ISSN 0005-1179, Automation and Remote Control, 2013, Vol. 74, No. 10, pp. 1679–1687. © Pleiades Publishing, Ltd., 2013.
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On Error Probability in a Multiple Access System under the Influence of “Tracking” Interference

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Received March 11, 2013

Abstract—We find an analytic expression for estimating error probability in a multiple access system that employs a signal-code construction based on q-ary codes, dynamically allocated frequency subranges, and a maximal energy receiver under the influence of “tracking” interference.

DOI: 10.1134/S0005117913100081

1. INTRODUCTION

One of the most important problems solved in modern communication systems design is how to protect the data transmitted over wireless channels from suppression. A traditional method for solving this problem includes the use of coded user separation methods, e.g., pseudorandom frequency hopping (PFH) [1]. However, systems with a classical version of the PFH technology become inefficient against reactive suppression strategies (the use of “tracking” interference and replaying previously recorded signals [2]). To solve this problem, the work [3] proposes a modification of a PFH-based system, namely a multiple access system that uses dynamically allocated subranges, frequency shift keying, and threshold reception (Dynamic Hopset Allocation Frequency Hopping OFDMA, DHA FH OFDMA). Dynamic allocation of a subrange for transmitting the next symbol significantly reduces the probability of this subrange being affected by replaying previously recorded signals. One of the most efficient methods of fighting “tracking” interference is non-coherent threshold reception. Therefore, it is most interesting to study a multiple access system of the said type that uses non-coherent reception [4]. In [5], a modification of such a system was proposed that uses external codes, which provided a possibility to use higher order modulation and allowed to replace symbol-by-symbol reception in [3, 4] with keyword decoding. Therefore, the signal-code construction in [5] potentially provides higher data transmission speed and better protection against suppression for the transmitted data.

The purpose of the present work is to find an analytic expression for estimating the error probability in a multiple access system that uses a signal-code construction based on q-ary codes, dynamically allocated frequency subranges, and maximum energy receiver under “tracking” interference.

Section 2 contains a brief description of the considered system, defines the reception criterion, and provides a method for reducing the problem at hand to the problem of computing central moments of the distributions of components in the decision statistic used by the detector (a method for computing these central moments is given in the Appendix). Section 3 shows the probabilistic transmission model in the considered system and proposes an analytic expression that lets us estimate the error probability. Section 4 concludes the paper.

2. A SIGNAL-CODE CONSTRUCTION BASED ON q-ARY CODES AND FREQUENCY SHIFT KEYING IN DYNAMICALLY ALLOCATED FREQUENCY SUBRANGES

Let us consider the transmission from many users "up" the channel (i.e., transmission to the base station) and let us correspondingly assume that all users transmit data asynchronously and without coordination. Besides, we will assume that the system implements optimal power control, i.e., the signal power from each user is chosen so that the power of the corresponding signal at the receiving end equals a certain predefined value P_U.

Suppose that the frequency band used in the system is partitioned into Q subchannels with the OFDM technology. Each user is assigned a subrange that includes q subchannels (in what follows we will without loss of generality assume that all elements of a vector corresponding to an OFDM symbol formed by the considered user are numbered from one to Q, the subrange used by this user includes elements with numbers from one to q, and the correspondence between vector elements and subchannels is given by a random permutation). Each subchannel of a certain subrange uniquely corresponds to a certain symbol from the field $GF(q)$ (to be definite, we will assume that the subchannel corresponding to the first element in the subrange corresponds to the symbol 0, the second element corresponds to symbol 1 and so on). We will assume that in each of the subranges, the user transmits a codeword in a q-ary $(\tilde{n}, k)$-code, sending one symbol per tick in a given subrange. A symbol is transmitted by sending a unit amplitude signal along the subchannel that corresponds to the q-ary symbol being transmitted.

Similar to the case of a classical PFH-based system, we will assume that the receiver has a permutation generator that is synchronized with the given user's permutation generator (similar to PFH, we assume that permutations are known to nobody except the "sender–receiver" pair). When receiving an OFDM symbol, the receiver first applies a permutation inverse to the one used by the receiver in constructing the OFDM frame and then computes signal energies on the output of each subchannel from the given subrange Thus, after receiving $\tilde{n}$ OFDM channels the receiver has a matrix consisting of $\tilde{n}$ columns and Q rows. Let us now go back to the subrange partition described above and let us consider only those q rows of this matrix that correspond to the subrange being used; we denote the corresponding submatrix by X.

In [5], the authors considered threshold non-coherent reception. In this work we consider maximal energy reception that works as follows.

We denote the set of positions in a submatrix X by $X(i, j)$, where i is the column index and j is the row index. Let V_z be a matrix of size $q \times \tilde{n}$ that corresponds to the codeword with index z. We introduce a mapping M that assigns a column with index g in matrix V_z an index j_g of a nonzero element in this column (note that only one such element corresponds to each of the columns of the V_z matrix):

$$M(V_z) = [j_1, j_2, \ldots, j_{\tilde{n}}]. \tag{1}$$

In maximal energy reception, we consider statistics of the following form:

$$y_z = \sum_{i=1}^{\tilde{n}} X^2(i, j_z(i)), \qquad \bar{\jmath}_z = M(V_z),$$

where $\bar{\jmath}_z$ is the vector of row indices where the matrix V_z contains nonzero elements.

Suppose that the codeword transmitted by the current user has index t. The correct maximal energy decoding condition can be written as follows: $y_t - y_z > 0$ for all $z \neq t$. A decoding error in maximal energy decoding can occur if the following inequality holds:

$$y_t - y_z \leqslant 0 \quad \text{for some } z \neq t. \tag{2}$$

The probability p_e that condition (2) holds can be estimated as

$$p_e < \sum_{z \neq t} P\left(\sum_{i=\overline{1,\tilde{n}}} \left(X^2\left(i, j_t\left(i\right)\right) - X^2\left(i, j_z\left(i\right)\right)\right) < 0\right), \tag{3}$$

where P is the probability of the fact that (2) holds for the zth word.

Note that due to the properties of the code used any two codewords coincide in at most $\tilde{n} - d$ positions. We denote by Θ the set of column indices that correspond to coinciding positions; by Ψ, the set of column indices that correspond to differing positions. Consider the following sum corresponding to some value of z:

$$S_z = \sum_{i=1}^{\tilde{n}} \left[X^2(i, j_t(i)) - X^2(i, j_z(i))\right] \tag{4}$$
$$= \sum_{i \in \Theta} \left[X^2(i, j_t(i)) - X^2(i, j_z(i))\right] + \sum_{i \in \Psi} \left[X^2(i, j_t(i)) - X^2(i, j_z(i))\right].$$

Since elements of the set Θ correspond to indices of coinciding elements, the first of two sums turns to zero, and we can write that

$$S_z = \sum_{i \in \Psi} X^2(i, j_t(i)) - \sum_{i \in \Psi} X^2(i, j_z(i)). \tag{5}$$

Condition (3) can now be rewritten as

$$p_e < \sum_{z \neq t} P\left(S_z < 0\right) = \sum_{z \neq t} P\left(\left(\sum_{i \in \Psi} X^2\left(i, j_t\left(i\right)\right) - \sum_{i \in \Psi} X^2\left(i, j_z\left(i\right)\right)\right) < 0\right). \tag{6}$$

Note that values that occur in sums in expression (5) are independent (but, as we will see from what follows, they do not have to all have an identical distribution), while the number of elements in each sum is at least d. If d is sufficiently large (in this case, d must surely be sufficiently large since otherwise the signal-code construction would not be able to ensure an acceptable level of protection from suppression), we can approximate the distribution of S_z statistics with a normal distribution.

Consequently, in order to estimate the probability p_e it suffices to estimate the expectation and variance of random values S_z. The expectation and variance of each of these values, in turn, depend on the number of positions that correspond to signals distributed according to a certain distribution. In Section 3 we will show how to get an estimate for the error probability in such a system on the example of a model of this multiple access system in the presence of "tracking" interference.

3. ESTIMATING THE ERROR PROBABILITY IN A SYSTEM OF THE CONSIDERED TYPE

Let us estimate the error probability in a system of the considered type. First we consider $\bar{X}_t = X(M(V_t))$, a vector of frequency–temporal positions in the received matrix V_t that correspond to the codeword transmitted by the current user. Here M is the mapping given by relation (1). The signal $\bar{v}^*$ transmitted by some authorized user can be influenced, first, by a background additive noise $\bar{\xi}$, second, by a signal $\bar{v}$ transmitted by another authorized user (this is called a collision), and third, by a "tracking" interference (i.e., interference transmitted in the same frequency subchannel where the current user is transmitting).

As we have already noted, central moments of distributions corresponding to each vector depend on central moments of the distributions of elements of these vectors. We introduce the following notation: we denote the expectation of each considered value by $\mathrm{E}(s, j, i)$; the variance, by $\mathrm{D}(s, j, i)$.

Here $s = 1$ means that a signal from the current user is present in the corresponding subchannel, $j = 1$ means that "tracking" interference is present, and $i = 1$ means that signals transmitted by other users are present in the subchannel ($s = 0$, $j = 0$, $i = 0$ denote the absence of signal from the current user, "tracking" interference, and other users respectively). The derivation of analytic expressions for central moments of various component distributions is given in the Appendix. Let us now consider the probabilistic model of transmission in the considered system.

Suppose that we know that "tracking" interference influences the signal transmitted by the current user with probability λ. Therefore, the influence of "tracking" interference on the transmitted signal can be viewed as a result of $n = \tilde{n} - \zeta$ sequential independent trials (here $0 \leqslant \zeta \leqslant (\tilde{n} - d)$ is the number of positions in which the transmitted codeword intersects with another codeword, where d is the minimal code distance), and in each trial the interference influence with probability λ and does not influence with probability $1 - \lambda$. Suppose, for definiteness, that υ out of n positions corresponding to the codeword transmitted by the current user have been under the influence of the interference. The probability of this event is

$$p_f(\upsilon, \lambda) = C_n^{\upsilon} \lambda^{\upsilon} (1 - \lambda)^{n-\upsilon}. \tag{7}$$

Besides, the current user's signal is influences by interference from signals transmitted by other active users. Since active users are not coordinated in their transmissions, and subchannels are chosen uniformly, the interaction with other users' signals can be viewed as a result of sequential independent trials in each of which the interference influences the signal with probability p_J and does not influence with probability $1 - p_J$. Since interaction with other users' signals and interference is independent, we can consider two independent series of trials one of which corresponds only to the ticks when "tracking" interference has influenced the signal (this series has length υ), and the other to all the rest (the length of this series is, consequently, $n - \upsilon$). The probability that in transmitting ρ symbols ($\rho \leqslant \upsilon$) the signal transmitted by the current user has been influenced by both "tracking" interference and signals transmitted by other active users, and in transmitting μ symbols the signal has been influenced by signals from other users only, can be written as

$$p'(\rho, \mu, \upsilon, p_J) = C_{\upsilon}^{\rho} p_J^{\rho} (1 - p_J)^{\upsilon-\rho} \times C_{n-\upsilon}^{\mu} p_J^{\mu} (1 - p_J)^{n-\upsilon-\mu}. \tag{8}$$

It has been shown in [3] that if the number of active users does not exceed half of the number of subchannels Q available to them then the probability of collision of multiplicity two is much larger than the probability of collisions with larger multiplicities. Therefore, in what follows we assume that only collisions of multiplicity two occur in the system (note that all derivations below can also be adapted to collisions of arbitrary multiplicity, but it will complicate the resulting formulas). Therefore, we can state that $p_J = 1 - (1 - \frac{1}{Q})^{K-1}$, where K is the number of active users. Consider the vector of frequency-temporal positions in the received matrix corresponding to a codeword other than the codeword transmitted by the current user:

$$\bar{X}_z = X(M(V_z)), \quad z \neq t.$$

The probability that only one user has been transmitting in a certain subchannel is

$$p_1 = (K - 1)\frac{1}{Q}\left(1 - \frac{1}{Q}\right)^{K-2}. \tag{9}$$

The probability that no user has been transmitting in a certain subchannel is

$$p_0 = \left(1 - \frac{1}{Q}\right)^{K-1}. \tag{10}$$

Consequently, the probability that more than one user has been transmitting in a certain subchannel (i.e., a collision has occurred) is

$$p_2 = 1 - p_1 - p_0.$$

Let us now find the probability of the fact that in a out of n positions only one user has been transmitting, while in b out of n positions, more than one. This probability is given by a polynomial distribution

$$p(n-a-b, a, b, p_0, p_1) = \frac{n!}{(n-a-b)!\, a!\, b!} p_0^{n-a-b} p_1^a p_2^b. \tag{11}$$

Finally, the probability of the fact that α positions out of a and β positions out of b have been subject to interference is

$$p''(\alpha, \beta, a, b, \lambda) = C_a^\alpha \lambda^\alpha (1-\lambda)^{a-\alpha} \times C_b^\beta \lambda^\beta (1-\lambda)^{b-\beta}. \tag{12}$$

Suppose that the first of two terms in the difference in formula (6) contains ρ terms characterized by the triple of parameters $(1,1,1)$, $\upsilon - \rho$ terms characterized by the triple of parameters $(1,1,0)$, μ terms characterized by the triple of parameters $(1,0,1)$, and $n-\upsilon-\mu$ terms characterized by the triple of parameters $(1,0,0)$. Suppose also that the second term contains β positions characterized by the triple of parameters $(1,1,1)$, α positions characterized by the triple of parameters $(1,1,0)$, $b-\beta$ positions characterized by the triple of parameters $(1,0,1)$, $a-\alpha$ positions characterized by the triple of parameters $(1,0,0)$, and $n-a-b$ positions characterized by the triple of parameters $(0,0,0)$. Then the expectation S_z, given by (5) equals

$$\begin{aligned} \mathrm{E}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n) = {} & ((\rho-\beta)\mathrm{E}(1,1,1)) + ((\upsilon-\rho-\alpha)\mathrm{E}(1,1,0)) \\ & + ((\mu-b+\beta)\mathrm{E}(1,0,1)) + (n-a-b)\mathrm{E}(0,0,0) \\ & + ((n-\upsilon-\mu-a+\alpha)\mathrm{E}(1,0,0)), \end{aligned} \tag{13}$$

and the variance is

$$\begin{aligned} \mathrm{D}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n) = {} & (\rho+\beta)\mathrm{D}(1,1,1) + (\upsilon-\rho+\alpha)\mathrm{D}(1,1,0) \\ & + (\mu+b-\beta)\mathrm{D}(1,0,1) + (n-a-b)\mathrm{D}(0,0,0) \\ & + (n-\upsilon-\mu+a-\alpha)\mathrm{D}(1,0,0). \end{aligned} \tag{14}$$

Consequently, we can estimate the error probability as

$$\begin{aligned} p(\upsilon, \mu, \rho, \alpha, \beta, a, b) \leqslant \sum_{i=1, i\neq t}^{q^k} \Bigg[& \sum_{\upsilon=0}^{n(i)} \sum_{\rho=0}^{\upsilon} \sum_{\mu=0}^{n(i)-\upsilon} \sum_{a=0}^{n(i)} \sum_{b=0}^{n(i)-a} \sum_{\alpha=0}^{a} \sum_{\beta=0}^{b} \Big(p_f(\upsilon, \lambda) \times p'(\rho, \mu, \upsilon) \\ & \times p(n(i)-a-b, a, b, p_0, p_1) \times p''(\alpha, \beta, a, b, \lambda) \\ & \times \int_{-\infty}^{0} f_N(x, \mathrm{E}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n(i)),\ \mathrm{D}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n(i)))\, dx \Big) \Bigg]. \end{aligned} \tag{15}$$

Here $f_N(x, \mathrm{E}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n),\ \mathrm{D}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n))$ is the probability density function of the normal distribution with expectation $\mathrm{E}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n(i))$ and variance $\mathrm{D}(\upsilon, \mu, \rho, \alpha, \beta, a, b, n(i))$, $n(i)$ is the weight of the ith codeword ($n(i) = w_h(\bar{v}_i)$), probability $p_f(\upsilon, \lambda)$ is given by (7), $p'(\rho, \mu, \upsilon)$ is given by (8), $p''(\alpha, \beta, a, b, \lambda)$ is given by (12), and $p(n(i)-a-b, a, b, p_0, p_1)$ is given by (11).

Note that the right-hand side of (15) has been found under the assumption that collision multiplicity is exactly two, which may influence the estimate's accuracy. However, the proposed approach can be generalized to collisions of any multiplicity.

4. CONCLUSION

In this work, we have proposed an approach that lets us find error probability estimates for maximal energy reception. The essence of out approach is to approximate the statistics in question with a normal law. Thus, finding probability estimates reduces to computing central moments of the components of these statistics.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project no. 12-07-31035 mol_a.

The author is grateful to the anonymous referees for their valuable comments.

APPENDIX

Consider the following case: suppose that signal $\bar{v}^*$ transmitted by some authorized user is under the influence, first, of the background additive noise $\bar{\xi}$, second, a signal $\bar{v}$ being transmitted by another authorized user (collision), and third, by "tracking" interference (i.e., interference transmitted in the same frequency subchannel where the current user is transmitting). We will assume that "tracking" interference $\bar{z}$ has amplitude βA and has a delay with respect to the signal transmitted by the current user by $\tau = \alpha T$, where T is the OFDM frame duration. Thus, the received signal has the form

$$\bar{X} = \bar{v}^* + \bar{v} + \bar{z} + \bar{\xi}.$$

Note that $\bar{v}^*$ and $\bar{v}$ are random vectors with amplitude A, $\bar{z}$ is a random vector with amplitude $\tilde{\mathrm{A}} = \beta\sqrt{1-\alpha}A = \gamma A$, and the vector $\bar{\xi}$ corresponds to a two-dimensional Gaussian process with zero mathematical expectation and mean-square deviation σ.

We denote $\bar{Y} = \bar{v}^* + \bar{\xi}$, $\bar{I} = \bar{v} + \bar{z}$. Then $\bar{X} = \bar{Y} + \bar{I}$. The value measured at the subchannel's output will therefore be given by

$$X^2 = Y^2 + I^2 + 2YI\cos\phi, \tag{A.1}$$

where ϕ is the angle between vectors $\bar{Y}$ and $\bar{I}$. Let us find the expectation of this value. By the theorems about numerical characteristics, $\mathrm{E}(X^2) = \mathrm{E}(Y^2) + \mathrm{E}(I^2) + \mathrm{E}(2YI\cos\phi)$. Note that amplitudes $Y = |\bar{Y}|$, $I = |\bar{I}|$ and phase ϕ are independent and uncorrelated, and therefore

$$\mathrm{E}(YI\cos\phi) = 2\mathrm{E}(Y)\mathrm{E}(I)\mathrm{E}(\cos\phi).$$

If ϕ is uniformly distributed, it means that

$$\mathrm{E}(\cos\phi) = \int\limits_0^{2\pi} \frac{\cos\phi}{2\pi}\, d\phi = 0. \tag{A.2}$$

The expression for the expectation (A.1) can be transformed into

$$\mathrm{E}(X^2) = \mathrm{E}(Y^2) + \mathrm{E}(I^2).$$

Note that the value Y^2 is distributed according to the noncentral χ^2 law with two degrees of freedom [1], and its central moments are well known. In particular, in this case $\mathrm{E}(Y^2) = 2\sigma^2 + A^2$ [6].

The expectation of the value I^2 is

$$\mathrm{E}(I^2) = \mathrm{E}(\gamma^2 A^2 + 2\gamma A^2\cos\phi + A^2) = (\gamma^2 + 1)A^2.$$

Consequently,

$$\mathrm{E}(1,1,1) = 2\sigma^2 + (\gamma^2+2)A^2. \tag{A.3}$$

Let us now find the variance of X^2. We first compute

$$\mathrm{E}((X^2)^2) = \mathrm{E}((Y^2+I^2)^2) + \mathrm{E}((2YI\cos\phi)^2) + \mathrm{E}((Y^2+I^2)2YI\cos\phi). \tag{A.4}$$

Note that the random variable $\cos\phi$ is uncorrelated with $\omega = (Y^2+I^2)2YI$. Therefore, it follows that

$$\mathrm{E}((Y^2+I^2)2YI\cos\phi) = \mathrm{E}((Y^2+I^2)2YI) \times \mathrm{E}(\cos\phi) = 0.$$

By the same reasoning,

$$\mathrm{E}((2YI\cos\phi)^2) = 4\mathrm{E}(Y^2)\mathrm{E}(I^2)\mathrm{E}(\cos^2\phi) = 4(2\sigma^2+A^2)(\gamma^2+1)A^2\mathrm{E}(\cos^2\phi).$$

By definition $\mathrm{E}(\cos^2\phi) = \frac{1}{2\pi}\int\limits_0^{2\pi}\cos^2\phi\,d\phi = \frac{1}{\pi}\int\limits_0^{\pi}\cos^2\phi\,d\phi$.

Note that

$$\int\limits_0^{\pi}\cos^2\phi\,d\phi = \frac{\pi}{2}, \tag{A.5}$$

which implies that $\mathrm{E}(\cos^2\phi) = \frac{1}{2}$, $\mathrm{E}((2YI\cos\phi)^2) = 2(2\sigma^2+A^2)(\gamma^2+1)A^2$.

Finally, the first term in (A.4) can be represented as

$$\mathrm{E}((Y^2+I^2)^2) = \mathrm{E}((Y^2)^2) + \mathrm{E}((I^2)^2) + 2\mathrm{E}(Y^2I^2),$$

and the entire expression (A.4) can be rewritten as

$$\mathrm{E}((X^2)^2) = \mathrm{E}((Y^2)^2) + \mathrm{E}((I^2)^2) + 4\mathrm{E}(Y^2I^2).$$

According to [6], $\mathrm{E}((Y^2)^2) = 4\sigma^4 + 4\sigma^2A^2 + (2\sigma^2+A^2)^2 = 8\sigma^4 + 8\sigma^2A^2 + A^4$.

By definition,

$$\begin{aligned}\mathrm{E}((I^2)^2) &= \mathrm{E}((\gamma^2A^2 + 2\gamma A^2\cos\phi + A^2)^2)\\ &= A^4\mathrm{E}(((\gamma^2+1)+2\gamma\cos\phi)^2) = A^4\gamma^4 + 4A^4\gamma^2 + A^4.\end{aligned}$$

We can finally write that

$$\mathrm{E}((X^2)^2) = 8\sigma^4 + 16\sigma^2A^2 + 6A^4 + A^4\gamma^4 + 8A^4\gamma^2 + 8\sigma^2\gamma^2A^2.$$

The variance (A.1) equals

$$\mathrm{D}(1,1,1) = \mathrm{E}((X^2)^2) - (\mathrm{E}(X^2))^2 = 4\sigma^4 + 8\sigma^2A^2 + 2A^4 + 4A^4\gamma^2 + 4\sigma^2\gamma^2A^2.$$

Let us now consider the case when the transmitted signal is only under the influence of "tracking" interference (i.e., situation described by the $(1,1,0)$ triple). The resulting signal has the form $\bar{X} = \bar{y} + \bar{z}$, where $\bar{y} = \bar{v}^* + \bar{\xi}$, $\bar{v}^*$ is a random vector with amplitude A, $\bar{z}$ is a random vector with amplitude γA, and $\bar{\xi}$ is a vector describing the two-dimensional Gaussian process

$$X^2 = |\bar{y}|^2 + |\bar{z}|^2 + 2|\bar{y}|\,|\bar{z}|\cos\alpha. \tag{A.6}$$

The expectation (A.6) is

$$\mathrm{E}(1,1,0) = \mathrm{E}(X^2) = \mathrm{E}(|\bar{y}|^2 + |\bar{z}|^2 + 2|\bar{y}|\,|\bar{z}|\cos\alpha),$$

Central moments of component distributions of deciding statistics

s	j	i	$\mathrm{E}(s,j,i)$	$\mathrm{D}(s,j,i)$
1	1	1	$2\sigma^2+(\gamma^2+2)A^2$	$4\sigma^4+8\sigma^2A^2+2A^4+4A^4\gamma^2+4\sigma^2\gamma^2A^2$
1	1	0	$2\sigma^2+\gamma^2A^2+A^2$	$4\sigma^4+4\sigma^2A^2+4\sigma^2\gamma^2A^2+2A^4\gamma^2$
1	0	1	$2\sigma^2+2A^2$	$4\sigma^4+8\sigma^2A^2+2A^4$
1	0	0	$A^2+2\sigma^2$	$4\sigma^4+4\sigma^2A^2$
0	0	0	$2\sigma^2$	$4\sigma^4$

where α is the angle between vectors $\bar{y}$ and $\bar{v}$ (since phases of vectors $\bar{y}$ and $\bar{v}$ are distributed uniformly on $[0,2\pi]$, the phase α is also uniformly distributed on $[0,2\pi]$). By (A.2) we get that

$$\mathrm{E}(X^2)=\mathrm{E}(|\bar{y}|^2)+\mathrm{E}(|\bar{z}|^2).$$

The value $|\bar{y}|^2$ is distributed according to the noncentral χ^2 law with two degrees of freedom. Consequently, the expectation (A.6) is

$$\mathrm{E}(1,1,0)=\mathrm{E}(X^2)=2\sigma^2+\gamma^2A^2+A^2.$$

Let us now find the expression for the value

$$\mathrm{E}((|X|^2)^2)=\mathrm{E}((|\bar{y}|^2+|\bar{z}|^2+2|\bar{y}|\,|\bar{z}|\cos\alpha)^2) \tag{A.7}$$
$$=\mathrm{E}((|\bar{y}|^2+|\bar{z}|^2)^2)+\mathrm{E}(4|\bar{y}|^2\,|\bar{z}|^2\cos^2\alpha)+\mathrm{E}((2|\bar{y}|\,|\bar{z}|)(|\bar{y}|^2+|\bar{z}|^2)\cos\alpha).$$

Taking (A.2) into account, we get from it that

$$\mathrm{E}(2|\bar{y}|\,|\bar{z}|(|\bar{y}|^2+|\bar{z}|^2)\cos\alpha)=0.$$

By (A.5),

$$\mathrm{E}(4|\bar{y}|^2\,|\bar{z}|^2\cos^2\alpha)=2\mathrm{E}(|\bar{y}|^2)\,\mathrm{E}(|\bar{z}|^2)=4\sigma^2\gamma^2A^2+2\gamma^2A^4.$$

The first term in expression (A.7) has the form

$$\mathrm{E}((|\bar{y}|^2+|\bar{z}|^2)^2)=\mathrm{E}(|\bar{y}|^4)+\mathrm{E}(|z|^4)+\mathrm{E}(2|\bar{y}|^2\times|\bar{z}|^2). \tag{A.8}$$

The first term of the sum in the right-hand side of expression (A.8) has the form [6]

$$\mathrm{E}(|\bar{y}|^4)=4\sigma^4+4\sigma^2A^2+(2\sigma^2+A^2)^2.$$

Thus, taking into account (A.5), we can say that

$$\mathrm{E}((|X|^2)^2)=\mathrm{E}((|\bar{y}|^2+|\bar{z}|^2+2|\bar{y}|\,|\bar{z}|\cos\alpha)^2)$$
$$=4\sigma^4+4\sigma^2A^2+(2\sigma^2+A^2)^2+\gamma^4A^4+4\sigma^2\gamma^2A^2+2\gamma^2A^4.$$

The variance (A.6) is

$$\mathrm{D}(1,1,0)=\mathrm{E}((X^2)^2)-(\mathrm{E}(X^2))^2=4\sigma^4+4\sigma^2A^2+4\sigma^2\gamma^2A^2+2A^4\gamma^2.$$

The situation when the received signal is influenced by a signal from another authorized user (i.e., a collision occurs) is a special case of this situation for $\gamma=1$, and, therefore,

$$\mathrm{E}(1,0,1)=2A^2+2\sigma^2,\qquad \mathrm{D}(1,0,1)=4\sigma^4+8\sigma^2A^2+2A^4.$$

The random value defined by the triple of parameters $(1, 0, 0)$ is distributed according to the non-central χ^2 law with two degrees of freedom, and, consequently, the corresponding central moments are given by expressions from [6]:

$$\mathrm{E}(1, 0, 0) = A^2 + 2\sigma^2, \qquad \mathrm{D}(1, 0, 0) = 4\sigma^4 + 4\sigma^2 A^2.$$

The random value defined by the triple of parameters $(0, 0, 0)$ is distributed according to the non-central χ^2 law with two degrees of freedom, and, consequently, the corresponding central moments are given by expressions from [6]:

$$\mathrm{E}(0, 0, 0) = 2\sigma^2, \qquad \mathrm{D}(0, 0, 0) = 4\sigma^4.$$

We summarize the resulting expressions for central moments in the following table.

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This paper was recommended for publication by A.V. Bernshtein, a member of the Editorial Board