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# Low-complexity *ad-hoc* non-linearities for blind multiuser detection of long-code code-division multiple access signals and asymptotic performance evaluation

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**Abstract:** In this study, monomials with odd integer powers are proposed for blind multiuser detection of code-division multiple access signals with long spreading codes, in the near–far situation. The performance of the new non-linear detectors are substantiated asymptotically and confirmed via simulations. It is demonstrated that the proposed non-linear detectors significantly outperform the matched filter detector in the block fading and Ricean fading channels, at the cost of a small increase in computational complexity. The optimum integer power value for the block fading channel with a limited interferer level, the Ricean channel with a given $K$ factor and asynchronous channel is determined. The proposed multiuser detectors are blind in the sense that they require neither training nor the spreading code of the interferers. The detectors also do not require long convergence time for decision making in contrast to conventional blind multiuser detectors.

## 1 Introduction

In a multiuser code-division multiple access (CDMA) system, the bit error rate performance of the conventional matched filter (MF) degrades severely in the near–far situation. Consequently, various multiuser detection schemes have been developed in the last two decades which are near–far resistant [1–5]. The optimal multiuser detector is developed in [6], which is a maximum-likelihood sequence detector. It is well known that the computational complexity of this detector grows exponentially with the number of active users [7]. Hence, the optimal multiuser detector is too complex for practical systems with even a moderate number of users. To address the complexity problem, a variety of suboptimal multiuser detectors have been proposed [1]. Linear multiuser detectors, such as decorrelator and minimum-mean-square error (MMSE) detector, are two important suboptimal methods that are investigated precisely in the literature [1, 8]. Decorrelating detector is near–far resistant [1]; however, it causes noise enhancement when the correlation matrix of codes is badly conditioned [3]. MMSE detector takes the background noise as well as the correlation between users into account to resolve the noise enhancement problem of the decorrelating detector [8]. Furthermore, MMSE multiuser detector can be implemented using adaptive algorithms to avoid matrix inversion which can be prohibitively complex when the number of users is large [9]. Non-blind adaptive multiuser detectors require fresh training sequences every time that a strong interferer enters into the system. This causes low bandwidth efficiency of the system [5], and consequently, a lot of blind multiuser detection methods are proposed; see [3, 4] for a comprehensive study of these methods. However, the blind implementation of the previously proposed adaptive methods has become ineffective [10], because practical systems such as IS-95 use long spreading codes and long spreading codes degrade the bit-interval cyclostationarity property of the CDMA signals [10]. Consequently, in all practical CDMA systems such as IS-95, the conventional MF detector is employed [11]. The use of long codes guarantees that all the users achieve the same performance, thus avoiding the unpleasant situation that there exist some preferred users [12]. For these reasons, signal processing techniques for long-code CDMA systems have received considerable attention [9–13]. In [9], a multiuser detection technique based on parallel interference cancellation (PIC) is proposed which acts on the long-code CDMA signals transmitted over an additive white Gaussian channel. The complexity of this technique is linear with the number of users and is independent of the system processing gain (PG) [9]. The channel estimation and multiuser detection schemes for long-code CDMA systems operating over a frequency-selective fading channel are designed in [11, 12]. The multiuser detection method in [12] has a computational complexity quadratic in PG, whereas the complexity of the algorithm in [11] is quadratic with the number of users. Both methods in [11, 12] rely on the transmission of a known training sequence [12]. A low-complexity blind channel estimation and symbol detection for long-code CDMA systems is proposed in [13], which acts on a slot of received symbols. It is demonstrated that for a large slot length, the complexity of the iteration of this algorithm is linear in the PG and linear in the number of active users in the CDMA system [13]. The scheme in [13] requires the knowledge of the codes of all active users which does not exist in the recently proposed decentralised CDMA systems [14].

In this paper, we propose low-complexity non-linear multiuser detectors for long-code CDMA systems which are blind in the sense that they require neither training nor the signatures of the interferers [5]. In fact, these detectors require only the desired user's signature and its timing, that is, the required knowledge in conventional MF [1, 5]. The computational complexity of the proposed multiuser detection method is a little more than that of the MF in a frequency-flat fading channel. Another advantage of the proposed scheme is that it does not require long convergence time and makes acceptable decisions from the first bit of the received signal, in contrast to the typical blind multiuser detectors which require long convergence time to yield appropriate performance [5].

The outline of the paper is as follows: After problem formulation in Section 2, we derive the maximum likelihood (ML) detector in a frequency-flat block fading [15–17] channel to compare the performance of the proposed detectors with that of the ML detector. Frequency-flat block fading channel [15, 16], also known as quasi-static fading channel [17, 18], is the first-order approximation to a continuously time-varying channel [16], which is a typical model for wireless communication systems with high speed data transmission or with slowly moving terminals [17]. In this model, the channel is assumed to stay constant over $N$ samples and independently changes to a new value for the next block of samples [15–18]. In Section 3, we derive the new detectors with *ad-hoc* non-linearities [19] which outperform the conventional MF detector and are robust to the time-varying parameters of the channel [19]. Then, we investigate the performance of the obtained non-linear detectors through asymptotic relative efficiency (ARE) [19–23] calculation of the proposed detectors. In Section 4, it is shown that probability of error evaluation of the proposed non-linear scheme is a cumbersome task that justifies the ARE calculation instead of probability of error evaluation. Section 5 presents the computational complexity of the proposed detectors. In Section 6, we prove that the proposed scheme also works in the Ricean fading and asynchronous channels and we obtain the optimal degree of the monomials for these cases. We present numerical results in Section 7. Finally, concluding remarks are presented in Section 8.

## 2 Problem formulation and ML detector design for block fading synchronous channel

We consider the output of the desired spread spectrum (SS) signal demodulator [2, p. 768] for one-bit duration, in the presence of a synchronous interferer. As in [10], we assume that the receiver knows the chip timing and signature of the desired signal. After the chip matched filtering of the received SS signal and sampling at the chip rate [2, p. 768, 10], the demodulator output for one-bit duration can be expressed as

$$y_k = s_k + i_k + v_k, \quad k = 1, 2, \ldots, N \tag{1}$$

where $s_k$, $i_k$ and $v_k$ denote samples of the desired SS signal, co-channel interference and the zero mean white Gaussian noise at the output of the demodulator, respectively. $N$ denotes the number of chips per every information bit, that is, the processing gain (PG) of the SS system. We assume that both the desired signal and interferer have binary phase shift keying (BPSK) modulation. In a frequency-flat block fading channel, channel gains remain constant for $N$ samples, and independently changes for the next block of $N$ samples [15–18]. Hence, we have $s_k = \pm A$ and $i_k = \pm I$ for a block of length $N$.

Since the long pseudo noise (PN) sequences of the users are truly random [10], $i_k$s are independent and identically distributed (IID) [10]. The sequences $s_k$, $v_k$ and $i_k$ are assumed to be mutually independent. Hence, the binary detection of the desired SS signal in the presence of a cochannel interference leads to the following binary hypothesis testing problem

$$\begin{aligned} H_1&: \quad y_k = A + w_k \\ H_0&: \quad y_k = -A + w_k, \quad k = 1, 2, \ldots, N \end{aligned} \tag{2}$$

where the conglomerate effect of additive noise and interference, that is, $v_k$ and $i_k$, is modelled in the observation noise $W$ with the definition $w_k = i_k + v_k$. Since $i_k$s and $v_k$s are independent from each other, the probability density function (PDF) of their sum is the convolution of their PDFs. Since the PDF of $i_k$ is $0.5[\delta(i_k - I) + \delta(i_k + I)]$, the PDF of $W$ is

$$f_{\mathrm{W}}(w_k) = \tfrac{1}{2}[\mathcal{N}_{\sigma^2}(w_k - I) + \mathcal{N}_{\sigma^2}(w_k + I)] \tag{3}$$

where $\mathcal{N}_{\sigma^2}(x)$ is defined as $\exp(-x^2/2\sigma^2)/\sqrt{2\pi\sigma^2}$ and $\sigma^2$ is the variance of the Gaussian component.

When the PDF of observation noise is known, it is possible to obtain ML detector for the problem in (2). Since the noise samples are IID variates, the test statistics for ML detector is

$$T_{\mathrm{ML}}(\boldsymbol{y}) = \sum_{k=1}^{N} \log \frac{f_{\mathrm{Y}}(y_k|H_1)}{f_{\mathrm{Y}}(y_k|H_0)} = \sum_{k=1}^{N} \log \frac{f_{\mathrm{W}}(y_k - A)}{f_{\mathrm{W}}(y_k + A)} \tag{4}$$

where $\boldsymbol{y} = (y_1, \ldots, y_N)$ and $\log(\cdot)$ denotes the natural logarithm function. We define $g_{\mathrm{ML}}(y) = \log[f_{\mathrm{w}}(y - A)/f_{\mathrm{w}}(y + A)]$ which expresses the required non-linearity for the ML detector. Using (4) and after some simplifications, we obtain the ML decision rule as follows

$$T_{\mathrm{ML}}(\boldsymbol{y}) = \sum_{k=1}^{N} \left( \frac{2A}{\sigma^2} y_k + \log\left[ \frac{\cosh\left(I(y_k - A)/\sigma^2\right)}{\cosh\left(I(y_k + A)/\sigma^2\right)} \right] \right) \underset{H_0}{\overset{H_1}{\gtrless}} 0 \tag{5}$$

where the threshold for decision making is determined by using the minimum probability of error criterion [24, p. 77]. In fact, since $H_0$ and $H_1$ are equally probable, the threshold is $\gamma = \log[P(H_0)/P(H_1)] = 0$ [24, p. 77].

## 3 Suboptimal detection scheme with *ad-hoc* non-linearity and performance analysis

It has been shown that reasonably configured *ad-hoc* non-linearities can be used to obtain suboptimal detectors in the presence of non-Gaussian noises [19]. These detectors have very low complexity and are robust to time-varying parameters of the channel [19]. We obtain such a non-linearity for the detection problem (2) in the presence of the noise with PDF in (3). The degree of non-Gaussianity of a random variable (RV) is measured by its Kurtosis relative to Gaussian variate, which is defined as $\gamma = E(w_k^4)/E^2(w_k^2) - 3$, [24, p. 382], where $E(\cdot)$ denotes the expectation operation. This parameter for the noise in (3) is $\gamma = -2I^4/(\sigma^2 + I^2)^2$, which is always negative, and consequently, the PDF for the noise in (3) has tails that fall off more quickly than the Gaussian PDF [24, p. 382]. Hence, the *ad-hoc* limiters in [19], which are proposed for heavy tail noises, are not applicable for detection in the noise with PDF in (3). To obtain our *ad-hoc* non-linear detectors, we consider that the noise PDF in (3) is even symmetric, and consequently, the non-linearity is an odd function. This is because

$$\begin{aligned} g_{\text{ML}}(-y) &= \log \frac{f_{\text{W}}(-y-A)}{f_{\text{W}}(-y+A)} \\ &= \log \frac{f_{\text{W}}(y+A)}{f_{\text{W}}(y-A)} = -g_{\text{ML}}(y) \end{aligned} \tag{6}$$

Hence, we propose the simplest odd hyperactive polynomial (that acts opposite to limiters), that is, $g(y) = y^m$ with odd $m$ values, as the suboptimal *ad-hoc* detector [25, 26]. Hence, the test statistics for the proposed detection scheme is

$$T_{\text{Suboptimal}}(\mathbf{y}) = \sum_{k=1}^{N} y_k^m \underset{H_0}{\overset{H_1}{\gtrless}} 0 \tag{7}$$

where the threshold for decision making is as in (5).

Now, we calculate the performance of this detector. We use ARE with respect to some reference detector as in [19–23]. ARE calculation is a succinct approach to compare the performance of two detectors and it is defined as $\text{ARE}_{2,1} = \lim_{A\to 0} \frac{N_1(P_e,\ A)}{N_2(P_e,\ A)}$, when $N_1 \to \infty$, $N_2 \to \infty$. $N_i(P_e, A)$ denotes the number of samples detector $i$ requires to achieve a given probability of error ($P_e$) with signal strength $A$. The expression for the ARE of a non-linear detector for a constant signal with respect to the linear detectors is obtained in [22], which is

$$\text{ARE}_{nd,ld} = \frac{\int_{-\infty}^{\infty} x^2 f_{\text{W}}(x)\mathrm{d}x \left[\int_{-\infty}^{\infty} g'(x) f_{\text{W}}(x)\mathrm{d}x\right]^2}{\int_{-\infty}^{\infty} g^2(x) f_{\text{W}}(x)\mathrm{d}x - \left[\int_{-\infty}^{\infty} g(x) f_{\text{W}}(x)\mathrm{d}x\right]^2} \tag{8}$$

For the above proposed non-linearity, $g(y) = y^m$ where $m$ is odd, we obtain a simple expression for ARE using (8) as

$$\text{ARE}_{nd,ld}(m,\ \sigma,\ I) = \frac{m^2 E(w_k^2)\left[E(w_k^{m-1})\right]^2}{E(w_k^{2m})} \tag{9}$$

The detector with non-linearity $g(y) = y^m$ has better performance than conventional MF when $\text{ARE}_{nd,ld}(m, \sigma, I) > 1$. Hence, for any noise whose PDF exhibits the even symmetry property and satisfies the inequality

$$m^2 E(w_k^2)\left[E(w_k^{m-1})\right]^2 > E(w_k^{2m}) \tag{10}$$

among its moments, the proposed detector in (7) has better performance with respect to the MF. Computation of (9) to determine ARE for observation noise in (3) is accomplished by computing the moments of $w_k$. For this purpose, we use the characteristic function of the observation noise PDF in (3), which is

$$\Phi_{\text{W}}(j\omega) = \cos(I\omega)\exp(-\sigma^2\omega^2/2) \tag{11}$$

Using Taylor series expansions of $\cos(u)$ and $\exp(u)$ in $\Phi_{\text{W}}(j\omega)$, we have

$$\Phi_{\text{W}}(j\omega) = \sum_{k=0}^{\infty}\sum_{n=0}^{\infty} (-1)^k \frac{(I\omega)^{2k}}{(2k)!}(-1)^n \frac{(\sigma\omega)^{2n}}{2^n n!} \tag{12}$$

Letting $m = k + n$ leads to

$$\begin{aligned} &\Phi_{\text{W}}(j\omega) \\ &= \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} (-1)^k(-1)^{(m-k)} \frac{(I\omega)^{2k}\quad(\sigma\omega)^{2(m-k)}}{(2\ k)!\,2^{(m-k)}(m-k)!}, \quad k \le m \\ &= \sum_{m=0}^{\infty} \frac{(-1)^m \sigma^{2m}}{2^m} \sum_{m=0}^{m} \frac{I^{2k}2^k}{(2\ k)!\sigma^{2k}(m-k)!}\omega^{2m} \end{aligned} \tag{13}$$

By expressing the uniformly convergent series for the characteristic function as $\Phi_{\text{W}}(j\omega) = \sum_{m=0}^{\infty} c_m \omega^{2m}$, the moments of $W$ are obtained by substituting (13) in $\mu_m = E(w^m) = (-j)^m \Phi_{\text{W}}^{(m)}(0)$. Hence, we obtain $\mu_{2m-1} = 0$ and

$$\mu_{2m} = (2\ m)!\left(\frac{\sigma^2}{2}\right)^m \sum_{k=0}^{m} \frac{I^{2k}2^k}{(2\ k)!\sigma^{2k}(m-k)!} \tag{14}$$

By expanding (14), it is straightforward to show that the largest degree of $I$ is $2m$, which is obtained for $k = m$. Similarly, the largest degree of $\sigma$ is $2m$, which is obtained for $k = 0$. Hence, by substituting (14) in (9) and factoring out $\sigma^{2m}$ in the numerator and denominator of (9), we have

$$\lim_{I/\sigma\to\infty, N\to\infty, A\to 0} \text{ARE}_{nd,ld}(m,\ \sigma,\ I) = m^2 \tag{15}$$

This establishes that the proposed non-linear detector outperforms the MF detector with a factor of $m^2$ in the limit case. Hence, we deduce that by increasing the degree of the non-linearity $g(y) = y^m$, the performance of the proposed suboptimal detector will be enhanced, for a strong interferer and weak signal, that is, near–far situation, when Gaussian noise power is fixed. To compute ARE for limited values of $I/\sigma$, we substitute (14) in (9) when $I$ and $\sigma$ are known. The ARE performance against $m$ is shown in Fig. 1 for $I = 13$, $I = 30$ and $\sigma^2 = 1$. As we see for $I = 13$ by increasing $m$, the value of ARE increases until reaching $m = 15$ and after that, ARE decreases. Hence, there is a maximum in ARE curve for limited values of $I/\sigma$, which corresponds to the optimal degree of the non-linearity. Meanwhile, we see in Fig. 1 for

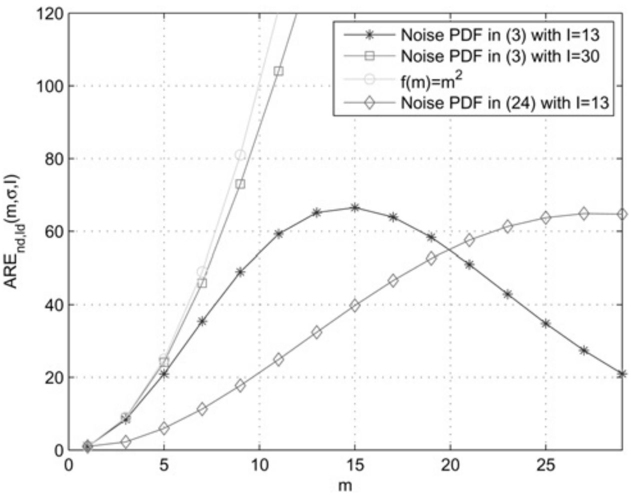


**Fig. 1** *$ARE_{nd,ld}$(m, σ, I) against m of the proposed detector in (7)*

$I = 30$ and $\sigma^2 = 1$, which mimics the case $I/\sigma \to \infty$, the ARE monotonically increases by increasing $m$ with a curve near to $m^2$ curve as shown in (15).

## 4 Remarks on probability of error calculation

It is worth to investigate the difficulties in $P_e$ performance evaluation of the MF, that is, detector in (7) with $m = 1$, and also our proposed non-linear detector. Assuming that the desired signal information bits are equally probable, and by considering the symmetry of observation noise PDF (3), $P_e$ is obtained from $P_e = P(s_k = A|s_k = -A)$ as follows

$$
\begin{aligned}
P_e &= \int_0^{\infty} f_{T_{\mathrm{MF}}(y)}(y|s_k = -A)\mathrm{d}y \\
&= \frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty} \Phi_{T_{\mathrm{MF}}(y)} \\
&\quad (j\omega|s_k = -A)\exp(-j\omega y)\mathrm{d}\omega\mathrm{d}y
\end{aligned}
\tag{16}
$$

where $T_{\mathrm{MF}}(y) = \sum_{k=1}^{N} y_k$. We calculate (16) using saddle point approximation [27, p. 125]. Since the samples of $y_k$ are IID RVs, we have $\Phi_{T_{MF}(y)}(j\omega|s_k = -A) = \Phi_{\mathrm{Y}}^{N}(j\omega|s_k = -A)$. Furthermore, considering (2), it is easy to show that $\Phi_{\mathrm{Y}}(j\omega/s_k = -A) = \exp(-jA\omega)\Phi_{\mathrm{W}}(j\omega)$. Hence, $\Phi_{\mathrm{Y}}(j\omega/s_k = -A) = \exp(-jA\omega)\cos(I\omega)\exp(-\sigma^2\omega^2/2)$ and we obtain $\Phi_{\mathrm{Y}}^{N}(u|s_k = -A)$ as follows

$$
\begin{aligned}
&\Phi_{\mathrm{Y}}^{N}(u|s_k = -A) \\
&\quad = (1/2)^N\left[\exp(-Au)\right]^N\left[\exp(Iu) + \exp(-Iu)\right]^N \\
&\qquad \times \exp(N\sigma^2u^2/2)
\end{aligned}
\tag{17}
$$

Using (17) in (16), $P_e$ is obtained; however, it is readily seen that this is not a viable approach for large values of $N$, for this reason, in order to estimate $P_e$, an approximation should be appealed. In fact binomial expansion of $[\exp(Iu) + \exp(-Iu)]^N$ requires large number of terms for large values of $N$. Saddle-point approximation is a numerically powerful tool for error probability calculation [27, p. 125]. The saddle-point approximation yields an approximation for (16) as follows [27, p. 125]

$$
P_e \simeq \frac{\exp(\Psi(u_0))}{\sqrt{2\pi\Psi''(u_0)}} \tag{18}
$$

where $\Psi(u) = \log\left(\Phi_Y^N(u|s_k = -A)\right) - \log(u)$ and $u_0$ is the positive root of $\Psi'(u) = 0$. The expression of $\Psi(u)$ is as follows

$$
\begin{aligned}
\Psi(u) &= N\log 0.5 - ANu + N\log \\
&\quad \times\left[\exp(Iu) + \exp(-Iu)\right] + \frac{N\sigma^2u^2}{2} - \log(u)
\end{aligned}
$$

and consequently, $\Psi'(u)$ is

$$
\Psi'(u) = -AN + IN\frac{e^{2Iu} - 1}{e^{2Iu} + 1} + N\sigma^2u - \frac{1}{u} \tag{19}
$$

The positive root of $\Psi'(u)$ can be evaluated numerically. For using (18), it is necessary to calculate $\Psi''(s)$, which is as follows

$$
\Psi''(u) = 4I^2N\frac{e^{2Iu}}{(e^{2Iu} + 1)^2} + N\sigma^2 + \frac{1}{u^2} \tag{20}
$$

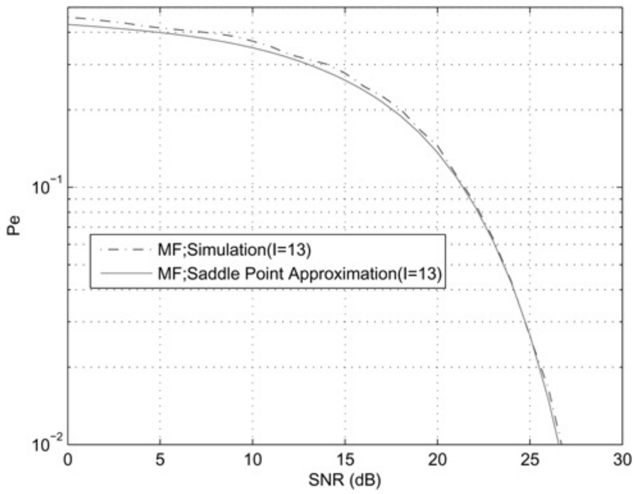


**Fig. 2** *Approximate $P_e$ using saddle point method in comparison to the simulated $P_e$ for $I = 13$, $\sigma^2 = 1$ and $N = 63$*

The approximate $P_e$ using saddle-point method in comparison to the simulated $P_e$ is shown in Fig. 2. As can be seen, the saddle-point approximation yields acceptable accuracy for $P_e$ calculation. For the proposed suboptimal detectors in (7), the $P_e$ calculation is a more protracted task because one has to evaluate the characteristic function of the non-linearity output, that is, $g(y) = y^m$, given $s_k = -A$. This calculation is necessary for using the saddle-point method to approximate the $P_e$ of the suboptimal detectors. Using the equation in [28, p. 161], the characteristic function evaluation of the non-linearity output leads to a mathematically intractable integral, which makes the use of saddle point approximation impossible. This discussion shows the benefits of the ARE calculation using (9) to compare the performance of the non-linear detectors with respect to linear detectors in a succinct and accurate manner.



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## 5 Computational complexity

In this section, we evaluate the computational complexity of the proposed detectors. The suboptimal detectors in (7) require $(m-1)N$ multiplications and $N-1$ additions for decision making about one bit. Since the MF requires $N-1$ additions for one bit decision making, the proposed detector requires only $(m-1)N$ more multiplications with respect to the conventional MF.

Now, we have to calculate the number of computations for evaluating ARE for a given $m$, using (9). First, we note that we can estimate the expected value of a RV from its received samples using an iterative method. In fact, we can estimate the expected value of the RV $Z$ from its samples, that is, $z_n$, as follows [29, p. 192]

$$\hat{E}_n(Z) = \left(\frac{n-1}{n}\right)\hat{E}_{n-1}(Z) + \left(\frac{1}{n}\right)z_n$$

where $\hat{E}_n(Z)$ denotes the estimate of expected value in the $n$th iteration. As we see, every iteration of the expected value estimator requires six floating point operations (FLOPS). Secondly, the evaluation of ARE in every iteration requires $m+5$ FLOPS for calculation of $\left[E\left(w_k^{m-1}\right)\right]^2$, 7 FLOPS for calculation of $E\left(w_k^2\right)$, $2m+5$ FLOPS for calculation of $E\left(w_k^{2m}\right)$ and 4 FLOPS to obtain the final result for the ARE value. Consequently, the overall FLOPS required for ARE evaluation is $3m+21$ per iteration, for a given $m$.

It is worth noting that one has access to the samples of the observation noise $w_k$ during the natural silent periods in the voice and data signals of the desired user [30, p. 474]. In these silence periods, the transmitter is kept inactive to reduce the overall interference in the system and also to increase the battery lifetime in a mobile station.

## 6 Performance evaluation in Ricean fading and asynchronous channels

For the Ricean fading channel, $A$ and $I$ are Ricean RVs with parameters $(\mu, \sigma_s)$ [2, 31]. The Rice factor is defined by $K = \mu^2/2\sigma_s^2$ and measures the fading severity of the channel. $\mu^2$ is the mean power of the LOS component and $2\sigma_s^2$ the mean power of the scattering components [31]. As we saw in Section 3, we have to calculate (9) to find out whether the performance of the proposed detectors is better than conventional MF or not. For this purpose, we can use (14) which corresponds to $E[w^{2m}/I]$ in the Ricean fading scenario. To obtain $E[w^{2m}]$, we use the equation $E[E(y/x)] = E[y]$ which leads to

$$E[w^{2m}] = (2\,m)!\left(\frac{\sigma^2}{2}\right)^m \times \sum_{k=0}^{m}\frac{2^k}{(2\,k)!\,\sigma^{2k}(m-k)!}\int_0^\infty I^{2k} f(I)\mathrm{d}I \qquad (21)$$

where $f(I)$ is the Ricean RV. Using the equation for the moments of Ricean PDF in [2, p. 51], we obtain

$$\int_0^\infty I^{2k} f(I)\mathrm{d}I = \left(2\sigma_s^2\right)^k \Gamma(1+k)\,{}_1F_1\left(-k,\ 1;\ -\frac{\mu^2}{2\sigma_s^2}\right) \qquad (22)$$

where ${}_1F_1(a,b;x)$ is the confluent hypergeometric function defined in [2, pp. 49–51]. Substituting (22) in (21) and using (9), we calculate the ARE performance of the proposed suboptimal detectors versus $m$ for different $K$ factor values, which is shown in Fig. 3. As we observe from this figure, for larger $K$ factor values more improvement is possible using the proposed detectors. We also note that for a given $K$ factor value, each ARE curve has a maximum that determines the best value for $m$ in the non-linearity, $g(y) = y^m$. $K$ factor estimation can be achieved using the method in [31].

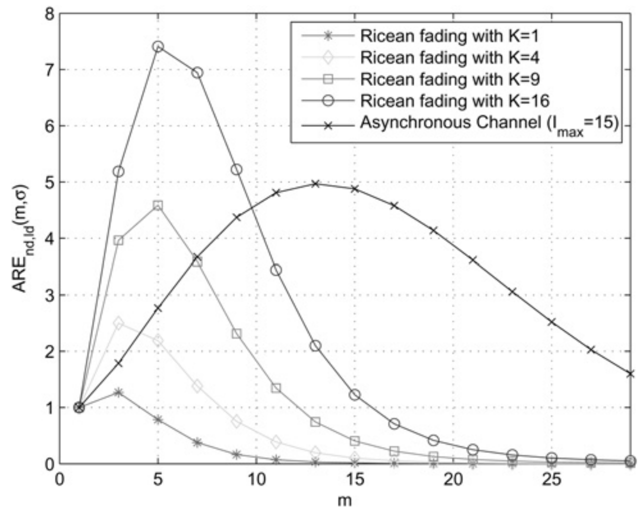


**Fig. 3** *$ARE_{nd,ld}(m, \sigma, I)$ for noise in (3) with $\sigma^2 = 1$, when $I$ is Ricean with different $K$ factor values and also asynchronous channel with $I_{max} = 15$*

For the asynchronous case, that is, when the chips of the different users are not aligned [2, p. 1039] [1, 10], there is a time-varying delay for the interferer chips with respect to the desired signal which can be modelled as a uniform RV in the interval $[0, T_c)$ [1, 10], where $T_c$ is the chip duration. In this case, there are exactly two consecutive chips from interferer that overlap a chip duration of the desired signal. When the consecutive chips of the interferer are similar at the desired signal chip MF integrator, the interferer component at the output of the chip MF has its maximum and minimum values $+I_{\max}$ and $-I_{\max}$, respectively. Since the PN sequence of the users are assumed truly random, we have $P(i_k = I_{\max}) = P(i_k = -I_{\max}) = 0.25$. Otherwise, when the consecutive chips of the interferer are different, it is reasonable to model the interferer component as a uniform RV in the interval $[-I_{\max}, I_{\max}]$, because the delay of the interferer is uniformly distributed. Since the different consecutive chips case happens with probability 0.5, the PDF of interferer at the output of the desired signal chip MF is

$$f(x) = \frac{1}{4}\delta(x - I_{\max}) + \frac{1}{4I_{\max}} \times \left[u(x+I_{\max}) - u(x-I_{\max})\right] + \frac{1}{4}\delta(x+I_{\max}) \qquad (23)$$

where $u(\cdot)$ is the unit step function and $\delta(\cdot)$ is the Dirac delta. Since the interferer is not phase synchronised for the carrier modulated signals, the interferer term must be multiplied to a random factor $\cos\phi$, where $\phi$ is a uniformly distributed RV in the interval $[0, 2\pi)$ [1, 32]. Hence, the observation noise $w_k$ in (2) can be written as $X\cos\phi + v_k$ for the asynchronous case, where the PDF of $X$ is given in (23). ARE curve for an asynchronous interferer with $I_{\max} = 15$, $\sigma^2 = 1$ is obtained based on $10^5$ samples of $w_k$, which is shown in Fig. 3. This curve predicts that the best value for $m$ is $m = 13$.

## 7 Numerical results

In this section, we perform some simulations to investigate the $P_e$ performance of the proposed detectors. We compare the performance of the proposed detectors with that of the ML detector which knows the parameters $A$, $I$ and $\sigma^2$. We depict $P_e$ performance of the different detectors in Fig. 4, for $I = 13$ and $\sigma^2 = 1$, when PG of the SS system is $N = 63$ and signal-to-noise ratio (SNR) is defined as $\text{SNR} = NA^2/2\sigma^2$. As we observe, the detector with non-linearity $g(y) = y^m$ outperforms the MF, and by increasing $m$ its performance improves until reaching $m = 15$ which agrees with the ARE curves in Fig. 1. In Fig. 5, we depict the $P_e$ performance of different detectors for $I = 30$, $\sigma^2 = 1$ and $N = 63$. As we expected from Fig. 1, for $I = 30$, by increasing $m$, the $P_e$ performance of the proposed detectors is enhanced. Notice that for limited values of $N$, this enhancement stops in an $m$ value because (15) is obtained for $N \to \infty$. Now, we present another scenario, where another interferer with unknown level $I$ enters into the system with model in (1). The observation noise PDF can be obtained as

$$f_U(w_k) = \frac{1}{4} \times \left[\mathcal{N}_{\sigma^2}(w_k - 2I) + 2\mathcal{N}_{\sigma^2}(w_k) + \mathcal{N}_{\sigma^2}(w_k + 2I)\right] \tag{24}$$

As we see the even property of the observation noise is still maintained for the PDF in (24), hence, it is plausible to use the proposed scheme for this case, too. We depict the $P_e$ performance of the proposed detectors in Fig. 6, which shows the capability of the proposed detectors in suppressing unknown interferers. As we observe, the performance of the proposed detectors for the observation noise with PDF in (24) monotonically increases by increasing $m$. This can be easily justified by calculating

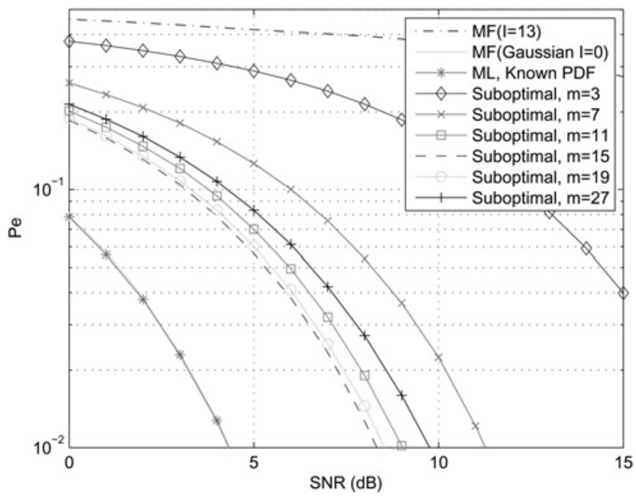


**Fig. 4** *$P_e$ against SNR for $I = 13$, $\sigma^2 = 1$, $N = 63$ and observation noise in (3)*

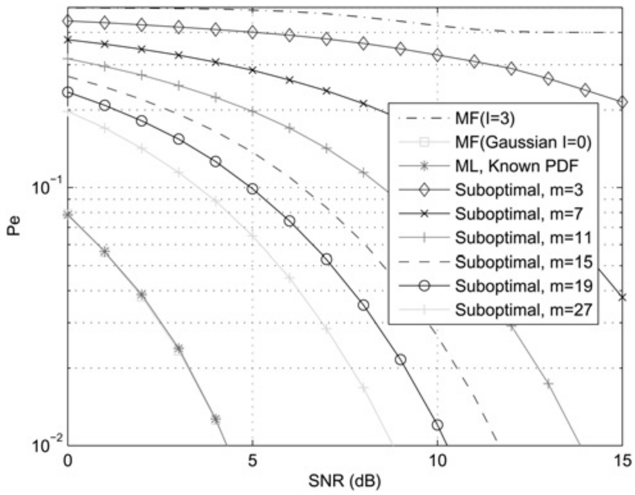


**Fig. 5** *$P_e$ against SNR for $I = 30$, $\sigma^2 = 1$, $N = 63$ and observation noise in (3)*

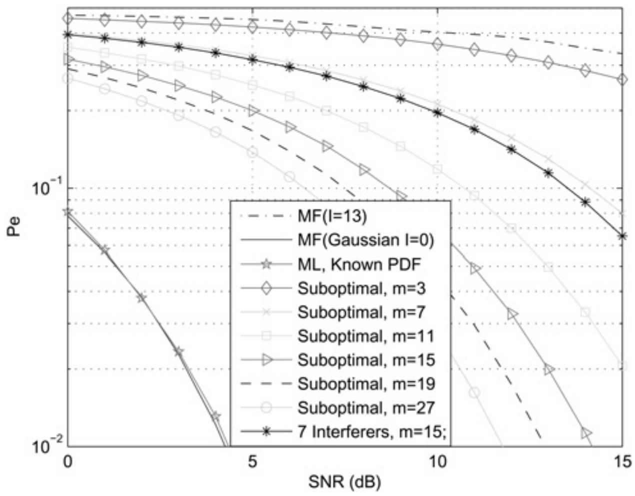


**Fig. 6** *$P_e$ against SNR for $I = 13$, $\sigma^2 = 1$, $N = 63$ and observation noise in (24) and also 7 interferers case*

ARE via (9) and using $10^5$ samples of the observation noise with PDF in (24). This is depicted in Fig. 1 for observation noise in (24) and $I = 13$ which agrees with $P_e$ curves in Fig. 6. Figs. 1 and 6 also show the capability of the proposed detectors in rejecting 7 interferers, two with level $I_1 = I_2 = 13$ and the others with unity power. Finally, we evaluate $P_e$ performance of the proposed detectors in a Ricean fading scenario with $K = 9$ and $\sigma_s^2 = 1$ and asynchronous channel with $I_{\max} = 15$. We depicted $P_e$ curves in Fig. 7. As we expected from Fig. 3, the non-linearities with degrees $m = 5$ and around $m = 13$ yield the best performance for the Ricean and asynchronous channels, respectively. It is worth noting that the definition of the ARE before (8) is for the cases with $N \to \infty$ and $A \to 0$. Consequently, when $N$ has limited value and $A$ is large, some small mismatches may happen in the ARE and $P_e$ curves.

## 8 Conclusions

In this paper, we have proposed monomials with odd integer powers for blind multiuser detection of CDMA signals with



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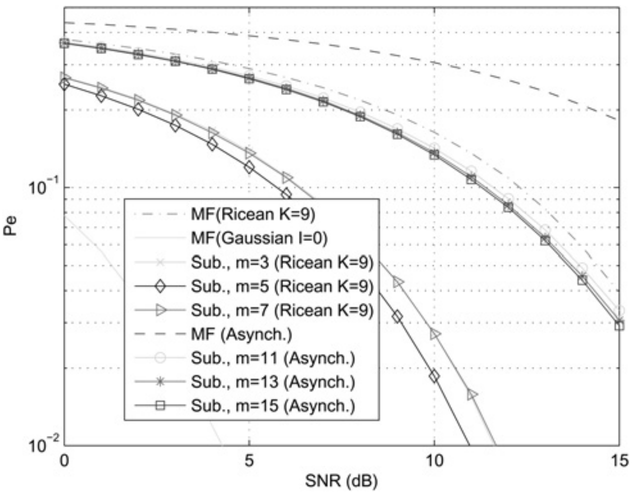


**Fig. 7** *$P_e$ against instantaneous SNR for Ricean faded interference with $K = 9$ and $N = 63$ and also asynchronous channel with $I_{max} = 15$*

long spreading codes. These non-linearities make the detectors robust against the strong interferer. The proposed non-linear detectors do not require neither training nor the spreading code of the interferers. Furthermore, the detectors do not require long convergence time for decision making. We have demonstrated analytically that the proposed detectors significantly outperform the MF detector at the cost of a small increase in computational complexity. We have also determined the optimum integer power values for some important communication channels.

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