www.ietdl.org

Published in IET Communications
Received on 24th June 2013
Revised on 22nd November 2013
Accepted on 13th December 2013
doi: 10.1049/iet-com.2013.0539

ISSN 1751-8628

Blind carrier frequency offset estimation for interleaved orthogonal frequency division multiple access uplink with multi-antenna receiver: algorithms and performance analysis

Weile Zhang, Qinye Yin

MOE Key Laboratory for Intelligent Networks and Network Security, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China
E-mail: wlzhang1984@gmail.com

Abstract: In this study, for interleaved orthogonal frequency division multiple access uplink with multi-antenna receiver, we propose two generalised carrier frequency offset estimators which, respectively, exploit the subspace theory and maximum-likelihood (ML) criterion. We find that, as long as the numbers of multipaths from two of the users are smaller than the number of antennas at the receiver, both the proposed estimators support fully loaded transmissions. We also derive the theoretical performance lower bound for the proposed ML estimator. The numerical results are then provided, which corroborate the proposed studies.

1 Introduction

Orthogonal frequency division multiple access (OFDMA) has recently received a considerable amount of interest [1, 2]. It is widely recognised that OFDMA inherits from orthogonal frequency division multiplexing (OFDM) the weakness of sensitivity to the effect of carrier frequency offset (CFO). The system performance of OFDMA relies heavily on the proper CFO estimation and compensation.

In OFDMA downlink transmissions, a single CFO exists between each transceiver pair, which makes the existing CFO estimators for OFDM directly applicable [3, 4]. However, for OFDMA uplink, multiple CFOs co-exist at the receiver, making the CFO estimation much more challenging. To perform the CFO estimation, various kinds of data-aided estimators [5, 6] as well as the blind estimators [7–15] have been developed. For example, by using the multiple signal classification technique [16], a frequency estimation scheme for interleaved OFDMA uplink that exploits the periodic structure of the signals from each user has been presented in [9]. Based on the observation of [9], several advancements have been proposed later [10–14]. However, [9–14] need null subcarriers or a longer cyclic prefix (CP) to build the noise space which decreases the bandwidth efficiency.

Recently, by exploiting the multi-antenna redundancy at the receiver, Zhang *et al.* [15] proposed two estimators for the interleaved OFDMA uplink based on the subspace theory and maximum-likelihood (ML) criterion, respectively. It was shown in [15] that the two estimators [15] can support fully loaded transmission, which provides higher bandwidth efficiency as compared with the estimators [9–14]. However, both the two estimators in [15] require that the number of antennas at the receiver should be larger than the maximum number of multipaths from all users, which severely limits their applicable scenarios.

In this paper, inspired by the work of [15], we further consider the more general scenarios that the number of multipaths from some users may be larger than the number of antennas at the receiver, and propose two generalised blind CFO estimators for the interleaved OFDMA uplink with multi-antenna receiver. The main contributions of this paper can be summarised as follows:

- Two generalised CFO estimators which, respectively, exploit the subspace theory and ML criterion are developed, referred to as 'GSSE' and 'GMLE', respectively. We find that, as long as the numbers of multipaths from two of the users are smaller than the number of antennas at the receiver, both the proposed estimators support fully loaded transmissions. This greatly relaxes the antenna number requirement at the receiver as compared with the work in [15].
- We derive the theoretical performance lower bound for the proposed GMLE. The numerical results are also provided, which demonstrate that GMLE behaves better than GSSE and almost achieves the analytical lower bound.

The rest of this paper is organised as follows. Section 2 formulates the problem. The proposed GSSE and GMLE are developed in Sections 3 and 4, respectively. The theoretical performance analysis is presented in Section 5.

Simulation results are given in Section 6 and conclusions are drawn in Section 7.

Notations: superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $[\cdot]^{\dagger}$ and $E[\cdot]$ represent conjugate, transpose, Hermitian, pseudo inverse and expectation, respectively; $\boldsymbol{j}=\sqrt{-1}$ is the imaginary unit; $\|\boldsymbol{X}\|$ denotes the Frobenius norm of $\boldsymbol{X}$, and diag$(\cdot)$ is a diagonal matrix with main diagonal $(\cdot)$; blkdiag$(\cdot)$ represents the block-diagonal matrix operator; $\mathbb{C}^{m\times n}$ defines the vector space of all $m\times n$ complex matrices; $\boldsymbol{I}_N$ is the $N\times N$ identity matrix; 0 represents an all-zero matrix with appropriate dimension; $\otimes$ stands for the Kronecker product; $\circ$ denotes the element-wise product; Tr$\{\cdot\}$ denotes the trace operation; MATLAB matrix representations are adopted, for example, $\boldsymbol{X}(r_1{:}r_2,\ c_1{:}c_2)$ denotes the submatrix of $\boldsymbol{X}$ with the rows from r_1 to r_2 and the columns from c_1 to c_2.

2 Problem formulation

Consider a multiuser OFDMA system with K users, N subcarriers and M antennas at the receiver. All subcarriers are sequentially indexed with $\{i\}$, $i=0,\ 1,\ \ldots,\ N-1$, and are equally divided into Q subchannels, each having $P=N/Q$ subcarriers. The qth subchannel consists of subcarriers with index set of $\{q,\ Q+q,\ \ldots,\ (P-1)Q+q\}$, $q=0,\ 1,\ \ldots,\ Q-1$. Each subchannel will be exclusively assigned to one user, and thus, no subchannel can be shared by more than one user. To ease the presentation, we assume the system is fully loaded in this paper, that is, $K=Q$.

Denote the normalised CFO of the kth user as $\xi^{(k)}=\Delta f^{(k)}/\Delta f$, where Δf is the subcarrier spacing and $\Delta f^{(k)}$ is the real CFO of the kth user. We assume $\xi^{(k)}\in(-0.5,\ 0.5)$. The channel impulse response from the kth user to the mth antenna can be modelled as $h_m^{(k)}(\tau)=\sum_{l=1}^{L^{(k)}}h_{l,m}^{(k)}\delta\left(\tau-\tau_l^{(k)}\right)$ where $L^{(k)}$ is the number of multipaths, whereas $h_{l,m}^{(k)}$ and $\tau_l^{(k)}$ are the complex amplitude and delay for the lth multipath, respectively. We assume $h_{l,m}^{(k)}$'s are independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and power $E\left\{|h_{l,m}^{(k)}|^2\right\}=1/L^{(k)}$ such that the total power is normalised, that is, $E\left\{\sum_{l=1}^{L^{(k)}}\left|h_{l,m}^{(k)}\right|^2\right\}=1$. Assume that the delays are rounded to the nearest sampling position and $\tau_l^{(k)}$'s are of integer values. We also consider the fading channels are constant over successive L_s block durations in this paper.

We define the user index set $\mathcal{C}=\left\{c_1,\ c_2,\ \ldots,\ c_{N_\mathcal{C}}\right\}$, where $L^{(c_i)}<M$, $\quad i=1,\ 2,\ \ldots,\ N_\mathcal{C}$, and $N_\mathcal{C}$ denotes the number of users whose multipath numbers are less than M. In this paper, we assume $N_\mathcal{C}\geq 2$, that is, there are at least two users whose multipath numbers are less than M. Here, we should note that the work in [15] requires that the multipath numbers of all users should be less than M. Thus, the antenna number requirement at the receiver in this paper is much more relaxed as compared with [15].

Assume the kth user occupies the $q^{(k)}$th subchannel and let $\boldsymbol{s}_g^{(k)}=\left[s_{0,g}^{(k)},\ s_{1,g}^{(k)},\ \ldots,\ s_{P-1,g}^{(k)}\right]^{\mathrm{T}}$ be the P information symbols of the kth user in the gth OFDMA block. Denote $\chi_g^{(k)}=\mathrm{e}^{\mathrm{j}\left(2\pi(g-1)(N+G)\xi^{(k)}/N\right)}$ as the phase shift of the kth user accumulated from the previous $g-1$ blocks with G being the length of CP. Then, in noise-free environment, the received signal at the mth receiving antenna element from the K users after CP removal can be expressed as [15]

$$\gamma_{m,g}^{(k)}(n)=\sum_{k=1}^{K}\frac{\chi_g^{(k)}}{\sqrt{N}}\sum_{p=0}^{P-1}\left(\sum_{l=1}^{L^{(k)}}h_{l,m}^{(k)}\mathrm{e}^{-\mathrm{j}\left(2\pi(pK+q^{(k)})\tau_l^{(k)}/N\right)}\right)s_{p,g}^{(k)}\mathrm{e}^{\mathrm{j}\left(\left(2\pi(pK+q^{(k)}+\xi^{(k)})/N\right)n\right)}$$
$$=\sum_{k=1}^{K}\sum_{l=1}^{L^{(k)}}h_{l,m}^{(k)}z_{l,n,g}^{(k)} \qquad (1)$$

where the expression in the bracket stands for the frequency-domain channel response at the $(pK+q^{(k)})$th subcarrier from the kth user to the mth antenna, and (see (1))

We further define (see equation at the bottom of the page)

$$\boldsymbol{h}_l^{(k)}=\left[h_{l,1}^{(k)},\ h_{l,2}^{(k)},\ \ldots,\ h_{l,M}^{(k)}\right]^{\mathrm{T}}\in\mathbb{C}^{M\times 1},\quad \boldsymbol{H}^{(k)}=\left[\boldsymbol{h}_1^{(k)},\ \boldsymbol{h}_2^{(k)},\ \ldots,\ \boldsymbol{h}_{L^{(k)}}^{(k)}\right]\in\mathbb{C}^{M\times L^{(k)}}$$
$$\boldsymbol{H}=\left[\boldsymbol{H}^{(1)},\ \boldsymbol{H}^{(2)},\ \ldots,\ \boldsymbol{H}^{(K)}\right]\in\mathbb{C}^{M\times L_{\mathrm{sum}}}$$

where $L_{\mathrm{sum}}=\sum_{k=1}^{K}L^{(k)}$. Stacking the received signals from all M antennas, we obtain the following space-domain snapshot with the presence of noise

$$\begin{aligned}\gamma_{n,g}&=\left[\gamma_{1,g}(n),\ \gamma_{2,g}(n),\ \ldots,\ \gamma_{M,g}(n)\right]^{\mathrm{T}}\\&=\boldsymbol{H}\boldsymbol{z}_{n,g}+\boldsymbol{n}_{n,g}\end{aligned} \qquad (2)$$

where $\boldsymbol{n}_{n,g}$ is a length-M additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_n^2\boldsymbol{I}_M$ at the nth sample in the gth OFDMA block, and

$$\boldsymbol{z}_{n,g}=\left[\left(\boldsymbol{z}_{n,g}^{(1)}\right)^{\mathrm{T}},\ \left(\boldsymbol{z}_{n,g}^{(2)}\right)^{\mathrm{T}},\ \ldots,\ \left(\boldsymbol{z}_{n,g}^{(K)}\right)^{\mathrm{T}}\right]^{\mathrm{T}}\in\mathbb{C}^{L_{\mathrm{sum}}\times 1}$$
$$\boldsymbol{z}_{n,g}^{(k)}=\left[z_{1,n,g}^{(k)},\ z_{2,n,g}^{(k)},\ \ldots,\ z_{L^{(k)},n,g}^{(k)}\right]^{\mathrm{T}}\in\mathbb{C}^{L^{(k)}\times 1}$$

3 Proposed GSSE

By defining

$$\bar{\boldsymbol{z}}_{p,g}^{(k)}=\begin{cases}\boldsymbol{z}_{p,g}^{(k)}, & L^{(k)}<M\\ \boldsymbol{H}^{(k)}\boldsymbol{z}_{p,g}^{(k)}, & L^{(k)}\geq M\end{cases} \qquad (3)$$

we can rewrite (2) into

$$\begin{aligned}\gamma_{p,g} &= \sum_{k=1}^{K} \boldsymbol{H}^{(k)} \boldsymbol{z}_{p,g}^{(k)} + \boldsymbol{n}_{p,g} \\ &= \sum_{k' \in \mathcal{C}} \boldsymbol{H}^{(k')} \bar{\boldsymbol{z}}_{p,g}^{(k')} + \sum_{k \notin \mathcal{C}} \boldsymbol{I}_M \bar{\boldsymbol{z}}_{p,g}^{(k)} + \boldsymbol{n}_{p,g} \end{aligned} \quad (4)$$

Treat $\theta^{(k)} = (q^{(k)} + \xi^{(k)})/K$ as the effective CFO of the kth user. For $t = 0, 1, \ldots, K-1$, there holds

$$\bar{\boldsymbol{z}}_{p+tP,g}^{(k)} = \mathrm{e}^{\mathrm{j}2\pi t\theta^{(k)}} \bar{\boldsymbol{z}}_{p,g}^{(k)} \quad (5)$$

and

$$\begin{aligned}\gamma_{p+tP,g} = &\sum_{k' \in \mathcal{C}} \mathrm{e}^{\mathrm{j}2\pi t\theta^{(k')}} \boldsymbol{H}^{(k')} \bar{\boldsymbol{z}}_{p,g}^{(k')} + \sum_{k \notin \mathcal{C}} \mathrm{e}^{\mathrm{j}2\pi t\theta^{(k)}} \boldsymbol{I}_M \bar{\boldsymbol{z}}_{p,g}^{(k)} \\ &+ \boldsymbol{n}_{p+tP,g} \end{aligned} \quad (6)$$

Stacking space-domain snapshot vectors from K equally spaced time samples (P samples apart) give the following length-MK vector

$$\boldsymbol{a}_{p,g} = \left[\gamma_{p,g}^{\mathrm{T}}, \ \gamma_{p+P,g}^{\mathrm{T}}, \ \ldots, \ \gamma_{p+N-P,g}^{\mathrm{T}}\right]^{\mathrm{T}} \quad (7)$$

Define

$$\boldsymbol{v}^{(k)} = \left[1, \ \mathrm{e}^{\mathrm{j}2\pi\theta^{(k)}}, \ \ldots, \ \mathrm{e}^{\mathrm{j}2\pi(K-1)\theta^{(k)}}\right]^{\mathrm{T}} \in \mathbb{C}^{K\times 1} \quad (8)$$

$$\boldsymbol{V}^{(k)} = \begin{cases} \boldsymbol{v}^{(k)} \otimes \boldsymbol{H}^{(k)}, & L^{(k)} < M \\ \boldsymbol{v}^{(k)} \otimes \boldsymbol{I}_M, & L^{(k)} \geq M \end{cases} \quad (9)$$

Let $\bar{L}^{(k)}$ equal $L^{(k)}$ if $L^{(k)} < M$ and M elsewhere, that is, $\bar{L}^{(k)} = \min\{L^{(k)}, M\}$. Then, we have $\boldsymbol{V}^{(k)} \in \mathbb{C}^{KM\times \bar{L}^{(k)}}$ and $\bar{\boldsymbol{z}}_{p,g}^{(k)} \in \mathbb{C}^{\bar{L}^{(k)}\times 1}$. Denote $\bar{L}_{\mathrm{sum}} = \sum_{k=1}^{K} \bar{L}^{(k)}$. From (6), we can further rewrite (7) into the following matrix-form

$$\boldsymbol{a}_{p,g} = \bar{\boldsymbol{V}} \boldsymbol{z}_{p,g} + \bar{\boldsymbol{n}}_{p,g} \quad (10)$$

where

$$\boldsymbol{V} = [\boldsymbol{V}^{(1)}, \ \boldsymbol{V}^{(2)}, \ \ldots, \ \boldsymbol{V}^{(K)}] \in \mathbb{C}^{MK\times \bar{L}_{\mathrm{sum}}}$$

$$\bar{\boldsymbol{z}}_{p,g} = \left[\left(\bar{\boldsymbol{z}}_{p,g}^{(1)}\right)^{\mathrm{T}}, \ \left(\bar{\boldsymbol{z}}_{p,g}^{(2)}\right)^{\mathrm{T}}, \ \ldots, \ \left(\bar{\boldsymbol{z}}_{p,g}^{(K)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \bar{L}_{\mathrm{sum}} \times 1$$

$$\bar{\boldsymbol{n}}_{p,g} = \left[\boldsymbol{n}_{p,g}^{\mathrm{T}}, \ \boldsymbol{n}_{p+P,g}^{\mathrm{T}}, \ \ldots, \ \boldsymbol{n}_{p+N-P,g}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{MK\times 1}$$

The correlation matrix of $\boldsymbol{a}_{p,g}$ can be computed as

$$\boldsymbol{R}_i = E\left\{\boldsymbol{a}_{p,g} \boldsymbol{a}_{p,g}^{\mathrm{H}}\right\} = \boldsymbol{V} \boldsymbol{R}_{\bar{z}\bar{z}} \boldsymbol{V}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_{MK} \quad (11)$$

where $\boldsymbol{R}_{\bar{z}\bar{z}} = E\left\{\bar{\boldsymbol{z}}_{p,g} \bar{\boldsymbol{z}}_{p,g}^{\mathrm{H}}\right\}$. Assume $L^{(k)} \leq P$ and $\tau_{L^{(k)}}^{(k)} < P$. Following the similar steps of Appendix B in [15], we obtain

$$\boldsymbol{R}_{\bar{z}\bar{z}} = \mathrm{blkdiag}\left\{\boldsymbol{R}_{\bar{z}\bar{z}}^{(1)}, \ \boldsymbol{R}_{\bar{z}\bar{z}}^{(2)}, \ \ldots, \ \boldsymbol{R}_{\bar{z}\bar{z}}^{(K)}\right\} \quad (12)$$

where

$$\boldsymbol{R}_{\bar{z}\bar{z}}^{(k)} = \begin{cases} \dfrac{1}{K} \boldsymbol{I}_{L^{(k)}}, & L^{(k)} < M \\ \dfrac{1}{K} \boldsymbol{H}^{(k)} \left(\boldsymbol{H}^{(k)}\right)^{\mathrm{H}}, & L^{(k)} \geq M \end{cases}$$

Owing to the random property of the wireless channels, the non-singularity of $\boldsymbol{R}_{\bar{z}\bar{z}}$ can be readily guaranteed.

Perform singular value decomposition (SVD) of $\boldsymbol{R}_i$, which gives

$$\boldsymbol{R}_i = \left[\boldsymbol{U}_s, \ \boldsymbol{U}_n\right] \Sigma_a \left[\boldsymbol{U}_s, \ \boldsymbol{U}_n\right]^{\mathrm{H}} \quad (13)$$

where $\boldsymbol{U}_s \in \mathbb{C}^{MK\times \bar{L}_{\mathrm{sum}}}$ and $\boldsymbol{U}_n \in \mathbb{C}^{MK\times (MK-\bar{L}_{\mathrm{sum}})}$ represent the signal and the noise space matrices, respectively.

We define the following parameterised Vandermonde vector $\mathbb{B}^{(k)}(\xi)$ with respect to ξ as

$$\mathbb{B}^{(k)}(\xi) = \left[1, \ \mathrm{e}^{\mathrm{j}\left(2\pi(q^{(k)}+\xi)/K\right)}, \ \ldots, \ \mathrm{e}^{\mathrm{j}\left(2\pi(K-1)(q^{(k)}+\xi)/K\right)}\right]^{\mathrm{T}} \quad (14)$$

where $\xi \in (-0.5, 0.5)$. The following 'Lemmas' are the key properties to design the generalised subspace-based CFO estimator.

Lemma 1: For the kth user with $L^{(k)} < M$, we have the following observations:

- For 'any' non-zero length-M vector $\boldsymbol{x}$, there holds

$$\left(\mathbb{B}^{(k)}(\xi^{(k)}) \otimes \boldsymbol{x}\right)^{\mathrm{H}} \boldsymbol{U}_n = \begin{cases} = 0, & \boldsymbol{x} \in \mathrm{Span}\{\boldsymbol{H}^{(k)}\} \\ \neq 0, & \boldsymbol{x} \notin \mathrm{Span}\{\boldsymbol{H}^{(k)}\} \end{cases} \quad (15)$$

- Given 'any' $M \times L^{(k)}$ matrix $\boldsymbol{X}$ with full column rank and $\xi \neq \xi^{(k)}$, there holds

$$\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{X}\right)^{\mathrm{H}} \boldsymbol{U}_n \neq 0 \quad (16)$$

Proof: When $\boldsymbol{x} \in \mathrm{Span}\{\boldsymbol{H}^{(k)}\}$, we know $\mathbb{B}^{(k)}\left(\xi^{(k)}\right) \otimes \boldsymbol{x} = \boldsymbol{v}^{(k)} \otimes \boldsymbol{x}$ belongs to the column space of $\boldsymbol{U}_s$. Thus, it is orthogonal to the noise space matrix $\boldsymbol{U}_n$. Moreover, when $\boldsymbol{x} \notin \mathrm{Span}\{\boldsymbol{H}^{(k)}\}$, according to Appendix C in [15], we know $\mathbb{B}^{(k)}\left(\xi^{(k)}\right) \otimes \boldsymbol{x}$ is linearly independent with the column vectors of $\boldsymbol{U}_s$. Thus, it cannot be orthogonal to $\boldsymbol{U}_n$. Then, we concentrate on the proof of (16) which can be obtained from the method of contradiction. According to Appendix D in [15], if $\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{X}\right)^{\mathrm{H}} \boldsymbol{U}_n = 0$ holds, the column space of $\boldsymbol{X} \in \mathbb{C}^{M\times L^{(k)}}$ should simultaneously belong to the column space of both $\boldsymbol{I}_M$ and each $\boldsymbol{H}^{(k')} \in \mathbb{C}^{M\times L^{(k')}}$, $k' \in \mathcal{C}$. However, when the number of users whose multipath number is less than M is not less than 2, that is, $N_{\mathcal{C}} \geq 2$, this has zero possibility because of the random property of the wireless channels. From this contradiction, we arrive at (16). Here it needs to be mentioned that, when $N_{\mathcal{C}} = 1$, that is, there is only one user whose multipath number is less than M, we have the equation $\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{X}\right)^{\mathrm{H}} \boldsymbol{U}_n = 0$ since $\boldsymbol{X}$ can simultaneously

IET Commun., 2014, Vol. 8, Iss. 7, pp. 1158–1168
doi: 10.1049/iet-com.2013.0539

belong to the column space of $\boldsymbol{I}_M$ and $\boldsymbol{H}^{(k)}$. Hence, $N_{\mathcal{C}} \geq 2$ is required in this paper to guarantee the validity of Lemma 1.

Lemma 2: For the kth user with $L^{(k)} \geq M$, there holds

$$\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} \begin{cases} = 0, & \xi = \xi^{(k)} \\ \neq 0, & \xi \neq \xi^{(k)} \end{cases} \tag{17}$$

Proof: Note that $\mathbb{B}^{(k)}(\xi^{(k)}) \otimes \boldsymbol{I}_M = \boldsymbol{v}^{(k)} \otimes \boldsymbol{I}_M$ serve as a submatrix of $\boldsymbol{V}$. Thus, we can readily obtain that it is orthogonal to the noise space matrix $\boldsymbol{U}_{\boldsymbol{n}}$, that is, $\left(\mathbb{B}^{(k)}(\xi^{(k)}) \otimes \boldsymbol{I}_M\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} = 0$. Then, we concentrate on $\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} \neq 0$ when $\xi \neq \xi^{(k)}$ which can be proved from the method of contradiction. According to Appendix D in [15], if $\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} = 0$, $\xi \neq \xi^{(k)}$, holds, the column space of $\boldsymbol{I}_M$ should simultaneously belong to the column space of each $\boldsymbol{H}^{(k')} \in \mathbb{C}^{M \times L^{(k')}}$, $k' \in \mathcal{C}$. However, this is impossible when $N_{\mathcal{C}} \geq 1$. From this contradiction, we arrive at $\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} \neq 0$ when $\xi \neq \xi^{(k)}$.

From the above two lemmas, we can design the CFO estimator as follows. For any non-zero length-M vector $\boldsymbol{x}$, we have

$$\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{x}\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} \boldsymbol{U}_{\boldsymbol{n}}^{\mathrm{H}} \left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{x}\right) = \boldsymbol{x}^{\mathrm{H}} \Pi^{(k)}(\xi) \boldsymbol{x} \tag{18}$$

where $\Pi^{(k)}(\xi) = \left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}} \boldsymbol{U}_{\boldsymbol{n}}^{\mathrm{H}} \left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M\right)$.

For the kth user with $L^{(k)} < M$, Lemma 1 indicates that $\Pi^{(k)}(\xi)$ drops its rank to $M - L^{(k)}$ if and only if $\xi = \xi^{(k)}$. Thus, the CFO estimate can be obtained similar to [15]. Specifically, for each trial ξ, we compute the M eigenvalues of the matrix $\Pi^{(k)}(\xi)$ denoted by $\kappa_1^{(k)}(\xi), \kappa_2^{(k)}(\xi), \ldots, \kappa_M^{(k)}(\xi)$ in ascending order, and then calculate the summation of the smallest $L^{(k)}$ eigenvalues of $\Pi^{(k)}(\xi)$ as the cost of current trial value. The CFO estimate for the kth user is the trial value that gives the minimum cost, that is

$$\hat{\xi}^{(k)} = \arg\min_{\xi} \sum_{i=1}^{L^{(k)}} \kappa_i^{(k)}(\xi) \tag{19}$$

On the other side, for the kth user with $L^{(k)} \geq M$, according to Lemma 2, we know $\left(\mathbb{B}^{(k)}(\xi) \otimes \boldsymbol{I}_M\right)^{\mathrm{H}} \boldsymbol{U}_{\boldsymbol{n}}$ equals zero matrix if and only if $\xi = \xi^{(k)}$. Thus, the CFO can be obtained by

$$\hat{\xi}^{(k)} = \arg\min_{\xi} \mathrm{Tr}\left\{\Pi^{(k)}(\xi)\right\} \tag{20}$$

Note that $\mathrm{Tr}\left\{\Pi^{(k)}(\xi)\right\} = \sum_{i=1}^{M} \kappa_i^{(k)}(\xi)$. Hence, combining both (19) and (20), the proposed GSSE can be described as the following unified form

$$\hat{\xi}^{(k)} = \arg\min_{\xi} g_k(\xi) \tag{21}$$

where the cost function is expressed as

$$g_k(\xi) = \sum_{i=1}^{\bar{L}^{(k)}} \kappa_i^{(k)}(\xi) \tag{22}$$

In noise-free environment, for both the situations with $L^{(k)} < M$ and $L^{(k)} \geq M$, one can imagine that, $g_k(\xi) = 0$ when $\xi = \xi^{(k)}$, whereas $g_k(\xi) > 0$ when $\xi \neq \xi^{(k)}$. Hence, by searching for the minimum point of $g_k(\xi)$, we could obtain the estimation for $\xi^{(k)}$. Moreover, for the kth user with $L^{(k)} < M$, it is seen from Lemma 1 that, after deriving CFO, $\left(\Pi^{(k)}(\hat{\xi}^{(k)})\right)^{\perp}$ can be considered as the estimate of $\boldsymbol{H}^{(k)}$ with certain ambiguity.

4 Proposed GMLE

The parameter set of interest for the kth user can be expressed as

$$\omega^{(k)} = \begin{cases} \left\{\xi^{(k)}, \boldsymbol{H}^{(k)}\right\} & L^{(k)} < M \\ \xi^{(k)} & L^{(k)} \geq M \end{cases} \tag{23}$$

The corresponding whole parameter set for the K users is given by $\omega = \{\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(K)}\}$. We rewrite (10) as the following parameterised equation

$$\boldsymbol{a}_{p,g} = \boldsymbol{V}(\omega)\bar{\boldsymbol{z}}_{p,g} + \bar{\boldsymbol{n}}_{p,g} \tag{24}$$

where

$$\boldsymbol{V}(\omega) = \left[\boldsymbol{V}^{(1)}(\omega^{(1)}), \boldsymbol{V}^{(2)}(\omega^{(2)}), \ldots, \boldsymbol{V}^{(K)}(\omega^{(K)})\right]$$

$$\boldsymbol{V}^{(k)}(\omega^{(k)}) = \begin{cases} \boldsymbol{v}^{(k)}(\xi^{(k)}) \otimes \boldsymbol{H}^{(k)} & L^{(k)} < M \\ \boldsymbol{v}^{(k)}(\xi^{(k)}) \otimes \boldsymbol{I}_M & L^{(k)} \geq M \end{cases}$$

$$\mathbf{v}^{(k)}(\xi^{(k)}) = \left[1, \mathrm{e}^{\mathrm{j}2\pi(q^{(k)}+\xi^{(k)}/K)}, \ldots, \mathrm{e}^{\mathrm{j}2\pi(K-1)(q^{(k)}+\xi^{(k)}/K)}\right]^{\mathrm{T}}$$

Concatenating the vectors $\boldsymbol{a}_{p,g}$, $p = 0, 1, \ldots, P-1$, $g = 1, 2, \ldots, L_s$, we obtain the following $MK \times PL_s$ matrix

$$\boldsymbol{A} = \left[\boldsymbol{A}_g, \boldsymbol{A}_{g+1}, \ldots, \boldsymbol{A}_{g+L_s-1}\right] = \boldsymbol{V}(\omega)\boldsymbol{Z} + \boldsymbol{N} \tag{25}$$

where $\boldsymbol{A}_g = [\boldsymbol{a}_{0,g}, \boldsymbol{a}_{1,g}, \ldots, \boldsymbol{a}_{P-1,g}]$, $\boldsymbol{Z}$ and $\boldsymbol{N}$ denote the corresponding signal and AWGN matrices, that is, $\boldsymbol{Z} = [\boldsymbol{Z}_g, \boldsymbol{Z}_{g+1}, \ldots, \boldsymbol{Z}_{g+L_s-1}]$ and $\boldsymbol{N} = \left[\bar{\boldsymbol{N}}_g, \bar{\boldsymbol{N}}_{g+1}, \ldots, \bar{\boldsymbol{N}}_{g+L_s-1}\right]$ with $\boldsymbol{Z}_g = \left[\bar{\mathrm{z}}_0, \bar{\mathrm{z}}_{1,g}, \ldots, \bar{\mathrm{z}}_{P-1,g}\right]$ and $\bar{\boldsymbol{N}}_g = [\bar{\boldsymbol{n}}_{0,g}, \bar{\boldsymbol{n}}_{1,g}, \ldots, \bar{\boldsymbol{n}}_{P-1,g}]$.

From (25), the ML parameter estimation of ω can be obtained by

$$\hat{\omega} = \arg\max_{\tilde{\omega}} \mathrm{Tr}\left\{\boldsymbol{A}^{\mathrm{H}} \boldsymbol{P}_V(\tilde{\omega}) \boldsymbol{A}\right\} \tag{26}$$

where $\tilde{\boldsymbol{x}}$ stands for the trial value of $\boldsymbol{x}$, and $\boldsymbol{P}_V(\tilde{\omega}) = \boldsymbol{V}(\tilde{\omega})\left[\boldsymbol{V}(\tilde{\omega})\right]^{\dagger}$ is the projection operator onto the space spanned by the columns of the matrix $\boldsymbol{V}(\tilde{\omega})$. The alternating projection algorithm [17] is then applied to reduce the multi-dimensional searching complexity. The estimation procedure consists of cycles and steps, where a cycle is made of K steps. Each step updates the parameter set of interest for a single user while keeping the other parameter sets constant at their most updated values. Without loss of generality, we follow the natural ordering

$k = 1, 2, \cdots, K$ in updating the users' parameters. Denote

$$\hat{\omega}_i^{(k)} = \begin{cases} \left\{ \hat{\xi}_i^{(k)}, \hat{\boldsymbol{H}}_i^{(k)} \right\} & L^{(k)} < M \\ \hat{\xi}_i^{(k)}, & L^{(k)} \geq M \end{cases} \tag{27}$$

as the estimate of $\omega^{(k)}$ at the ith cycle. Define $\hat{\omega}_{|k,i} = \left\{ \hat{\omega}_{i+1}^{(1)}, \ldots, \hat{\omega}_{i+1}^{(k-1)}, \hat{\omega}_i^{(k+1)}, \ldots, \hat{\omega}_i^{(K)} \right\}$ and use $\boldsymbol{P}_V(\tilde{\omega}^{(k)}, \hat{\omega}_{|k,i})$ to indicate the functional dependence of $\boldsymbol{P}_V$ on $\left\{ \hat{\omega}_{i+1}^{(1)}, \ldots, \hat{\omega}_{i+1}^{(k-1)}, \tilde{\omega}^{(k)}, \hat{\omega}_i^{(k+1)}, \ldots, \hat{\omega}_i^{(K)} \right\}$. At the kth step of the $(i+1)$th cycle, the alternating projection algorithm updates the estimate of $\omega^{(k)}$ by solving the following minimisation problem [15 eq. (34)–(39)]

$$\hat{\omega}_{i+1}^{(k)} = \arg \min_{\tilde{\omega}^{(k)}, \tilde{\boldsymbol{X}}} \left\| \boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i}) \boldsymbol{V}^{(k)}(\tilde{\omega}^{(k)}) \tilde{\boldsymbol{X}} - \boldsymbol{A} \right\|^2 \tag{28}$$

where $\boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i}) = \boldsymbol{I}_{MK} - \boldsymbol{B}(\hat{\omega}_{|k,i}) \left[\boldsymbol{B}(\hat{\omega}_{|k,i}) \right]^{\dagger}$ with $\boldsymbol{B}(\hat{\omega}_{|k,i}) \in \mathbb{C}^{MK \times \left(\bar{L}_{\text{sum}} - \bar{L}^{(k)} \right)}$ is a submatrix of $\boldsymbol{V}(\hat{\omega}^{(k)}, \hat{\omega}_{|k,i})$ formed by deleting the corresponding columns of $\boldsymbol{V}^{(k)}(\hat{\omega}^{(k)})$, and $\tilde{\boldsymbol{X}} \in \mathbb{C}^{\bar{L}^{(k)} \times PL_s}$ serves as a trial matrix that aims to minimise the above cost function.

According to the value of $L^{(k)}$, we can further rewrite (28) into

$$\hat{\omega}_{i+1}^{(k)} = \arg \min_{\tilde{\xi}^{(k)}, \tilde{\boldsymbol{H}}^{(k)}, \tilde{\boldsymbol{X}}} \left\| \boldsymbol{\Pi}_i^{(k)}\left(\tilde{\xi}^{(k)}\right) \tilde{\boldsymbol{H}}^{(k)} \tilde{\boldsymbol{X}} - \boldsymbol{A} \right\|^2, \quad L^{(k)} < M \tag{29}$$

$$\hat{\xi}_{i+1}^{(k)} = \arg \min_{\tilde{\xi}^{(k)}, \tilde{\boldsymbol{X}}} \left\| \boldsymbol{\Pi}_i^{(k)}\left(\tilde{\xi}^{(k)}\right) \tilde{\boldsymbol{X}} - \boldsymbol{A} \right\|^2, \quad L^{(k)} \geq M \tag{30}$$

where

$$\boldsymbol{\Pi}_i^{(k)}\left(\tilde{\xi}^{(k)}\right) = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i}) \left(\boldsymbol{v}^{(k)}\left(\tilde{\xi}^{(k)}\right) \otimes \boldsymbol{I}_M \right) \tag{31}$$

First, we consider the case of $L^{(k)} < M$. Following the steps similar to [15, eq. (43)–(50)], we know the minimisation problem of (29) in fact falls into the well-known low rank matrix approximation problem. Let SVD of $\Pi_i^{(k)}\left(\tilde{\xi}^{(k)}\right)$ be

$$\boldsymbol{\Pi}_i^{(k)}\left(\tilde{\xi}^{(k)}\right) = \boldsymbol{U}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \Sigma_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \left(\boldsymbol{V}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \right)^{\mathrm{H}} \tag{32}$$

where the diagonal entries of $\Sigma_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \in \mathbb{C}^{M \times M}$ are the non-zero singular values, $\boldsymbol{U}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \in \mathbb{C}^{MK \times M}$ and $\boldsymbol{V}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \in \mathbb{C}^{M \times M}$ denote the left and right singular vector matrices that correspond to the M non-zero singular values, respectively.

Define $\Xi_i^{(k)}\left(\tilde{\xi}^{(k)}\right) = \left(\boldsymbol{U}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \right)^{\mathrm{H}} \boldsymbol{R} \boldsymbol{U}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right)$ where $\boldsymbol{R} = \boldsymbol{A}\boldsymbol{A}^{\mathrm{H}}$. Denote the M eigenvalues of $\Xi_i^{(k)}\left(\tilde{\xi}^{(k)}\right)$ in ascending order and the corresponding eigenvectors by $\lambda_{l,i}^{(k)}\left(\tilde{\xi}^{(k)}\right)$ and $\nu_{l,i}^{(k)}\left(\tilde{\xi}^{(k)}\right)$, $l = 1, 2, \ldots, M$, respectively. According to [15], the CFO estimation for the kth user at the $(i+1)$th cycle can be expressed as

$$\hat{\xi}_{i+1}^{(k)} = \arg \max_{\tilde{\xi}^{(k)}} \sum_{l=M-L^{(k)}+1}^{M} \lambda_{l,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \tag{33}$$

The channel estimation for the kth user at this step can be then obtained from [15]

$$\hat{\boldsymbol{H}}_{i+1}^{(k)} = \boldsymbol{V}_{\Pi,i}^{(k)}\left(\hat{\xi}_{i+1}^{(k)}\right) \times \left(\Sigma_{\Pi,i}^{(k)}\left(\hat{\xi}_{i+1}^{(k)}\right) \right)^{-1} \left[\boldsymbol{\nu}_{M-L^{(k)}+1,i}^{(k)}\left(\hat{\xi}_{i+1}^{(k)}\right), \ldots, \boldsymbol{\nu}_{M,i}^{(k)}\left(\hat{\xi}_{i+1}^{(k)}\right) \right] \tag{34}$$

Next, we consider the case of $L^{(k)} \geq M$. In this situation, we can rewrite (30) into

$$\begin{aligned} \hat{\xi}_{i+1}^{(k)} &= \arg \min_{\tilde{\xi}^{(k)}} \left\| \left(\boldsymbol{I}_{MK} - \Pi_i^{(k)}\left(\tilde{\xi}^{(k)}\right) \Pi_i^{(k)}\left(\tilde{\xi}^{(k)}\right)^{\dagger} \right) \boldsymbol{A} \right\|^2 \\ &= \arg \max_{\tilde{\xi}^{(k)}} \left\| \left(\boldsymbol{U}_{\Pi,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \right)^{\mathrm{H}} \mathbf{A} \right\|^2 \\ &= \arg \max_{\tilde{\xi}^{(k)}} \operatorname{Tr}\left\{ \Xi_i^{(k)}\left(\tilde{\xi}^{(k)}\right) \right\} \end{aligned} \tag{35}$$

Note that $\operatorname{Tr}\left\{ \Xi_i^{(k)}\left(\tilde{\xi}^{(k)}\right) \right\} = \sum_{l=1}^{M} \lambda_{l,i}^{(k)}\left(\tilde{\xi}^{(k)}\right)$. Thus, combining both (33) and (35), an unified form of the proposed GMLE can be described as follows

$$\hat{\xi}_{i+1}^{(k)} = \arg \max_{\tilde{\xi}^{(k)}} G_i^{(k)}\left(\tilde{\xi}^{(k)}\right) \tag{36}$$

where the utility function is expressed as

$$G_i^{(k)}\left(\tilde{\xi}^{(k)}\right) = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \lambda_{l,i}^{(k)}\left(\tilde{\xi}^{(k)}\right) \tag{37}$$

5 Performance lower bound of GMLE

In this section, we assume that after the iteration of infinite cycles, to estimate the CFO of the kth user using (36), the effect from the parameter estimation error of the other users can be neglected. We then derive the analytical mean square error (MSE) for this simplified situation. Thus, the following analysed MSE can be considered as the performance lower bound of the GMLE.

We define $\Lambda_i^{(k)}(\xi) = \Pi_i^{(k)}(\xi)\left(\Pi_i^{(k)}(\xi)\right)^{\mathrm{H}} = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i}) \Gamma^{(k)}(\xi)(\Gamma^{(k)}(\xi))^{\mathrm{H}} \boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i})$, where $\Gamma^{(k)}(\xi) = \boldsymbol{v}^{(k)}(\xi) \otimes \boldsymbol{I}_M$. Denote the MK eigenvalues of $\Lambda_i^{(k)}(\xi)$ in descending order and the corresponding eigenvectors by $\mu_{l,i}^{(k)}(\xi)$ and $\boldsymbol{e}_{l,i}^{(k)}(\xi)$, $i = 1, 2, \ldots, MK$, respectively. Following the proof of Appendix E in [15], we know $\Lambda_i^{(k)}(\xi)$ has rank of M. Then, there holds $\boldsymbol{U}_{\Pi,i}^{(k)}(\xi) = \left[\boldsymbol{e}_{1,i}^{(k)}(\xi), \boldsymbol{e}_{2,i}^{(k)}(\xi), \ldots, \boldsymbol{e}_{M,i}^{(k)}(\xi) \right]$, and $\mu_{l,i}^{(k)}(\xi) = 0$, $l = M+1, \ldots, MK$. In the following, we omit the parameterised notation (ξ) for presentation clarity.

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doi: 10.1049/iet-com.2013.0539

We obtain (see equation at the bottom of the page)

where $\boldsymbol{D} = (\mathrm{j}2\pi/K)\mathrm{diag}(0, 1,\ldots, K-1) \otimes \boldsymbol{I}_M$.

From Appendix of this paper, we know the first and second derivatives of $G_i^{(k)}(\xi)$ with respect to ξ are given by

$$\frac{\partial G_i^{(k)}}{\partial \xi} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{l,i}^{(k)} \tag{38}$$

(see (39))

where

$$\mathbb{Q}_i^{(k)} = \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \boldsymbol{R}\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1} + \left(\mu_i^{(k)}\right)^{-1}\left(\Omega_i^{(k)}\right)^{\mathrm{H}} \boldsymbol{R}\boldsymbol{U}_{\Pi,i}^{(k)} \tag{40}$$

$$\Omega_i^{(k)} = \left(\boldsymbol{I}_{MK} - \boldsymbol{U}_{\Pi,i}^{(k)}\left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\right)\frac{\partial \Lambda_i^{(k)}}{\partial \xi}\boldsymbol{U}_{\Pi,i}^{(k)} \tag{41}$$

$$\mu_i^{(k)} = \mathrm{diag}\left\{\mu_{1,i}^{(k)}, \mu_{2,i}^{(k)}, \ldots, \mu_{M,i}^{(k)}\right\} \tag{42}$$

(see (43))

For the kth user, we denote $\Pi_{\boldsymbol{i}}^{(k)} = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\omega_{|k,\boldsymbol{i}}\right)\left(\boldsymbol{v}^{(k)}(\xi^{(k)}) \otimes \boldsymbol{I}_M\right)$ and $\Lambda_{\boldsymbol{i}}^{(k)} = \Pi_{\boldsymbol{i}}^{(k)}\left(\Pi_{\boldsymbol{i}}^{(k)}\right)^{\mathrm{H}}$, where $\omega_{k,\ \boldsymbol{i}} = \{\omega^{(1)}, \ldots, \omega^{(k-1)}, \omega^{(k+1)}, \ldots, \omega^{(K)}\}$ is formed by the perfect knowledge of the parameters of the other users, except the kth user. Denote the M non-zero eigenvalues of $\Lambda_{\boldsymbol{i}}^{(k)}$ in descending order and the corresponding eigenvectors by $\mu_{l,\boldsymbol{i}}^{(k)}$ and $\boldsymbol{e}_{l,\boldsymbol{i}}^{(k)}$, $l = 1, 2, \ldots, M$, respectively. Denote $\boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)} = \left[\boldsymbol{e}_{1,\boldsymbol{i}}^{(k)}, \boldsymbol{e}_{2,\boldsymbol{i}}^{(k)}, \ldots, \boldsymbol{e}_{M,\boldsymbol{i}}^{(k)}\right]$ as the matrix formed by the eigenvectors that correspond to the non-zero eigenvalues of $\Lambda_{\boldsymbol{i}}^{(k)}$. The eigenvalues in ascending order and the corresponding eigenvectors of $\left(\boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)}\right)^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{i}}\boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)}$ are expressed by $\lambda_{l,\boldsymbol{i}}^{(k)}$ and $\nu_{l,\boldsymbol{i}}^{(k)}$, $l = 1, 2, \ldots, M$, respectively.

The lower bound of the MSE of the CFO estimation for the kth user can be expressed as [4]

$$\mathrm{MSE}_{\mathrm{ML,LB}}\{\xi^{(k)}\} = \left.\frac{E\left\{\left(\partial G_\infty^{(k)}/\partial\xi\right)^2\right\}}{\left(E\left\{\partial^2 G_\infty^{(k)}/\partial^2\xi\right\}\right)^2}\right|_{\xi=\xi^{(k)}} \tag{44}$$

where $G_\infty^{(k)}$ stands for the utility function of $G_i^{(k)}$ after infinite iterative cycles. As mentioned earlier, we ignore the effect of the parameter estimation error from the other users in this section. Thus, there holds

$$\begin{cases} \boldsymbol{U}_{\Pi,\infty}^{(k)} = \boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)} \\ \mu_\infty^{(k)} = \mu_{\boldsymbol{i}}^{(k)} = \mathrm{diag}\left\{\mu_{1,\boldsymbol{i}}^{(k)}, \mu_{2,i}^{(k)}, \ldots, \mu_{M,\boldsymbol{i}}^{(k)}\right\} \\ \Omega_\infty^{(k)} = \Omega_{\boldsymbol{i}}^{(k)} = \left(\boldsymbol{I}_{MK} - \boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)}\left(\boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)}\right)^{\mathrm{H}}\right)\frac{\partial\Lambda_{\boldsymbol{i}}^{(k)}}{\partial\xi}\boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)} \\ \frac{\partial\Lambda_{\boldsymbol{i}}^{(k)}}{\partial\xi} = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\omega_{|k,\boldsymbol{i}}\right)\boldsymbol{D}\Gamma^{(k)}\left(\boldsymbol{\Pi}_{\boldsymbol{i}}^{(k)}\right)^{\mathrm{H}} + \boldsymbol{\Pi}_{\boldsymbol{i}}^{(k)}(\Gamma^{(k)})^{\mathrm{H}}\boldsymbol{D}^{\mathrm{H}}\boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\omega_{|k,\boldsymbol{i}}\right) \end{cases}$$

We denote $\boldsymbol{R} = \boldsymbol{R}_{\boldsymbol{i}} + \Delta\boldsymbol{R}$ and $\nu_{l,\infty}^{(k)} = \nu_{l,\boldsymbol{i}}^{(k)} + \Delta\nu_{l,\infty}^{(k)}$, where $\Delta\boldsymbol{R}$ and $\Delta\nu_{l,\infty}^{(k)}$ stand for the corresponding deviations from their expectation. Then, according to (38), we have (see (45) in next page)

Based on the fact that $\left(\boldsymbol{U}_{\Pi,\boldsymbol{i}}^{(k)}\right)^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{i}}\Omega_{\boldsymbol{i}}^{(k)} = 0$, we can simplify $E\{(\partial G_\infty^{(k)}/\partial\xi)^2\}$ into (see (46) and (47) in next page)

The derivation from (46) to (47) is based on the fact of [18, eq. (12)]

$$E\left\{\boldsymbol{x}_1^{\mathrm{H}}\Delta\boldsymbol{R}\boldsymbol{x}_2\boldsymbol{x}_3^{\mathrm{H}}\Delta\boldsymbol{R}\boldsymbol{x}_4\right\} = \frac{1}{PL_s}\boldsymbol{x}_1^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{i}}\boldsymbol{x}_4\boldsymbol{x}_3^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{i}}\boldsymbol{x}_2 \tag{48}$$

$$\frac{\partial\Lambda_i^{(k)}}{\partial\xi} = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\hat{\omega}_{|k,i}\right)\boldsymbol{D}\Gamma^{(k)}\left(\boldsymbol{\Pi}_i^{(k)}\right)^{\mathrm{H}} + \boldsymbol{\Pi}_i^{(k)}(\Gamma^{(k)})^{\mathrm{H}}\boldsymbol{D}^{\mathrm{H}}\boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\hat{\omega}_{|k,i}\right)$$

$$\frac{\partial^2\Lambda_i^{(k)}}{\partial\xi^2} = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\hat{\omega}_{|k,i}\right)\left(2\boldsymbol{D}\Gamma^{(k)}(\Gamma^{(k)})^{\mathrm{H}}\boldsymbol{D}^{\mathrm{H}} + \boldsymbol{D}\boldsymbol{D}\Gamma^{(k)}(\Gamma^{(k)})^{\mathrm{H}} + \Gamma^{(k)}(\Gamma^{(k)})^{\mathrm{H}}\boldsymbol{D}^{\mathrm{H}}\boldsymbol{D}^{\mathrm{H}}\right)\boldsymbol{P}_{\boldsymbol{B}}^{\perp}\left(\hat{\omega}_{|k,i}\right)$$

$$\frac{\partial^2 G_i^{(k)}}{\partial\xi^2} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} 2\Re\left\{\left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}}\mathbb{M}_i^{(k)}\nu_{l,i}^{(k)} + \sum_{z=1}^{M-\bar{L}^{(k)}}\frac{\left|\left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}}\mathbb{Q}_i^{(k)}\nu_{z,i}^{(k)}\right|^2}{\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}}\right\} \tag{39}$$

$$\begin{aligned} \mathbb{M}_i^{(k)} = {} & \left(\mu_i^{(k)}\right)^{-1}\left(\Omega_i^{(k)}\right)^{\mathrm{H}}\boldsymbol{R}\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1} - \Xi_i^{(k)}\left(\mu_i^{(k)}\right)^{-1}\left(\Omega_i^{(k)}\right)^{\mathrm{H}}\frac{\partial\Lambda_i^{(k)}}{\partial\xi}\boldsymbol{U}_{\Pi,i}^{(k)}\left(\mu_i^{(k)}\right)^{-1} \\ & + \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\boldsymbol{R}\left(\boldsymbol{I}_{MK} - \boldsymbol{U}_{\Pi,i}^{(k)}\left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\right)\left(\frac{\partial\Lambda_i^{(k)}}{\partial\xi}\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-2} + \frac{\partial^2\Lambda_i^{(k)}}{\partial\xi^2}\boldsymbol{U}_{\Pi,i}^{(k)}\left(\mu_i^{(k)}\right)^{-1}\right) \\ & - 2\left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\boldsymbol{R}\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1}\left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\frac{\partial\Lambda_i^{(k)}}{\partial\xi}\boldsymbol{U}_{\Pi,i}^{(k)}\left(\mu_i^{(k)}\right)^{-1} \end{aligned} \tag{43}$$

On the other hand, from (39), we obtain (see (49))

Note that $\left(\Omega_i^{(k)}\right)^{\mathrm H}\left(\partial\Lambda_i^{(k)}/\partial\xi\right)\boldsymbol U_{\Pi,i}^{(k)}=\left(\Omega_i^{(k)}\right)^{\mathrm H}\Omega_i^{(k)}$. Then, combining both (47) and (49), the MSE lower bound of the CFO estimation for the kth user can be finally given by (see (50))

6 Simulations

In this section, we assess the proposed CFO estimation algorithms from computer simulations. The total number of subcarriers is taken as $N=64$ and are divided into $Q=4$ subchannels. The QPSK is adopted. The normalised CFO of each user is randomly generated from −0.35 to 0.35. The root MSE (RMSE) of the normalised CFO estimation is adopted as the figure of merit. $M=8$ and $L_s=8$ are assumed in the following simulations unless otherwise stated. The estimates of GSSE are taken as the initial values of GMLE. For comparison, we include the CFO estimation scheme of [10], referred to as 'ESPRIT'. For fairness, the multiple receive antenna diversity is exploited in ESPRIT as described in [9, eq. (28)]. The number of users in ESPRIT is three since at least one subchannel should be reserved for null subcarriers in ESPRIT. Note that the results of the estimators of [15] are not included here, since they are equivalent to the proposed GSSE and GMLE in this paper when the multipath numbers of all users equal a same value less than the antenna number.

We start by investigating the convergence rate of the proposed GMLE. In Fig. 1, we depict the CFO estimation performance of GMLE as the cycle number increases. The signal-to-noise ratio (SNR) is taken as 20 dB. The analytical lower bound is also included in this figure by the dotted curves as the benchmark. We denote $\boldsymbol L=[L^{(1)}, L^{(2)}, L^{(3)}, L^{(4)}]$ and include several different cases of $\boldsymbol L$ in this figure. Bearing in mind that GMLE estimator is initialised by the GSSE estimator, the results explicitly demonstrate the performance improvement of GMLE introduced by the iterative procedure. We see that, for the included different scenarios, the RMSEs of GMLE quickly decline with first few cycles and almost approach the corresponding analytical lower bounds. These indicate the validity of our method and correctness of the analytical results.

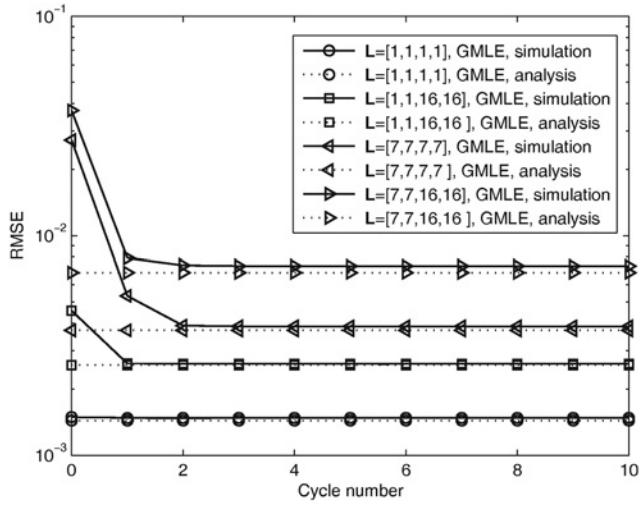

Fig. 1 *CFO estimation RMSE convergence process of GMLE (SNR = 20 dB)*

In the second example, we consider $L^{(1)}=L^{(2)}$ and $L^{(3)}=L^{(4)}$ without loss of generality. We increase $L^{(k)}$ ($k=3, 4$) from 1 to 16, and show the performance evolution of both GSSE and GMLE in Fig. 2 when SNR equals 20 dB. From these results, we can make the following observations.

$$\begin{aligned} E\left\{\left(\frac{\partial G_\infty^{(k)}}{\partial\xi}\right)^2\right\} = E\Bigg\{\Bigg(&\sum_{l=M-\bar{L}^{(k)}+1}^{M}\left(\nu_{l,i}^{(k)}+\Delta\nu_{l,\infty}^{(k)}\right)^{\mathrm H}\left(\left(\mathbf{U}_{\Pi,i}^{(k)}\right)^{\mathrm H}(\boldsymbol R_i+\Delta\boldsymbol R)\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1}\right.\,.. \\ &\left.+\left(\mu_i^{(k)}\right)^{-1}\Omega_i^{(k)}(\boldsymbol R_i+\Delta\boldsymbol R)\boldsymbol U_{\Pi,i}^{(k)}\right)\left(\nu_{l,\mathrm i}^{(k)}+\Delta\nu_{l,\infty}^{(k)}\right)\Bigg)^2\Bigg\} \end{aligned} \tag{45}$$

$$E\left\{\left(\frac{\partial G_\infty^{(k)}}{\partial\xi}\right)^2\right\}\simeq E\left\{\left(\sum_{l=M-\bar{L}^{(k)}+1}^{M}\left(\nu_{l,i}^{(k)}\right)^{\mathrm H}\left(\left(\boldsymbol U_{\Pi,i}^{(k)}\right)^{\mathrm H}\Delta\boldsymbol R\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1}+\left(\mu_i^{(k)}\right)^{-1}\Omega_i^{(k)}\Delta\boldsymbol R\boldsymbol U_{\Pi,i}^{(k)}\right)\nu_{l,i}^{(k)}\right)^2\right\} \tag{46}$$

$$\simeq\sum_{l=M-\bar{L}^{(k)}+1}^{M}\frac{2\sigma_n^2}{PL_s}\Re\left\{\lambda_{l,i}^{(k)}\left(\nu_{l,i}^{(k)}\right)^{\mathrm H}\left(\mu_i^{(k)}\right)^{-1}\left(\Omega_i^{(k)}\right)^{\mathrm H}\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1}\nu_{l,i}^{(k)}\right\} \tag{47}$$

$$E\left\{\frac{\partial^2 G_\infty^{(k)}}{\partial\xi^2}\right\}\simeq-\sum_{l=M-\bar{L}^{(k)}+1}^{M}2\Re\left\{\lambda_{l,i}^{(k)}\left(\nu_{l,i}^{(k)}\right)^{\mathrm H}\left(\mu_i^{(k)}\right)^{-1}\left(\Omega_i^{(k)}\right)^{\mathrm H}\frac{\partial\Lambda_i^{(k)}}{\partial\xi}\boldsymbol U_{\Pi,i}^{(k)}\left(\mu_i^{(k)}\right)^{-1}\nu_{l,i}^{(k)}\right\} \tag{49}$$

$$\mathrm{MSE}_{\mathrm{ML,LB}}\{\xi^{(k)}\}=\frac{\left(\sigma_n^2/2PL_s\right)}{\sum_{l=M-\bar{L}^{(k)}+1}^{M}\lambda_{l,i}^{(k)}\left(\nu_{l,i}^{(k)}\right)^{\mathrm H}\left(\mu_i^{(k)}\right)^{-1}\left(\Omega_i^{(k)}\right)^{\mathrm H}\Omega_i^{(k)}\left(\mu_i^{(k)}\right)^{-1}\nu_{l,i}^{(k)}} \tag{50}$$

doi: 10.1049/iet-com.2013.0539

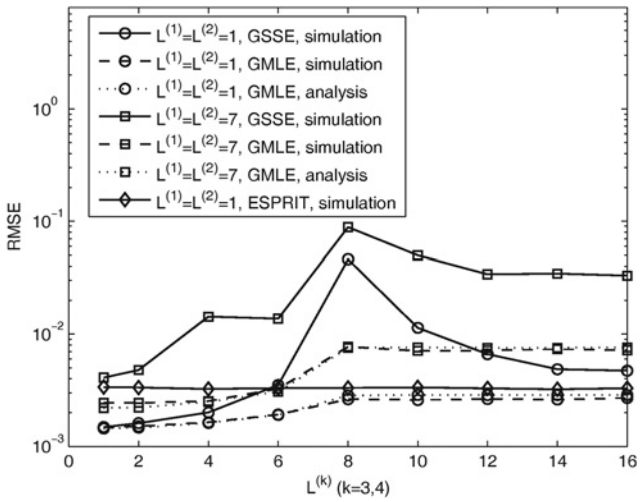

Fig. 2 *CFO estimation RMSE performance of GSSE and GMLE against $L^{(k)}$, $k = 3, 4$ (SNR = 20 dB)*

First, as expected, ESPRIT has the advantage of insensitivity to the value of $L^{(k)}$, whereas both GSSE and GMLE suffer from the performance degradation as the multipath number increases when $L^{(k)} \leq 8$, $k = 3, 4$, which coincides with the observations in [15]. It is seen that, GMLE always behaves better than GSSE when $L^{(k)}$ ($k = 3, 4$) are increased from 1 to 16, especially when $L^{(k)}$ ($k = 3, 4$) are around 8. Moreover, we observe that the performance of GMLE is gradually degraded when $L^{(k)}$ is increased from 1 to 8 and is basically unchanged when $L^{(k)}$ becomes even larger. Thus, this indicates GMLE also has the advantage of being more insensitive to $L^{(k)}$ as compared with GSSE.

Second and more interestingly, we can observe that GSSE behaves worst when $L^{(k)}$ ($k = 3, 4$) equal 8 and its performance can be improved when $L^{(k)}$ ($k = 3, 4$) become larger than 8. This may be explained as follows. When $L^{(k)} \geq M$, the increase of multipath number would improve the non-singularity of the correlation matrices $\boldsymbol{R}_{\bar{z}\bar{z}}^{(k)}$ statistically, $k = 3, 4$, which is beneficial to the estimation performance of GSSE.

Third, it is seen that, the simulation results of GMLE also match the corresponding analytical curves of lower bound, which indicates the correctness of theoretical analysis.

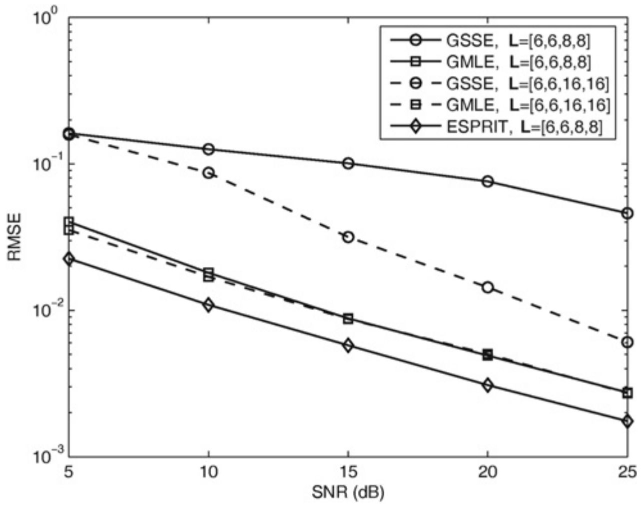

Fig. 3 *CFO RMSE performance comparison between GSSE and GMLE as a function of SNR*

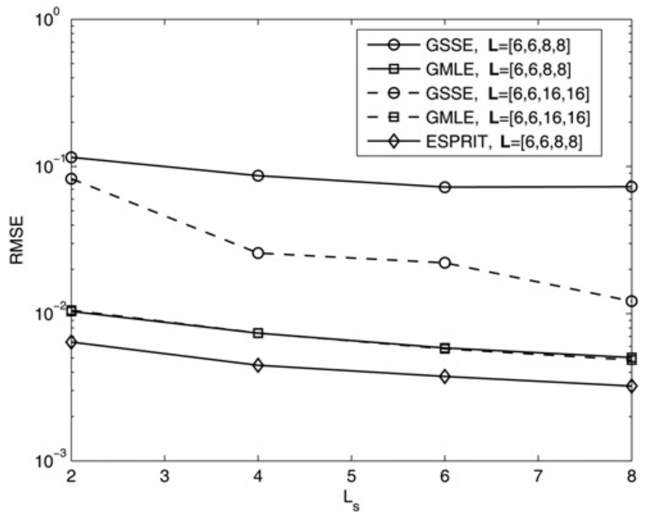

Fig. 4 *CFO RMSE performance comparison between GSSE and GMLE as a function of L_s*

In Fig. 3, we show the performance comparison between GSSE and GMLE as a function of SNR with different configurations of multipath numbers. The comparison clearly demonstrates the superiority of GMLE as compared with GSSE. We also see that ESPRIT achieves better performance than both GSSE and GMLE in the investigated scenarios. However, we should note that the system is fully loaded in both GSSE and GMLE, whereas one subchannel should be reserved in ESPRIT. Thus, our estimators support higher bandwidth efficiency as compared with ESPRIT.

Next, we show the performance comparison among the different estimators as L_s increases from 2 to 8 in Fig. 4. The SNR is assumed to be 20 dB in this example. The comparison again indicates the superiority of GMLE as compared with GSSE. It is also seen that both GMLE and ESPRIT suffer from only slight performance degradation when fewer block durations are available.

Fig. 5 shows the QPSK and 16QAM symbol error rate (SER) performance comparison between the proposed GMLE and ESPRIT estimator. We assume the multi-antenna receiver employs the maximal-ratio combining decoder after the CFO compensation. The channel responses are assumed to be known to the receiver.

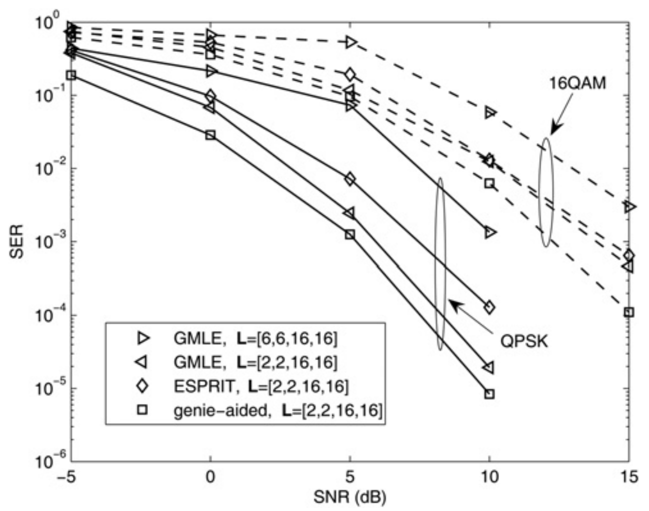

Fig. 5 *SER performance comparison as a function of SNR*

We consider two block durations are adopted in this example. The results with perfect CFO estimation are also plotted as the benchmark, referred to as 'genie-aided' in this figure. We can observe the performance gap between our method and the genie-aided results. However, it is seen that the proposed method can achieve comparable SER performance with the ESPRIT estimator.

7 Conclusions

We have developed two generalised blind CFO estimators for the interleaved OFDMA uplink with multi-antenna receiver. We consider the more general scenarios that the number of multipaths from some users may be larger than the number of antennas at the receiver. We have found that, as long as the numbers of multipaths from two of the users are smaller than the number of antennas at the receiver, the proposed estimators support fully loaded transmissions. Both the theoretical and numeral results are provided, which corroborate the proposed studies.

8 Acknowledgments

This work is supported by the National Natural Science Foundation of China under grants no. 61302069, 61071125, 61071216, 61172093, 61172092 and 61102081, China Postdoctoral Science Foundation grant no. 2013M530425, and Shaanxi Province Postdoctoral Research Funding.

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10 Appendix

In the following, we omit the parameterised notation (ξ) for presentation clarity. The first derivative of $G_i^{(k)}$ with respect to ξ is expressed as

$$\frac{\partial G_i^{(k)}}{\partial \xi} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \frac{\partial \Xi_i^{(k)}}{\partial \xi} \nu_{l,i}^{(k)} \tag{51}$$

where

$$\frac{\partial \Xi_i^{(k)}}{\partial \xi} = \left(\frac{\partial \boldsymbol{U}_{\Pi,i}^{(k)}}{\partial \xi}\right)^{\mathrm{H}} \boldsymbol{R} \boldsymbol{U}_{\Pi,i}^{(k)} + \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \boldsymbol{R} \frac{\partial \boldsymbol{U}_{\Pi,i}^{(k)}}{\partial \xi}$$

We have

$$\begin{aligned} \frac{\partial \boldsymbol{e}_{l,i}^{(k)}}{\partial \xi} = & \sum_{j \neq l}^{M} \boldsymbol{e}_{j,i}^{(k)} \frac{\left(\boldsymbol{e}_{j,i}^{(k)}\right)^{\mathrm{H}} \left(\partial \Lambda_i^{(k)} / \partial \xi\right) \boldsymbol{e}_{l,i}^{(k)}}{\mu_{l,i}^{(k)} - \mu_{j,i}^{(k)}} \\ & + \left(\boldsymbol{I}_{MK} - \boldsymbol{U}_{\Pi,i}^{(k)} \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\right) \frac{\left(\partial \Lambda_i^{(k)} / \partial \xi\right) \boldsymbol{e}_{l,i}^{(k)}}{\mu_{l,i}^{(k)}} \end{aligned} \tag{52}$$

Consequently, we have

$$\frac{\partial \boldsymbol{U}_{\Pi,i}^{(k)}}{\partial \xi} = \boldsymbol{U}_{\Pi,i}^{(k)} \boldsymbol{E}_i^{(k)} + \Omega_i^{(k)} \left(\mu_i^{(k)}\right)^{-1}$$

where $\mu_i^{(k)}$ is defined in (43) and $\boldsymbol{E}_i^{(k)} \in \mathbb{C}^{M \times M}$ is constructed as follows. We have

$$\boldsymbol{E}_i^{(k)}(l, j) = \frac{\left(\boldsymbol{e}_{j,i}^{(k)}\right)^{\mathrm{H}} \left(\partial \Lambda_i^{(k)} / \partial \xi\right) \boldsymbol{e}_{l,i}^{(k)}}{\lambda_{j,i}^{(k)} - \lambda_{l,i}^{(k)}}$$

when $l \neq j$, and $\boldsymbol{E}_i^{(k)}(l, l) = 0$, $l = 1, 2, \ldots, M$. Hence, (51) can be rewritten as (see (53) in next page)

Bearing in mind that $\boldsymbol{E}_i^{(k)} + \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} = 0$, we simplify (53) into

$$\frac{\partial G_i^{(k)}}{\partial \xi} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{l,i}^{(k)} \tag{54}$$

where $\mathbb{Q}_i^{(k)} = \Sigma_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} + \left(\mu_i^{(k)}\right)^{-1} \left(\Sigma_i^{(k)}\right)^{\mathrm{H}}$ and $\Sigma_i^{(k)} = \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \boldsymbol{R} \Omega_i^{(k)}$. Then, we arrive at (38).

Next, based on (51), the second derivative of $G_i^{(k)}$ can be given by

$$\frac{\partial^2 G_i^{(k)}}{\partial \xi^2} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \frac{\partial \mathbb{Q}_i^{(k)}}{\partial \xi} \nu_{l,i}^{(k)} + (\nu_{l,i})^{\mathrm{H}} \mathbb{Q}_i^{(k)} \frac{\partial \nu_{l,i}^{(k)}}{\partial \xi} + \left(\frac{\partial \nu_{l,i}^{(k)}}{\partial \xi}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{l,i}^{(k)} \tag{55}$$

We obtain (see (56))

where

$$\frac{\partial \Sigma_i^{(k)}}{\partial \xi} = \left(\frac{\partial \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}}{\partial \xi}\right)^{\mathrm{H}} \boldsymbol{R}\Omega_i^{(k)} + \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \mathbf{R} \frac{\partial \Omega_i^{(k)}}{\partial \xi} = \mathbb{H}_i^{(k)} + \Sigma_i^{(k)} \boldsymbol{E}_i^{(k)} + \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)} \tag{57}$$

(see (58 and 59))

By substituting (57) into (56), we have (see (60))

where

$$\frac{\partial (\mu_i^{(k)})^{-1}}{\partial \xi} = -((\mu_i^{(k)})^{-1} (\boldsymbol{U}_{\Pi,i}^{(k)})^{H} \frac{\partial \Lambda_i^{(k)}}{\partial \xi} \boldsymbol{U}_{\Pi,i}^{(k)} (\mu_i^{(k)})^{-1}) \circ \boldsymbol{I}_M$$

Moreover, note that (see equation at the bottom of the page)

Then, there holds

$$\left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \frac{\partial \nu_{l,i}^{(k)}}{\partial \xi} = \sum_{z \neq l}^{M} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{z,i}^{(k)} \left(\nu_{z,i}^{(k)}\right)^{\mathrm{H}} \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \nu_{l,i}^{(k)} + \frac{\left|(\nu_{l,i}^{(k)})^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{z,i}^{(k)}\right|^2}{\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}} \tag{61}$$

On the other side, we obtain (see (62) in next page)

Substituting both (60) and (62) into (55), we arrive at

$$\frac{\partial G_i^{(k)}}{\partial \xi} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \left((\nu_{l,i}^{(k)})^{\mathrm{H}} \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \boldsymbol{R}\Omega_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} \nu_{l,i}^{(k)} + \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\mu_i^{(k)}\right)^{-1} \left(\Omega_i^{(k)}\right)^{\mathrm{H}} \boldsymbol{R}\boldsymbol{U}_{\Pi,i}^{(k)} \nu_{l,i}^{(k)} + \lambda_{l,i}^{(k)} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\boldsymbol{E}_i^{(k)} + \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}}\right) \nu_{l,i}^{(k)}\right. \tag{53}$$

$$\left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \frac{\partial \mathbb{Q}_i^{(k)}}{\partial \xi} \nu_{l,i}^{(k)} = 2\,\Re \left\{ \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\frac{\partial \Sigma_i^{(k)}}{\partial \xi}\right) \left(\mu_i^{(k)}\right)^{-1} \nu_{l,i}^{(k)} + \nu_{l,i}^{(k)} \Sigma_i^{(k)} \frac{\partial \left(\mu_i^{(k)}\right)^{-1}}{\partial \xi} \nu_{l,i}^{(k)} \right\} \tag{56}$$

$$\mathbb{H}_i^{(k)} = \left(\mu_i^{(k)}\right)^{-1} \left(\Omega_i^{(k)}\right)^{\mathrm{H}} \boldsymbol{R}\Omega_i^{(k)} + \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \boldsymbol{R} \left(\boldsymbol{I} - \boldsymbol{U}_{\Pi,i}^{(k)} \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}}\right) \left(\frac{\partial \Lambda_i^{(k)}}{\partial \xi} \Omega_i^{(k)} (\mu_i^{(k)})^{-1} + \frac{\partial^2 \Lambda_i^{(k)}}{\partial \xi^2} \boldsymbol{U}_{\Pi,i}^{(k)}\right) - \left(\Sigma_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} + \Xi_i^{(k)} (\mu_i^{(k)})^{-1} \left(\Omega_i^{(k)}\right)^{\mathrm{H}}\right) \frac{\partial \Lambda_i^{(k)}}{\partial \xi} \boldsymbol{U}_{\Pi,i}^{(k)} \tag{58}$$

$$\frac{\partial^2 \Lambda_i^{(k)}}{\partial \xi^2} = \boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i}) \left(2D\Gamma^{(k)}(\Gamma^{(k)})^{\mathrm{H}} D^{\mathrm{H}} + DD\Gamma^{(k)}(\Gamma^{(k)})^{\mathrm{H}} + \Gamma^{(k)}(\Gamma^{(k)})^{\mathrm{H}} D^{\mathrm{H}} D^{\mathrm{H}}\right) \boldsymbol{P}_{\boldsymbol{B}}^{\perp}(\hat{\omega}_{|k,i}) \tag{59}$$

$$\left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \frac{\partial \mathbb{Q}_i^{(k)}}{\partial \xi} \nu_{l,i}^{(k)} = 2\,\Re \left\{ \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{H}_i^{(k)} (\mu_i^{(k)})^{-1} \nu_{l,i}^{(k)} + \nu_{l,i}^{(k)} \Sigma_i^{(k)} \frac{\partial \left(\mu_i^{(k)}\right)^{-1}}{\partial \xi} \nu_{l,i}^{(k)} \right\} + 2\,\Re \left\{ \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\Sigma_i^{(k)} \boldsymbol{E}_i^{(k)} + \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)}\right) \left(\mu_i^{(k)}\right)^{-1} \nu_{l,i}^{(k)} \right\} \tag{60}$$

$$\left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \frac{\partial \nu_{l,i}^{(k)}}{\partial \xi} = \sum_{z \neq l}^{M} \left(\nu_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{z,i}^{(k)} \frac{\left(\nu_{z,i}^{(k)}\right)^{\mathrm{H}} \left(\partial \Xi_i^{(k)} / \partial \xi\right) \nu_{l,i}^{(k)}}{\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}}$$

$$(\nu_z^{(k)})^{\mathrm{H}} \frac{\partial \Xi_i^{(k)}}{\partial \xi} \nu_{l,i}^{(k)} = \left(\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}\right) \left(\nu_{z,i}^{(k)}\right)^{\mathrm{H}} \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \nu_{l,i}^{(k)} + \left(\nu_{z,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \nu_{l,i}^{(k)}$$

(see (63))

Note that (see equation at the bottom of the page)

where

$$\boldsymbol{M}_i^{(k)} = \left(\left(\mu_i^{(k)}\right)^{-1} \left(\boldsymbol{U}_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \left(\partial \Lambda_i^{(k)}/\partial \xi\right) \boldsymbol{U}_{\Pi,i}^{(k)} \left(\mu_i^{(k)}\right)^{-1} \right) \circ (1_{M\times 1} - \boldsymbol{I}_M)$$

Then, we can rewrite $\left(\partial^2 G_i^{(k)}/\partial \xi^2\right)$ as (see (64))

By substituting $\mathbb{H}_i^{(k)}$ from (58) into (64), we can arrive at (39) after some simple manipulations.

This completes the proof.

$$\sum_{l=M-\bar{L}^{(k)}+1}^{M} \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \frac{\partial \boldsymbol{\nu}_{l,i}^{(k)}}{\partial \xi} + \left(\frac{\partial \boldsymbol{\nu}_{l,i}^{(k)}}{\partial \xi}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \boldsymbol{\nu}_{l,i}^{(k)} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} \left[\sum_{z=1}^{M-\bar{L}^{(k)}} 2 \frac{\left| \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \boldsymbol{\nu}_{z,i}^{(k)} \right|^2}{\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}} + 2\,\Re\left\{ \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \boldsymbol{\nu}_{l,i}^{(k)} \right\} \right] \tag{62}$$

$$\begin{aligned} \frac{\partial^2 G_i^{(k)}}{\partial \xi^2} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} & 2\,\Re\left\{ \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{H}_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} \boldsymbol{\nu}_{l,i}^{(k)} + \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)} \frac{\partial \left(\mu_i^{(k)}\right)^{-1}}{\partial \xi} \boldsymbol{\nu}_{l,i}^{(k)} \right\} \\ & + 2\,\Re\left\{ \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\Sigma_i^{(k)} \boldsymbol{E}_i^{(k)} + \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)} \right) \left(\mu_i^{(k)}\right)^{-1} \boldsymbol{\nu}_{l,i}^{(k)} + \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \boldsymbol{\nu}_{l,i}^{(k)} \right\} \\ & + \sum_{z=1}^{M-\bar{L}^{(k)}} 2 \frac{\left| (\boldsymbol{\nu}_{l,i}^{(k)})^{\mathrm{H}} \mathbb{Q}_i^{(k)} \boldsymbol{\nu}_{z,i}^{(k)} \right|^2}{\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}} \end{aligned} \tag{63}$$

$$\begin{aligned} & 2\,\Re\left\{ \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\Sigma_i^{(k)} \boldsymbol{E}_i^{(k)} + \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)} \right) \left(\mu_i^{(k)}\right)^{-1} \boldsymbol{\nu}_{l,i}^{(k)} + \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \left(\boldsymbol{E}_i^{(k)}\right)^{\mathrm{H}} \boldsymbol{\nu}_{l,i}^{(k)} \right\} \\ & = 2\Re\left\{ \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)} \left(\boldsymbol{E}_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} - \left(\mu_i^{(k)}\right)^{-1} \boldsymbol{E}_i^{(k)} \right) \boldsymbol{\nu}_{l,i}^{(k)} \right\} \\ & = -2 \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \Sigma_i^{(k)} \boldsymbol{M}_i^{(k)} \boldsymbol{\nu}_{l,i}^{(k)} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 G_i^{(k)}}{\partial \xi^2} = \sum_{l=M-\bar{L}^{(k)}+1}^{M} & 2\,\Re\left\{ \left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \left(\mathbb{H}_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} - \boldsymbol{\Sigma}_i^{(k)} \left(\mu_i^{(k)}\right)^{-1} \left(U_{\Pi,i}^{(k)}\right)^{\mathrm{H}} \frac{\partial \Lambda_i^{(k)}}{\partial \xi} U_{\Pi,i}^{(k)} \left(\mu_i^{(k)}\right)^{-1} \boldsymbol{\nu}_{l,i}^{(k)} \right\} \\ & + \sum_{z=1}^{M-\bar{L}^{(k)}} 2 \frac{\left| \left(\left(\boldsymbol{\nu}_{l,i}^{(k)}\right)^{\mathrm{H}} \mathbb{Q}_i^{(k)} \boldsymbol{\nu}_{z,i}^{(k)} \right|^2}{\lambda_{l,i}^{(k)} - \lambda_{z,i}^{(k)}} \end{aligned} \tag{64}$$

doi: 10.1049/iet-com.2013.0539