www.ietdl.org

Published in IET Communications
Received on 29th May 2013
Revised on 17th January 2014
Accepted on 3rd March 2014
doi: 10.1049/iet-com.2013.0444

ISSN 1751-8628

Optimal power allocation and relay selection for multiple code division multiple access peer-to-peer communication

Banafsheh Lashkari[1,2]*, Mehrzad Biguesh*[1]*, Saeed Gazor*[2]

[1]*School of Electrical and Computer Engineering (ECE), Shiraz University, Shiraz, Iran*
[2]*Department of ECE, Queens University, Kingston, Ontario, Canada*
E-mail: biguesh@shirazu.ac.ir

Abstract: The authors consider a half-duplex asynchronous code division multiple access cooperative network with N source–destination (SD) pairs and a number of relay nodes where the nodes of all pairs have to exchange data in two hops via assistance of one of the available relays. In this study, they minimise the total transmit power and derive the closed-form solution for choosing the best relay, the best relay gain and the transmit powers of all sources where some predefined signal-to-interference plus noise-ratios (SINRs) are guaranteed. Interestingly, the feasibility condition of the problem depends only on the required SINRs, the number of SD pairs and the maximum cross-correlation of users' codes. They suggest two control procedures for admitting or dropping of users to the network to satisfy the feasibility condition. For a reciprocal environment, the best relay and its gain are proved to remain unchanged for reversing the communication directions. In addition, the authors' power control algorithm can be directly applied to the case of two-hop two-way relaying. Computer simulations are used to demonstrate the system performance.

1 Introduction

In relay communication, one or more relaying nodes receive signal originated from the source and forward it to the destination. The main goal of such a cooperation is to enhance the communication data rate, reliability and power efficiency [1].

Relay communication has attracted lots of attention in recent years and various methods have been proposed for relaying the transmitted signal [2–5]. Amplify and forward (AF) is a method in which the relay node simply transmits a scaled copy of its received signal and this method is considered by lots of researches on relay communications. For better performance gain in AF method, channel information is required to adjust the transmit power and phase at transmitting nodes. Regarding power adjustment in AF relay communications, the early works have studied relaying in simple structures consisting of one source–destination (SD) pair and one relay node [6, 7]. Adjusting power in network configurations composed of several SD pairs and several relay nodes has been focused on by the recent works (e.g. [8–18]). Minimising sum of transmitting powers in multiple user systems elevates the power efficiency of the network and, in this way, increases the network lifetime [10, 18–20].

Using multiple relay nodes can add to the diversity order on one hand [11] and impose additional complexity on the other hand [15]. When the medium is shared by many transmitting nodes, that is, multiple sources or multiple relay nodes or both, combination of multi-path signals at destination may result in attenuating the quality of communication. One of the means to harness this issue is beamforming [12–15, 17, 21]. In reality, the use of relay beamformers is restricted by constraints, for example, perfect time synchronisation of relays retransmission and full channels states requirement. Another means to suppress the destructive effect of multi signal interference is orthogonal signalling schemes such as distributed space time coding (DSTC) [3–5], time (or frequency) division relaying [2, 8–11], relay selecting [11, 22] etc. The idea of DSTC proposed for MIMO systems is extended to be applied in relay networks [3, 4]. Indeed, the implementation of orthogonal and quasi-orthogonal codes without the sacrifice of data rate is possible only for, respectively, two and four relay nodes [5]. For time-division schemes, transmission period is divided to several time slots (TS) where each of these TSs is assigned to one or more specific transmitting nodes and subsequently multiple received signals are combined at destination. This mechanism is specifically useful for networks with a few nodes. Particularly, the protocol does not take into account the channel spectral efficiency and, last but not least, the transmission time management of system nodes makes this protocol less practical.

As an alternative to DSTC [22–24], relay selection methods are favoured for simplicity of implementation where low error rates can be gained because of more yielding use of power [11, 22]. A variety of relay selection strategies have been discussed in [25] and the references there in, among which the best relay selection achieves full diversity order and outperforms the DSTC scheme of [3] for most of network sizes [26]. Using a relay to serve multiple users is also considered in the literature for various objectives (e.g. [19, 20, 27–31]). In [20], the authors proposed a relaying scenario where the SD pairs share non-orthogonal CDMA in the uplink phase and the relay performs digital encoding and broadcasts. They investigated iterative power control and multiuser detection problem to minimise the network total transmit powers and to provide receivers quality of service (QoS) requirements. The source-sum-power minimisation problem is considered in [19] over disjoint frequency bands without interference where, in effect, the system is decomposed into multiple orthogonal channels. In [31], the pricing and power control algorithms are studied when there exist one relay and multiple SD pairs. A generic application for using one relay node to transmit data between multiple users can be in rich scattering environments where a relay is deployed for helping base station in uplink mode and assisting the users in downlink transmission.

In this paper, we consider a wireless *ad hoc* network with multiple SD pairs and multiple relay nodes. We assume that both nodes of all pairs have data for exchange and CDMA is used for symbol transmission between each SD pair. Our system operates in asynchronous fading channels (as in [32, 33]), therefore the users spreading codes interfere with each other at the receiver points. A two-hop relaying method with best relay selection is considered to communicate data both in forward and reverse directions, using AF scheme. The main contributions of this paper are 3-fold:

- We propose a scheme to allocate the power among the set of sources and the best relay such that the sum of transmit powers in the whole network is minimised, whereas a target level of signal-to-interference plus noise-ratio (SINR) is satisfied at each destination.
- The necessary and sufficient condition for the feasibility of the problem is analysed in terms of the maximum cross-correlation of users' codes, the target SINR values and the number of SD pairs. Using this condition, we propose two procedures for admitting of new SD pairs and for dropping of pairs in case that the problem is infeasible.
- We prove that if all the communication directions of SD nodes are switched, the solution for the best relay node, the relay's gain and the optimal sum of transmit powers remain unchanged.

Throughout this paper, bold face small letters and bold face capital letters are used to show vectors and matrices, respectively. The identity matrix is shown by $\boldsymbol{I}$, and $\boldsymbol{1}$ represents the unit vector. δ_{i-j} denotes the Kronecker delta function. $E\{\cdot\}$ is used to show the statistical mean, $\lceil\cdot\rceil$ represents the ceiling function, and superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{*}$ denote transposition, hermitian and complex conjugate, respectively. The convolution of functions f and g is written as $f * g$.

The reminder of this paper is organised as follows. In Section 2, we present the system model. Power allocation and relay selection and also the discussion on system feasibility are developed in Section 3. Section 4 is devoted to simulation results and the conclusions are drawn in Section 5.

2 System model

Consider a network with N SD node pairs $\{(\mathcal{A}_1, \mathcal{B}_1), \ldots, (\mathcal{A}_N, \mathcal{B}_N)\}$ and M relay nodes $\{\mathcal{R}_1, \ldots, \mathcal{R}_M\}$ where all

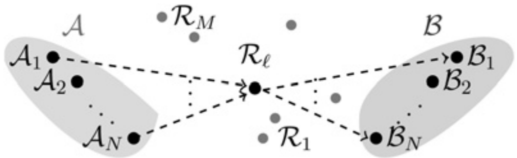

Fig. 1 *Relay network with N SD node pairs $\{(\mathcal{A}_1, \mathcal{B}_1), \ldots, (\mathcal{A}_N, \mathcal{B}_N)\}$ and M relay nodes, where relay ℓ is selected to transmit data between each SD pair node*

terminals in the network are equipped with an antenna (see Fig. 1). To simplify the system complexity, we assume that only one of the relays has to carry data between all node pairs in this system. That is, the set of node pairs $\{(\mathcal{A}_k, \mathcal{B}_k)\}_{k=1}^{N}$ have to exchange (transmit and receive) their data via the assistance of one relay node, let us say by $\mathcal{R}_\ell$. CDMA technique is used in our assumed system and each pair of users is assigned a unique CDMA spreading code.

We consider a quasi-static environment and assume that the direct channel gain between $\{\mathcal{A}_k\}_{k=1}^{N}$ and $\{\mathcal{B}_{k'}\}_{k'=1}^{N}$ is negligible. First, we focus on communication from $\{\mathcal{A}_1, \ldots, \mathcal{A}_N\}$ to $\{\mathcal{B}_1, \ldots, \mathcal{B}_N\}$. Data transmission from $\mathcal{A}_k$ to its corresponding receive node $\mathcal{B}_k$ is performed in two transmission hops. During the first hop of transmission, the source nodes $\{\mathcal{A}_k\}_{k=1}^{N}$ simultaneously transmit their data to relays $\{\mathcal{R}_k\}_{k=1}^{M}$. During the second hop of transmission, relay node $\mathcal{R}_\ell$ processes and broadcasts the combined received signals from N source nodes for the destination nodes; that is, in the second transmission hop, the selected relay amplifies and forwards its received signal to $\{\mathcal{B}_k\}_{k=1}^{N}$. For such transmission, the received signals at $\mathcal{R}_\ell$ and $\{\mathcal{B}_k\}_{k=1}^{N}$ are, respectively, given by

$$r_\ell(t) = \sum_{i=1}^{N} h_{\ell i}(t) * s_i(t) d_i + n_\ell(t) \tag{1}$$

$$y_{k\ell}(t) = \sqrt{w_\ell}\, g_{k\ell}(t) * r_\ell(t) + \nu_k(t) \tag{2}$$

Here, d_i denotes the data of $\mathcal{A}_i$, $s_i(t)$ is the normalised spreading code signal assigned to the ith SD pair with duration T_{s} where $\int_{T_{\mathrm{s}}} |s_i(t)|^2\,\mathrm{d}t = 1$, $h_{\ell i}(t)$ is the impulse response of the channel from $\mathcal{A}_i$ to $\mathcal{R}_\ell$ and $g_{k\ell}(t)$ denotes the impulse response of channel between $\mathcal{R}_\ell$ and $\mathcal{B}_k$. w_ℓ is the relay power gain, and $n_\ell(t)$ and $\nu_k(t)$ are, respectively, independent additive zero-mean Gaussian noise components at relay $\mathcal{R}_\ell$ and $\mathcal{B}_k$. It is reasonable to assume that the data of different source nodes are also uncorrelated, that is, $E\{d_i d_j^*\} = p_i \delta_{i-j}$ for $i, j = 1, \ldots, N$ where p_i represents the transmitted power from $\mathcal{A}_i$. We assume that each channel response of $\{h_{\ell k}(t)\}$ and $\{g_{k\ell}(t)\}$ introduces a gain and a delay [34, p-405], [35], that is, $h_{\ell k}(t) = \alpha_{\ell k}\delta(t - \tau_{\ell k})$ and $g_{k\ell}(t) = \beta_{k\ell}\delta(t - \tau'_{k\ell})$. Thus, the received signal $y_{k\ell}(t)$ can be expressed as

$$\begin{aligned} y_{k\ell}(t) = {} & \sqrt{w_\ell}\beta_{k\ell}\left(\sum_{i=1}^{N} \alpha_{\ell i} s_i(t - \tau_{\ell i} - \tau'_{k\ell}) d_i\right) \\ & + \sqrt{w_\ell}\beta_{k\ell} n_\ell(t - \tau'_{k\ell}) + \nu_k(t) \end{aligned} \tag{3}$$

At destination $\mathcal{B}_k$, the signal originated from $\mathcal{A}_k$ is decoded by correlating the received signal with a perfectly synchronised copy of the corresponding spreading code over a symbol period. De-spreading of $y_{k\ell}(t)$ at $\mathcal{B}_k$

generates the following statistics

$$\begin{aligned}\tilde{y}_{k\ell} &= \int_{T_s} y_{k\ell}(t)s_k(t-\tau_{\ell k}-\tau'_{k\ell})\,dt \\ &= \underbrace{\sqrt{w_\ell}\beta_{k\ell}\alpha_{\ell k}d_k}_{\text{desired signal}} + \underbrace{\sqrt{w_\ell}\beta_{k\ell}\sum_{\substack{i=1\\ i\neq k}}^{N}\alpha_{\ell i}\varrho_{ik\ell}d_i}_{\text{multiple access interference}} \\ &\quad + \underbrace{\sqrt{w_\ell}\beta_{k\ell}\tilde{n}_\ell + \tilde{v}_k}_{\text{noise}}\end{aligned} \tag{4}$$

where $\varrho_{ij\ell} \stackrel{\text{def}}{=} \int_{T_s} s_i(t-\tau_{\ell i})s_j(t-\tau_{\ell j})\,dt$ denotes the cross-correlation between delayed spreading waveforms assigned to source i and source j while $\mathcal{R}_\ell$ is chosen. $\tilde{n}_\ell$ and $\tilde{v}_k$ are zero-mean noises, respectively, corresponding to $n_\ell(t)$ and $v_k(t)$, where $E\{|\tilde{n}_\ell|^2\} = \sigma_n^2$ and $E\{|\tilde{v}_k|^2\} = \sigma_v^2$. If $\{\varrho_{ij\ell}\}$ were all zero for all i, j, ℓ, then the signals of different SD pairs would be separable. However in this scheme, the multiple access interference is occurred at the final destinations because of the cross-correlation among the spreading codes, that is, this CDMA cooperative network is not equivalent to multiple parallel channels.

For the assumed system, the total transmit power from nodes $\{\mathcal{A}_k\}_{k=1}^N$ is $\sum_{i=1}^N p_i$ and, using (1), the transmit power from $\mathcal{R}_\ell$ is $w_\ell(\boldsymbol{\alpha}_\ell^T\boldsymbol{p} + \sigma_n^2)$, where $\boldsymbol{p} \stackrel{\text{def}}{=} [p_1 \dots p_N]^T$, $\boldsymbol{\alpha}_\ell \stackrel{\text{def}}{=} \left[|\alpha_{\ell 1}|^2 \dots |\alpha_{\ell N}|^2\right]^T$ and $\boldsymbol{\beta}_\ell \stackrel{\text{def}}{=} \left[|\beta_{1\ell}|^2 \dots |\beta_{N\ell}|^2\right]^T$. Hence, the total transmit power in our assumed system can be written as

$$P_\ell = \boldsymbol{1}^T\boldsymbol{p} + w_\ell\boldsymbol{\alpha}_\ell^T\boldsymbol{p} + w_\ell\sigma_n^2 \tag{5}$$

In the above equation, term $\boldsymbol{1}^T\boldsymbol{p} + w_\ell\boldsymbol{\alpha}_\ell^T\boldsymbol{p}$ stands for the sum power spent by $\{\mathcal{A}_1, \dots, \mathcal{A}_N, \mathcal{R}_\ell\}$ to amplify the data for communication between $\{\mathcal{A}_k\}_{k=1}^N$ and $\{\mathcal{B}_k\}_{k=1}^N$ and $w_\ell\sigma_n^2$ is the power of noise received and retransmitted from $\mathcal{R}_\ell$. In this paper, our objective is to select relay $\mathcal{R}_\ell$ among relay set $\{\mathcal{R}_k\}_{k=1}^M$, and compute relay gain w_ℓ and transmit power of all sources $\{p_1, \dots, p_N\}$ such that some predefined QoS is guaranteed for all SD pairs. The SINR is considered as the link QoS, and our optimisation criterion is the total transmit power P_ℓ. Let $\tilde{\gamma}_{\ell k}$ denote the SINR at $\mathcal{B}_k$ when $\mathcal{R}_\ell$ is selected to establish the links. Using these notations, we solve the following optimisation problem

$$\begin{aligned}\{\ell^{\text{opt}}, \boldsymbol{p}^{\text{opt}}, w_\ell^{\text{opt}}\} &= \arg\min_{\ell,\boldsymbol{p},w_\ell} P_\ell(\boldsymbol{p}, w_\ell) \\ &\quad \text{s.t. } \tilde{\gamma}_{\ell k} \geq \gamma_k \text{ for } k = 1, \dots, N\end{aligned} \tag{6}$$

where γ_k is the target SINR for the link between the kth SD pair. The solution of this problem minimises the total power consumption and provides a guaranteed QoS. However in this scheme, those pairs with poor channel conditions are disadvantaged and may require significantly more power.

3 Power allocation and relay selection

3.1 Solving optimisation problem (6)

Optimisation problem (6) is non-convex and thus cannot be solved by conventional optimising methods. To solve this problem, we first assume that the ℓth relay node is the selected relay and then use the results to choose the best relay, that is, we first solve the following problem

$$\begin{aligned}\{\boldsymbol{p}^{\text{opt}}, w_\ell^{\text{opt}}\} &= \arg\min_{\boldsymbol{p},w_\ell} P_\ell(\boldsymbol{p}, w_\ell) \\ &\quad \text{s.t. } \tilde{\gamma}_{\ell k} \geq \gamma_k \text{ for } k = 1, \dots, N\end{aligned} \tag{7}$$

Using (4), the received SINR at the kth destination is expressed by

$$\tilde{\gamma}_{\ell k} = \frac{w_\ell|\beta_{k\ell}|^2|\alpha_{\ell k}|^2 p_k}{\rho_\ell w_\ell|\beta_{k\ell}|^2\sum_{\substack{i=1\\ i\neq k}}^N |\alpha_{\ell i}|^2 p_i + w_\ell|\beta_{k\ell}|^2\sigma_n^2 + \sigma_v^2} \tag{8}$$

where $\rho_\ell \stackrel{\text{def}}{=} \max_{\forall i\neq j}|\varrho_{ij\ell}|^2 < 1$ is a small number. We can rewrite (7) as

$$\begin{aligned}\{\boldsymbol{p}^{\text{opt}}, w_\ell^{\text{opt}}\} &= \arg\min_{\boldsymbol{p},w_\ell} P_\ell(\boldsymbol{p}, w_\ell) \\ &\quad \text{s.t. } (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{p} \geq (\frac{1}{w_\ell}\boldsymbol{b}_v + \boldsymbol{b}_n)\end{aligned} \tag{9}$$

where $\boldsymbol{A}$, $\boldsymbol{b}_v$ and $\boldsymbol{b}_n$ in this problem are defined as follows

$$[\boldsymbol{A}]_{kq} \stackrel{\text{def}}{=} \begin{cases}\gamma_k\rho_\ell\dfrac{|\alpha_{\ell q}|^2}{|\alpha_{\ell k}|^2}, & k \neq q \\ 0, & k = q\end{cases} \tag{10}$$

$$[\boldsymbol{b}_v]_k \stackrel{\text{def}}{=} \frac{\gamma_k\sigma_v^2}{|\alpha_{\ell k}|^2|\beta_{k\ell}|^2} \tag{11}$$

$$[\boldsymbol{b}_n]_k \stackrel{\text{def}}{=} \frac{\gamma_k\sigma_n^2}{|\alpha_{\ell k}|^2} \tag{12}$$

Note that $\boldsymbol{A}$ is a non-negative matrix, and all elements of $\boldsymbol{b}_v$ and $\boldsymbol{b}_n$ are positive. We can solve (9) by solving [36, pp-133]

$$\{w_\ell^{\text{opt}}\} = \arg\min_{w_\ell} P_\ell(\tilde{\boldsymbol{p}}(w_\ell), w_\ell) \tag{13}$$

where

$$\begin{aligned}\tilde{\boldsymbol{p}}(w_\ell) &= \arg\min_{\boldsymbol{p}} P_\ell(\boldsymbol{p}, w_\ell) \\ &\quad \text{s.t. } (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{p} \geq \left(\frac{1}{w_\ell}\boldsymbol{b}_v + \boldsymbol{b}_n\right)\end{aligned} \tag{14}$$

The optimisation in (9) requires a joint search over $\boldsymbol{p}$ and w_ℓ. In (14), we first find the optimal value for $\boldsymbol{p}$ in terms of w_ℓ and then using (13) we find w_ℓ.

It is easy to show that the solution to (14) should make all the constraints active [37, 38], that is, solution $\tilde{\boldsymbol{p}}(w_\ell)$ must satisfy $(\boldsymbol{I} - \boldsymbol{A})\tilde{\boldsymbol{p}}(w_\ell) = (1/w_\ell)\boldsymbol{b}_v + \boldsymbol{b}_n$. To prove this claim, assume that one of the elements of $(\boldsymbol{I} - \boldsymbol{A})\tilde{\boldsymbol{p}}(w_\ell)$ is greater than the corresponding element of $(1/w_\ell)\boldsymbol{b}_v + \boldsymbol{b}_n$, then the SINR of the corresponding SD pair is above the desired target. In that case, we can reduce the source power of that pair to exactly meet the SINR of the corresponding SD pair. It is obvious that the total power is reduced and all other SINRs are increased as the power of one of the interferers is reduced. Thus, $\tilde{\boldsymbol{p}}(w_\ell)$ can be obtained as

$$\tilde{\boldsymbol{p}}(w_\ell) = (\boldsymbol{I} - \boldsymbol{A})^{-1}\left(\frac{1}{w_\ell}\boldsymbol{b}_v + \boldsymbol{b}_n\right) = \frac{1}{w_\ell}\boldsymbol{\zeta}_v + \boldsymbol{\zeta}_n \tag{15}$$

IET Commun., 2014, Vol. 8, Iss. 11, pp. 1892–1901
doi: 10.1049/iet-com.2013.0444

where $\zeta_\nu \overset{\text{def}}{=} (I - A)^{-1} b_\nu$ and $\zeta_n \overset{\text{def}}{=} (I - A)^{-1} b_n$. From the Perron–Frobenius theorem [39], we note that (15) is a feasible solution for $\tilde{p}(w_\ell)$ if and only if $(I - A)^{-1}$ exists and all its elements are positive. If any element of $(I - A)^{-1}$ is negative, then the network cannot support all of the users. In such a case, we drop some of the SD pairs in the process of admission control as described in Section 3.2. We continue to solve (9), assuming that the feasibility of network is verified. Substituting (15) back into (14), we have

$$P_\ell(\tilde{p}(w_\ell), w_\ell) = w_\ell(\alpha_\ell^{\text{T}} \zeta_n + \sigma_n^2) + \frac{1}{w_\ell} 1^{\text{T}} \zeta_\nu + 1^{\text{T}} \zeta_n + \alpha_\ell^{\text{T}} \zeta_\nu \tag{16}$$

Now, by differentiating (16) with respect to w_ℓ and equating the result to zero, the solution to (13) is obtained as

$$w_\ell^{\text{opt}} = \sqrt{\frac{1^{\text{T}} \zeta_\nu}{\alpha_\ell^{\text{T}} \zeta_n + \sigma_n^2}} \tag{17}$$

Substituting (17) in (15), we obtain

$$p^{\text{opt}} = \sqrt{\frac{\alpha_\ell^{\text{T}} \zeta_n + \sigma_n^2}{1^T \zeta_\nu}} \zeta_\nu + \zeta_n \tag{18}$$

Substituting (17) in (16), the power consumption using the ℓth relay is a function of α_ℓ and β_ℓ given by

$$P_\ell^{\text{opt}} = 1^{\text{T}} \zeta_n + \alpha_\ell^{\text{T}} \zeta_\nu + 2\sqrt{1^{\text{T}} \zeta_\nu (\alpha_\ell^{\text{T}} \zeta_n + \sigma_n^2)} \tag{19}$$

Therefore the best relay is selected from the following problem

$$\ell^{\text{opt}} = \arg \min_{\ell} P_\ell^{\text{opt}} \tag{20}$$

In a similar manner, the optimal power allocation and relay selection can be obtained for transmission from the set of source nodes $\{\mathcal{B}_k\}_{k=1}^N$ to the set of destination nodes $\{\mathcal{A}_k\}_{k=1}^N$. The following lemma gives a better understanding about the effect of channel gains of the first hop and the second hop on the optimal relay gain and the optimal power consumption. Hereafter, without loss of generality, the index ℓ that has been used for the best relay which forwards its received signal to the destinations is dropped in our notations.

Theorem 1: For a reciprocal communication environment, and assuming that the variances of the additive noise are equal at all nodes, that is, $\sigma_n^2 = \sigma_\nu^2$, then the total optimal power consumption $P^{\text{opt}}(\alpha, \beta)$, the selected relay $\ell^{\text{opt}}(\alpha, \beta)$ and the optimal relay gain $w^{\text{opt}}(\alpha, \beta)$ all remain invariant if the direction of all communications are reversed, that is, $P^{\text{opt}}(\alpha, \beta) = P^{\text{opt}}(\beta, \alpha)$, $\ell^{\text{opt}}(\alpha, \beta) = \ell^{\text{opt}}(\beta, \alpha)$ and $w^{\text{opt}}(\alpha, \beta) = w^{\text{opt}}(\beta, \alpha)$.

Proof: From (19) and using the equations for $[\zeta_n]_i$ and $[\zeta_\nu]_i$ derived in Appendix 1, we observe that $P^{\text{opt}}(\alpha, \beta)$ is composed of the following terms

$$1^{\text{T}} \zeta_n = \frac{\sigma_n^2}{\rho(1 - 1^{\text{T}} u)} \sum_{k=1}^N \frac{u_k}{|\alpha_k|^2} \tag{21a}$$

$$\alpha^{\text{T}} \zeta_\nu = \frac{\sigma_\nu^2}{\rho(1 - 1^{\text{T}} u)} \sum_{k=1}^N \frac{u_k}{|\beta_k|^2} \tag{21b}$$

$$1^{\text{T}} \zeta_\nu = \frac{\sigma_\nu^2}{\rho} \sum_{k=1}^N \frac{u_k}{|\alpha_k|^2} \left(\frac{1}{|\beta_k|^2} + \frac{1}{1 - 1^{\text{T}} u} \sum_{j=1}^N \frac{u_j}{|\beta_j|^2} \right) \tag{21c}$$

$$\alpha^{\text{T}} \zeta_n + \sigma_n^2 = \frac{\sigma_n^2 1^{\text{T}} u}{\rho(1 - 1^{\text{T}} u)} + \sigma_n^2 \tag{21d}$$

where $u_i \overset{\text{def}}{=} (\rho\gamma_i / 1 + \rho\gamma_i)$ and $u \overset{\text{def}}{=} [u_1, \ldots, u_N]^{\text{T}}$. For $\sigma_n^2 = \sigma_\nu^2$ and from (21), we conclude that $P^{\text{opt}}(\alpha, \beta) = P^{\text{opt}}(\beta, \alpha)$. Consequently, $\ell^{\text{opt}}(\alpha, \beta) = \ell^{\text{opt}}(\beta, \alpha)$. In addition using (17) and (21), we obtain equation for w^{opt} as follows

$$|w^{\text{opt}}|^2 = \frac{\sigma_\nu^2 / \sigma_n^2}{1^{\text{T}} u / (1 - 1^{\text{T}} u) + \rho} \sum_{k=1}^N \frac{u_k}{|\alpha_k|^2} \times \left(\frac{1}{|\beta_k|^2} + \frac{1}{1 - 1^{\text{T}} u} \sum_{j=1}^N \frac{u_j}{|\beta_j|^2} \right) \tag{22}$$

Thus for $\sigma_n^2 = \sigma_\nu^2$, we have $w^{\text{opt}}(\alpha, \beta) = w^{\text{opt}}(\beta, \alpha)$. □

Remark 1: This is an advantage of the proposed scheme when both nodes in all pairs desire to exchange (transmit and receive) information among each other. In this case, from the symmetry of above solution we see that the solution for relay selection and transmission gain remain optimal if all pairs switch the direction of their communications.

Remark 2: The power control algorithm proposed in Section 3.1 can be directly applied to the case of two-hop two-way relaying where all nodes $\{\mathcal{A}_1, \ldots, \mathcal{A}_N\} \cup \{\mathcal{B}_1, \ldots, \mathcal{B}_N\}$ simultaneously transmit to the relay in the first hop, and the relay node amplifies and broadcasts its received signals to all nodes in the second hop. The proposed methods in this paper are applicable for two-way relaying by considering $\{\mathcal{A}_1, \ldots, \mathcal{A}_N\} \cup \{\mathcal{B}_1, \ldots, \mathcal{B}_N\}$ simultaneously as the transmitting nodes and the receiving nodes.

A benefit of this system is its low-cost, low-complexity architecture with reduced amount of required feedback information. The required operations at relay include radio frequency (RF) demodulation to the baseband, sampling, analogue-to-digital conversion, storing the samples, processing the digital signal, digital-to-analogue conversion and RF modulation back to the bandpass. In contrast to the scheme in this paper, the relay in the joint demodulation and forwarding method (proposed in [20]) is required to perform accurate synchronisation and detection on its received signal for each user pair, that is, some circuits or processing must be allocated to each user pair which makes the relay device more complex. Another advantage of the scheme in this paper over [20] is that the optimal power control procedure in (17) and (18) has relatively simple solution given in closed form, whereas in [20] the power control procedure involves constructing iterative algorithms using the receiver updates. Compared with multi relay systems using distributed beamforming, for example, in [13–15], the overhead of precise synchronisation and phase adjustment at relays is eliminated for our system. The relay selection and power control can be performed by a central unit. The information exchange from this central unit consists of collecting all the channels gains, transmitting the index of selected relay and its transmit power to the relays and informing the transmitter nodes about their allocated transmit powers. From Theorem 1, it follows that the selected relay node and the corresponding amplification gain of both forward and reverse communication paths are identical. Hence concerning the relay, the amount of data feedback is reduced by half.

3.2 Feasibility of problem (6) and admission control

In this section, we study the requirements for the feasibility of problem (6). To this aim, we use the following theorem.

Theorem 2: The problem (6) is feasible if and only if

$$\sum_{k=1}^{N} \frac{1}{1+\left(1/\rho\gamma_k\right)} < 1 \tag{23}$$

Proof: From Lemma 1 in Appendix 2, it is straightforward that we must have $|\boldsymbol{I}-\boldsymbol{A}|>0$. To simplify the feasibility condition

$$|\boldsymbol{I}-\boldsymbol{A}| = \begin{vmatrix} 1 & \dfrac{-\rho\gamma_1|\alpha_2|^2}{|\alpha_1|^2} & \cdots & \dfrac{-\rho\gamma_1|\alpha_N|^2}{|\alpha_1|^2} \\ \dfrac{-\rho\gamma_2|\alpha_1|^2}{|\alpha_2|^2} & 1 & \cdots & \dfrac{-\rho\gamma_2|\alpha_N|^2}{|\alpha_2|^2} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{-\rho\gamma_n|\alpha_1|^2}{|\alpha_N|^2} & \dfrac{-\rho\gamma_N|\alpha_2|^2}{|\alpha_N|^2} & \cdots & 1 \end{vmatrix} > 0 \tag{24}$$

we take a factor $\rho\gamma_k/|\alpha_k|^2$ from the kth row and a factor $|\alpha_q|^2$ from the qth column of (24) and obtain

$$|I-A| = \left|\mathrm{diag}\left(1+\frac{1}{\rho\gamma_1},\ \ldots,\ 1+\frac{1}{\rho\gamma_N}\right) - \boldsymbol{1}\boldsymbol{1}^{\mathrm{T}}\right| \prod_{k=1}^{N}\rho\gamma_k \tag{25}$$

Using $|\boldsymbol{A}+\boldsymbol{x}\boldsymbol{y}^{\mathrm{T}}| = |\boldsymbol{A}|(1+\boldsymbol{y}^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{x})$, we obtain

$$|\boldsymbol{I}-\boldsymbol{A}| = \left(1-\sum_{k=1}^{N}\frac{1}{1+\left(1/\rho\gamma_k\right)}\right)\prod_{k=1}^{N}(\rho\gamma_k+1) \tag{26}$$

Since $\rho\gamma_k+1 \geq 1$, the condition (26) is equivalent to (23). □

Remark 3: Interestingly, this feasibility condition is independent of the channels state. The channel of course has impact on the total required power. The left side of the above inequality increases with increase in ρ, in any of γ_k or by the addition of a new SD pair. This means that by using an improved set of codes (reduction in ρ) or by requiring less target SINRs (reduction in γ_k) or number of users an unfeasible problem may become feasible. Based on this, we propose a very simple procedure for admission of new SD pairs.

Remark 4: This method allows a very simple admission control. A new additional SD pair with a given target SINR of $\gamma_{(N+1)}$ can only be admitted only if by adding the new term $1/(1+1/(\rho\gamma_{(N+1)}))$ to the left side of (23), the result remains smaller than one. For the vanishing SD pairs, the corresponding term must be deducted. This means that the quantity $1/[1+(1/\rho\gamma_k)]$ simply represents a feasibility index where each pair contributes in to the reduction of the feasibility.

Remark 5: If the problem is not feasible, that is, $\sum_{k=1}^{N}\frac{1}{1+(1/\rho\gamma_k)} > 1$, some of the SD pairs cannot attain their target SINR value. The remedy is to drop a selected number of pairs. For example, one may drop the users which require higher target SINR in order to support the maximum number of SD pairs.

Remark 6: For the special case, where all SD pairs require an identical QoS, that is, $\gamma_1 = \ldots = \gamma_N = \gamma$, the feasibility condition (23) is simplified to

$$\rho\gamma(N-1) < 1 \tag{27}$$

This means that the network can support up to $N_{\max} = \lceil 1/\rho\gamma \rceil$ SD pairs. Thus, the network sum rate is upper bounded by

$$R(\gamma) = \frac{N}{2}\log_2(1+\gamma) \leq \frac{1}{2}\left\lceil\frac{1}{\rho\gamma}\right\rceil \log_2(1+\gamma)\ \text{bits/s/Hz} \tag{28}$$

From the above, it is easy to show that $R(\gamma)$ is upper bounded by $1/(2\rho\ \ln 2)$.

4 Simulation results

In our realisations, the channel coefficients $\{h_{\ell k}\}$ and $\{g_{k\ell}\}$ are generated as zero-mean circularly symmetric complex Gaussian random variables with variances σ_h^2 and σ_g^2, respectively. The values of σ_h^2 and σ_g^2 directly quantify the expected value of channel gains $|h_{\ell k}|^2$ and $|g_{k\ell}|^2$. Therefore they can be regarded as a measure of channels quality. We assume that the required SINRs at the destination nodes are identical for all users, that is, $\gamma_1 = \ldots = \gamma_N = \gamma$. In these simulations, each of the average values is obtained over 1000 runs of the program. In all simulations except Fig. 8, we set $\sigma_n^2 = \sigma_\nu^2 = 1$. The performance of our CDMA cooperative network is dependent on the properties of its users' spreading code. Gold codes and Kasami codes are among the most common chip sequences, and the values of ρ associated to sequences of length $2^n - 1$ generated from each of these codes are, respectively, $[(2^{(n+2)/2}+1/2^n-1)^2]$ and $[(2^{n/2}+2)/(2^n-1)]^2$ [34, p-397, 398].

Fig. 2 compares the performance of four different schemes in a network with four SD node pairs. Here, the average minimum transmit power against users minimum required data rate is plotted. For each transmission between SD pairs, eight TSs each with length T_{o} are considered. It is worth mentioning that the total transmission time from the source nodes to the relays is identical for all these schemes and it is equal to $4T_{\mathrm{o}}$. These schemes are as follows:

- In Scheme 1, we assume that eight TSs, each with length T_{o} are available for transmission between SD pairs. Using

doi: 10.1049/iet-com.2013.0444

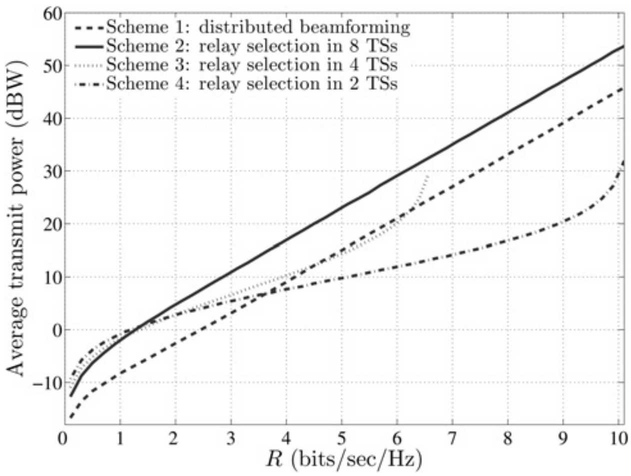

Fig. 2 *Average transmit power with respect to minimum data rate, for $N = 4$, $M = 20$, $\rho = 0.01$ and $\sigma_h^2 = \sigma_g^2 = 10$ dB using different schemes*

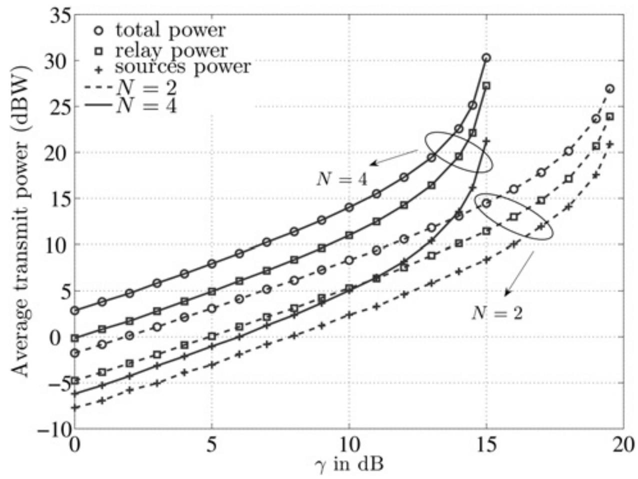

Fig. 3 *Average transmit power, average relay transmit power and average source nodes transmit power with respect to required SINR, for $M = 20$, $\rho = 0.01$ and $\sigma_h^2 = \sigma_g^2 = 10$ dB*

time-division strategy, two TSs are devoted to each of the SD node pairs for communication. In the first TS, the first source node transmits its data to the relay nodes. The relays retransmit their received signals to the proper destination in another TS. In the same manner, the other SD node pairs communicate during the specified TSs. In this scheme, the relay nodes AF their received signals from each source node using complex weights. The transmitting nodes send their data with proper powers and optimal beamforming is performed at relays so that the QoS at destination nodes is maximised.

- In Scheme 2, again eight TSs, each with length T_o are considered. For each SD node pair, two TSs are dedicated and one relay node is selected to establish a link for data communication. In the first TS, the first source node transmits its data to the relay nodes. In the second TS, the best relay is selected for conveying data of the first source node to its destination. Analogously, the other SD pairs select the best relay and communicate during the specified TSs. In each phase of transmission, the optimal relay gain and transmit power of the source node are found.
- In Scheme 3, we consider four TSs, each with length $2T_\text{o}$. In this scheme, two TSs are used for communication between two pairs of SD nodes. In the first TS, two of the source nodes upload data to the relay nodes. The best relay is selected to retransmit its received signal in the second TS. In the third TS, the remaining source nodes send data to the relay nodes and so on. Thus in each phase of transmission, we deal with two SD node pairs and optimally find the relay gain and source transmit powers.
- In Scheme 4, there are two TSs each with length $4T_\text{o}$. In the first TS, all source nodes transmit data to the relay nodes. The selected best relay node consequently retransmits its signal to the destination node.

We assume that the SD pairs are required to have a minimum data rate given as

$$R = \frac{d}{2}\log_2(1+\gamma) \tag{29}$$

Here, d is the number of SD pairs that share a TS and it is assumed that the interference has Gaussian distribution. From (29), γ can be computed by $2^{2R/d} - 1$. In Schemes 3 and 4, the power allocation problem becomes feasible when the users data rates are, respectively, less than $\log_2[1 + (1/\rho)]$ and $2\log_2[1 + (1/3\rho)]$. In these simulations, we consider a network with 20 relay nodes. It is assumed that $\sigma_h^2 = \sigma_g^2 = 10$ dB and $\rho = 0.01$ is chosen. For Gold codes and Kasami codes, $\rho = 0.01$ is obtained for sequences of lengths $2^7 - 1$ and $2^9 - 1$, respectively. Furthermore we let $T_\text{o} = 1$ sec. It can be seen from this figure that for small values of R, Scheme 1 has the lowest average transmit power and the difference between Schemes 2 to 4 is trivial. However, beyond a certain value of R, the average total transmit power in Scheme 4 is remarkably lower as compared with the other schemes. Moreover, Scheme 4 is capable of providing larger data rates in comparison with Scheme 3.

Fig. 3 illustrates the average value of total transmit power, the relay transmit power and the source transmit power for two networks with various number of SD node pairs. For both networks, it is assumed that $\sigma_h^2 = \sigma_g^2 = 10$ dB, and the number of relays is 20. $\rho = 0.01$ is selected. It can be observed from this figure that, for both networks, the average relay power is about 3 dBW below the average total power consumption of the network, and the rest of total power is equally divided between the source nodes. Hence, the average transmit power of each source node is $10\log_{10} N$ (in dBW) below the relay transmit power.

In Fig. 4, we study the effect of ρ, that is quantified by the maximum cross-correlation between two distinct codes, on the network average total transmit power. Here, a network with two SD pairs and 20 relay nodes is considered. We let $\sigma_h^2 = \sigma_g^2 = 10$ dB and ρ has various values. It can be observed from this figure that for moderate to large values of average transmit power, the QoS at the destination nodes is significantly promoted by the decrease of ρ. In addition, the system becomes able to provide higher levels of target SINRs for the users, as ρ decreases. For example, the network in this example cannot support $\gamma = 10$ dB when ρ is ≥ 0.1. However, the curves for $\rho = 0.05$, 0.02 and 0.01 illustrate that the network can support this target SINR value when ρ decreases.

In Fig. 5, the average of transmit power against γ is compared for $\rho = 0.01$, $M = 20$, $\sigma_h^2 = \sigma_g^2 = 10$ dB and

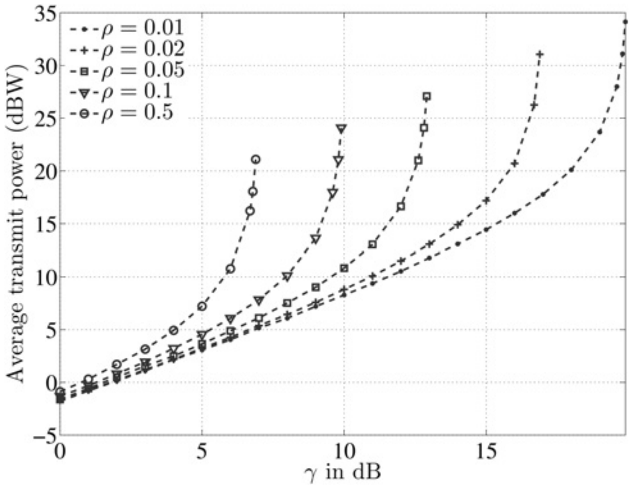

Fig. 4 *Average transmit power of network with respect to required SINR and ρ, for N = 2, M = 20 and* $\sigma_h^2 = \sigma_g^2 = 10$ *dB*

various number of SD pairs, that is, $N = 1, \ldots, 5$ and 8. This figure shows that for $N \geq 2$, the network becomes infeasible as γ in dB tends to $20 - 10\log_{10}(N-1)$. Moreover, the network total transmit power for a given target SINR increases as the number of SD pairs increases.

We investigate the effect of channel condition on total power consumption in a reciprocal environment for two SD pairs, $\rho = 0.01$, $\sigma_h^2 = 10$ dB and various values of σ_g^2. Fig. 6 illustrates the optimal average of power consumption against the system required target SINR, γ. From this figure, it follows that the average power consumption of the network is increasing in γ and decreasing in σ_g^2. We have reversed the selected values of σ_h^2 and σ_g^2 in the second set of experiments, that is, σ_h^2 obtains various values and $\sigma_g^2 = 10$ dB. The average total transmit power, as a function of σ_g^2 and σ_h^2, remains the same when all the communication directions are changed.

Fig. 7 shows the average minimum transmit power of the network against γ for various number of relay nodes. In our simulations, a network with two SD node pairs is considered. We let $\sigma_h^2 = \sigma_g^2 = 10$ dB and $\rho = 0.01$ is selected. It can be seen that as the number of relay nodes

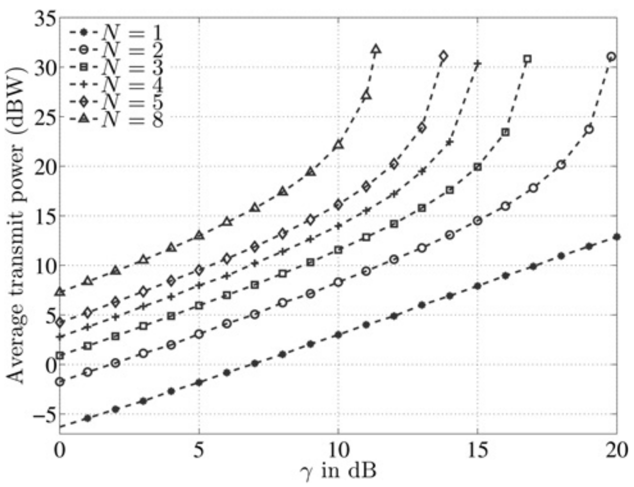

Fig. 5 *Average transmit power of network with respect to required SINR and N, for ρ = 0.01, M = 20 and* $\sigma_h^2 = \sigma_g^2 = 10$ *dB*

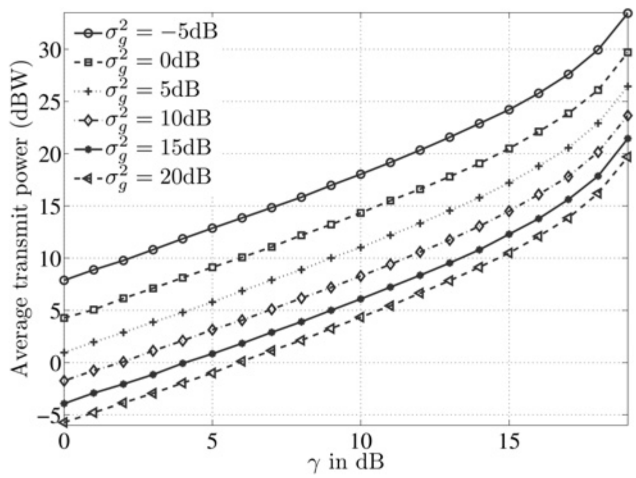

Fig. 6 *Average transmit power against required SINR, for N = 2, ρ = 0.01 and* $\sigma_h^2 = 10$ *dB. Reversing the selected values of* σ_h^2 *and* σ_g^2 *gives the same curve*

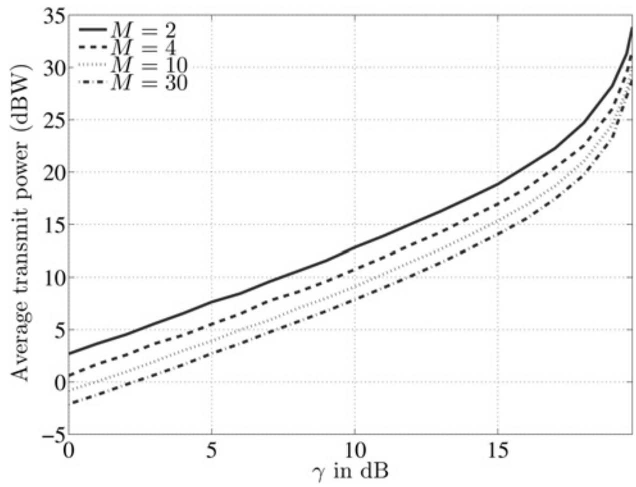

Fig. 7 *Average minimum transmit power of network with respect to required SINR and number of relay nodes, for N = 2, ρ = 0.01 and* $\sigma_h^2 = \sigma_g^2 = 10$ *dB*

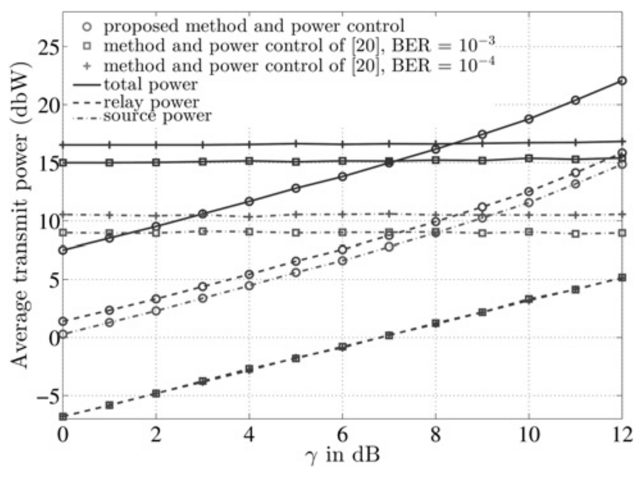

Fig. 8 *Comparison of average transmit power, average relay transmit power and average source nodes transmit power with respect to required SINR and relay decoding BER requirement in system of [20]*

doi: 10.1049/iet-com.2013.0444

increases, because of the additional diversity added to the system, the total power consumption is decreased.

Now, we compare the performance of our asynchronous method with the two-way synchronous one in [20]. We set $\sigma_h^2 = \sigma_g^2 = 10$ dB, $\sigma_n^2 = 10$ and $\sigma_\nu^2 = 1$, and select the best relay among 20 available nodes. In this simulation, the spreading codes for the synchronised method in [20] are generated in the same way as in [20] with length of 40 and $\rho = 0.01$. We consider four nodes $\mathcal{A}_1, \mathcal{B}_1, \mathcal{A}_2, \mathcal{B}_2$ communicating in pairs and in two TSs via the selected relay. In the first TS, all nodes transmit simultaneously to the relay node and the relay forwards a signal to all the destinations simultaneously in the second TS. Since in the first TS they all transmit, we shall substitute $\boldsymbol{\alpha}_\ell = \left[|\alpha_{\ell 1}|^2, |\alpha_{\ell 2}|^2, |\beta_{\ell 1}|^2, |\beta_{\ell 2}|^2\right]^{\mathrm{T}}$ as the energy gain between the sources and the relay in our equations. Moreover, we use $\boldsymbol{\beta}_\ell \stackrel{\text{def}}{=} \left[|\beta_{\ell 1}|^2, |\beta_{\ell 2}|^2, |\alpha_{\ell 1}|^2, |\alpha_{\ell 2}|^2\right]^{\mathrm{T}}$ as all nodes receive in the second TS. In [20], the relay decodes the data of sources, re-encodes them and forwards them to destinations in the second TS. We set values 10^{-3} and 10^{-4} as the relay required bit error rate (BER) for the method in [20]. Fig. 8 demonstrates the average value of total transmit power, the relay transmit power and the sources transmit power with respect to the required SINR at destination nodes for these systems. Using the method in [20], the source transmit power depends on a pre-set BER value for the decoding at the relay and is independent of the target SINR γ. However, the total transmit power in the method of Chen and Yener [20] is a function of both the target SINR and relay BER values. When γ has small values, a big fraction of total transmit power in the system of Chen and Yener [20] is consumed to meet the requirements of relay decoding. Compared with the method in [20] for relay decoding BER values 10^{-3} and 10^{-4}, we observe that our proposed method requires less total power in average when, respectively, $\gamma \leq 7$ and 8.38 dB. In terms of average source transmit power and for the same relay BER values, our proposed system has better performance when $\gamma \leq 8$ and 9.15 dB.

Fig. 9 compares the end-to-end BER in our method and the proposed method by Chen and Yener [20] for various values of BER at the relay with respect to γ. Assuming Gaussian distribution for interference component and using binary modulation, the end-to-end BER in our proposed method

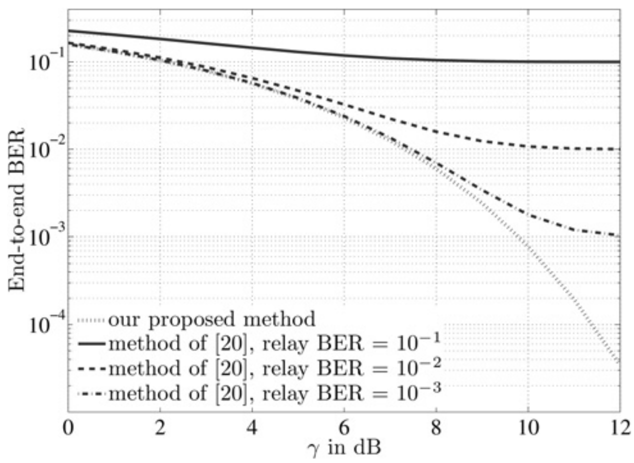

Fig. 9 *Comparison of the end-to-end BER with respect to required SINR and relay decoding BER requirement in method of [20] and our proposed method*

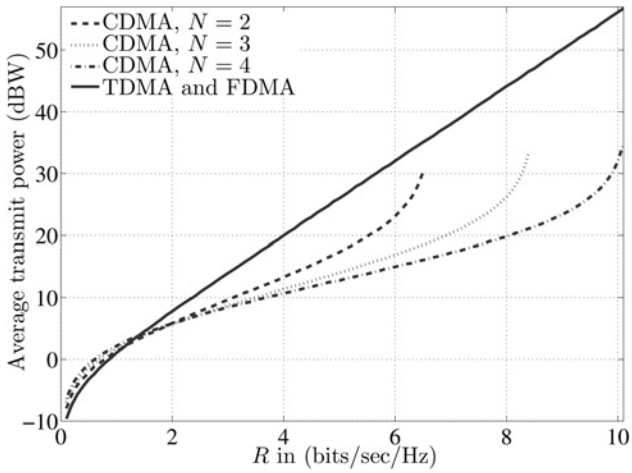

Fig. 10 *Comparison of average transmit power with respect to sum data rate in our proposed CDMA technique and systems using TDMA and FDMA techniques*

and in the method of Chen and Yener [20] are, respectively, approximated by $Q(\sqrt{\gamma})$ and $Q(\sqrt{\gamma}) + P_e(1 - 2Q(\sqrt{\gamma}))$ where P_e stands for the required BER value at relay and $Q(x) \stackrel{\text{def}}{=} \int_x^\infty \left(\mathrm{e}^{-u^2/2}/\sqrt{2\pi}\right) \mathrm{d}u$. It can be observed that the end-to-end BER in our method is always better than that of the proposed method in [20]. The proposed method of Chen and Yener [20] can reach the BER performance of our system when the relay decodes signal with negligible error probability.

In Fig. 10, we compare the proposed CDMA method, time division multiple access (TDMA) and frequency division multiple access (FDMA) techniques in terms of the required average transmit power against the minimum sum data rate (in bits/s/Hz) defined by $R = N\epsilon_1\epsilon_2 \log_2(1 + \gamma)$, where ϵ_1 and ϵ_2 are the fractions of time and bandwidth assigned to each SD pair for data transmission, respectively. For simplicity, we model the interference as an additive white Gaussian noise. In our scheme, we have two TSs that is, $\epsilon_1 = \frac{1}{2}$ and the whole bandwidth is used by all users that is, $\epsilon_2 = 1$. Thus, we have $R = (N/2)\log_2(1 + \gamma)$. In TDMA, the transmission time is divided $2N$ TSs, that is, $\epsilon_1 = 1/(2N)$, and the whole bandwidth is used by all users, that is, $\epsilon_2 = 1$. Thus, we have $R = (1/2)\log_2(1 + \gamma)$. The expression of R in FDMA is the same as in TDMA. In Fig. 10, we considered $N = 2$, 3 and 4, $\sigma_g^2 = \sigma_h^2 = 10$ dB, $\rho = 0.01$ and the relay is selected from 20 nodes. For $R < 1.3$ bits/s/Hz, the TDMA and FDMA techniques require less power. In contrast, the proposed method results in significant power saving as R and N increase.

5 Conclusion

We considered relay networks with N set of SD pairs $(\mathcal{A}_k, \mathcal{B}_k)|_{k=1}^N$ and M relay nodes $\{\mathcal{R}_k\}_{k=1}^M$, where both nodes of pairs $(\mathcal{A}_k, \mathcal{B}_k)|_{k=1}^N$ have data to send for each other. We assumed that CDMA is employed by the users and the communication channels are asynchronous. Transmission of data from each $\mathcal{A}_k$ to its corresponding $\mathcal{B}_k$, and vice versa, is carried out in two hops via the assistant of one relay node. At the first hop, the set of N source nodes simultaneously uploads their coded data to the relays in the system. Then, a relay node amplifies and broadcasts the superposed multiple received signals to the destination nodes. We proposed and studied an optimisation problem for finding the sources transmit powers and relay gain such that the total transmit power in the network is minimised and at the same time a certain SINR is met at the destination nodes. The closed-form solution to the optimisation problem was derived. Using the solution of this problem, the relay whose path yields the least transmit power is selected. We proved that the best relay node, the optimal transmit powers and the gain of selected relay are the same for transmission from $\{\mathcal{A}_1, \ldots, \mathcal{A}_N\}$ to $\{\mathcal{B}_1, \ldots, \mathcal{B}_N\}$ and for transmission from $\{\mathcal{B}_1, \ldots, \mathcal{B}_N\}$ to $\{\mathcal{A}_1, \ldots, \mathcal{A}_N\}$ for the proposed optimisation problem. To obtain the possibility of the proposed problem for satisfying the required SINR $\gamma_k|_{k=1}^{N}$, a necessary and sufficient relation in terms of the cross-correlation value of users codes, γ_k, and N was obtained. The results and the system performance were verified using computer simulations.

6 Acknowledgment

The authors would like to thank Iran Telecommunication Research Center (ITRC) for their financial support of this research.

7 References

1 Kramer, G., Gastpar, M., Gupta, P.: 'Cooperative strategies and capacity theorems for relay networks', *IEEE Trans. Inf. Theory*, 2005, **51**, (9), pp. 3037–3063
2 Laneman, J.N., Tse, D.N., Wornell, G.W.: 'Cooperative diversity in wireless networks: efficient protocols and outage behavior', *IEEE Trans. Inf. Theory*, 2004, **50**, (12), pp. 3062–3080
3 Laneman, J.N., Wornell, G.W.: 'Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks', *IEEE Trans. Inf. Theory*, 2003, **49**, (10), pp. 2415–2425
4 Anghel, P.A., Leus, G., Kavehl, M.: 'Multi-user space-time coding in cooperative networks'. Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP), 2003, vol. 4, pp. 73–76
5 Jing, Y., Jafarkhani, H.: 'Using orthogonal and quasi-orthogonal designs in wireless relay networks', *IEEE Trans. Inf. Theory*, 2007, **53**, (11), pp. 4106–4118
6 Hasna, M.O., Alouini, M.S.: 'Optimal power allocation for relayed transmissions over Rayleigh-fading channels', *IEEE Trans. Wirel. Commun.*, 2004, **3**, (6), pp. 1999–2004
7 Deng, X., Haimovich, A.M.: 'Power allocation for cooperative relaying in wireless networks', *IEEE Commun. Lett.*, 2005, **9**, (11), pp. 994–996
8 Lashkari, B., Biguesh, M., Gazor, S.: 'Joint beamforming and power control in a spatial cooperative multiplexing network with multiple source-destination pairs'. Proc. IEEE 25th Queen's Biennial Symp. Communications (QBSC), 2010, pp. 408–411
9 Li, Y., Vucetic, B., Zhou, Z., Dohler, M.: 'Distributed adaptive power allocation for wireless relay networks', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (3), pp. 948–958
10 Phan, K.T., Le-Ngoc, T., Vorobyov, S.A., Tellambura, C.: 'Power allocation in wireless multi-user relay networks', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (5), pp. 2535–2545
11 Zhao, Y., Adve, R., Lim, T.J.: 'Improving amplify-and-forward relay networks: optimal power allocation versus selection', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (8), pp. 3114–3123
12 Fazeli-Dehkordy, S., Shahbazpanahi, S., Gazor, S.: 'Multiple peer-to-peer communications using a network of relays', *IEEE Trans. Signal Process.*, 2009, **57**, (8), pp. 3053–3062
13 Bornhorst, N., Pesavento, M., Gershman, A.B.: 'Distributed beamforming for multiuser peer-to-peer and multi-group multicasting relay networks'. Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP), 2011, pp. 2800–2803
14 Zhang, Y., Li, X., Amin, M.: 'Distributed beamforming in multi-user cooperative wireless networks'. Proc. IEEE Fourth Int. Conf. Communications and Networking, China, 2009, pp. 1–5
15 Chen, H., Gershman, A.B., Shahbazpanahi, S.: 'Distributed peer-to-peer beamforming for multiuser relay networks'. Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP), 2009, pp. 2265–2268
16 Maric, I., Yates, R.D.: 'Bandwidth and power allocation for cooperative strategies in Gaussian relay networks', *IEEE Trans. Inf. Theory*, 2010, **56**, (4), pp. 1880–1889
17 Ding, Z., Chin, W.H., Leung, K.K.: 'Distributed beamforming and power allocation for cooperative networks', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (5), pp. 1817–1822
18 Devarajan, R., Jha, S.C., Phuyal, U., Bhargava, V.K.: 'Energy-aware resource allocation for cooperative cellular network using multi-objective optimization approach', *IEEE Trans. Wirel. Commun.*, 2012, **11**, (5), pp. 1797–1807
19 Kim, I., Kim, D.: 'Minimizing source-sum-power consumption in multi-sensor single-relay networks', *IEEE Trans. Commun.*, 2011, **59**, (9), pp. 2362–2366
20 Chen, M., Yener, A.: 'Multiuser two-way relaying: detection and interference management strategies', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (8), pp. 4296–4305
21 Jing, Y., Jafarkhani, H.: 'Network beamforming using relays with perfect channel information', *IEEE Trans. Inf. Theory*, 2009, **55**, (6), pp. 2499–2517
22 Bletsas, A., Shin, H., Win, M.Z.: 'Cooperative communications with outage-optimal opportunistic relaying', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (9), pp. 3450–3460
23 Bletsas, A., Khisti, A., Reed, D.P., Lippman, A.: 'A simple cooperative diversity method based on network path selection', *IEEE J. Sel. Areas Commun.*, 2006, **24**, (3), pp. 659–672
24 Talwar, S., Jing, Y., Shahbazpanahi, S.: 'Joint relay selection and power allocation for two-way relay networks', *IEEE Signal Process. Lett.*, 2011, **18**, (2), pp. 91–94
25 Jing, Y., Jafarkhani, H.: 'Single and multiple relay selection schemes and their achievable diversity orders', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (3), pp. 1414–1423
26 Beres, E., Adve, R.: 'Selection cooperation in multi-source cooperative networks', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (1), pp. 118–127
27 Joung, J., Sayed, A.H.: 'Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems', *IEEE Trans. Signal Process.*, 2010, **58**, (3), pp. 1833–1846
28 Chen, M., Yener, A.: 'Power allocation for F/TDMA multiuser two-way relay networks', *IEEE Trans. Wirel. Commun.*, 2010, **9**, (2), pp. 546–551
29 Amah, A.U.T., Klein, A.: 'Pair-aware transceive beamforming for non-regenerative multi-user two-way relaying'. Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP), 2010, pp. 2506–2509
30 Leow, C.Y., Ding, Z., Leung, K.K., Goeckel, D.L.: 'On the study of analogue network coding for multi-pair, bidirectional relay channels', *IEEE Trans. Wirel. Commun.*, 2011, **10**, (2), pp. 670–681
31 Ren, S., van der Schaar, M.: 'Pricing and distributed power control in wireless relay networks', *IEEE Trans. Signal Process.*, 2011, **59**, (6), pp. 2913–2926
32 Vardhe, K., Reynolds, D., Valenti, M.C.: 'The performance of multi-user cooperative diversity in an asynchronous CDMA uplink', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (5), pp. 1930–1940
33 Song, S., Zhang, Q.: 'Design collaborative systems with multiple AF-relays for asynchronous frequency-selective fading channels', *IEEE Trans. Commun.*, 2009, **57**, (9), pp. 2808–2817
34 Goldsmith, A.: 'Wireless communications' (Cambridge University Press, 2005)
35 Lashkari, B., Biguesh, M., Gazor, S.: 'Opportunistic communications at the downlink of mobile systems in the presence of inaccurate channel information'. Proc. IEEE 25th Queen's Biennial Symp. Communications (QBSC), 2008, pp. 356–359
36 Boyd, S., Vandenberghe, L.: 'Convex optimization' (Cambridge University Press, 2004)
37 Boche, H., Schubert, M.: 'A general duality theory for uplink and downlink beamforming'. Proc. IEEE 56th Vehicular Technology Conf. (VTC), 2002, vol. 1, pp. 87–91
38 Biguesh, M., Shahbazpanahi, S., Gershman, A.B.: 'Robust downlink power control in wireless cellular systems', *EURASIP J. Wirel. Commun. Netw.*, 2004, **2**, pp. 261–272
39 Pillai, S.U., Suel, T., Cha, S.: 'The Perron–Frobenius theorem: some of its applications', *IEEE Signal Process. Mag.*, 2005, **22**, (2), pp. 62–75

doi: 10.1049/iet-com.2013.0444

8 Appendix 1: computation of ζ_n and ζ_v

From (10) we obtain $A = \rho \begin{bmatrix} \frac{\gamma_1}{|\alpha_1|^2} \\ \vdots \\ \frac{\gamma_N}{|\alpha_N|^2} \end{bmatrix} \left[|\alpha_1|^2, \ \ldots, \ |\alpha_N|^2\right] -$ $\mathrm{diag}(\rho\gamma_1, \ \ldots, \ \rho\gamma_N)$, which leads to

$$[(\boldsymbol{I} - \boldsymbol{A})^{-1}]_{ij} = \begin{cases} \dfrac{u_i|\alpha_j|^2}{(1+\rho\gamma_j)|\alpha_i|^2(1-\boldsymbol{1}^{\mathrm{T}}\boldsymbol{u})}, & i \neq j \\ \dfrac{1}{1+\rho\gamma_i}\left(1 + \dfrac{u_i}{(1-\boldsymbol{1}^{\mathrm{T}}\boldsymbol{u})}\right), & i = j \end{cases} \tag{30}$$

Using (30), the ith element of $\boldsymbol{\zeta}_n$ and $\boldsymbol{\zeta}_v$ is written as

$$[\boldsymbol{\zeta}_n]_i = \sum_{j=1}^{N} [(\boldsymbol{I} - \boldsymbol{A})^{-1}]_{ij}[\boldsymbol{b}_n]_j = \frac{\sigma_n^2 u_i}{\rho|\alpha_i^2|(1-\boldsymbol{1}^{\mathrm{T}}\boldsymbol{u})} \tag{31}$$

$$\begin{aligned}[\boldsymbol{\zeta}_v]_i &= \sum_{j=1}^{N} [(\boldsymbol{I} - \boldsymbol{A})^{-1}]_{ij}[\boldsymbol{b}_v]_j \\ &= \frac{\sigma_v^2 u_i}{\rho|\alpha_i^2|}\left(\frac{1}{|\beta_i|^2} + \frac{1}{1-\boldsymbol{1}^{\mathrm{T}}\boldsymbol{u}}\sum_{k=1}^{N}\frac{u_k}{|\beta_k|^2}\right)\end{aligned} \tag{32}$$

9 Appendix 2: lemma used in the proof of Theorem 2

Lemma 1: Consider a matrix $\boldsymbol{A}$ with non-negative elements and zero on its diagonal. Then, all elements of $\boldsymbol{B}_n^{-1} \overset{\mathrm{def}}{=} (\boldsymbol{I}_n - \boldsymbol{A})^{-1}$ are non-negative if and only if all leading principal minors of $\boldsymbol{B}_n$ are positive.

Proof: We use mathematical induction. We first prove the sufficiency. For $(n-1)$, the lemma is trivially valid. Now, let us assume that the statement is valid for $(n-1)$-dimensional matrices, (i.e. if all elements of $\boldsymbol{B}_{n-1}^{-1}$ are non-negative, then all leading principal minors of $\boldsymbol{B}_{n-1}$ are positive) and write $\boldsymbol{B}_n$ as

$$\boldsymbol{B}_n = \begin{bmatrix} \boldsymbol{B}_{n-1} & -\boldsymbol{x} \\ -\boldsymbol{y}^{\mathrm{T}} & 1 \end{bmatrix} \tag{33}$$

where $\boldsymbol{x} \overset{\mathrm{def}}{=} [[\boldsymbol{A}]_{1,n} \ldots [\boldsymbol{A}]_{n-1,n}]^{\mathrm{T}}$ and $\boldsymbol{y} \overset{\mathrm{def}}{=} [[\boldsymbol{A}]_{n,1} \ldots [\boldsymbol{A}]_{n,n-1}]^{\mathrm{T}}$ have non-negative elements. Using the block-wise matrix inversion formula (For invertible matrices $\boldsymbol{A}$ and $(\boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})$, we have (see equation at the bottom of the page))
and (33), we obtain $\boldsymbol{B}_n^{-1}$ as

$$\boldsymbol{B}_n^{-1} = \begin{bmatrix} \boldsymbol{B}_{n-1}^{-1} + k_n\boldsymbol{B}_{n-1}^{-1}\boldsymbol{x}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{B}_{n-1}^{-1} & k_n\boldsymbol{B}_{n-1}^{-1}\boldsymbol{x} \\ k_n\boldsymbol{y}^{\mathrm{T}}\boldsymbol{B}_{n-1}^{-1} & k_n \end{bmatrix} \tag{34}$$

where $k_n \overset{\mathrm{def}}{=} (1 - \boldsymbol{y}^{\mathrm{T}}\boldsymbol{B}_{n-1}^{-1}\boldsymbol{x})^{-1}$. Assuming that the elements of $\boldsymbol{B}_n^{-1}$ are positive for every $\boldsymbol{x}$ and $\boldsymbol{y}$ with positive elements, it follows that $k_n > 0$ and $\boldsymbol{B}_{n-1}^{-1}$ are component wise positive. Using (33), we have

$$|\boldsymbol{B}_n| = (1 - \boldsymbol{y}^{\mathrm{T}}\boldsymbol{B}_{n-1}^{-1}\boldsymbol{x})|\boldsymbol{B}_{n-1}| = k_n^{-1}|\boldsymbol{B}_{n-1}| \tag{35}$$

From $|\boldsymbol{B}_{n-1}| > 0$ and $k_n > 0$, we conclude that $|\boldsymbol{B}_n| > 0$. Using the induction assumption, all the leading principal minors of $\boldsymbol{B}_{n-1}$ are positive. From this result and $|\boldsymbol{B}_n| > 0$, we conclude that all the leading principal minors of $\boldsymbol{B}_n$ are positive. □

For $n = 1$, we have $\boldsymbol{B}_1 = 1$ and the necessity is trivial. We assume now that the necessity condition of lemma is valid for $n-1$ and all leading principal minors of $\boldsymbol{B}_n$ are positive. Hence all elements of $\boldsymbol{B}_{n-1}^{-1}$ are non-negative. Using $|\boldsymbol{B}_{n-1}| > 0$ and $|\boldsymbol{B}_n| > 0$ in (35), we conclude that $k_n > 0$. From (34), we can easily deduce that all elements of $\boldsymbol{B}_n^{-1}$ are positive. □

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{A}^{-1} + \boldsymbol{A}^{-1}\boldsymbol{B}(\mathbf{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1} & -\boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1} \\ -(\boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1} & (\boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1} \end{bmatrix}$$