\right\}$ and $U_{\max} = \max_{i=1,\dots,n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\}$.

Step 2

(Search procedure) Determine the optimal U value that maximizes the objective in P3.1 by applying the bisection method (Bazaraa and Shetty [2]) within the search interval.

Table 1 Characteristics for the various operations

Operation number	Type of operation	Length (L_i (in))	Diameter (D_i (in))
1	Turning	8	3
2	Turning	5	2.6
3	Turning	3	2.6
4	Turning	5	2.5
5	Drilling	3	0.5
6	Drilling	1.3	0.35
7	Tapping	1	0.5

Step 3
Determine the optimal feed rate and spindle speed at each stage by using Eqs. 14 and 15.

5 The efficiency range

Let us define the efficiency range of economical cutting conditions as follows (this definition is an extension of Hitomi's definition [7]):

$$E = \begin{cases} \left[U_c^*,\ U_p^*\right] \text{ if } U_c^* < U_p^* \\ U_p^* \qquad\quad \text{if } U_c^* = U_p^* \\ i = \left[U_p^*,\ U_c^*\right] \text{ if } U_c^* > U_p^* \end{cases} \tag{25}$$

where U_c^* is the optimal solution for the expected cost per unit criterion and U_p^* is the optimal solution for the expected cycle time criterion. In the following properties 4–6, we prove that the efficiency range is $E = \left[U_p^*,\ U_c^*\right]$ and that the optimal solution for the maximum expected profit rate criterion is within the efficiency range.

Property 4 $U_p^* \leq\ U_c^*$, that is, $E = \left[U_p^*,\ U_c^*\right]$.

Proof In Sections 2 and 3, we proved that the expected cycle time and cost per unit are single-variable (U), convex, non-differentiable functions. Thus, the minimum expected cycle time occurs at either a differentiable or a non-differentiable point. Since, at any differentiable point, $\frac{dE(cost(U))}{dU} < Co \times \frac{dE(Tp(U))}{dU}$, if U_p^* is at a differentiable point, then $U_c^* >\ U_p^*$. Otherwise, if U_p^* is at a non-differentiable point, then $\lim\limits_{U \to U_p^{*-}} \frac{dE(Tp(U))}{dU} < 0$ and $\lim\limits_{U \to U_p^{*-}} \frac{dE(cost(U))}{dU} < Co \times \frac{dE(Tp(U))}{dU} < 0$. Hence, we can conclude that $U_p^* \leq\ U_c^*$.

Property 5 There is at least one point where a positive profit rate occurs within the efficiency range.

Proof Under the assumption that there is at least one point of positive profit rate (otherwise, the optimal policy is to do nothing), it is obvious that the solution that minimizes the expected cost per unit is also a solution with a positive profit rate. Since the point of minimum expected cost per unit is within the efficiency range, then the property is proved.

Property 6 The optimal solution for the expected profit rate criterion, U_Ω^*, is within the efficiency range.

Proof Outside the efficiency range, a value of U can be found in one of two ranges:

1. The range where $U \leq U_p^*$. In this range, $E(cost(U + \Delta)) < E(cost(U))$ and $E(Tp(U + \Delta)) < E(Tp(U))$. If the profit rate is negative, then, according to property 5, there is at least one point where a positive profit rate occurs within the efficiency range. Otherwise, if the profit rate is positive, it is worthwhile to increase U to at least U_p^*, which reduces

Table 2 Tool-life parameters for the various operations

Operation number	Tool material	n_i	m_i	Constant (K_i)
1	Uncoated carbide	0.25	0.29	2.18×10^{-9}
2	Uncoated carbide	0.25	0.29	7.69×10^{-10}
3	Uncoated carbide	0.25	0.29	4.61×10^{-10}
4	Uncoated carbide	0.25	0.29	6.57×10^{-10}
5	High-speed still	0.1	0.5	5.87×10^{-22}
6	High-speed still	0.1	0.5	7.18×10^{-24}
7	High-speed still	0.1	0.5	1.96×10^{-22}

Table 3 Cutting constraints

Operation number	Spindle speed limitation ($N_{\min_i}$ (rev/min))	Feed limit ($f_{\max_i}$ (in/rev))	Feed rate (in/min) limits	
			Lower	Upper
1	80	0.03	1	50
2	80	0.03	1	50
3	80	0.03	1	60
4	80	0.03	1	80
5	40	0.01	0.2	20
6	40	0.01	0.2	25
7	40	0.01	0.2	25

Table 4 Economic and time parameters

Operation number	Te_i (min)	Td_i (min)	Ct_i ($)
1	0.25	0. 5	2
2	0.235	0. 5	2
3	0.25	0. 5	2
4	0.25	0. 5	2
5	0.225	2	4
6	0.225	2	4
7	0.225	2	4

Table 5 Optimal cutting conditions for the three objective functions under the failure replacement strategy

Criteria parameter	Expected cycle time	Expected cost per unit	Expected profit rate
U^* (min)	**0.701**	**1.215**	**0.768**
Hf_1^* (in/min)	17.76	8.29	15.44
N_1^* (rpm)	591.87	276.40	514.52
Hf_2^* (in/min)	10.74	5.10	9.38
N_2^* (rpm)	358.00	170.11	312.53
Hf_3^* (in/min)	6.66	3.11	5.79
N_3^* (rpm)	221.95	103.65	192.95
Hf_4^* (in/min)	11.10	5.18	9.65
N_4^* (rpm)	369.92	172.75	321.57
Hf_5^* (in/min)	6.31	3.03	5.52
N_5^* (rpm)	630.84	303.10	552.20
Hf_6^* (in/min)	2.73	1.31	2.39
N_6^* (rpm)	273.37	131.34	239.29
Hf_7^* (in/min)	2.10	1.01	1.84
N_7^* (rpm)	210.28	101.03	184.07
Ex. cycle time (min)	**0.851**	1.230	0.867
Ex. cost ($/unit)	0.770	**0.307**	0.568
Ex. profit ($/min)	4.972	3.815	**5.112**

the expected cycle time and the expected cost per unit, and, therefore, increases the expected profit rate.

2. The range where $U \geq U_c^*$. In this range, $E(cost(U+\Delta)) > E(cost(U))$ and $E(Tp(U+\Delta)) > E(Tp(U))$. If the profit rate is negative, then, according to property 5, there is at least point where a positive profit rate occurs within the efficiency range. Otherwise, if the profit rate is positive, it is worthwhile to reduce U to at least U_c^*. This procedure reduces the expected cycle time and the expected cost per unit, and, therefore, increases the expected profit rate.

6 Numerical example

In this section, we illustrate our method by using a numerical example. Due to difficulties in obtaining data from a practical process, we took the relevant data from the numerical example given by Agapiou [1], as described in the following.

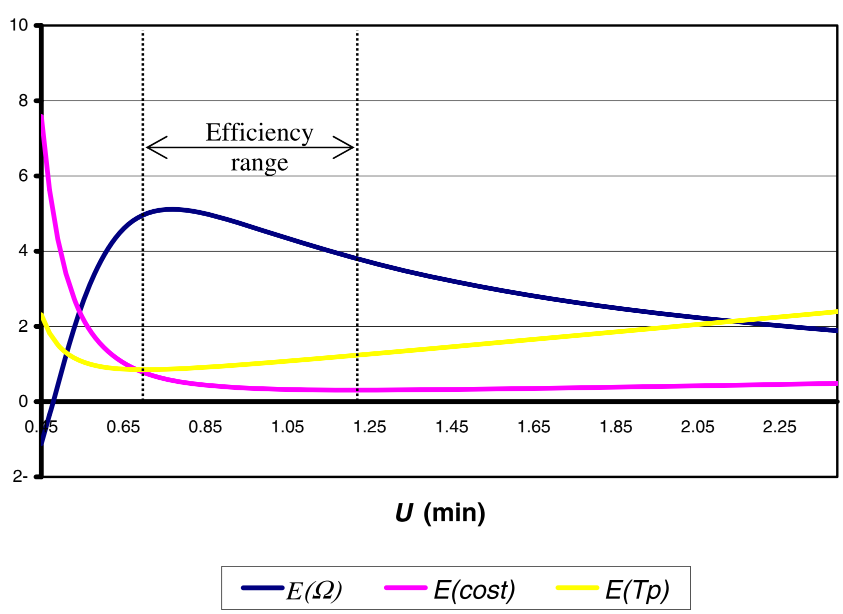
A simple part made of gray cast iron material is considered (the part is presented in Fig. 1 in Agapiou's paper). The machining process of this part is simulated in a transfer line consisting of seven stations. The processing operations and the part processing dimensions are given in Table 1 (note that the original data in Agapiou's paper is given in mm, but we convert it into in).

The first four stations provide a turning operation and the following two stations accomplish the drilling operation. The final operation is done in the tapping station. The tool-life data for all of the operations are provided in Table 2 (the conversion needed from Agapiou's data [1] to ours is given by the following relations: $n_i = -\frac{1}{a_{1i}}, \quad m_i = -\frac{a_{2i}}{a_{1i}}$, and $C = R^{n_i}$).

The feed, feed rate, and spindle speed limitations are given in Table 3 (again, some adjustments were required), and Table 4 provides the tool failure cost data (Ct_i), the tool replacement time (Td_i), and the sum of the tool retract time, load, unload, and set-up time per component (Te_i) in each stage. We assume that the revenue per component is r=5 and that the operation cost is Co=0.2 $/min.

From the data given above, the search interval is lower bounded by $U_{\min} = \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\max_i}} + Te_i\right\} = \max\left\{\frac{8}{50}+0.25,\ \frac{5}{50}+0.235,\ \frac{3}{60}+0.25,\ \frac{5}{80}+ 0.25,\ \frac{3}{20}+0.225,\ \frac{1.3}{25}+0.225,\ \frac{1.3}{25}+0.225,\ \frac{1}{25}+0.225\right\} = 0.41$ and upper bounded by $U_{\max} = \max_{i=1,\ldots,n}\left\{\frac{L_i}{Hf_{\min_i}} + Te_i\right\} =$

Fig. 1 The expected cycle time, cost per unit, and profit rate as a function of U



$\max\left\{\frac{8}{1}+0.25,\ \frac{5}{1}+0.235,\ \frac{3}{1}+0.25,\ \frac{5}{1}+0.25,\ \frac{3}{1}+0.225,\ \frac{1.3}{0.2}+0.225,\ \frac{1.3}{0.2}+0.225,\ \frac{1}{0.2}+0.225\right\}=8.25.$

The results of the numerical example for the three objective functions (obtained by applying algorithms 1–3) are summarized in Table 5.

The efficiency range of the economical cutting condition is $E=\{0.701,\ 1.215\}$. Since the optimal profit rate is positive, the optimal solution for the maximum expected profit rate criterion is within the efficiency range. Figure 1 depicts the expected cycle time, cost per unit, and profit rate as a function of the decision variable (U). As shown in the figure, the three functions are insensitive to changes in U (therefore, they are also insensitive to changes in the feed rates) within the close neighborhood of the optimal solution. This finding was observed for a wide range of numerical examples examined.

7 Summary

This paper presented and solved models to determine the optimal cutting feed rates and spindle speeds for the multistage machining system under the failure replacement strategy (FRS) by taking into account the cutting constraints for three different objective functions, which are: minimum expected cycle time, minimum expected cost per unit, and maximum expected profit rate. Although all of these three problems are multivariable, each was converted to a single-variable problem. The first two objective functions were shown to be convex, while the third objective function was shown to be unimodal. Thus, for each criterion, the optimal solution was obtained by using a one-dimensional search procedure, independent of the number of stages. The efficiency range in which the optimal solutions for the three objective functions can be found was described and analyzed. The optimal feed rate of any stage in a multistage machining system is found to be an upper bound to the optimal feed rate of each stage that operates as a stand-alone cutting machine. Finally, a numerical example illustrated that the three objective functions are insensitive to changes in the feed rate within the close neighborhood of the optimal solution.

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