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Frequency planning for clustered jointly processed cellular multiple access channel

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Abstract: Owing to limited resources, it is hard to guarantee minimum service levels to all users in conventional cellular systems. Although global cooperation of access points (APs) is considered promising, practical means of enhancing efficiency of cellular systems is by considering distributed or clustered jointly processed APs. The authors present a novel 'quality of service (QoS) balancing scheme' to maximise sum rate as well as achieve cell-based fairness for clustered jointly processed cellular multiple access channel (referred to as CC-CMAC). Closed-form cell level QoS balancing function is derived. Maximisation of this function is proved as an NP hard problem. Hence, using power-frequency granularity, a modified genetic algorithm (GA) is proposed. For inter site distance (ISD) < 500 m, results show that with no fairness considered, the upper bound of the capacity region is achievable. Applying hard fairness restraints on users transmitting in moderately dense AP system, 20% reduction in sum rate contribution increases fairness by upto 10%. The flexible QoS can be applied on a GA-based centralised dynamic frequency planner architecture.

1 Introduction

Global access points (AP) cooperation which has been studied in classical Wyner model [1], extended with fading [2] and distance dependent pathloss [3], is considered too complex to implement. Recently, localised joint processing of APs has been proposed in the framework of isolated groups [4], local message passing [5, 6] and limited backhaul [7]. In [8], the concept of rate splitting from interference channel was applied to clustered jointly processed APs. In such schemes, inter cluster interference from users outside cluster is the only dominant type of interference. Since edge cells are more prone to interference from neighbouring cells, the level of interference at the edge and centre of a cluster is not the same [9]. Hence, techniques involving strongest channel coefficients like dynamic clustering [10] are proposed to address the varying degree of interference levels. In this work however, we consider geographically fixed clusters. To conform to current cellular networks, each cluster processing unit (CU) (analogous to base station controller (BSC)) handles a collection of APs (analogous to base transceiver station (BTS)); each geographically fixed cluster hence behaves as a network multiple-input multiple-output (MIMO) (analogous to uplink of coordinated multipoint transmission reception concept in fourth generation [11] but with interference between clusters).

Similar to conventional cellular networks, frequency planning can be considered an effective method to control the interference situation. Here, we refer to such systems as cluster cooperative cellular multiple access channels or (CC-CMAC) [12]. We consider frequency planning for CC-CMAC in order to maximise network utility using a derived formulation of per cell sum rate. We consider 'balancing of per cell sum rate' within cell users in a seven cell cluster surrounded by six first tier interfering clusters and so on. In this context, per cell sum rate balancing takes place at the PU of the cluster. Cumulatively, the network wide balancing of per cell sum rate for users transmitting in a CC-CMAC is referred to as 'quality of service (QoS) balancing function'. This balancing approach is useful in quantifying cell utility for a range of per cell sum rate conditions. The proposed QoS-based utility function is used to study resource allocation problem in cluster-based CMAC. Two cases of interest are: (a) maximising sum rate and (b) achieving cell-based fairness.

We take a cell-based approach as in [13] and assume a medium-term time scale corresponding to cell-level load variations. The short term variations related to user mobility and instantaneous channel conditions are assumed to be handled by each cluster's radio resource management functionality.

The rest of the paper is organised as follows. Section 2 introduces the concept of CC-CMAC and bin allocation over AP-based fixed clusters and presents the system model and architecture. Performance measure of CC-CMAC is presented in Section 3. Here, the closed-form representation of cell-based QoS balancing function for CC-CMAC is derived. In Section 4, the solution to this formulation is proved to be NP hard. Section 5 discusses the solution framework using a modified heuristic, that is genetic algorithm (GA). The GA implementation and effect of power-frequency granularity and fairness are discussed in Section 6. The application of CC-CMAC in a practical

GA-based architecture, optimising QoS balancing function, with architectural complexity analysis is presented in Section 2.2. We conclude in Section 8.

Notation: Lower and upper case boldface symbols denote vectors and matrices, respectively; Math Curl represents the set notation, $(.)^{\dagger}$ denotes the Hermitian transpose, $|\cdot|$ represents the cardinality of a set, diag is the diagonal of the matrix, tr represents the matrix trace and $\mathbb{E}[.]$ represents the expectation operator.

2 System model

2.1 Transmitted signal model

The uplink capacity of CC-CMAC is analysed using bin allocation. Bins are disjoint equal width frequency bands with a flat transmit power spectral density used over them [13]. Specifically, a hexagonal grid of N cells is assumed which is divided into Q fixed, equal and identical clusters. $\mathcal{N}_q$ is the set of all cells belonging to cluster q, where $q = 1, \ldots, Q$, and $|\mathcal{N}_q| = N/Q$. Similarly, $\overline{\mathcal{N}_q}$ is the set of all cells not belonging to cluster q where $|\overline{\mathcal{N}_q}| = N - (N/Q)$. Noise is additive white Gaussian noise (AWGN). For transmission over a given bin b, the $|\mathcal{N}_q| \times 1$ received signal vector $\boldsymbol{y}^q$ for the qth cluster can be modelled as

$$\boldsymbol{y}^q = \boldsymbol{H}^q \boldsymbol{x}^q + \widehat{\boldsymbol{z}}^q \quad (1)$$

$$\widehat{\boldsymbol{z}}^q = \hat{\boldsymbol{H}}^q \hat{\boldsymbol{x}}^q + \boldsymbol{z}^q \quad (2)$$

where the $|\mathcal{N}_q| \times K|\mathcal{N}_q|$ channel matrix, $\boldsymbol{H}^q = \left[\boldsymbol{H}_1^q, \ldots, \boldsymbol{H}_{|\mathcal{N}_q|}^q\right]$, contains complex gain matrices for K users per cell, within the $|\mathcal{N}_q|$ cells which are located within qth cluster and $|\mathcal{N}_q| \times K\left(N - |\mathcal{N}_q|\right)$ channel matrix, $\hat{\boldsymbol{H}}^q = \left[\hat{\boldsymbol{H}}_1^q, \ldots, \hat{\boldsymbol{H}}_{N-|\mathcal{N}_q|}^q\right]$, contains complex gain matrices for all $N - |\mathcal{N}_q|$ cells out of the qth cluster. Similarly, $\boldsymbol{x}^q = \left[\boldsymbol{x}_1^q, \ldots, \boldsymbol{x}_{|\mathcal{N}_q|}^q\right]^{\mathrm{T}}$ is a $K|\mathcal{N}_q| \times 1$ transmit symbol vector for all cells within qth cluster and $\hat{\boldsymbol{x}}^q = \left[\hat{\boldsymbol{x}}_1^q, \ldots, \hat{\boldsymbol{x}}_{N-|\mathcal{N}_q|}^q\right]^{\mathrm{T}}$ is a $K\left(N - |\mathcal{N}_q|\right) \times 1$ transmit symbol vector for all cells outside cluster q. $\boldsymbol{z}^q$ represents the $|\mathcal{N}_q| \times 1$ independent complex circularly symmetric AWGN vector at receiver. The transmitters are subject to power constraint $\mathrm{tr}\left(\mathbb{E}\left[\boldsymbol{x}_i^q\left(\boldsymbol{x}_i^q\right)^{\dagger}\right]\right) \leq \mathrm{KP}_i$ and $\mathrm{tr}\left(\mathbb{E}\left[\hat{\boldsymbol{x}}_j^q\left(\hat{\boldsymbol{x}}_j^q\right)^{\dagger}\right]\right) \leq \mathrm{KP}_j$, where cell i and cell j are located within and outside cluster, q respectively. We assume that the users have same per user power constraint. Hence, $P_i = P_j = P_n$, $\forall n = 1, \ldots, N$. This has applications in energy constrained transmitter, where power is restricted over the entire spectrum.

Let $a_{n,b}$ be the nth row and bth column entry for allocation matrix, $\boldsymbol{A}$. Allocation matrix $\boldsymbol{A}$ represents allocation of bth bin to users in nth cell (non-zero integer implies allocation; 0 implies otherwise). If allocation matrix for users transmitting in cell 1 in Fig. 1 is produced it will have values of $P(1,1)$ at $a_{1,1}$ and so on. The maximum bins (discrete frequency allocation intervals) that are implemented are B bins. For V quantisation bits per bin,

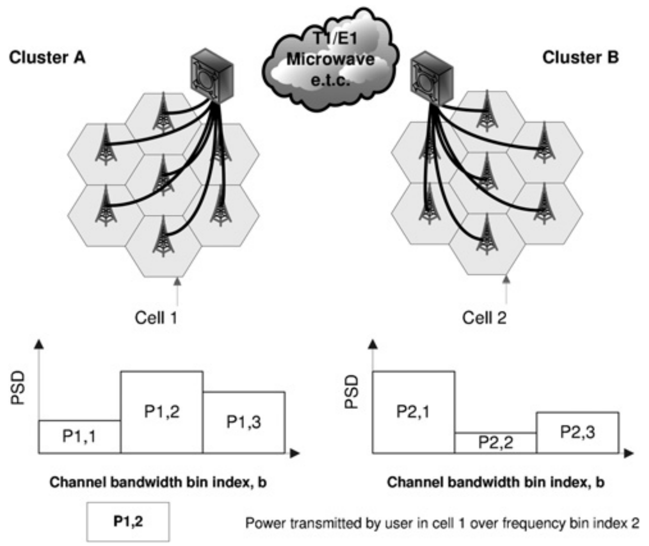

Fig. 1 *Demonstration of uplink transmission by users in cells from two adjacent clusters. Here the scenario is shown across three bins which are allocated power to cater interference in FC as per CC-CMAC. Here the backhaul is also shown*

$\boldsymbol{A} \in \{1, \ldots, 2^V - 1\}^{N \times B}$ represents joint bin and power allocation to the N cells in the system. Here, $\mathcal{N}_b(\boldsymbol{A})$ represents the set of cells which have been allocated the bth bin. A simple scenario for two cells transmitting to APs in two different clusters is explained in Fig. 1. We consider allocating sets of frequency bins to each cell in a cluster. A simple case of two interfering cells with all users in each cell allocated similar bin allocation is shown in the bottom of Fig. 1.

2.2 System architecture

Consider a conventional cellular network. The BSC handles the base-band signal processing and encoding/decoding of users served by individual BTS. The collection of BTS served by a BSC handles the base band signal processing of all user transmissions associated with the cluster of BTS served by BSC. Owing to the nature of radio propagation, user transmission from other BSC clusters is also received at BTS; this is considered as interference. In a similar way, CC-CMAC considers limited cooperation using clustering at APs. It consists of a cluster of APs which treats signals from users within the cluster as useful signal and signal from users located outside the cluster as interference. Hence, to maximise performance, two types of coordination activities take place between APs. These are referred to as ‘inter cluster’ and ‘intra cluster’ coordination:

- *Inter cluster coordination*: A basic coordination level exists between different clusters and is handled at the Network wide central processing unit. This unit handles the CSI strategies allowing the centralised unit to coordinate signaling strategies such as power allocation and spectrum assignment. The analysis on frequency allocation which is the scope of this paper achieves a network wide criterion based on this iterative feedback model.
- *Intra cluster coordination*: All APs within a cluster fully share their CSI and user data. Combined use of several AP antennas belonging to different cells to send or receive multiple user datastreams mimics transmission over MIMO)

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channel and is conventionally referred to as MIMO cooperation or MU-MIMO.

Information theoretic models (Wyner's GC-MAC model) considered in evaluating CC-CMACs architecture do not assume any physical layer (CDMA etc.) a priori. However, since the objective of the model under study is to maximise a criterion which handles both inter cluster and intra cluster coordination, any resource allocation at cell user level should ensure how much power is allocated to each cell user at each of the different sub bands. To maximise resource usage, the allocation to each cell within a cluster will not be orthogonal. In that sense, inter cluster coordination is close to orthogonal frequency division multiple access (OFDMA) in which multiple cells within each cluster are separated in the frequency domain.

3 Performance measure of CC-CMAC

As per ITU Standard X.902, QoS is defined as the ability of a network to guarantee a set of quality requirements on a single or group of users. High QoS improves level of service from the operator's point of view. In the context of bin allocation, QoS refers to the sum rate generated because of bins allocated to users within a cell. This indirectly determines QoS for users belonging to a network. Hence, QoS is rate centric rather than service and application specific. An acceptable QoS would mean that the users in a cell have a data rate which is acceptable, whereas a non-acceptable QoS would mean that users in a cell have a data rate which is not acceptable to the operator as per requirements of the business. If the operator's objective is to maximise system throughput, only the cells with good channel conditions are given preferential treatment. The cells with poor channel condition are not given equal share of bin resources as compared with the cells with good channel conditions. To solve this disparity, a fair metric can be framed on the level of user groups. Tradeoff between sum rate and fairness helps satisfy service requirement of users which motivates the use of utility in communication theoretic framework [14]. This is discussed in the next section.

3.1 Network wide QoS balancing framework

Mobile network operators would like to ascertain network wide efficiency criterion to meet the objectives of profit maximisation. Hence, the need for a system wide efficiency criterion. Utility optimisation is a useful tool to measure system performance against user satisfaction criterion [15, 16]. Using sum rate as a function of utility function is a common approach. Utility can also address a wide range of fairness conditions as in [14]. It has been associated with particular choice of bandwidth or power allocation and can be measured using a composite function known as network utility maximisation (NUM) function [14, 17, 18]. Depending on the type of resource allocation, the N cells within a cellular system can be paired independently. Hence, total network utility decomposes into the sum of user utilities. Therefore as in [19]

$$U(R(x)) = \sum_{n=1}^{N} \sum_{k=1}^{K} u^{n,k}\left(R^{n,k}(x)\right) \tag{3}$$

Here, $u^{n,k}$ is the utility for kth user in nth cell, $R^{n,k}(x)$ is the sum rate because of transmission from kth user in nth cell, x determines transmission allocation and U is global utility function.

Per cluster analysis does not give insight into actual user contributions and per user sum rate would require additional feedback overheads for CC-CMAC. The short term variations related to user's mobility and instantaneous channel conditions are assumed to be handled by each cluster's radio resource management functionality. As far as provision of service levels (QoS provisioning) is concerned, cell-based service provision helps to maintain differentiated levels of service.

In this section we determine this efficiency in terms of QoS balancing of per cell sum rate. For a given fairness coefficient γ, $U_\gamma(\boldsymbol{A})$ is the system level QoS balance metric which is broken down to cell-based QoS balance metric in (4). Mathematically

$$U_\gamma(\boldsymbol{A}) = \frac{1}{N} \sum_{q=1}^{Q} \sum_{n \in \mathcal{N}_q} u_\gamma\left(R_{n,q}(\boldsymbol{A})\right)$$
$$\forall b = 1, \ldots, B; \quad \forall n = 1, \ldots, N \tag{4}$$

Here $u_\gamma(R_{n,\,q}(\boldsymbol{A}))$ is the QoS balancing metric for users in nth cell within the qth cluster. Here for a given A, $(R_{n,\,q}(\boldsymbol{A}))$ is the sum rate because of transmission over bands specified by $\boldsymbol{A}$ from the nth cell within the qth cluster. Our objective is to

$$\begin{aligned} &\max \; U_\gamma(\boldsymbol{A}) \\ &\text{s.t.} \quad R_{n,q}(\boldsymbol{A}) \in \mathbb{R}^+ \end{aligned} \tag{5}$$

subject to the power constraints as embodied in $\boldsymbol{A}$. Combining (4) and (5), the optimisation problem is to maximise network wide QoS balancing function. Hence

$$\max_A \overbrace{\frac{1}{N} \sum_{q=1}^{Q} \left(\overbrace{\sum_{n \in \mathcal{N}_q} \left[u_\gamma\left(R_{n,q}(\boldsymbol{A})\right) \right]}^{\text{cell-based-QoS-balancing-qth-cluster}} \right)}^{\text{System-wide-cell-based-QoS-balancing}} \tag{6}$$

$$\text{s.t.} \quad R_{n,q}(\boldsymbol{A}) \in \mathbb{R}^+ \tag{7}$$

$$\sum_{b=1}^{B} P_{n,b} \le P$$
$$\forall n = 1, \ldots, N; \forall b = 1, \ldots, B$$

where P is the maximum per user transmit power over all the allocated bins. The value of $P_{n,b}$ varies across bins and across different cell users within a cluster. This is because QoS balancing needs to account for two forms of interference, that is inter cluster and intra cluster interferences. This has implications for considering such a formulation for macro-cells in conjunction with pico cells and femto cells. Since our formulation considers fixed cluster and is not a hierarchical-based clustering, further topics likes HetNet can be modelled but are not explored in detail for in this paper. The bin-based CC-CMAC formulation has been discussed in [11] and performance detailed in [12]. In this paper, the authors build on the material from their previous work.

3.2 Complexity and simulation-based sum rate for CC-CMAC

The concept of joint processing of signals to evaluate sum rate was introduced in Wyner's Gaussian CMAC (GCMAC) model [1]. Letzepis extended classical work of Wyner, to produce a log det formulation for measuring capacity of jointly processed cellular networks with free space path loss. The sum rate representation for transmission received in qth geographical fixed cluster of APs as in [11] was represented using the expectation of per cell sum rate formulation: $R_q = \log\det(\boldsymbol{S_q}/\boldsymbol{S_{nq}})$. The useful signal is represented in numerator ($\boldsymbol{S_q}$) and interference as well as noise is shown in denominator ($\boldsymbol{S_{nq}}$).

Recalling R_q as the sum rate generated because of signal transmission from users across all clusters but received at APs in the qth cluster [11] can be represented as

$$R_q(\boldsymbol{A}) = \frac{1}{B}\sum_{b=1}^{B}\mathbb{E} \times \left[\log\det\left(\frac{\sigma_z^2\boldsymbol{I} + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})}P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger}{\sigma_z^2\boldsymbol{I} + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})\cap\overline{\mathcal{N}_q}}P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger}\right)\right] \tag{8}$$

This formulation was used to generate random snapshots for channel conditions and simulated using Monte Carlo-based iterations. In order to reduce complexity, a closed-form representation of per cell sum rate for qth cluster $\mathbb{E}\left[R_q(\boldsymbol{A})\right]$ accommodated in our proposed architecture (Fig. 10 is now considered. As per simulation results of $\mathbb{E}\left[R_q(\boldsymbol{A})\right]$ against $P_n(\mathbf{A})$; $\forall n$, $R_q(\boldsymbol{A})$ is proved to be a concave function [12]. Recall that inter cluster coordination requires exchange of CSI over the backhaul. This coordination can be strong provided a large set of users per cell (multiuser diversity) coexist in the system. The effect of Jensen's inequality can be approximated by identifying $K\to\infty$. The Jensen's inequality [20] can hence be applied. As per law of large numbers, $\mathbb{E}[X/Y] = \mathbb{E}[X]/\mathbb{E}[Y]$, since large K implies a deterministic X and Y. This gives the following

$$\mathbb{E}\left[R_q(\boldsymbol{A})\right] \le \frac{1}{B}\sum_{b=1}^{B}\log \left[\mathbb{E}\left(\frac{\det\left(\sigma_z^2\boldsymbol{I}_{N/Q} + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})}P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right)}{\det\left(\sigma_z^2\boldsymbol{I}_{N/Q} + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})\cap\overline{\mathcal{N}_q}}P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right)}\right)\right] \tag{9}$$

Further, using $\log(AB^{-1}) = \log(A) - \log(B)$ the following can be deduced: (see (10))

Consider a fast fading scenario. For transmission from kth user in nth cell to AP in cell n_q located within cluster q, each fading coefficient is represented by $g_{n,k}^{n_q}$ and pathloss is identified by $s_{n,k}^{n_q}$, respectively. The channel coefficient is represented by $h_{n,k}^{n_q} = g_{n,k}^{n_q}\odot s_{n,k}^{n_q}$ where $\odot$ represents the Hadamard product. The transmission is modelled as a time-varying ergodic process. Assuming a large number of users per cell, that is $K\to\infty$, as per 'law of large numbers', $1/K\sum_{k=1}^{K}|g_{n,k}^{n_q}|^2 \to 1$ for $\forall n$, $\forall q$. Using complex matrices for fading, product of complex fading coefficients with its complex conjugate is equal to power which is normalised to unity. Hence

$$\mathbb{E}\left[g_{n,k}^{n_q}g_{n,k}^{n_q\dagger}\right] = \mathbb{E}\left[\left(g_{n,k}^{n_q}\right)^2\right] = 1 \tag{11}$$

Moreover, the expectation of the product of a complex fading coefficient with the complex conjugate of a different fading coefficient but following the same PDF is the square of the expected value of an individual fading coefficient. Hence

$$\mathbb{E}\left[g_{n,k}^{n_q}g_{n',k'}^{n_q\dagger}\right] = \left[\mu_g\right]^2 \tag{12}$$

here $k'\neq k$ and $n'\neq n$. μ_g is the expected value of an individual fading coefficient. In the case of the Rayleigh-based flat fading, $\mu_g = 0$ as in [2].

Further, define $\boldsymbol{g}_n^{n_q} = \left[g_{n,1}^{n_q}, g_{n,2}^{n_q}, \ldots, g_{n,K}^{n_q}\right]$ and $\boldsymbol{s}_n^{n_q} = \left[s_{n,1}^{n_q}, s_{n,2}^{n_q}, \ldots, s_{n,K}^{n_q}\right]$ as $1\times K$ complex fading vector and $1\times K$ deterministic pathloss vector for transmission from users in nth cell and received at the AP in the n_q^{th} cell within the qth cluster.

Considering only the diagonal entries of the estimation for covariance of $\boldsymbol{H}_n^q$, the following can be deduced

$$\begin{aligned}\operatorname{diag}\left(\mathbb{E}\left[\boldsymbol{H}_n^q\boldsymbol{H}_n^{q\dagger}\right]\right) &= \mathbb{E}\left[\left(\boldsymbol{s}_n^1\odot\boldsymbol{g}_n^1\right)\left(\boldsymbol{s}_n^1\odot\boldsymbol{g}_n^1\right)^\dagger, \ldots, \left(\boldsymbol{s}_n^{|\mathcal{N}_q|}\odot\boldsymbol{g}_n^{|\mathcal{N}_q|}\right)\right.\\ &\quad\left.\times\left(\boldsymbol{s}_n^{|\mathcal{N}_q|}\odot\boldsymbol{g}_n^{|\mathcal{N}_q|}\right)^\dagger\right]\end{aligned} \tag{13}$$

$$= \mathbb{E}\left[\left(\boldsymbol{s}_n^1\boldsymbol{s}_n^{1\dagger}\right)\odot\left(\boldsymbol{g}_n^1\boldsymbol{g}_n^{1\dagger}\right), \ldots, \left(\boldsymbol{s}_n^{|\mathcal{N}_q|}\boldsymbol{s}_n^{|\mathcal{N}_q|\dagger}\right)\odot\left(\boldsymbol{g}_n^{|\mathcal{N}_q|}\boldsymbol{g}_n^{|\mathcal{N}_q|\dagger}\right)\right] \tag{14}$$

$$= \left[\mathbb{E}\left[\boldsymbol{s}_n^1\boldsymbol{s}_n^{1\dagger}\right], \ldots, \mathbb{E}\left[\boldsymbol{s}_n^{|\mathcal{N}_q|}\boldsymbol{s}_n^{|\mathcal{N}_q|\dagger}\right]\right] \tag{15}$$

$$= \left[\overline{\varsigma_n^1}, \ldots, \overline{\varsigma_n^{|\mathcal{N}_q|}}\right] \tag{16}$$

Here $\operatorname{diag}(\cdot)$ is the diagonal entry of the matrix in the argument. Equation (15) is derived from (14) after combining with (11). Further, $\overline{\varsigma_n^q} = \left[\overline{\varsigma_n^1}, \ldots, \overline{\varsigma_n^{|\mathcal{N}_q|}}\right]$ is the $1\times|\mathcal{N}_q|$ deterministic vector representing average pathloss coefficient, $\overline{\varsigma_n^{n_q}}$ experienced by users in nth cell transmitting

$$\begin{aligned}\mathbb{E}[R_q(\boldsymbol{A})] \le \frac{1}{B}\sum_{b=1}^{B}&\left[\log\left\{\mathbb{E}\left[\det\left(\sigma_z^2\boldsymbol{I}_{N/Q} + \sum_{n\in\mathcal{N}_b(\mathbf{A})}P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right)\right]\right\}\right.\\ &\left.-\log\left\{\mathbb{E}\left[\det\left(\sigma_z^2\boldsymbol{I}_{N/Q} + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})\cap\overline{\mathcal{N}_q}}P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right)\right]\right\}\right]\end{aligned} \tag{10}$$

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to all APs in qth cluster. Hence, $\overline{\varsigma_n^{n_q}} \triangleq 1/K \sum_{k=1}^{K} \left(\varsigma_{n,k}^{n_q}\right)^2$ $\forall n_q = 1, \ldots, |\mathcal{N}_q|$. Extending the above to the formulation for power and bin allocation, the following can be deduced for asymptotically large number of users

$$\mathrm{diag}\left(\mathbb{E}\left[P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right]\right) = P_n(\boldsymbol{A})\overline{\varsigma_n^q} \tag{17}$$

Recalling (10) and decomposing the RHS log argument, it is known that

$$W_1 = \mathbb{E}\left[\det\left(\sigma_z^2 \boldsymbol{I}_{N/Q} + \sum_{n\in\mathcal{N}_b(\mathbf{A})} P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right)\right] \tag{18}$$

$$W_2 = \mathbb{E}\left[\det\left(\sigma_z^2 \boldsymbol{I}_{N/Q} + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})\cap\overline{\mathcal{N}_q}} P_n(\boldsymbol{A})\boldsymbol{H}_n^q\left(\boldsymbol{H}_n^q\right)^\dagger\right)\right] \tag{19}$$

Plugging (17) in (19) (see (20))

Applying similar concept to argument of second log on RHS of (10), and summing up for all Q clusters, the closed-form sum rate formulation for CC-CMAC, is as follows

$$\begin{aligned}\bar{R}(\boldsymbol{A}) \le \sum_{q=1}^{Q}\Bigg\{\frac{1}{B}\sum_{b=1}^{B}\Bigg[&\log\left(\prod_{n_q\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})} P_n(\boldsymbol{A})\overline{\varsigma_n^{n_q}}\right]\right) \\ &-\log\left(\prod_{n_q\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})\cap\overline{\mathcal{N}_q}} P_n(\boldsymbol{A})\overline{\varsigma_n^{n_q}}\right]\right)\Bigg]\Bigg\}\end{aligned} \tag{21}$$

This concludes the derivation. (see (22))

3.3 Time complexity analysis: simulation against closed form

Algorithmic efficiency is computed using complexity analysis. Fig. 2 shows that the average Monte Carlo-based simulation matches closely to the closed-form representation for the CC-CMAC. This justifies use of closed-form representation in subsequent analysis.

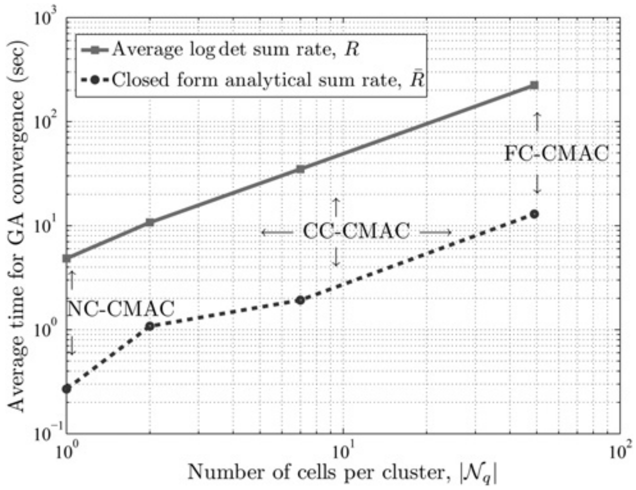

Fig. 2 *Time complexity comparison between $\mathbb{E}\left[R_q(\boldsymbol{A})\right]\forall q$ & $\bar{R}$ (21) as implemented using GA for uplink transmission undergoing flat fading and pathloss in bin-based cellular MAC design. Here $B = 5$, $K = 60$. For log det simulation, D is averaged over 1000 runs. For comparison the time complexity of NC-CMAC ($Q = 49$) and FC-CMAC ($Q = 1$) are also shown*

3.4 Cell-based QoS balancing function

U_γ from (4) depends on u_γ, which is a function of cell-based sum rate, $R_{n,q}(\boldsymbol{A})$. The allocation matrix is a function of N cells and total of B bins. We first derive a closed-form representation of per cluster sum rate from the iterative simulation-based formula. The closed-form expression has the advantage of reduced complexity and signaling cost as compared with the averaged simulation-based sum rate expression. The jointly processed sum rate for qth cluster is then decoded to analyse the sum rate contribution by users in every cell located within qth cluster. This is implemented using minimum mean square estimation based soft interference cancellation (MMSE SIC) framework [21]. It is implemented for CC-CMAC in the following section.

3.4.1 Derivation of closed-form cell-based QoS balancing function: We further perform SIC [21] on the cells within each cluster such that the sum rate of users within a cluster can be evaluated. In order to decode the

$$W_1 = \det\begin{bmatrix} \sigma_z^2 + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})} P_n(\boldsymbol{A})\overline{\varsigma_n^1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \sigma_z^2 + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})} P_n(\boldsymbol{A})\overline{\varsigma_n^{|\mathcal{N}_q|}} \end{bmatrix} \tag{20}$$

$$\begin{aligned}&\bar{R}_{\pi(o),q}(\boldsymbol{A}) = \tilde{\bar{R}}_{\pi(o),q}(\boldsymbol{A}) - \sum_{i=1}^{o-1}\bar{R}_{\pi(i),q}(\boldsymbol{A}) \\ &\text{where} \\ &\tilde{\bar{R}}_{\pi(o),q}(\boldsymbol{A}) = \left[\log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{n\in\mathcal{N}_b(\boldsymbol{A})} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right) - \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{\substack{n\in\mathcal{N}^+_{\pi(o),b} \\ o\in\mathcal{N}_q}} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right)\right]\end{aligned} \tag{22}$$

cell-based ordering from the cluster-based sum rate, we need to first consider closed-form of system wide sum rate of all cells represented by $\bar{R}(\boldsymbol{A})$ (21) from [13].

We consider MMSE-SIC detection [21] on the received signal at qth cluster (i.e. $R_q(\boldsymbol{A})$) with decoding ordered as follows: 1, 2, 3, …, N/Q. For a given bin allocation, b, we first detect the signal from cell 1 treating the signals from all the other cells within the cluster as interference, and then subtract the contribution of this interference from this signal. The detection process is then repeated for cell with index 2 up till index N/Q. Here it is assumed that $|N/Q| = 7$. Defining $\pi(n)$ as the permutation of cells in set $\mathcal{N}_q$ with $\pi(1)$ being the first decoded cell and $\pi(7)$ as the last decoded cell. Generalising for cell decoding order π we have per cell sum rate for oth cell transmitting over bth bin in qth cluster defined as in (22).

Formulating (22), $\bar{R}_{\pi(o),q}(\boldsymbol{A})$ defines the signal from all cluster users transmitting over bin, b. $\tilde{\bar{R}}_{\pi(o),q}(\boldsymbol{A})$ defines the interference from users transmitting in cluster different from the reference cluster as well as signals from users in cells within the same cluster but not decoded. We further decode the per cell sum rate for qth cluster by deriving (23) from (22), where σ_z^2 is defined previously and $\mathcal{N}^+_{\pi(o),b} = \mathcal{N}_b(\boldsymbol{A}) \cap \overline{\mathcal{N}_q} \cup \mathcal{N}^+_{\pi(o)}$. Here the non-decoded cells within the cluster, are represented by the set, $\mathcal{N}^+_{\pi(o)} = \{\pi(o+1), \ldots, \pi(N/Q)\}$.

Lemma 1: For CC-CMAC using MMSE-SIC, the sum rate for oth cell in the qth cluster, has an ergodic sum rate given by

$$\bar{R}_{\pi(o),q}(\boldsymbol{A}) = \begin{cases} f(\mathcal{N}_b(\boldsymbol{A})) - f\left(\mathcal{N}^+_{\pi(1),b}\right) & \text{if } o = \pi(1) \\ f\left(\mathcal{N}^+_{\pi(o-1),b}\right) - f\left(\mathcal{N}^+_{\pi(o),b}\right) & \text{otherwise} \\ \forall o \in \mathcal{N}_q & \end{cases} \tag{23}$$

where, for simplicity it is assumed that

$$f(\mathcal{N}_b(\boldsymbol{A})) = \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{n\in\mathcal{N}_b} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right) \text{ and}$$

$$f\left(\mathcal{N}^+_{\pi(o),b}\right) = \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{\substack{n\in\mathcal{N}^+_{\pi(o),b}\\ o\in\mathcal{N}_q}} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right)$$

and that $1/B\sum_{b=1}^{B}(.)$ can be ignored from RHS of (23) for simplicity.

Proof: Here we attempt to derive (23) from (22). For $o = 1$, and from (22), it is easy to show the sum rate for first decoded cell as

$$\bar{R}_{\pi(1),q}(\boldsymbol{A}) = \tilde{\bar{R}}_{\pi(1),q}(\boldsymbol{A}) = f(\mathcal{N}_b(\boldsymbol{A})) - f\left(\mathcal{N}^+_{\pi(1),b}\right)$$

Similarly, for $o = 2$

$$\begin{aligned}\bar{R}_{\pi(2),q}(\boldsymbol{A}) &= \tilde{\bar{R}}_{\pi(2),q}(\boldsymbol{A}) - \bar{R}_{\pi(1),q}(\boldsymbol{A}) \\ &= \underline{f(\mathcal{N}_b(\boldsymbol{A}))} - f\left(\mathcal{N}^+_{\pi(2),b}\right) \\ &\quad - \left[\underline{f(\mathcal{N}_b(\boldsymbol{A}))} - f\left(\mathcal{N}^+_{\pi(1),b}\right)\right] \\ &= \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{\substack{n\in\mathcal{N}^+_{\pi(1),b}\\ o\in\mathcal{N}_q}} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right) \\ &\quad - \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{\substack{n\in\mathcal{N}^+_{\pi(2),b}\\ o\in\mathcal{N}_q}} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right) \\ &\;\;\vdots\end{aligned} \tag{24}$$

Similarly for $o = N/Q$

$$\begin{aligned}&R_{\pi(N/Q),q}(\boldsymbol{A}) \\ &= \tilde{\bar{R}}_{\pi([N/Q]),q}(\boldsymbol{A}) - \bar{R}_{\pi([N/Q]-1),q}(\boldsymbol{A}) \\ &= \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{\substack{n\in\mathcal{N}^+_{\pi([N/Q]-1),b}\\ o\in\mathcal{N}_q}} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right) \\ &\quad - \log\left(\prod_{m\in\mathcal{N}_q}\left[\sigma_z^2 + \sum_{\substack{n\in\mathcal{N}^+_{\pi([N/Q]),b}\\ o\in\mathcal{N}_q}} P_{n,b}(\boldsymbol{A})\overline{\varsigma_n^m}\right]\right)\end{aligned}$$

Generalising the above results by replacing cell indices with global variable o, one can get second part of (23). This sums up the proof. □

4 NP Hardness for BAP in CC-CMAC

The aim of the BAP in fixed CC-CMAC is to assign bin resources to multiple cells such as to maximise QoS balancing function. In order to select a solution for BAP, it is imperative to determine complexity of the problem. In this section BAP is proven to be NP hard. This hence requires a selection of heuristic technique to solve the problem. In this section, it is assumed that QoS balancing function maximises sum rate of all users. The sum rate maximisation problem is a subset of the general QoS balancing function (which is a function of cell-based sum rate). The well-studied MI-FAP is mapped from literature [22] to the BAP problem as per following definition.

Definition 1: (BAP using multiple bin for CC-CMAC): The achievable sum rate because of transmissions from users in the Q clusters using B bins, and received by the APs in the Q clusters is defined as in (21).

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Theorem 1: Solution to the BAP using multiplebBin for CC-CMAC with $Q \gg 2$ is NP hard.

Proof: This section will highlight important steps which are detailed in [12]. Consider the clustering problem as the communication theoretic analogy of graph partitioning problem. This is defined as the division of total vertices, $\mathcal{V}$ into disjoint sets represented by $\mathcal{V}_q$ such that the number of edges whose end points are in Q–1 different vertices subsets are minimised.

Vertices of graph play role of transmitters in CC-CMAC and edges model point to point link between nodes. Define a cut across sets within and without cluster q and define the conductance of cut as a measure of cluster quality. This is considered analogous to system efficiency of CC-CMAC.

$\mathcal{U}_q$ is the set of transmitters in qth cluster. Since users are colocated with receivers, $|\mathcal{U}_q| = |\mathcal{V}_q|$. These terms are used interchangeably denoting transmission and reception nodes in graph theoretic framework. Sum rate contribution because of transmission from users within qth cluster is denoted by, R_a which is as follows

$$R_a = \log\left(\prod_{v\in\mathcal{V}_q}\left[\sigma_z^2 + \sum_{u\in\mathcal{U}_q} P_u \overline{\varsigma_u^v}\right]\right) \tag{25}$$

where σ_z^2 is the noise variance and P_u is the maximum transmit power for users in u. Similarly, sum rate contribution because of transmission from users outside qth cluster is denoted by, R_b. Hence

$$R_b = \log\left(\prod_{v\in\mathcal{V}_q}\left[\sigma_z^2 + \sum_{u\notin\mathcal{U}_q} P_u \overline{\varsigma_u^v}\right]\right) \tag{26}$$

Both R_a and R_b can be regarded as specific instances of $\bar{R}(\boldsymbol{A})$ (21). These can be evaluated from (21) using MMSE-SIC techniques [21].

4.1 Active receivers in single cluster ($|\mathcal{V}_q| = |\mathcal{N}_q|$)

Consider $\mathcal{W}$ as the set of all edges. $|\mathcal{W}|$ increases linearly with $|\hat{\mathcal{U}}|$. Further denote $\mathcal{W}_q^{\text{intra}}$ and as the set of graph edges with both endpoints lie within qth cluster. Similarly. $\mathcal{W}_q^{\text{inter}}$ is the set of edges with one endpoint in and one out of qth cluster. Further, $\mathcal{W}_q = \mathcal{W}_q^{\text{intra}} \cup \mathcal{W}_q^{\text{inter}}$. Hence, the following mapping function can be formulated

$$\begin{aligned} R_a(\widehat{\mathcal{U}}) &\mapsto |\mathcal{W}_q^{\text{intra}}| \\ R_b(\widehat{\mathcal{U}}) &\mapsto |\mathcal{W}_q^{\text{inter}}| \end{aligned} \tag{27}$$

where $\mapsto$ refers to the mapping between number of interfering edges to the sum rate contribution because of transmission from users within $R_a(\widehat{\mathcal{U}})$ and outside $R_b(\widehat{\mathcal{U}})$ the cluster.

4.2 Active receivers in multiple clusters ($|\mathcal{V}_q| > |\mathcal{N}_q|$)

Assuming identical clusters, and using $\min(|\overline{\mathcal{W}_q^{\text{intra}}}|, |\mathcal{W}_q|) = |\mathcal{W}_q|$.

Also as in [23], take subset $\mathcal{S}$ of $\mathcal{V}$ and define a cut $(\mathcal{S}, \mathcal{V}/\mathcal{S})$. Here, for BAP in CC-CMAC, the cut is represented by $(\mathcal{N}_q, \overline{\mathcal{N}_q})$ where $\mathcal{N}_q \subseteq \mathcal{N}$ and $\mathcal{N}$ is the set of all cells within the system. Applying concept of graph clustering from [23], the following can be deduced

$$\begin{aligned} \phi(\mathcal{N}_q) &= \frac{|\mathcal{W}_q^{\text{inter}}|}{\min(|\overline{\mathcal{W}_q^{\text{intra}}}|, |\mathcal{W}_q|)} \\ &= \frac{|\mathcal{W}_q^{\text{inter}}|}{|\mathcal{W}_q|} \end{aligned} \tag{28}$$

From [23], it is known that conductance of graph cluster, q, that is $\phi(\mathcal{N}_q)$ will be smallest conductance within that cluster. Further the conductance of graph is minimum conductance over all possible clusters, q. Applying to CC-CMAC this would imply a spectral efficiency measure over all clusters Q. Hence

$$\phi(G) = \min_{\mathcal{N}_q \subseteq \mathcal{N};\forall q} \phi(\mathcal{N}_q) \tag{29}$$

$$= \max_{\mathcal{N}_q \subseteq \mathcal{N};\forall q}\left(\frac{1}{\phi(\mathcal{N}_q)}\right) \tag{30}$$

$$\begin{aligned} &= \max_{\forall q}\left(\frac{R_a + R_b}{R_b}\right) \\ &= \max_{\forall q\in\{1,2\}}\left(\frac{\log\left(\prod_{v_1\in\mathcal{V}_q}\left[\sigma_z^2 + \sum_{n\in\mathcal{N}} P_n \overline{\varsigma_n^{v_1}}\right]\right)}{\log\left(\prod_{v_1\in\mathcal{V}_q}\left[\sigma_z^2 + \sum_{n\in\mathcal{N}\cap\overline{\mathcal{N}_q}} P_n \overline{\varsigma_n^{v_1}}\right]\right)}\right) \end{aligned} \tag{31}$$

Equation (30) follows from (29). In terms of rate contribution as a measure of conductance, (30) can be expanded to represent (31).

4.3 Multiple bin allocation for reception at receivers within multiple clusters

In a multiple cluster multiple bin BAP, (31) can be extended from single bin to multiple bins formation, that is from $B = 1$ to $B \gg 1$ and average over a bin. The number of receivers $|\mathcal{V}_q|$ increases for $Q \gg 2$. This suits the requirement of seven clusters and seven cells per cluster formation as implemented for Definition 1. Being a subset of BAP general problem, if (29) is proved NP hard, then the generalised BAP for CC-CMAC is also NP hard. From [22, 23], solving (29) is proved to be NP hard. The proof assumes that given $(\alpha - \epsilon)$ clustering, maximising α or graph conductance for no inter cluster conductance (ϵ) is proven to be NP hard. This is equivalent to saying that given interference from across the cluster is 0, what is the maximum value of $|\mathcal{W}_q^{\text{inter}}|/|\mathcal{W}_q|$. Since numerator and denominator are both dependent on the weighted edge of inter cluster subgraph, the solution to (29) is hence proved to be NP hard. Here, (31) is equivalent to (29). Hence, (31) is also proven as NP hard.

Since BAP for multiple bin allocation in CC-CMAC is a more general case of (31), Definition 1 is also NP hard. QoS balancing function is a function of sum rate, that is depends on (21) and therefore Definition 1. Optimising the network QoS balancing function problem as defined in Section 3.1 is therefore an NP hard problem. □

5 GA-based implementation of QoS balancing function in CC-CMAC

GA is a powerful optimisation tool widely used in solving channel allocation problems including dynamic channel allocation [14, 24] and bin allocation in CMAC [13].

To optimise efficiency of allocation for CC-CMAC users, (6) is used as an objective function to a modified GA. The allocation is optimised over a number of generations. In each generation, the allocation matrix encoded onto a number of chromosome strings is evaluated. The crossover technique is selected such that the crossover point separates allocation for different users. Mutation is implemented by flipping the alleles [25] to any of the 2^V–1 alternate power states. The termination criteria is determined by the number of generations over which the efficiency is near constant. These are summarised in Fig. 3 and detailed as in [12].

In the following section, we introduce the different steps in optimising GA to solve frequency planning for CC-CMAC.

5.1 GA Framework for resource allocation in CC-CMAC

The modification of GA with respect to generic string encoding, using fitness function, crossover technique and mutation takes place as shown in Fig. 3. It is explained in terms of the different stages as well as through the pseudocode as follows.

5.1.1 Fitness function-cell-based QoS balancing function: To optimise efficiency of resource allocation for CC-CMAC users, we consider network utility based on per cell sum rate QoS balancing function as fitness function. Since GA is optimised over a number of generations we consider the least complex form of fitness function to optimise our algorithm.

Using the analytical derivation from Section 3.2, the closed-form representation of $\mathbb{E}[R_q(\boldsymbol{A})]$ over all q is given by (21). Average of per cell sum rate is evaluated using 1000 Monte Carlo-based iterations. These represent random fading and user distribution snaps. A for loop is conventionally used to average over these states. Fig. 2 shows that simulation-based analysis has complexity to the order of n^2. This explains the increasing gradient with increasing cells per cluster, $|\mathcal{N}_q|$ for simulated sum rate in Fig. 2. For the closed-form representation, these loops are no longer needed as the fading states are approximated using the law of large numbers. Further, Fig. 2 shows that the order of complexity increases logarithmically in the order non-cooperative (NC-CMAC), CC-CMAC and full-cooperative (FC-CMAC) cellular MAC. This can be explained because of the reduced search space from

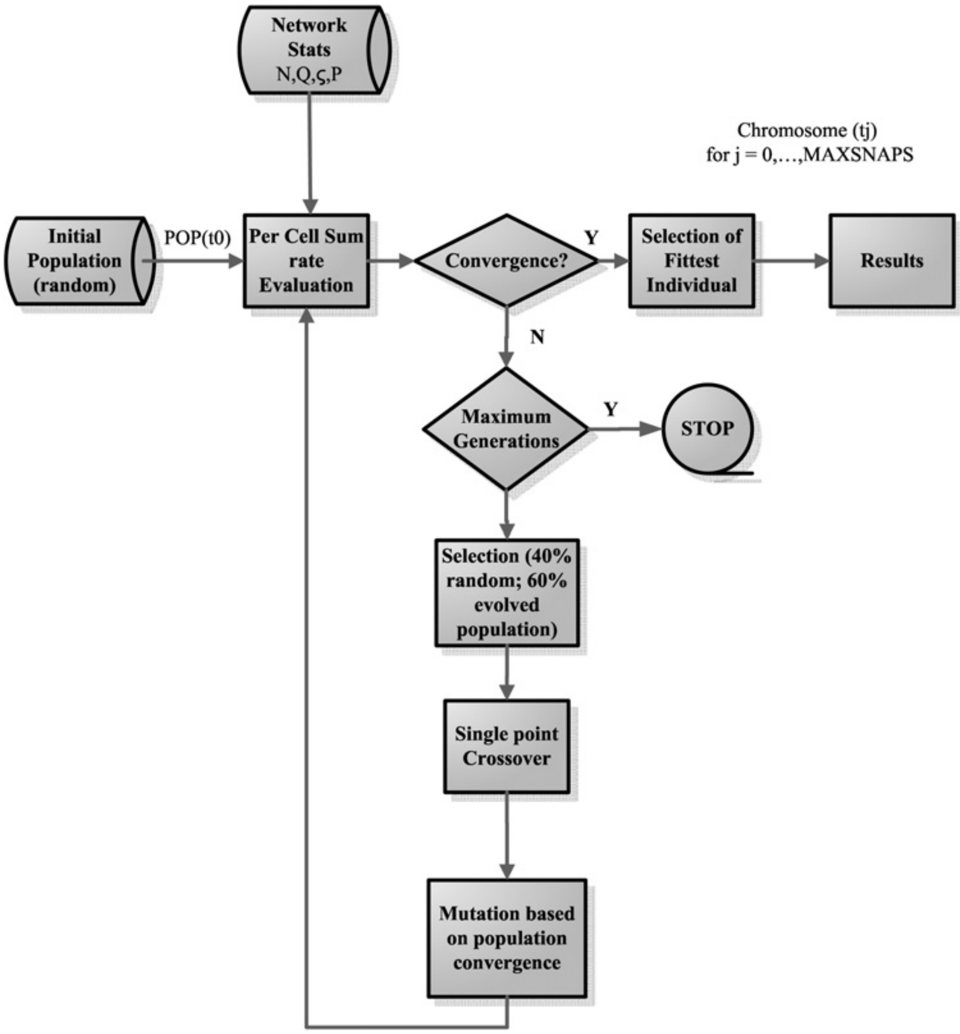

Fig. 3 *GA Flowchart for optimising bin-based allocation in CC-CMAC. POP(t0) represents the population at reference time, t0. MAXSNAPS is the maximum number of time snaps*

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FC-CMAC(2^{BN}) to CC-CMAC($2^{B|\mathcal{N}_q|}$). This gives another motivation for choosing CC-CMAC which can be implemented for a range of practically realisable receivers in an AP cooperation-based cellular system framework.

Here (23) is considered as the fitness function as per our analysis.

5.1.2 Encoding of bin allocation matrix: The method of representation (encoding) for a chromosome has a major impact on GAs performance. Binary string encoding is selected since it helps to select from a large number of possibilities using few trials. Compared with a binary string of 5 a binary string of 10 has 32 times larger schemata. The string length ($B \times N$) represents total number of genes. Each gene represents allocation of a specific bin to a specific cell for CC-CMAC. Further, binary strings were chosen for simplicity of operation [25]. In this work a 0 signifies no allocation whereas a non-zero allele indicates an allocation.

Fig. 4 shows the effect of evolutionary frequency allocation scheme for cell-based QoS balancing function. An initial group of individuals consisting of M chromosomes is first created by mapping the QoS balancing function optimised bin allocation to a chromosome structure.

5.1.3 Block sized crossover: This type of crossover puts restriction on the location of the plane of crossover. This is required since the allocation on either side of the plane should belong to different cells. GA would otherwise descend to premature convergence [12]. This scheme is depicted in Fig. 5. Here Parent 1 and Parent 2 pass their characteristics to Child 1 and Child 2. This is shown by a direct mapping from Parent 1 to Child 1 and Parent 2 to Child 2. Beyond the plane of crossover, genes are swapped for the remaining chromosome such that Child 1 receives Parent 2s genes and vice versa. The above is verified after creating test points in the Matlab simulation. Secondly, content of the chromosome is verified before and after the crossover operation. Moreover, the bins allocated to each user are summed up. This should be the same before and after the crossover operation.

5.1.4 Elitism: Fitness function is a non-negative figure of merit [26] used to quantify the 'best fit' amongst the population. Survival of the fittest translates to discarding the chromosomes which are unfit. In the bin allocation problem (BAP), bins are allocated such that they maximise sum rate. This is done by reshaping encoded matrix from the chromosome string to a bin allocation table. This allocation matrix denotes the input matrix for (21). After GA has reached maximum generations, the encoded matrix is converted back to chromosome strings. As can be observed, the fitness function (9) is a summation of user rates in all Q clusters. In each cluster, the sum rate is of the form $R_q(\boldsymbol{A}) = 1/B\sum_{b=1}^{B} \log\det\{\boldsymbol{I}/\boldsymbol{J}\}$. Here B is constant over a given run and depends on allocation $\boldsymbol{A}$ dimensions. $\boldsymbol{I}$ represents the received signal strength as observed by the receivers in qth cluster, from all transmitters in the system (i.e. inside and outside the qth cluster) and $\boldsymbol{J}$ is received signal strength of the transmissions from outside the cluster q. The set of conditions under which $\boldsymbol{J}$ is minimised and $\boldsymbol{I}$ is maximised, hence determines the 'best fit' solution to the GA under study.

5.1.5 Selection: The concept of elitism was implemented with a modified version of De Jong's elitist model [27]; the best member of current population is forced to become member of the next population. This helps maximise sum rate over all generations. The fitter the parents, the higher the chances that they are selected.

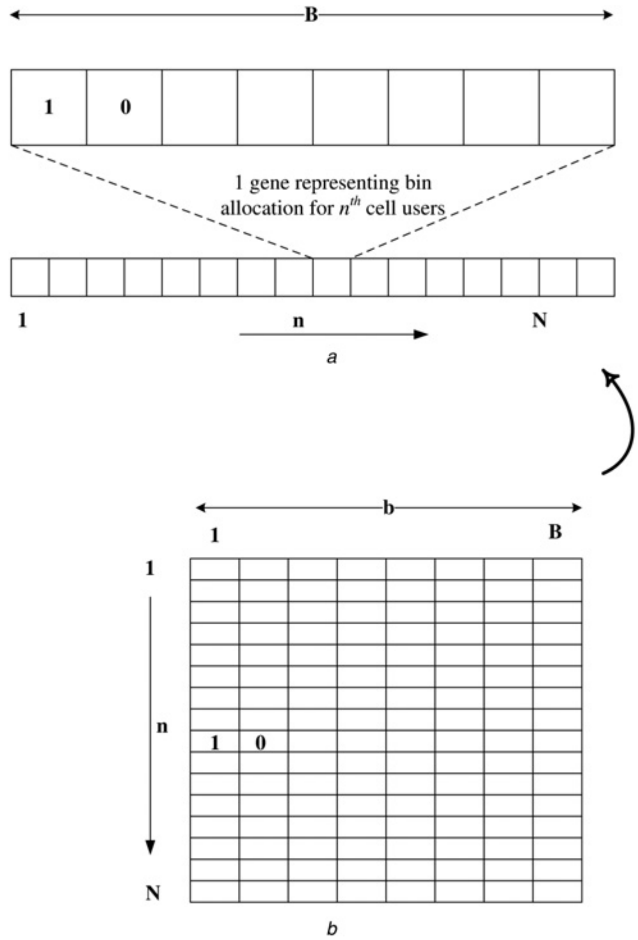

Fig. 4 *Effect of evolutionary frequency allocation scheme for cell-based QoS balancing function*

a Chromosome representation of bins allocated to the *n*th cell as per our GA optimisation
b Bin allocation table, $\boldsymbol{A}$ for all cells and its corresponding bin value, b is shown. $a_{n,b} = 1$ means that the bth bin has been assigned to the nth cell. $a_{n,b} = 0$ implies that no such allocation takes place

5.1.6 Crossover: Crossover is the main genetic operator which preserves inherit characteristics from each parent using a 'cut-catenate' technique. Single point crossover is the simplest of all crossover techniques [24]; hence it is adopted in this analysis. However, Fig. 5 shows that point of crossover cannot bisect bins allocated to a single user (shaded group of 3 bins). If that is the case, it will result in the algorithm converging prematurely.

The sort of single point crossover used depends on restriction on the location of the plane of crossover. This is required since the allocation on either side of the plane should belong to different cells. GA would otherwise descend to premature convergence [12]. This scheme is depicted in Fig. 5. Here Parent 1 and Parent 2 pass their characteristics to Child 1 and Child 2. This is shown by a direct mapping from Parent 1 to Child 1 and Parent 2 to Child 2. Beyond the plane of crossover, genes are swapped for the remaining chromosome such that Child 1 receives Parent 2s genes and vice versa. The above is verified after creating test points in the Matlab simulation. Secondly, content of the chromosome is verified before and after the crossover operation. Moreover, the bins allocated to each

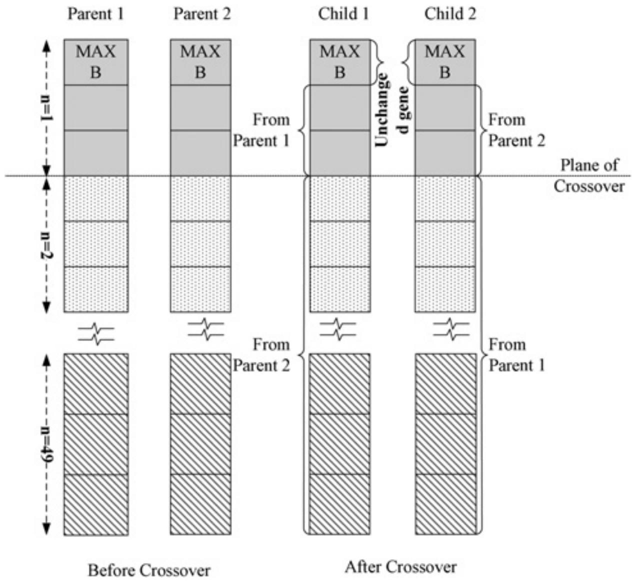

Fig. 5 *Modified GA: block size crossover operation for a CC-CMAC*

```
Input: PopSize; P_m; P_c; Gen
Output: BxN vector (System wide bin allocation)
Generate PopSize random solutions; save in Pop;
while No consistent fitness value in last 10 Gen do
    Select PopU=PopSize/2 (elitism);
    for j=1 to (Pop-PopU)/2 do
        randomly select 2 solutions Xa and Xb from Pop;
        generate Xc and Xd by block sized crossover with prob P_c
        save Xc and Xd to Popl;
    end
    for j=1 to (Pop-PopU)/2 do
        mutate solution Xj with prob P_m × Pop;
        save Xj as Xj';
        if Xj' is unfeasible then
            repeat mutation with Xj';
        end
        Update Xj with Xj' in Popl
    end
    Update Pop = PopU+Popl;
end
return best BxN vector from Pop
```

Fig. 6 *GA-based bin allocation for CC-CMAC*

user are summed up. This should be the same before and after the crossover operation. Crossover takes place with a set probability, $\mathcal{P}_c$. If $\mathcal{P}_c = 0$, then the new chromosome population is a copy of the old. If $\mathcal{P}_c = 1.0$ all the offspring are made by crossover. In our analysis, $\mathcal{P}_c$ is in the range of 0.75–0.90.

5.1.7 Variable mutation: Mutation is a genetic operator which transforms individual chromosomes by randomly changing allele (inverting bit positions) of some genes. The operation is carried out on the allocated bins and varied as per sum rate gradient in order to respond the random nature of user positioning. Mutation takes place with a very low bit probability, to prevent the GA from becoming a random search operation. Probability of mutation on a bit, $\mathcal{P}_m$ is hence in the range 0.0001–0.01.

5.1.8 Termination condition: The termination condition specifies whether the algorithm needs to continue searching or stop. When no further bin allocation maximises sum rate and the population has converged, the GA terminates. In this implementation, similar fitness values over consecutive generation indices satisfies termination condition. The GA will terminate if the fitness value is consistent for the last ten consecutive generations (see Fig. 6).

5.2 Effect of fairness on QoS balancing function

The max–min fairness is a tractable and flexible fairness model that helps to compare a range of fairness conditions. It is known that rate region for (23) is not convex in general. Since class of utilities depending on per cell sum rate should have a convex formulation as their input, further conditions are required to optimise the fairness formulation using cell-based sum rate.

Using function $h(x)$ of the form $(e^x – 1)^{-1}$ satisfies the conditions for maximising minimum sum rate contribution of users in a cell such that any further increase will likely to decrease sum rate allocated to higher rate cell users [12].

Lemma: If h(x) is differentiable increasing negative & concave function than given $x \geq 0$, the solution of $U_\gamma(h(x))$ approaches max–min fair vector for $\gamma \to 10$.

From definition of log-concave, we know that a function is log-concave if log f is concave. We know that cumulative Gaussian probability function is log-concave. Applying the same to the fairness formulation introduced, it can be shown that [12]

$$u_\gamma\Big(R_{m,q}(\boldsymbol{A})\Big) = \frac{-\Big[h\Big(R_{m,q}(\boldsymbol{A})\Big)\Big]^{-\gamma}}{\gamma} \max - \min F \qquad (32)$$

Here, $u_\gamma(R_{m,\,q}(\boldsymbol{A}))$ is the service balancing function for users in mth cell in qth cluster using fairness coefficient of γ. Simplifying, the following can be deduced

$$u_\gamma\Big(R_{m,q}(\boldsymbol{A})\Big) = \frac{1}{\gamma}\left[\frac{-1}{(\mathrm{e}^{(R_{m,q}(\boldsymbol{A}))} - 1)}\right]^{\gamma} \max - \min F \qquad (33)$$

Based on the above, it is imperative to compare fairness with the system efficiency. Having a fair distribution of resources will reduce the numerator for R. However, the sum rate contribution of edge cells will increase since they now have more resources allocated to them. Since the number of edge cells (6) is far greater than the non-edge cells (1), there is an increase in interference to receivers in adjacent clusters. This reduces the over all sum rate further. However, the CDF determines in two stages the effect of fairness and improvement of group performance (cell users) within a cluster.

- The 10th percentile is used to gauge performance of users in cell edge.
- The 90th percentile is used to gauge performance of users in cell centre.

Since the fairness described is strict and effects the edge cell users most, 10th percentile is generally regarded as the determining metric for qualifying edge cell users since their sum rate contribution is the least [12].

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5.3 GA simulation

A total of 16 simulations were carried out. The total number of users K, transmitting in each cell was modified such that:

- Four sets of experiment were carried out using varying population size, M for $K > 100$
- Four sets of experiment with different crossovers ($M^{-1} \leq \mathcal{P}_c \leq M$), $K > 100$
- Four sets of experiment were carried out with varying M, $K < 100$
- Four sets of experiment with different crossovers ($M^{-1} \leq \mathcal{P}_c \leq M$), $K < 100$

The above parameters were optimised by tuning to gain stability within the experiment (consistency in values over the last ten generations), and the efficiency achieved (difference in sum rates between first and last completed generation cycle). A further set of eight experiments over similar sets as above but reduced iteration. These experiments were carried out to compare two forms of fairness constraints as available in literature including Jains fairness index (JFI) and max–min fairness index (MMF). The results are discussed in Section 6.

To sum up, the bin allocation is optimised over a number of generations. 100 chromosomes are randomly generated. For each set of ten non-consistent generation blocks, in each generation, the allocation matrix is encoded onto a number of chromosome strings and each string is evaluated against an objective criteria (optimisation of cell-based QoS balancing function). The crossover technique is selected such that the crossover point separates allocation for different users. Mutation is implemented by flipping the alleles [25] to any of the 2^V–1 alternate power states. Here V is the total number of bits encoded for bin allocation to every user. The termination criteria is determined by the number of generations over which the efficiency is near constant. The process is summarised in Algorithm 1.

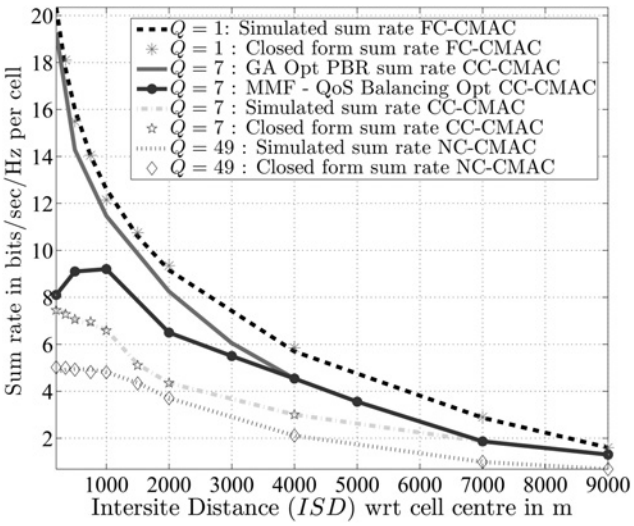

Fig. 7 *Effect of ISD on per cell sum rate because of GA optimised partial bin reuse (PBR) and max–min fairness via joint power and bin allocation. Here $V = 5$, $B = 10$, AP density is varied from ISD = 200 to 9000 m, $N = 49$, $Q = 7$, $\eta = 3.5$, $L_o = 31.5$ dB, and $\sigma_o^2 = 16.9$ dBm/Hz over 5 MHz bandwidth. For comparison, full reuse as implemented on NC-CMAC ($Q = 49$) and FC-CMAC ($Q = 1$) are also shown*

6 Results and discussion

Fig. 7 shows the effect of ISD gain on V and B using sum rate as metric GC-CMAC. For high density APs, the efficiency of GA optimised allocation in CC-CMAC approaches the upper bound. For less dense systems, the difference between GA optimised and full reuse allocation schemes reduces to nearly 0. The given parameters make GA suitable for dense urban centres. GA option is compared with fairness based metric (min-max algorithm). This is explored in the next section.

6.1 Impact of fairness on QoS: max–min fairness

For $D = 1000$ m, sum rate contribution because of hard fairness optimised QoS balancing function is shown in Fig. 7. Corresponding CDF plot is shown in Fig. 9. For nominally dense APs, the sum rate is reduced by about 20%. However, the minimum rate of disadvantaged users is increased by 10%. At the 10th-percentile reference, sum rate for MMF optimised allocation is 5.1 as compared with 4.7 for maximised sum rate. Hence, 10% users have rates which are at least 0.5 bps/Hz/cell higher than that due to QoS balancing function without fairness. This gap is further increased if the cross over points are varied along with the population size.

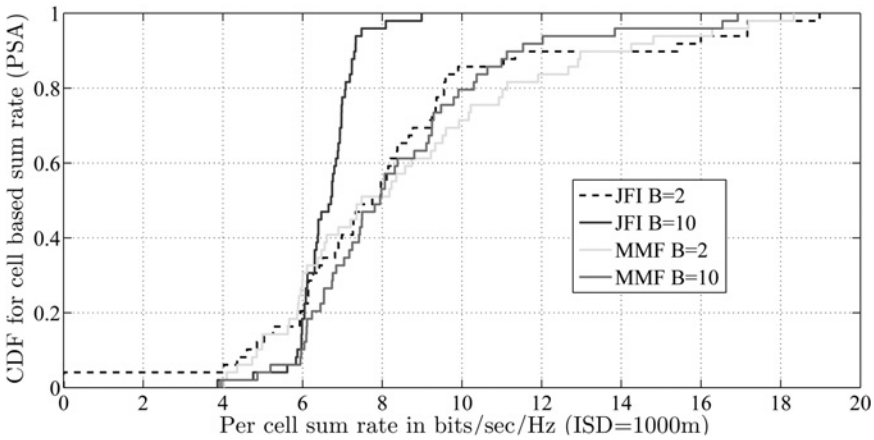

Fig. 8 *Comparison between JFI and MMF. The per cell sum rate degrades for JFI's lower fairness coefficient and smaller number of bins. The MMF is said to exhibit better performance as its slope about median is simply a function of per cell sum rate when bin granularity is increased. Here for $B \geq 2$ MMF fares better than JFI which is a common metric for measuring fairness*

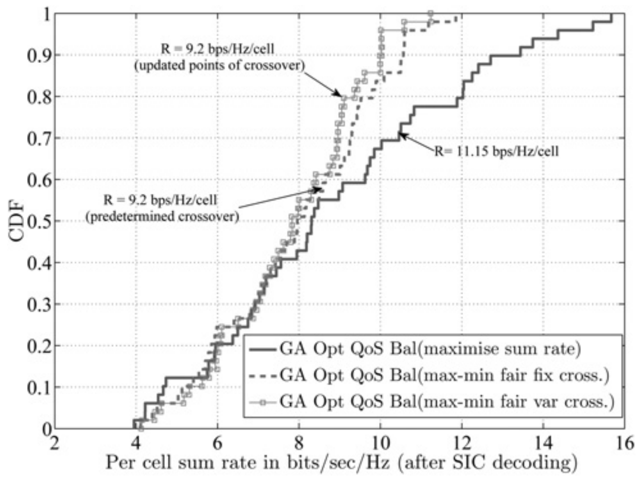

Fig. 9 *Effect of γ on CDF of joint power and bin allocation. Here $V=5$, $B=10$, $ISD=1000$, $N=49$, $Q=7$, $\eta=3.5$, $L_o=31.5$ dB, and $\sigma_o^2=16.9\,dBm/Hz$ over 5 MHz bandwidth. For comparison the effect of block sized crosssover (both predetermined and variable) are shown*

That is the cross over points are not predetermined. This effect is shown in Fig. 9 and discussed in next paragraph.

In lower percentile CDFs, lower sum rate contributors are less prone to interference from other low sum rate contributing cells in alternate clusters. This is not the same for higher cell sum rate contributors which are more prone to interference. Using variable crossover points, the resource allocation becomes more flexible and scant resources are allocated to the very few users. This improves the situation as depicted for 10th percentile users. The increasing sum rate contribution leads to interference which overshadows the advantage in flexible bin allocation because of variable crossover points. Hence, the CDFs converge for percentiles greater than the median.

A similar set of experiments were run as described earlier to compare fairness indices and granularity. Fig. 8 shows that at ISD of 1000 m and for limited value of B implementing MMF for higher coefficient results in greater contribution of per cell sum rate from 10th percentile users than JFI using the same fairness coefficient (Fig. 8). The median for MMF is also more in line with the average per cell sum rate which is not affected by changing the degree of fairness ($\eta=2$–10). This explains the choice of

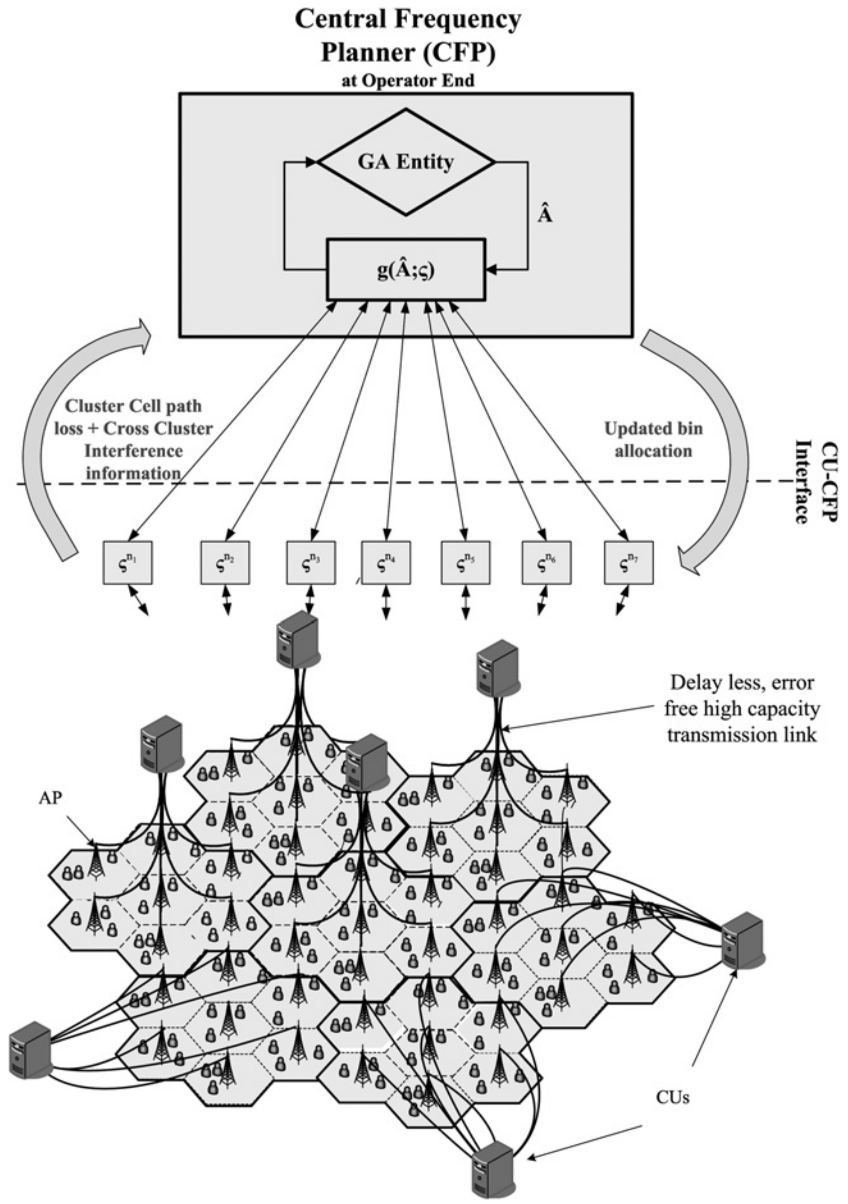

Fig. 10 *GA architecture as implemented for QoS balancing formulation for a CC-CMAC*

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MMF as fairness metric in the final evaluation of QoS balancing function Fig. 8.

7 Analysis of GA optimised allocation design

As shown in Fig. 10, GA-based network QoS balancing function is represented by $g(\cdot)$. Here for first run, $\widehat{\boldsymbol{A}}$ consists of randomly generated bin allocation matrix, $\boldsymbol{A}$. Subsequently, once the termination condition is met, the GA optimisation entity assigns $\widehat{\boldsymbol{A}}$ with GA optimised partial bin reuse type allocation. The cell path loss information is in the form of $\overline{\varsigma_1^q}$ for the transmission from cell indexed by 1 and whose signal is received at the AP in the qth cell. This information can be represented with minimum algorithmic complexity with the help of closed-form framework of bin-based allocation in CC-CMAC. This is explored in Section 5.1.1. The filtering function is represented by $g(\cdot)$ which takes $\widehat{\boldsymbol{A}}$ and $\overline{\varsigma_n^q}$ as inputs. It assigns the updated bin allocation $\widehat{\boldsymbol{A}}$ to the network. Further, the updated $\overline{\varsigma_n^q}$ is sent to central frequency planner (CFP). Here for the first run, CFP processes $\widehat{\boldsymbol{A}}$ which consists of randomly generated bin allocation matrix, $\boldsymbol{A}$. Subsequently, the network entity assigns the fittest solution to $\widehat{\boldsymbol{A}}$ fedback to the cell users. These entries are the result of heuristic solution framework which are discussed in Section 5. In this context, $\overline{\varsigma_n^q}$ is collected every hour so as to represent the changing dynamics of user profiles. This has applications in Collingwood circle design, and random non-homogeneous user traffic profile as described in [12].

7.1 Signalling analysis

Dimensions of channel matrices reveal complexity requirement for cell path loss feedback. This is used to measure the signalling overhead for different cellular MAC. For FC-CMAC, average cell-based path loss information from each of N user groups to the N APs in system are represented by the N^2 bits for transferring 1 bit per cell. Owing to power granularity each cell information now requires V bits for transmission. Hence, the total signalling requirement is VN^2 bits for FC-CMAC. It is worth noting that $N = Q|\mathcal{N}_q|$ for any cellular MAC. For CC-CMAC, the signalling requirement is for $VQ|\mathcal{N}_q|^2$ bits per snapshot. CC-CMAC requires Q times less signalling overhead. This is one of the motivations for choosing CC-CMAC.

7.2 Complexity

The search space for joint power and bin allocation depends on B, V, and N as per the following relationship 2^{NVB}. Hence, for fixed N and V, the complexity rises to the order of 2^B. The complexity is hence a function of the chromosome length (total bins allocated). As a motivation to reduced chromosome length, it is possible to reduce B whereas at the same time increase V such as not to affect the increase in sum rate. This approach makes the algorithm less computationally intensive and GA becomes feasible for modern OMC-Rs and centralised frequency planners as in Fig. 10.

8 Conclusions

A novel QoS balancing framework for CC-CMAC is proposed in this work. The aim is to increase spectral efficiency as well as achieve cell-based fairness for given per user power constraint. This is implemented by deriving cell-based QoS balancing function. Using analogy with graph conductance, QoS balancing problem is proved to be NP hard. Using joint frequency and power granularity, this problem is formulated as an objective function input to a GA and compared for a range of ISDs and bin and power quantisation states. Results show that maximising sum rate CC-CMAC can help achieve the upper bound of the capacity region in highly dense AP scenario. Using max–min fairness in moderately dense AP scenarios, reduced sum rate (improvement over full reuse), can effect a slight increase in fairness. A practical centralised frequency planner for implementing modified GA is proposed. This framework has application for both high priority (HP) and best effort (BE) customers [28] alike. Current research work under way looks into tapered cellular architectures and extension to comply with further restriction as per a multi-objective optimisation design.

9 References

1 Wyner, A.D.: 'Shannon-theoretic approach to a Gaussian cellular multiple-access channel', *IEEE Trans. Inf. Theory*, 1994, **40**, (6), pp. 1713–1727
2 Somekh, O., Shamai, S.: 'Shannon-theoretic approach to a Gaussian cellular multiple-access channel with fading', *IEEE Trans. Inf. Theory*, 2000, **46**, (4), pp. 1401–1425
3 Letzepis, N., Grant, A.: 'Information capacity of multiple spot beam satellite channels'. Proc. Sixth Australian Communications Theory Workshop, 2005, 2–4 February 2005, pp. 168–174
4 Marsch, P., Fettweis, G.: 'A framework for optimizing the uplink performance of distributed antenna systems under a constrained backhaul'. Proc. IEEE Int. Conf. Communications (ICC'07), June 24–28, 2007, pp. 975–979
5 Aktas, E., Evans, J., Hanly, S.: 'Distributed decoding in a cellular multiple-access channel', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (1), pp. 241–250
6 Choi, W., Andrews, J.G.: 'Downlink performance and capacity of distributed antenna systems in a multicell environment', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (1), pp. 69–73
7 Somekh, O., Zaidel, B.M., Shamai, S.: 'Sum rate characterization of joint multiple cell-site processing', *IEEE Trans. Inf. Theory*, 2007, **53**, (12), pp. 4473–4497
8 Levy, N., Shamai, S.: 'Clustered local decoding for Wyner-type cellular models', *IEEE Trans. Inf. Theory*, 2009, **55**, (11), pp. 4967–4985
9 Shen, Z., Andrews, J.G., Evans, B.L.: 'Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints', *IEEE Trans. Wirel. Commun.*, 2005, **4**, (6), pp. 2726–2737
10 Papadogiannis, A., Gesbert, D., Hardouin, E.: 'A dynamic clustering approach in wireless networks with multi-cell cooperative processing'. Proc. IEEE Int. Conf. Communications (ICC'08), May 19–23 2008, pp. 4033–4037
11 Majid, M.I., Imran, M.A., Hoshyar, R.: 'Cell based fair resource allocation in fixed clustered cellular systems using a genetic algorithm'. Proc. IEEE 21st Int. Symp. Personal, Indoor and Mobile Radio Communications, (PIMRC 2010), 2010
12 Majid, M.I.: 'Frequency planning for clustered jointly processed cellular MAC'. PhD thesis, University of Surrey, Guildford, UK, 2010
13 Majid, M.I., Imran, M.A., Hoshyar, R.: 'Optimization of uplink sum-rate for bin based clustered cellular system using a genetic algorithm'. Proc. Sixth Int. Wireless Communications and Mobile Computing Conf, (IWCMC'10), 2010, pp. 1016–1020
14 Song, G., Li, Y.: 'Cross-layer optimization for OFDM wireless networks-part I: theoretical framework', *IEEE Trans. Wirel. Commun.*, 2005, **4**, (2), pp. 614–624
15 Low, S.H., Lapsley, D.E.: 'Optimization flow control. Basic algorithm and convergence', *IEEE/ACM Trans. Netw.*, 1999, **7**, (6), pp. 861–874
16 Cheng, H.T., Zhuang, W.: 'An optimization framework for balancing throughput and fairness in wireless networks with QoS support', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (2), pp. 584–593
17 Palomar, D.P., Chiang, M.: 'A tutorial on decomposition methods for network utility maximization', *IEEE J. Sel. Areas Commun.*, 2006, **24**, (8), pp. 1439–1451

18 Chiang, M., Low, S.H., Calderbank, A.R., Doyle, J.C.: ‘Layering as optimization decomposition: a mathematical theory of network architectures’, *Proc. IEEE*, 2007, **95**, (1), pp. 255–312
19 Gesbert, D., Kiani, S.G., Gjendemsj, A., Oien, G.E.: ‘Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks’, *Proc. IEEE*, 2007, **95**, (12), pp. 2393–2409
20 Cover, T.M., Thomas, J.A.: ‘Elements of information theory’ (John Wiley & Sons, 2006, 2nd ed.)
21 Tse, D., Viswanath, P.: ‘Fundamentals of wireless communications’ (University Press, Cambridge, 2005)
22 Colombo, G., Allen, S.: ‘A decomposed approach for the minimum interference frequency assignment’, *Linkage in Evol. Comput.*, 2008, **157**, pp. 389–417
23 Kannan, Vempala, S., Vetta, A.: ‘On clusterings: good, bad and spectral’, *J. ACM*, 2004, **51**, (3), pp. 497–515
24 Thilakawardana, D., Moessner, K., Tafazolli, R.: ‘Darwinian approach for dynamic spectrum allocation in next generation systems’, *IET Commun.*, 2008, **2**, (6), pp. 827–836
25 Goldberg, D.E.: ‘Genetic algorithms in search, optimization and machine learning’ (Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA, 1989)
26 Mitchell, M.: ‘An introduction to genetic algrithms’ (MIT Press, 1998)
27 Man, K.T.K.F., Kwong, S.: ‘Genetic Algorithms’ (Springer-Verlag, London, 2001)
28 Bashar, S., Ding, Z.: ‘Admission control and resource allocation in a heterogeneous OFDMA wireless network’, *IEEE Trans. Wirel. Commun.*, 2009, **8**, (8), pp. 4200–4210

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