How to better understand the diverse mathematical thinking of learners

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In this article Jessica Hunt explores the use of clinical interviews to gain a deep understanding of students' knowledge. Examples of clinical interviews are provided and advice for planning, giving and interpreting the results of interviews is also included.

How can primary school teachers use assessment as a foundation from which to increase access and opportunities for success in learning and understanding mathematics for all children, especially those who struggle to understand mathematics? One way to do this is to utilise assessment as a gauge and diagnosis of a child's present and potential knowledge. In this article, we describe clinical interviews in terms of how they can be used to gain an understanding of children's mathematical thinking with examples from our own work to illustrate three stages (i.e., planning, conducting, and interpreting) of clinical interviews. In this way, we provide teachers with a practical framework to uncover children's current and potential levels of understanding of a mathematical concept. First, notions of assessment are defined through clinical interviews as part of a formative, diagnostic assessment framework from which teachers can determine children's current and potential mathematical understandings. Next, we take a look at an example of a clinical interview constructed to uncover children's understanding of fraction concepts, illustrated with examples from research showing how to prepare and give clinical interviews. Final thoughts are given as a catalyst to further assist teachers in using aspects of clinical interviews as a basis for instruction and intervention.

What are clinical interviews and why use them?

Like standardised forms of assessment, clinical interviews are not novel. Clinical interviews were developed as a means to explore evidence of children's thinking (Ginsburg, 1997). That is, they help teachers uncover how children think and solve problems, develop and test inferences about children's thought processes, and interpret thinking or understanding in the context of children as holistic individuals. In this way, clinical interviews provide a respectful, flexible means to better understand not only what a child's present performance in mathematics content is, but also why the child may be performing as observed or assessed. A teacher's understanding of the 'why' behind children's thinking provides guidance when planning instruction and interventions to cultivate mathematical conceptions (Vygotsky, 1978). Thus, using clinical interviews as part of teachers' instructional toolboxes can help them begin instruction from a rich picture of children's diverse mathematical thinking such that we can help them progress in their understanding.

Clinical interviews can compliment information from standardised assessments to provide teachers with a holistic account of a child's present and potential understanding of mathematics. Clinical interviews are flexible and include the following defining features (Ginsburg):

a) An initial plan for problem tasks, the mathematical tools students may use, and the questions (e.g., How did you solve it? Why?) an educator might utilise to gain insight into children's thinking of a specific mathematics content;
b) A flexible plan for respectful implementation; that is, a means in which to interpret children's thinking and interpret/probe the meaning of children's responses along with an anticipation of the need to alter to more fully understand and be respectful of children's thinking; and
c) A way in which to document what was found out about the child's thinking to guide the planning of instruction.

In the following paragraphs, we will use these key features as a framework (see Figure 1) to discuss how to plan for, give, and interpret results of clinical interviews.

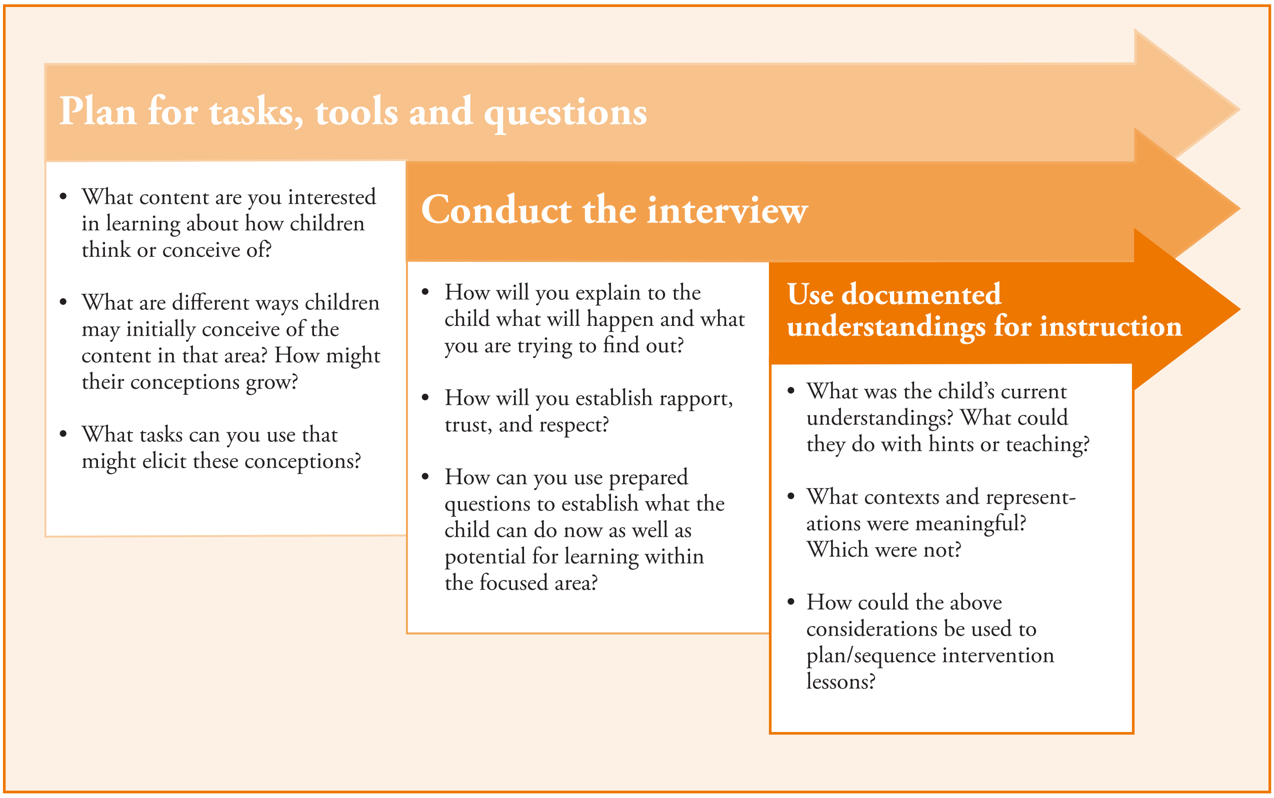

Figure 1. A framework for planning and conducting the clinical interview.

Table 1. Curriculum standards across Years 2, 3, and 4 addressed in our interviews.

Year	Standards (Main and Associated)
Year 1	**Main:** Recognise and describe one-half as one of two equal parts of a whole (ACMNA016).
Year 2	**Main:** Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (ACMNA033).
Year 3	**Main:** Model and represent unit fractions including $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}$ and their multiples to a complete whole (ACMNA058).
Year 4	**Main:** Count by quarters, halves and thirds, including with mixed numerals (ACMNA078). Compare and order common unit fractions (ACMNA102). **Associated:** Investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9 (ACMNA074).

Planning, conducting, and interpreting clinical interviews

Plan for Tasks, Tools, and Questions

Before a clinical interview can take place, teachers must have a plan for the tasks and mathematical tools children will work with, as interviews should be directed by children's thinking within topics the educator believes are important to study. This requires the teacher to know: a) what content/standards/skills they want to talk to the child about to gain information about the child's current understandings and b) some idea, or theory, about what children's notions of content/standard/skill begin and evolve over time. A practical starting place for teachers to consider in terms of the content are the curriculum standards used in their schools (i.e., *Australian Curriculum*, Australian Curriculum, Assessment and Reporting Authority, 2012).

For example, the clinical interview illustrated within this article (i.e., notions of unit and non-unit fractions as quantities) addresses concepts and skills that fall under standards in Years 2, 3, and 4 (see Table 1). We looked at standards across year levels to get a holistic sense of what we might look for in terms of children's understanding within a developmental progression (i.e., prerequisite, current, and potential conceptions, representations, and operational aspects of unit fractions and their multiples). These standards can be found here: (http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1).

Curriculum gives teachers a snapshot of children's understanding at different points in time (i.e., years). To better understand how children's knowledge grows from year to year, it can also be helpful to consult related or different resources that supply a more extended conversation of how thinking might evolve in the targeted area. For example, resources such as Empson and Levi's (2011) work on children's problem solving strategies for equal sharing problems (equally sharing some number of same sized objects among some number of people, where the result is a unit fractional quantity) provide a picture of how children's knowledge may grow while also addressing their notion of fractional quantity in terms of parts and wholes as well as in terms of division and multiplication.

For instance, consider a child's strategy in solving 14 sticks of clay shared by 4 people. In a beginning strategy, the child might draw out 14 sticks of clay and four people, share, or 'deal out', the sticks of clay, one by one, to each person until there were two left and notice they had two 'leftovers'. At this point, the child might:

- Say there are not enough to share the rest with no attempt to partition the remaining sticks.
- Partition each leftover stick in half, creating four halves, and give one to each sharer.
- Partition each leftover stick into fourths, creating eight fourths, and give two fourths to each sharer.

Each of these scenarios gives teachers a window into the child's understanding of and ability to "Model and represent unit fractions including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$ and their multiples to a complete whole". For instance, the child who cuts each leftover stick in halves might be asked, "How much does each person get?" If the child says, "four" (i.e., 3 whole sticks of clay and 1 half stick of clay), the interviewer might ask, "Four what? Four wholes?" along with, "How many pieces did you cut those leftover sticks into? What do we called each piece? Why? How many halves fit back inside one whole?"

Conversely, a child who cuts each leftover stick into fourths and calls the quantity "Three and two more" might be asked how to better name the "two more" (i.e., "You cut this stick into four pieces. What is each of those pieces called? Why? How many fourths fit back inside one whole? What might we call the total share?"). In a more advanced strategy, a child might partition each stick into four. Similar questions and/or observations can be utilised by the teacher to assess the child's propensity to not only model the fourths but also to reproduce the whole in their responses (e.g., Did the child count by fourths as they quantified? Did the child see that four-fourths recreated one whole?).

Once teachers identify resources that yield a picture of how children's thinking looks differently over time, they can prepare clear and specific problem tasks to elicit this thinking. For example, if designing tasks to elicit understanding of addition, Ginsburg (1997) says: "Do not simply ask, 'How do you add?' A more effective approach would be to present the child with an addition problem to be solved and

Open	• How do you know? • How would you show it? • Can you help me picture it? (stay silent) • Can you do it out loud? • How did you figure it out?
Mid	• What do you mean? • I didn't quite understand what you meant. Can you help me? (Repeat child's words back to them) • Say more about that. (stay silent)
Focused	• 'On the fly' questions on child's specific strategy. • Offer a counter suggestion (e.g., I worked with a child the other day and he/she said... What do you think about that?)

Figure 2. Open-ended and specific questioning in clinical interviews.

then ask about the methods just used in dealing with those specific numbers (p. 123)". Problem tasks should be varied in nature (i.e., contextualised versus numeric) and placed in everyday, familiar situations. Have mathematical tools, such as linking cubes, counters, or other tangible objects available for children to use, if they elect, to show their thinking. The child's actions on the objects along with gestures and other nonverbal communication are windows into understanding.

Problem tasks are written to elicit thinking; questions are prepared to accompany tasks and encourage the child to elaborate and provide full explanation of their thoughts. Prepare open-ended and focused questions to accompany each task, progressing from open-ended toward more specific questions. Sometimes repeating back a child's words to them encourages them to say more. Figure 2 provides teachers with ideas of open-ended and more specific questions to consider when planning a clinical interview, with the more open-ended ones variations of the essential question, "How did you solve it?"

For example, in our work, we prepared a series of six initial tasks based in story contexts that would produce solutions involving unit and non-unit fractions. We took care to make the first task one in which we believed children would be successful: in some cases, we simply asked children to count as high as they could to provide an accessible task. We provided children with linking cubes, rectangular papers that can be torn or cut, and pencil/paper as mathematical tools they could utilise in their thinking. We prepared a series of open-ended and specific questions to utilise as children solved each problem. This became a starting point from which we began each interview (see Figure 3). Here, we provided a space such that we could jot down notes about children's strategies, representations (i.e., tool used), gestures/talk, and any additional talk or questions used during the interview. This starting point is flexible; teachers may use all or some of the prepared tasks. Also, additional tasks that were not initially prepared but seemed appropriate given the child's thought processes can be used during the interview. Teachers are not limited in clinical interviews and should be ready to alter questions 'on the fly' to learn as much as possible about the child's conceptual learning related to the goal.

Conduct the Interview

When tasks, tools and questions are ready, it is time to conduct the interview. Teachers can begin by explaining to the child what will happen in the interview and why it is important.

The best way to do this is to state, simply and honestly, the purpose for the interview. Getting any child, especially those who may struggle to understand mathematics, to show you their thinking can be difficult. For instance, children might believe that it is not their role to show their thinking in mathematics but instead to mimic the thinking of the teacher. These perceptions could affect the way in which the child initially interacts with the mathematics tasks and the educator. An example of how to begin interviews with children is supplied in Figure 4.

Sample Problem Task	Questions to Ask (general)	Questions to Ask (specific)	Child's Representations /Strategies	Child's Gestures/ Talk	Teacher Talk/ Interactions
Lidia and Jerome shared 5 soft tacos so that each of them got the same amount to eat. How many tacos did Lidia and Jerome each eat if they finished all of the tacos?	How did you figure it out? Can you help me picture it? What if they want to [eat/share] all of it (if shares not exhausted)? Say more about what you meant there. They want to share this, too (if the shares are not exhausted). What if they each want the same amount (if shares are uneven)? I worked with a child earlier and they showed me this… what do you think?	How much did each person receive (what was their share)? Can you count that out for me? [If child quantifies as whole number] Four what? Four wholes? You split those into [two, four] pieces. What do we call each piece? Are halves and fourths the same thing/size? Why/why not? How many [halves/fourths] fit back inside the whole?			
14 sticks of clay are shared among 4 children for a project. How much clay does each child receive?					
4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?					

Figure 3. A sample preparation for a clinical interview on unit/non-unit fractions.

Hi my name is ______________. I like to learn about how kids just like you think about math problems. I call the problems "math riddles". Can you help me out? We are going to work with several math riddles. I will give you several to work with. You may think some of my riddles are really easy, and you may think others are really tough. Solve each one the best that you can. As you think about each one, you can use any of these tools [reference the counters, linking cubes, paper rectangles, base ten materials, pencil/paper] that you like. You can also use your fingers as tools! There are a lot of different ways to think about each math riddle and I am not looking for a "correct" or "incorrect" answer. I am just interested in how you solve them. If you are not sure how to solve one, just let me know. Sometimes I might ask you questions so that I can understand what you mean a little better. Are you ready to get started?

Figure 4. Starting the interview.

Determining learning potential

The questions prepared for the clinical interview are used during implementation to determine the child's current understanding. Yet, often, teachers wish to also assess learning potential. In these instances, teachers can use multiple types of scaffolds (hints or even instances of teaching) to clarify or even push children's understanding toward a new skill and see what they do. (See Figure 5).

- Ask for help: play the role of another child and ask the child to help you understand how to solve the math problems.
- Be authentically excited about the child's responses; encourage/ acknowledge effort, not right or wrong. Be specific with feedback (e.g., You counted all the way to 100; you shared everything equally; I understand just what you did there.)
- Be kind; be genuine; be respectful of and acknowledge child's way of 'doing'.
- Listen: let the child do the majority of the talking.
- Rephrase questions.
- Alter or completely change tasks.
- Suggest alternate representations or problem contexts.

(Ginsberg, 1997)

Figure 5. Respecting children's thinking by building rapport.

We found that if children did not respond to a rephrasing of a question or a change in representation, we could plan for hints that included phrases in the provided examples as, "What do you know about soft tacos? What's your favorite kind? Can you picture them on a plate in front of you? Would you want to leave that leftover taco on the plate if you were really hungry or would you want to eat it, too? How about your friend, if they wanted to share it with you?" Hints do not give away answers, but they do scaffold the question to make it more real for the child. In other instances, a simple change of representation or context opened access to the tasks for children who other appeared to not know how to solve the problem (Hunt & Empson, 2014). Another way to determine learning potential is to provide brief instruction within the interview to see how the child responds. If their propensity to explain and justify their thinking increases, this shows the child's potential to learn with guidance. For example, one child we worked with used an effective problem solving strategy with some types of sharing problems but not with others. Directing her attention toward the effective strategy in the new problems was helpful. Other times, more direct information is provided to see if the child can utilise it to solve a similar task.

From our clinical interviews, we obtained a rich insight into children's thinking, both what they could conceive of currently (with and without changes in context or representations, hints, or short instructional interactions) as well as where learning could potentially go. We interviewed some children who had a firm grasp of situations resulting in unit fractions (i.e, a demonstrated conceptual and procedural accuracy of grade level fractional knowledge); these children utilised pictorial and symbolic representations and appropriate fraction language to engage with the tasks regardless of contexts presented and could write number sentences for the quantities that resulted (e.g., $2\frac{1}{2}$ for 5 tacos shared by 2 children). These same children used concrete materials to conceive of situations involving non-unit fraction quantities and had trouble quantifying the result; contexts where children considered different types of pizzas seemed to help them (e.g., 4 share 3 pizzas with each pizza being a different flavor). This information helped us to suggest instruction for this student that used similar tasks to begin instruction and then move to multiple contexts later. We provide an example of completed interview notes for one child as an illustration on the following page.

Final thoughts: use documented understandings for instruction

Although we teach a group of students in our classrooms, learning is done by individuals. Our assessment tasks should help us teach individual students to further diagnose the presenting instructional concerns, monitor

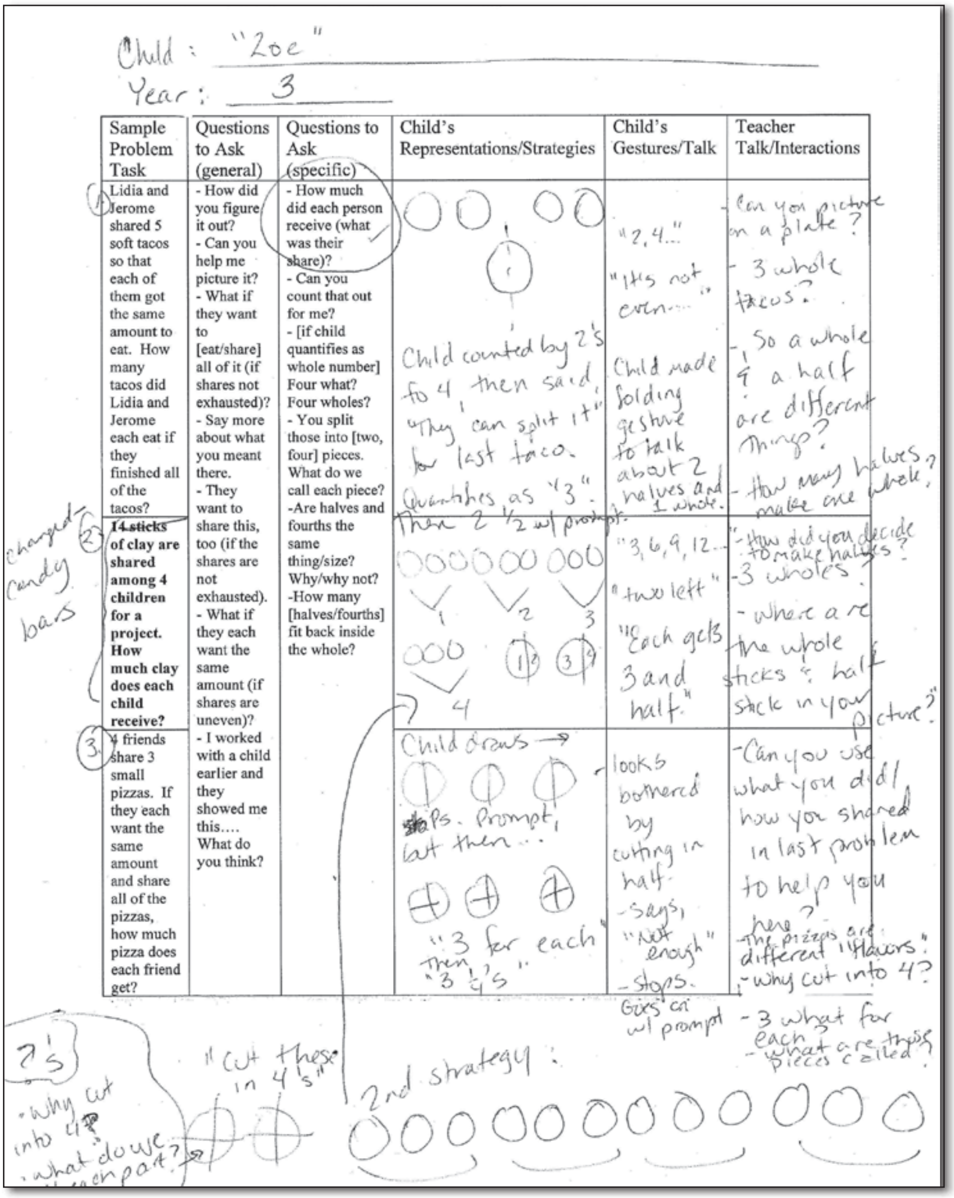

Figure 6. Example of completed interview notes.

student progress, and summarise the outcomes for a specific period of time (e.g., length of intervention, grading period, annual progress, etc.). We use this to identify a specific area of potential learning. Through our interactions with our students, especially students who are struggling in mathematics, we are clarifying, defining, and seeking to understand their specific knowledge, thinking, and conceptual understandings to provide better instruction.

Utilising clinical interviews as formative mathematics assessment for all children gives teachers a means in which to richly understand children's present and potential understandings. The valuable information gleaned from such an interview serves as a basis from which to bolster access and opportunities for success in learning and understanding mathematics for all learners.

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