

Convex optimisation-based joint channel and power allocation scheme for orthogonal frequency division multiple access networks

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Abstract: This study concerns joint channel and power allocation scheme for multi-user orthogonal frequency division multiple access system. The author's highlight is margin adaptive (MA) resource allocation problem namely minimising the total transmit power of users with rate requirement constraints. MA is generally provable NP-hard; the typical methods are either to relax and round, or to fix the transmission mode of users (e.g. modulation and coding). Differently, they reorganise MA problem with only power variables left and design a novel relaxation scheme to enable the convexity. The polynomial-time algorithm-interior-point method-is employed to solve the relaxation problem and the theoretical complexity is further presented. Simulation results demonstrate that the author's scheme can provide high energy efficiency compared with the existing methods, 100% relative error bounds with respect to the optimum in most cases, and low computational complexity.

1 Introduction

Two kinds of joint channel and power allocation problems have mainly raised concern in orthogonal frequency division multiple access (OFDMA) networks. One is to maximise system capacity with total power constraint while the counterpart is to minimise the total transmit power as well as meeting user rate requirements [1]. They are called rate adaptive (RA) problem and margin adaptive (MA) problem, respectively. Recently, MA has been redrew more and more attention in small cell networks [2]. Thanks to minimising the total transmit power, small cell network can mitigate inter-cell interference and conserve energy at the same time, both are current issues.

MA is generally shown to be strongly NP-hard [3] because of the binary variables (channel allocation indicators) and OFDMA constraints (i.e. any two users cannot share the same channel). In the existing literature, the solutions either directly relax the original problem, or resort to discrete methods. The method in [4] (Method A) first relaxes channel allocation indicators into continuous variables and allocates the channel to whom with the largest value [4]. After that, transmit power is compensated for guaranteeing user rate requirements. The method in [2] (Method B) first determines the user's transmission mode (modulation and coding) based on the average channel conditions and then correspondingly regulates the transmit power to meet the signal-to-interference-plus-noise ratio (SINR) target of the selected mode. Another computationally efficient approach is to allocate channels using the proportionally fair algorithm and control the power of water-filling algorithm (PF + WF).

Owing to the separate optimisation of channel and power, multi-user channel variations cannot be sufficiently exploited and hence the performance loss is expected.

In this paper, we design a novel relaxation scheme to enable the convexity of MA problem. To be specific, we first reformulate MA problem so that only power variables are left. Using this expression, we loosen OFDMA constraints (equations) into the inequations by introducing a tolerant error constant, which shows to be free of performance loss from the engineering perspective. In order to get a standard convex expression, we attach a series of auxiliary constraints in the optimisation model. The convex counterpart is solved using interior-point method, a polynomial-time algorithm. We provide the self-concordant barrier functions (elaborate later) of all the constraints on the convex counterpart and further derive the theoretical complexity. Our method jointly optimises channel and power and hence sufficiently exploits user channel variations. Benefit from this, the algorithm costs less power compared with the above-mentioned approaches, which is shown in simulation results.

The rest of this paper is outlined as follows. Section 2 presents the system model and the formulation of MA problem. We elaborate the novel relaxation method in Section 3. Simulation results are arranged in Section 4, followed by the conclusion in Section 5.

2 System model and problem formulation

We consider an OFDMA network with a base station as the central controller. There exist U users indexed by

$u \in \mathcal{U} = \{1, 2, \dots, U\}$, where \mathcal{U} denotes user set. K channels are assumed; the band of each channel is denoted by B_k . Similarly, $k \in \mathcal{K} = \{1, 2, \dots, K\}$ and \mathcal{K} denotes the index of channels and channel set. $\chi_u^k \in \{0, 1\}$ indicates whether user u occupies channel k ; $\chi_u^k = 1$ if occupies, and 0 otherwise.

The SINR of the k th channel of user u is expressed as

$$\gamma_u^k = \frac{P_u^k |h_u^k|^2}{\sigma_0^2 + I_u^k} = \alpha_u^k P_u^k \quad (1)$$

by setting $\alpha_u^k = (|h_u^k|^2)/(\sigma_0^2 + I_u^k)$, where h_u^k and P_u^k denote the channel gain between the base station and user u in the k th channel and the transmit power in this link, respectively. I_u^k and σ_0^2 represent the interference and noise power, which are assumed to be constant since the scenario of interest is a single cell. Thus, the rates of user $u \in \mathcal{U}$ are the sum of all rates in allocated channels, given by

$$\mathcal{R}_u = \sum_{k \in \mathcal{K}} B_k \log(1 + \alpha_u^k P_u^k \chi_u^k) \quad (2)$$

All of three significant factors in a typical radio link, path loss, shadowing and channel variations in frequency (f) and time (t) domains, are taken into account. In this sense, the channel model is given by

$$L(\text{dB}) = a + b \log(d) + X_s + X_t(t, f) \quad (3)$$

where a and b are constants determined by particular path loss models, and d is the distance between the transmitter and the receiver. X_s accounts the effect of shadowing. Multipath effect on frequency variations (f) and time-varying fading (t) are modelled as a Rayleigh fading and a multi-tap filter, respectively; they are jointly considered in $X_t(t, f)$.

In MA problem, each user is profiled with a rate requirement [1], for example, R_u^{target} of user u . The objective of MA is to minimise the total transmit power in the sense that it can be formulated as follows

$$\text{MA: } \min_{\{P_u^k\}, \{\chi_u^k\}} \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} P_u^k \chi_u^k$$

$$\text{s.t. } \mathcal{R}_u \geq R_u^{\text{target}}, \quad \forall u \in \mathcal{U} \quad (4a)$$

$$\sum_{u \in \mathcal{U}} \chi_u^k \leq 1, \chi_u^k \in \{0, 1\}, \quad \forall k \in \mathcal{K} \quad (4b)$$

$$P_u^k \in \{p | 0 \leq p \leq p_{\max}\}, \quad \forall u \in \mathcal{U} \quad (4c)$$

where (4a), (4b) and (4c) represent rate constraints, OFDMA constraints and maximum power constraints, respectively.

Based on the recent work [3], MA is strongly NP-hard unless a special case of $U=K$. The non-trivial property comes from the binary variables $\{\chi_u^k\}$ and OFDMA constraints (4b). To handle them, we devise a novel relaxation scheme in the sequel.

3 Relaxation scheme to solve MA

We first reformulate MA with only power variables $\{P_u^k\}$ and reach the following problem (ReMA)

$$\min_{\{P_u^k\}} \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} P_u^k$$

$$\text{s.t. } R_u^{\text{target}} - \mathcal{R}_u \leq 0, \quad \forall u \in \mathcal{U} \quad (5a)$$

$$\sum_{k \in \mathcal{K}} P_u^k P_v^k = 0, \quad \forall u \neq v, \quad u, v \in \mathcal{U} \quad (5b)$$

$$P_u^k \in \{p | 0 \leq p \leq p_{\max}\}, \quad \forall u \in \mathcal{U} \quad (5c)$$

where R_u is given by (6) instead of (2)

$$\mathcal{R}_u = \sum_{k \in \mathcal{K}} B_k \log(1 + \alpha_u^k P_u^k) \quad (6)$$

Note that the OFDMA constraints (4b) are replaced by (5b). In fact, the OFDMA constraints can also be represented as

$$P_u^k P_v^k = 0, \quad \text{for all } u \neq v, \quad \text{and } k \in \mathcal{K} \quad (7)$$

We employ the form of (5b) in that it is more efficient in terms of the complexity. The complexity of interior-point method is dependent highly on the number of inequations [5] (also see Theorem 1). We will later loosen OFDMA constraints into inequations so the number of ‘new’ OFDMA constraints is directly related to the complexity. By using the expression of (5b), the number of inequations with respect to OFDMA constraints is $(U-1)U/2$ which does not associate with the number of channels [but the number of inequations will be $(U-1)UK/2$ if we use (7)].

As we know, a standard convex problem requires the objective and constraint functions to be convex, and the equation constraint functions to be affine [5]. However, the constraint function of (5b), that is, the left-side function of ‘=’, is non-affine. To enable the convexity, we introduce a tolerant error ϵ to make the equation unequal and then exploit the convexity of the constraint function at the ‘new’ inequality constraint.

The first step of getting the in equations can be obtained by

$$\sum_{k \in \mathcal{K}} P_u^k P_v^k \leq \epsilon, \quad \forall u \neq v, \quad u, v \in \mathcal{U} \quad (8)$$

In view of engineering, the introduction of the tolerant error is reasonable in the sense that a small deviation of transmit power is allowed and exists. For instance, ϵ can be designed as the transmit power error of a transmitter. Consider a case of two channels and two users. Suppose $\mathbf{p}_1 = [10, 0]$, $\mathbf{p}_2 = [0, 10]$ (\mathbf{p}_u denotes the vector containing user u 's transmit power at all channels) is a solution of MA or ReMA, in which case the constraint (5b) is strictly satisfied. If the relaxed constraint (8) is used, the result could be replaced by $\mathbf{p}_1 = [10, 0.5 \times 10^{-4}]$ and $\mathbf{p}_2 = [0.5 \times 10^{-4}, 10]$ if $\epsilon = 10^{-3}$. To get a feasible solution of our original tight problem, we can allow the user with largest transmit power to transmit at a certain channel while cutting down the transmit power of others by setting $p_1^2 = 0$ and $p_2^1 = 0$.

After loosening equality constraints into inequations, the constraint function of (8) is still not convex (its Hessian matrix is not semi-positive definite since the components

are all 0 on the diagonal and exist 1 on other places). Then, we further replace p_u^k by $e^{q_u^k} = p_u^k$ for the reason that the exponential function can make the product to be *linear* and its summation operation remains convex.

Unfortunately, since $\log(1 + e^x)$ is convex the constraint function in (5a) will be concave after the replacement. To fix this problem, we reserve p_u^k in (5a) and add auxiliary constraints relating q_u^k to p_u^k . $\{e^{q_u^k} - p_u^k \leq 0\}$ are introduced as they reserve the convexity of the constraint functions.

Finally, after the relaxation and adding the auxiliary constraints we reach the convex counterpart of MA as follows

$$\text{CoxMA: } \min_{\{q_u^k\}, \{p_u^k\}} \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} p_u^k$$

$$\text{s.t. } \varphi(\mathbf{p}_u) \triangleq R_u^{\text{target}} - \mathcal{R}_u \leq 0, \quad \forall u \in \mathcal{U} \quad (9a)$$

$$\phi(\mathbf{q}_u, \mathbf{q}_v) \triangleq \sum_{k \in \mathcal{K}} e^{q_u^k + q_v^k} - \epsilon \leq 0, \quad \forall u \neq v \quad (9b)$$

$$e^{q_u^k} - p_u^k \leq 0, \quad \forall u \in \mathcal{U}, k \in \mathcal{K} \quad (9c)$$

$$p_u^k - p_{\max} \leq 0, \quad \forall u \in \mathcal{U}, k \in \mathcal{K} \quad (9d)$$

Indeed, the introduction of new auxiliary constraints has almost no effects on the solution. As we know, the optimisation problem $x^* = \text{argmin} f(x)$ is equivalent to $[x^*, t^*] = \text{argmin} t, x \in \{x | f(x) \leq t\}$ [5]. Admittedly, the equivalence of $e^{q_u^k}$ and p_u^k is obtained only when q_u^k is able to come down to $-\infty$, which is unreachable in practical implementation. Thus, there must exist a gap to the optimum. However, q_u^k does not need to be too small. As ϵ is tolerant error, it is meaningless to reserve the result less than it. From this point, we simply request for the algorithm being capable of making q_u down to the same order of $\log \epsilon$. For example, if ϵ is 10^{-6} , the value will be -6 .

CoxMA is a smooth convex problem with non-linear constraints, and a well-known approach is leveraging interior-point (IP) methods [6]. The idea of IP is to approximate the solution of a convex problem with non-linear constraints by solving a sequence of unconstrained optimisation problems (called barrier problems in general). In a barrier problem, ‘barrier functions’ reflect the cost of the constraints because of their deviation in the objective function. A typical barrier function is the logarithmic barrier function that is $-\ln(-f(x))$.

The theoretical convergency of IP needs the barrier functions to be self-concordant [7]. Based on the existing literature [8], the self-concordant barrier functions of constraints (9b)–(9d) can be

$$\hat{\phi}(\mathbf{q}) = - \sum_{\forall u \neq v} \ln(-\phi(\mathbf{q}_u, \mathbf{q}_v)) \quad (10)$$

$$\hat{\psi}(\mathbf{p}, \mathbf{q}) = - \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} (\ln(p_u^k) + \ln(\ln(p_u^k) - q_u^k)) \quad (11)$$

$$\hat{\omega}(\mathbf{p}) = - \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} \ln(p_{\max} - p_u^k) \quad (12)$$

However, the self-concordant barrier function of (9a) is unknown. To provide it, a straightforward way is to use logarithmic barrier function and prove the logarithmic barrier function to be self-concordant. However, proving whether a general function is self-concordant is NP-hard [9]. With the help of [8], we can alternatively investigate

whether the constraint functions themselves possess some particular properties; if they have, the logarithmic barrier function associated with these constraint functions is self-concordant. Based on that, we present the self-concordant barrier function of (9a) in Lemma 1 and the proof below the lemma.

Lemma 1: The barrier function of constraint (9a) that satisfies self-concordant condition is

$$\hat{\varphi}(\mathbf{p}) \triangleq - \sum_{u \in \mathcal{U}} \ln(-\varphi(\mathbf{p}_u)) \quad (13)$$

Proof: Based on Theorem 9.1.1 in [8], $-\ln(f(x))$ is self-concordant if $f(x)$ falls into two conditions:

- $f(x)$ is concave meaning that

$$\nabla^2 f(x)[h, h] \leq 0, \quad \forall x \in \mathbf{R}^n, \forall h \in \mathbf{R}^n \quad (14)$$

- There exists a constant $\beta \geq 0$, such that

$$\nabla^3 f(x)[h, h, h] \leq -3\beta \nabla^2 f(x)[h, h] \quad (15)$$

$f(x)$ can be defined as $f(x) = \sum_{k \in \mathcal{K}} B_k \log(1 + \alpha^k x^k) - R^{\text{target}}$ by removing the subscript u and replacing the variables $\{p_u^k\}$ by $\{x^k\}$ in constraint (9a), where $x = \{x^k\}$. From $f(x)$, we learn that its variables are separated and the derivatives (any order) only have non-zero values along the direction of the variables themselves. More specifically, to check whether $f(x)$ is qualified with respect to condition (14), only direction $[x^k, x^k]$ is required. In this case, $\nabla^2 f(x)[h, h] = -((\alpha^k)^2) / ((1 + \alpha^k x^k)^2) \leq 0$. Likewise, for condition (15), it can be satisfied by setting $\beta \geq \alpha^k$ through deriving the second and third derivatives along the directions $[x^k, x^k]$ and $[x^k, x^k, x^k]$. Moreover, self-concordance is reserved by summation [8], and hence we complete the proof by summing $-\ln(-\varphi(\mathbf{p}_u))$. \square

Note that the proof is motivated by the observation that $f(x)$ is separated. Since it is non-trivial to show whether $-\ln(f(x))$ is self-concordant, a fairly good alternative is to investigate the property of $f(x)$.

Based on (10)–(13), we derive the ‘barrier problem’ of CoxMA for certain penalty t , expressed as

$$\min_{\{q_u^k\}, \{p_u^k\}} t \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} p_u^k + \hat{\varphi}(\mathbf{p}) + \hat{\phi}(\mathbf{q}) + \hat{\psi}(\mathbf{p}, \mathbf{q}) + \hat{\omega}(\mathbf{p}) \quad (16)$$

As we have mentioned, the principle of IP is to solve a sequence of barrier problems (16) with fixed penalty t and enlarge the penalty incrementally to approximate the optimum. Formally, it has two kinds of iterations-outer iteration and inner iteration-which can be summarised as follows:

- **Outer iteration:** update t by $t = ct$ till $\vartheta/t \leq \zeta$ [5].
- **Inner iteration:** given t , solve problem (16) to obtain ζ_{in} -solution $(\mathbf{p}^*(t), \mathbf{q}^*(t))$ through Newton’s method with the input of (\mathbf{p}, \mathbf{q}) in last **Inner iteration**

where $\vartheta = ((U(U - 1))/2) + 2UK$, that is, the number of inequalities in CoxMA, c denotes the step size of updating t , ζ_{in} and ζ_{ou} are the termination criteria of inner iterations

and outer iterations, respectively. c addresses a tradeoff between outer and inner iteration; a large c indeed accelerates outer iteration but may not be a good starting point for the next inner iteration. A practical setting of c is about 10–20 [5]. In addition, ζ_{in} can be set larger than ζ_{ou} . Such an inexact solution from inner iteration still generates a sequence of points converging to the optimal as $t \rightarrow \infty$.

As we have provided all self-concordant barrier functions of all the constraints, from [7] we know that our proposed method can converge to ζ_{ou} -optimality with respect to CoxMA and the theoretical complexity can be directly obtained. We conclude it in Theorem 1.

Theorem 1: Our proposed algorithm converges to ζ_{ou} -solutions to CoxMA and its complexity is

$$\mathcal{O}(1)\sqrt{\vartheta}\ln\left(\frac{\vartheta}{t_0\zeta_{ou}} + 1\right)$$

where t_0 denotes the initial point of t and $\mathcal{O}(1)$ is a constant depending only on the step size c and ζ_{in} .

As we can see, the complexity is dependent mainly on the number of inequality constraints (ϑ), which shows the reason why we replace OFDMA constraint by (5b) instead of (7).

Let us recapitulate what we have done and provide an insight into design parameters. To deal with OFDMA constraints, we first introduce tolerant errors making them to be inequities and replace $\{p_u^k\}$ by $\{e^{q_u^k}\}$ to reserve the convexity. We have taken an example to show the tolerant error (ϵ) has almost no impact on the practical decision. This viewpoint can also be supported by the fact that interior-point method only obtains ζ_{ou} -solutions, thus whether we introduce tolerant errors does not make a substantial difference with respect to the solution. In practice, we can set ζ_{ou} being equal to or the same order of ϵ . In spite of the increasing complexity with the decrease of ϵ , the effect is very small as ϵ appears within logarithmic function.

4 Simulation results

An OFDMA small cell network is chosen as our simulation scenario, and the system-level parameters refer to [10]. Owing to hardware limitations, the simulation includes only a limited number of users and channels; in particular, up to ten channels are employed. The rate requirement rate is 384 kbps which is the typical demand of the video stream. Moreover, channel model is based on [11]. Interference is assumed a constant being equal to $10\sigma_0^2$.

We compare our proposed scheme with three above-mentioned methods: Method A, Method B and PF + WF in order. To gauge the optimality, we also present the optimal solutions obtained by exhaustive searching. The performance metrics are total transmit power and the complexity. In addition, we evaluate the instant property of our proposed scheme with respect to the optimal solutions in 50 realisations.

As we can see in Fig. 1, all the methods cost more power with the increase of the number of users. Comparably, our proposed method keeps a low level in terms of total power even being served a large number of users in the network. As for Method A, the transmit power increases significantly when the number of users becomes large. The solutions in

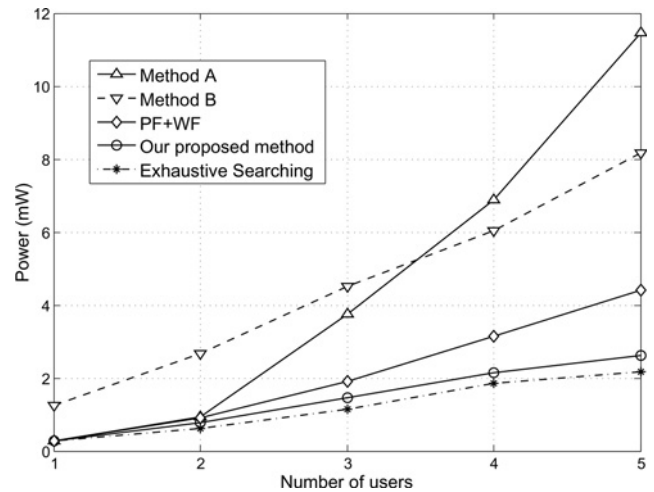


Fig. 1 Power with the increase of the number of users

relaxation procedure of Method A can be regarded as the time share of channels and the largest one is rounded to 1. Such a scheme does not consider the overall performance of users. If users' channel conditions are unbalanced, it will cause considerable performance loss. Since the transmission mode is fixed in Method B, channel variations cannot be sufficiently exploited. As for PF + WF, the performance loss is because of variables' separate optimisation.

Fig. 2 illustrates the theoretical complexity of all methods; our proposed method acts out the best performance. The complexity of Method A is $\mathcal{O}(KL^\dagger)$, $L^\dagger \gg K$, U [1], and we set $L^\dagger = 10K$ in our simulation. Method B's complexity is $\mathcal{O}(V^2E \log V)$ [8], where $V \simeq U + K$ and $E \simeq UK$. As for PF + WF, the complexity is $\mathcal{O}(UK)$. In the exhaustive searching, each channel can only be allocated to one user or none of them, thus there exist $U + 1$ possibilities for each channel. The total number of the exhaustive searching is $(U + 1)^K$ as we have K channels. Note that all presented complexities of three methods are in theoretically worst case in that practical results depend heavily on network conditions such as user location and channel conditions.

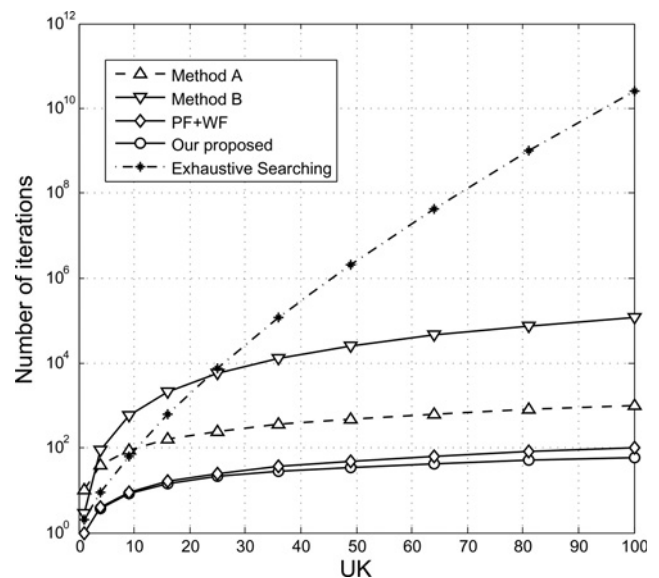


Fig. 2 Complexity of three methods with the increase of UK when $U = K$

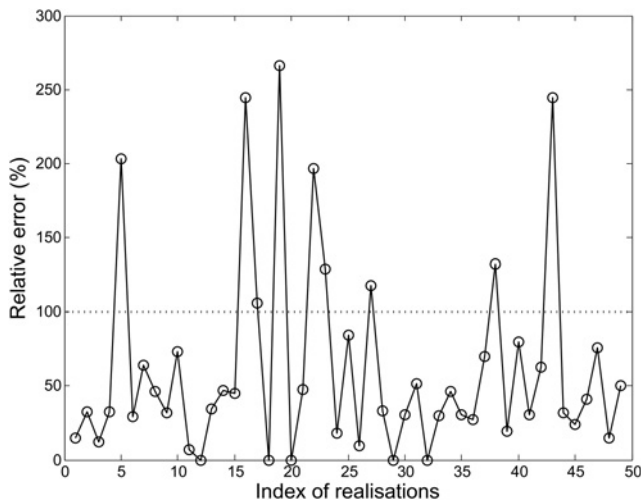


Fig. 3 Relative error of our proposed method to the optimal solution in 50 particular realisations

Finally, we investigate the performance of our proposed method in particular simulation realisations in Fig. 3, where the relative error with respect to the optimal solution is assessed. The relative error is defined as the ratio of the difference between the result from our proposed method and the optimal to the optimal. As we can see, in most cases (41 of 50 realisations) our proposed method obtains 100% relative error bounds.

5 Conclusion

We focus on a joint channel and power allocation problem for OFDMA networks, where the objective is to minimise the total power of users under the constraints of rate requirements. The problem redrew the attention since it is beneficial to interference avoidance and energy efficiency in small cells. We propose a novel relaxation scheme to solve this NP-hard problem. We first reform this optimisation problem with only power variables and then loosen it into a convex counterpart. The convex relaxation problem is

solved efficiently using interior-point method. As we jointly optimise users' channel and power, unlike existing methods, channel variations can be sufficiently exploited and hence a considerable improvement in terms of energy-saving is obtained.

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