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Low-complexity joint regularised equalisation and carrier frequency offsets compensation scheme for single-carrier frequency division multiple access system

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Abstract: Since the conventional zero-forcing receiver does not operate satisfactorily in interference-limited environments, because of its noise amplification; the statistics of the transmitted data and the additive noise are required for the minimum-mean-square error receiver; the potential of the regularised receiver is proposed and investigated in this study to cope with these problems. In this study, the authors introduce an efficient low-complexity joint regularised equalisation and carrier frequency offset compensation (LJREC) scheme for single-carrier frequency division multiple access system. The proposed LJREC scheme avoids the noise amplification problem and the estimation of the signal-to-noise ratio and the interference matrices of other users. From the obtained simulation results, the proposed scheme enhances the system performance with lower complexity and sufficient robustness to the estimation errors.

1 Introduction

Orthogonal frequency-division multiple access (OFDMA) is an effective multicarrier modulation scheme that has received significant attention in recent years because of its resistance to multipath fading. However, the OFDMA has its disadvantages: high peak-to-average power ratio (PAPR), high sensitivity to carrier frequency offset (CFO) and a need for an adaptive or coded scheme to overcome the spectral nulls in the channel [1–3]. Recently, single-carrier FDMA (SC-FDMA) system which utilises single-carrier modulation and frequency domain equalisation is a technique that has similar throughput and essentially the same overall structure as the OFDMA system [4–6]. One advantage over the OFDMA system is that the SC-FDMA signal has lower PAPR because of its inherent single-carrier structure [6]. The SC-FDMA system has attracted much attention as an alternative to the OFDMA system, especially in uplink communications, where a lower PAPR benefits the mobile terminal in terms of transmit power efficiency [6]. Although the SC-FDMA system has a number of attractive features, it also has some disadvantages of the OFDMA system. One major disadvantage is its sensitivity to CFOs. In the uplink of an SC-FDMA system, all the users transmitting in the same block should be time and frequency aligned with the other users. Frequency mismatches among the uplink users as well as between the uplink users and the base station cause power leakage among the subcarriers. This leakage has two effects, namely inter-carrier interference (ICI), and multi-user interference (MUI) [1, 7, 8]. Several CFO compensation and interference cancellation schemes have been proposed for the OFDMA [2, 3, 9, 10]. The zero-forcing (ZF), the single-user and the circular convolution schemes are the three common CFO compensation schemes for the uplink OFDMA systems [3, 9]. In the single-user and circular convolution schemes, the MUI after CFO compensation is still large, especially for large CFO values. The conventional ZF equaliser enhances the noise and degrades the system performance [7, 9].

A number of studies have addressed the CFOs compensation issue in the SC-FDMA uplink [1, 7, 8]. A joint MMSE (JMMSE) equaliser was proposed in [1] to jointly perform equalisation and CFOs compensation processes. It was shown that the JMMSE scheme provides better performance than that of the single-user [2] and the circular convolution schemes [3]. Low-complexity equalisation and CFOs compensation scheme for multi-input–multi-output SC-FDMA systems is derived and investigated in [7]. In [8], a joint equalisation and CFOs compensation (JEC) scheme was suggested and investigated to obtain the initial estimation for each user. To further eliminate the MUI, the authors in [8] combined JEC with parallel interference cancellation. They iteratively design the equaliser to suppress the remaining MUI at each stage and obtain better estimation. In the MMSE schemes in [1, 8], the estimation of the SNR and the interference matrices of other users is required, which requires more complexity. Also, the conventional ZF equaliser amplifies the noise and applying it for the SC-FDMA system is not practical [9].

Hence, introducing an efficient low-complexity practical scheme for the uplink SC-FDMA system is an important issue, which is the main objective of this paper.

The difference between the proposed LJREC and that in [7] is that the idea of the regularisation is proposed and studied for the single-input–single-output SC-FDMA system and not for the MIMO SC-FDMA as that in [7]. Also, the equalisation process in this paper is performed in one step. On the other hand, the equalisation process in [7] is performed in two steps and the idea of the regularisation is only applied in the second step.

In this paper, we propose an efficient low-complexity joint equalisation and CFO compensation for the SC-FDMA system, namely the LJREC scheme, to cope with the problems associated with the ZF and the MMSE schemes. It represents the correlation matrices of the noise and of the interference signals by a single-identity matrix multiplied by a regularisation parameter. The regularisation parameter is used to avoid the noise amplification and to reduce the ICI and the MUI. Moreover, to further reduce the complexity, the proposed equaliser avoids the direct matrix inversion by exploiting the banded structure of the interference matrix of the desired user. Simulation results have demonstrated that the proposed LJREC enhances the system performance with lower complexity and sufficient robustness to the estimation errors.

The rest of this paper is organised as follows. In Section 2, the considered SC-FDMA system model in the presence of CFOs is presented. In Section 3, the proposed LJREC scheme is discussed. The computational complexity of the proposed LJREC scheme is discussed in Section 4. Section 5 presents the simulation results to verify the effectiveness of the proposed LJREC scheme. Conclusions are drawn in Section 6.

2 SC-FDMA system model in the presence of CFO

Consider an uplink SC-FDMA system, where U mobile users simultaneously transmit data to a base station. The structure of the uplink SC-FDMA system with the proposed LJREC scheme is shown in Fig. 1. In the SC-FDMA transmitter, the encoded signals are modulated and then transformed into the frequency domain via an N-point DFT. Then, the subcarriers are mapped in the frequency domain. There are two popular subcarriers mapping in the SC-FDMA system [6]. The first one is the localised FDMA (LFDMA), where each user's subcarriers are occupying a dedicated part of the usable spectrum. The second one is the interleaved FDMA (IFDMA), where the user's subcarriers are interleaved to each other. After that, the inverse discrete Fourier transform (IDFT) is performed, and a cyclic prefix (CP) is added to the resulting signal. Finally, the resulting signal is transmitted through the wireless channel. The transmitted signal from the uth user can be formulated as follows [1]

$$\bar{\boldsymbol{x}}^u = \boldsymbol{P}_{\text{add}}\boldsymbol{\Psi}_M^{-1}\boldsymbol{M}_T^u\boldsymbol{\Psi}_N\boldsymbol{d}^u \tag{1}$$

where $\boldsymbol{d}^u$ is an $N \times 1$ vector containing the modulated symbols of the uth ($u = 1, 2, \dots, U$) user. $\boldsymbol{\Psi}_N$ is the $N \times N$ DFT matrix. $\boldsymbol{M}_T^u$ is an $M \times N$ subcarriers mapping matrix of the uth user. $\boldsymbol{\Psi}_M^{-1}$ is the $M \times M$ IDFT matrix. $\boldsymbol{P}_{\text{add}}$ is an $(M + N_C) \times M$ matrix, which adds a CP of length N_C. The entries of $\boldsymbol{M}_T^u$ and $\boldsymbol{P}_{\text{add}}$ are given in [1].

At the receiver side, assuming perfect time synchronisation, the received signal after the removal of the

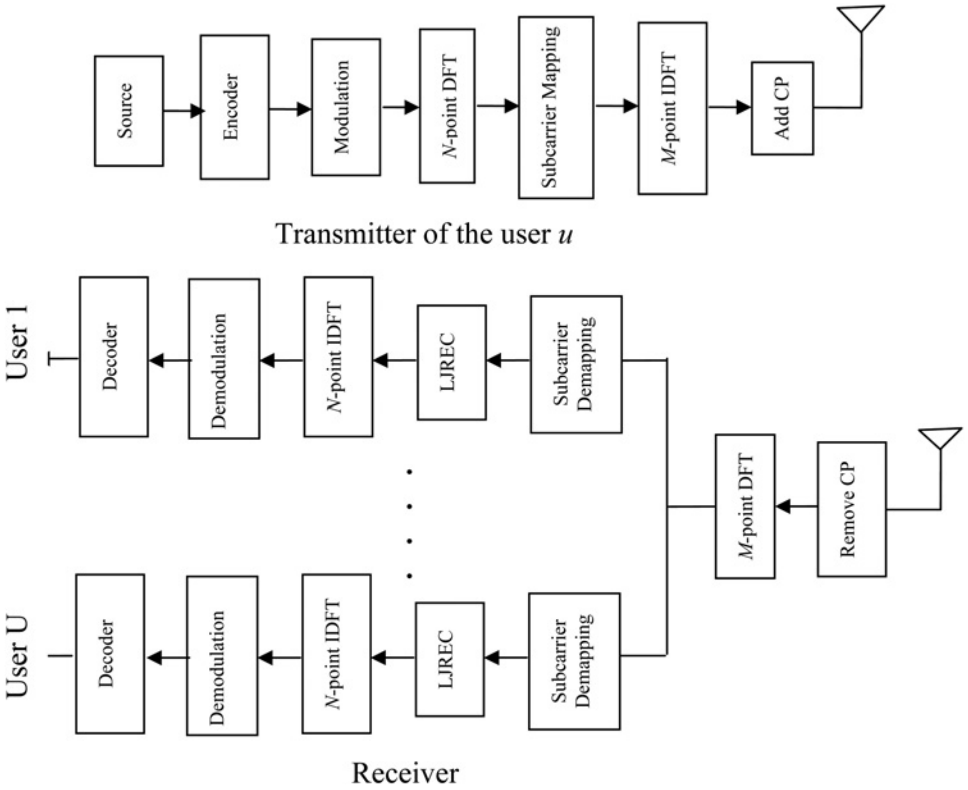

Fig. 1 *Structure of the SC-FDMA system over a frequency selective channel*

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CP can be written as follows [1]

$$\boldsymbol{r} = \sum_{u=1}^{U} \boldsymbol{E}^u \boldsymbol{H}^u \boldsymbol{x}^u + \boldsymbol{n} \tag{2}$$

where $\boldsymbol{E}^u$ is an $M \times M$ diagonal CFO matrix of the uth user. $\boldsymbol{n}$ is an $M \times 1$ vector containing the noise. $\boldsymbol{H}^u$ is an $M \times M$ circulant matrix describing the channel of the uth user. $\boldsymbol{x}^u = \boldsymbol{\Psi}_M^{-1} \boldsymbol{M}_T^u \boldsymbol{\Psi}_N \boldsymbol{d}^u$ is an $N \times 1$ vector containing the transmitted signal of the uth user after the DFT. The received signal is transformed into the frequency domain via an M-point DFT as follows [1]

$$\boldsymbol{R} = \sum_{u=1}^{U} \boldsymbol{\Pi}_{\text{cir}}^u \boldsymbol{\Lambda}^u \bar{\boldsymbol{X}}^u + \boldsymbol{N} \tag{3}$$

where $\boldsymbol{\Pi}_{\text{cir}}^u = \boldsymbol{\Psi}_M \boldsymbol{E}^u \boldsymbol{\Psi}_M^{-1}$ is a circulant matrix representing the interference from the uth user. $\bar{\boldsymbol{X}}^u$ is an $M \times 1$ vector representing the transmitted samples from the uth user after the mapping process. $\boldsymbol{N}$ is the DFT of $\boldsymbol{n}$. The simplification of (3) depends on the fact that $\boldsymbol{H}^u = \boldsymbol{\Psi}_M^{-1} \boldsymbol{\Lambda}^u \boldsymbol{\Psi}_M$, where $\boldsymbol{\Lambda}^u$ is an $M \times M$ diagonal matrix containing the DFT of the circulant sequence of $\boldsymbol{H}^u$. Then, the demapping, CFO compensation, equalisation, IDFT and demodulation processes are applied. After the demapping process, the received signal from the kth user is given by

$$\boldsymbol{R}_d^k = \boldsymbol{\Pi}_d^k \boldsymbol{\Lambda}_d^k \boldsymbol{X}^k + \sum_{u=1, u \neq k}^{U} \boldsymbol{\Pi}_r^u \boldsymbol{\Lambda}_d^u \boldsymbol{X}^u + \boldsymbol{N}_d \tag{4}$$

where $\boldsymbol{\Pi}_d^k = \boldsymbol{M}_R^k \boldsymbol{\Pi}_{\text{cir}}^k \boldsymbol{M}_T^k$ is an $N \times N$ matrix describing the interference with the kth user's data. $\boldsymbol{M}_R^k$ is an $N \times M$ matrix representing the subcarriers demapping process for the kth user. $\boldsymbol{\Pi}_r^u = \boldsymbol{M}_R^k \boldsymbol{\Pi}_{\text{cir}}^u \boldsymbol{M}_T^u$ is an $N \times N$ matrix describing the interference from the uth user. $\boldsymbol{\Lambda}_d^u = \boldsymbol{M}_R^u \boldsymbol{\Lambda}^u \boldsymbol{M}_T^u$ is an $N \times N$ diagonal matrix representing the channel of the uth user. $\boldsymbol{X}^u = \boldsymbol{\Psi}_N \boldsymbol{d}^u$ is an $N \times 1$ vector representing the signal after the DFT. $\boldsymbol{N}_d = \boldsymbol{M}_R^k \boldsymbol{N}$ is an $N \times 1$ vector containing the noise after the demapping process. The estimated time domain symbols of the kth user can be written in terms of interferences as follows [1]

$$\hat{\boldsymbol{d}}^k = \boldsymbol{\Gamma}^k \boldsymbol{d}^k + \boldsymbol{\Omega}^k \boldsymbol{d}^k + \sum_{u=1, u \neq k}^{U} \boldsymbol{\Delta}^u \boldsymbol{d}^u + \boldsymbol{n} \tag{5}$$

The structures of all the components of (5) are given as follows

$$\boldsymbol{\Gamma}^k = \text{diag}\left(\boldsymbol{\Psi}_N^{-1} \boldsymbol{W}_{\text{LJREC}}^k \boldsymbol{\Pi}_d^k \boldsymbol{\Lambda}_d^k \boldsymbol{\Psi}_N\right) \tag{6}$$

$$\boldsymbol{\Omega}^k = \boldsymbol{\Psi}_N^{-1} \boldsymbol{W}_{\text{LJREC}}^k \boldsymbol{\Pi}_d^k \boldsymbol{\Lambda}_d^k \boldsymbol{\Psi}_N - \boldsymbol{\Gamma}^k \tag{7}$$

$$\hat{\boldsymbol{n}} = \boldsymbol{\Psi}_N^{-1} \boldsymbol{W}_{\text{LJREC}}^k \boldsymbol{M}_R^k \boldsymbol{N} \tag{8}$$

$$\boldsymbol{\Delta}^i = \boldsymbol{\Psi}_N^{-1} \mathbf{W}_{\text{LJREC}}^k \boldsymbol{\Pi}_r^u \boldsymbol{\Lambda}_d^u \boldsymbol{\Psi}_N \tag{9}$$

where $\boldsymbol{W}_{\text{LJREC}}^k$ is the proposed LJREC scheme. The proposed LJREC scheme will be discussed in detail in Section 3. From (5), it is clear that only the first term contains the desired data, the second term is due to the ISI, the third term is an MUI and the fourth term is a noise.

3 Proposed LJREC scheme

In this section, the proposed LJREC scheme is investigated. Equation (4) can be rewritten as follows

$$\boldsymbol{R}_P^k = \boldsymbol{\Pi}_P^k \boldsymbol{X}^k + \boldsymbol{N}_P \tag{10}$$

where $\boldsymbol{\Pi}_P^k = \boldsymbol{\Pi}_d^k \boldsymbol{\Lambda}_d^k$ is an $N \times N$ matrix representing the interference because of both the channel and the CFO. $\boldsymbol{N}_P = \sum_{u=1, u \neq k}^{U} \boldsymbol{\Pi}_R^u \boldsymbol{X}^u + \boldsymbol{N}_d$ is the MUI plus noise matrix. $\boldsymbol{\Pi}_R^u = \boldsymbol{\Pi}_r^u \boldsymbol{\Lambda}_d^u$. Based on the least squares criteria, the joint ZF (JZF) solution of (10) is given as

$$\boldsymbol{W}_{\text{JZF}}^k = \left(\boldsymbol{\Pi}_P^{kH} \boldsymbol{\Pi}_P^k\right)^{-1} \boldsymbol{\Pi}_P^{kH} \tag{11}$$

The JZF solution in (11) is derived without taking into account both the noise and the MUI [1]. Hence, this solution suffers from the noise enhancement problem, which degrades the system performance. To avoid this problem, the solution of (10) must be derived based on the minimum-mean-square error (MMSE) criteria as follows [1]

$$\boldsymbol{W}_{\text{JMMSE}}^k = \left(\boldsymbol{\Pi}_P^{kH} \boldsymbol{\Pi}_P^k + \sum_{u=1, u \neq k}^{U} \boldsymbol{\Pi}_R^{uH} \boldsymbol{\Pi}_R^u + (1/\text{SNR}) \boldsymbol{I}_N\right)^{-1} \boldsymbol{\Pi}_P^{kH} \tag{12}$$

The JMMSE solution in (12) is derived by taking into account both the noise and the MUI [1]. However, from (12), it is clear that the MMSE solution requires the estimation of the SNR and the interference matrices, which are not known at the receiver. This indicates that the complexity of the JMMSE scheme in [1] is greater than that of the JZF scheme. Thus, introducing an efficient low-complexity receiver by avoiding the problems associated with the MMSE and the ZF receiver is an important issue. To do this, we propose an efficient regularised receiver scheme, named an LJREC scheme. The proposed LJREC scheme uses a constant regularisation parameter rather than the SNR and the interference matrices as in the JMMSE scheme. The main objectives of this parameter are to avoid the noise enhancement problem in the JZF solution and thus provide better performance than that of the JZF solution and to avoid the estimation of the SNR and the interference matrix of the other users in the JMMSE scheme. It is also used to reduce the effect of the ICI and the MUI. The regularisation parameter must be optimised through simulation to provide better performance. The LJREC scheme can be expressed by the following equation

$$\boldsymbol{W}_{\text{LJREC}}^k = \left(\boldsymbol{\Pi}_P^{kH} \boldsymbol{\Pi}_P^k + \alpha \boldsymbol{I}_N\right)^{-1} \boldsymbol{\Pi}_P^{kH} \tag{13}$$

where α is the regularisation parameter. From (13), it is clear that the estimation of the SNR and the interference matrices is not required. Hence, the complexity of the LJREC scheme is lower than that of the JMMSE scheme in (12). The proposed LJREC scheme requires a single DFT stage for all the users rather than a single DFT stage for each user as in [9], since it is performed in the frequency domain. Note that if $\alpha = 0$, the LJREC is reduced to the JZF equaliser in (11). The optimum solution of the proposed scheme is the JMMSE solution in (12). On the other hand, the condition that would make the LJREC equal to the JMMSE solution is to replace $\alpha \boldsymbol{I}_N$ in (13) by $\sum_{u=1,u\neq k}^{U} \boldsymbol{\Pi}_R^{u\,H}\boldsymbol{\Pi}_R^{u} + (1/\mathrm{SNR})\boldsymbol{I}_N$. The optimum solution of the proposed scheme depends on the estimation of the SNR and the interference matrices, which requires high complexity. Thus, the proposed LJREC with a constant α is better to use in the practical receivers because of its good performance and lower complexity.

4 Complexity evaluation

As explained in Section 3, the LJREC scheme represents the correlation matrices of the noise and of the interference signals by a single-identity matrix multiplied by a regularisation parameter and thus the calculation of the equaliser parameters is simplified. Moreover, the calculation of the interference matrix inversion requires a computational complexity of the order of $O(N)$, since the interference matrix of the desired signal is approximated as a banded matrix [1]. In [1], it was shown that the multiplication of two banded matrices requires approximately $2N(2r^2+1)$ operations, where r is the bandwidth of the banded matrix. In the JMMSE scheme, the step $\boldsymbol{\Pi}_P^{k\,H}\boldsymbol{\Pi}_P^{k} + \sum_{u=1,u\neq k}^{U} \boldsymbol{\Pi}_R^{u\,H}\boldsymbol{\Pi}_R^{u}$ requires about $2NU(2r^2+1)$ operations. However, in the JLREC scheme, the step $\boldsymbol{\Pi}_P^{k\,H}\boldsymbol{\Pi}_P^{k}$ requires about $2N(2r^2+1)$ operations, which are much lower than that required for the JMMSE scheme, especially when the number of users is large. Moreover, the proposed LJREC scheme also avoids the estimation of the SNR and of the interference correlation matrices and thus it is further reducing the computational complexity in the receiver.

5 Simulation results

In this section, we evaluate the bit error rate (BER) performance of the SC-FDMA system with the proposed receiver via computer simulations. For the comparison purpose, the JMMSE, the JZF, the single-user and the circular-convolution schemes are simulated for the uplink SC-FDMA system.

5.1 Simulation setup

In our simulations, we have employed quadrature phase shift keying (QPSK) for the modulation and the demodulation schemes, and set the number of users $U=4$, where all the users are assigned the same transmit power. The number of subcarriers per-user $N=32$, therefore, the IDFT size is $M=128$ and the length of the CP is set to be 20. The channel model used for the simulations is the vehicular A model [11]. A convolutional code with rate 1/2, constraint length 7 and octal generator polynomial (133,171) is used. Each frequency offset is a random variable with uniform distribution in the period $[-\varepsilon_{\max}, \varepsilon_{\max}]$, where $\varepsilon_{\max}$ is the maximum normalised CFO.

5.2 Regularisation parameter measurement

Here, the best value of the regularisation parameter α of the proposed scheme is determined for different SNR values and different CFOs values. The BERs of the proposed LJREC scheme against the regularisation parameter α and the SNR for the LFDMA and the IFDMA systems are shown in Figs. 2 and 3, respectively. The CFOs of all the users are chosen randomly in the interval $[-0.15, 0.15]$. It is clearly seen that the best choice of α lies between $\alpha=0.1$ and $\alpha=0.01$ for both the LFDMA and the IFDMA systems, regardless of the SNR values.

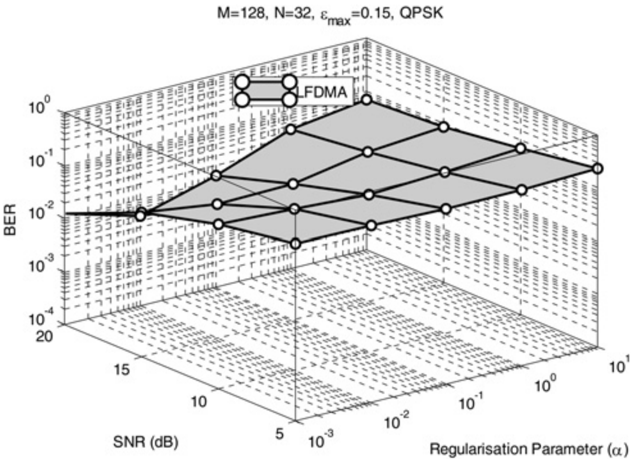

Fig. 2 *Variation of the BER with both SNR and α for the LFDMA system with the proposed LJREC scheme at $\varepsilon_{max}=0.15$*

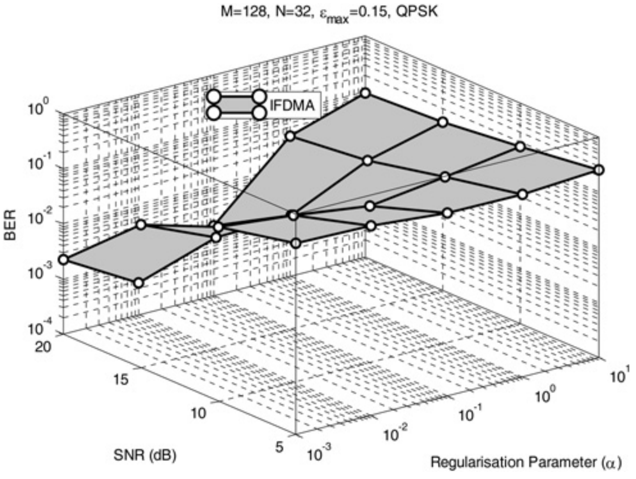

Fig. 3 *Variation of the BER with both SNR and α for the IFDMA system with the proposed LJREC scheme at $\varepsilon_{max}=0.15$*

The best value of α is also determined for different CFOs values. The results are shown in Figs. 4 and 5. SNR = 15 dB is considered. It is clear that the best choice of α lies also between $\alpha=0.1$ and $\alpha=0.01$ for both the LFDMA and the IFDMA systems, regardless of the CFOs values. Thus, we will use $\alpha=0.01$ in the next experiments.

Figs. 6 and 7 investigate the performance of the JLREC and the JMMSE schemes in various combinations of the SNR and the maximum value of normalised CFO, $\alpha=0.01$ is considered. It is clearly seen that the proposed JLREC scheme does not substantially impair the performance of the IFDMA and the LFDMA systems when compared with that of the JMMSE scheme over wide ranges of the SNR and the maximum value of the normalised CFO. This indicates that the proposed JLREC is an efficient low-complexity scheme for future wireless communications.

5.3 BER performance

The performance comparison of the BER of the proposed LJREC scheme is presented in Fig. 8, where the results

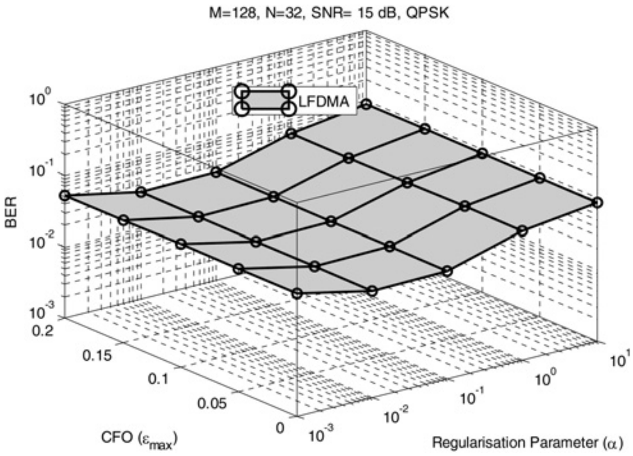

Fig. 4 *Variation of the BER with both ε_{max} and α for the LFDMA system with the proposed LJREC scheme at SNR = 0.15 dB*

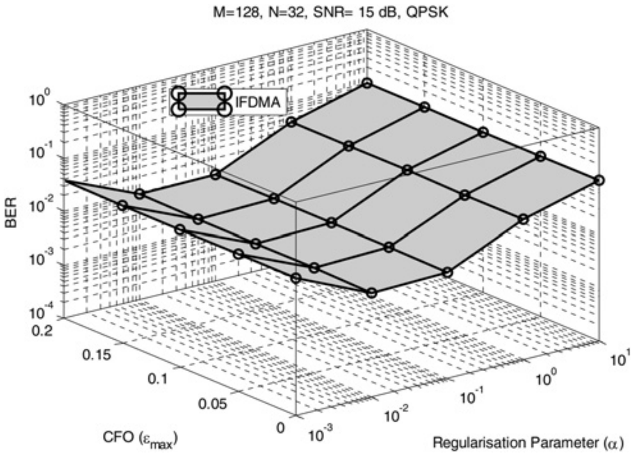

Fig. 5 *Variation of the BER with both ε_{max} and α for the IFDMA system with the proposed LJREC scheme at SNR = 15 dB*

without compensation, without CFO, with JZF and with JMMSE, are given as references. $\alpha = 0.01$, $\varepsilon_{\max} = 0.15$ and ½ rate convolutional code are used. From this figure, one can see that both the LJREC and the JMMSE schemes completely eliminate the effect of the CFOs for the LFDMA system. It is also clear that the proposed LJREC scheme provides the same BER as that of the JMMSE scheme at high SNR, because the interference is dominant at high SNR. When the SNR is low, there is a slight difference in the BER performance between the proposed LJREC scheme and the JMMSE scheme. Fig. 8 demonstrates also that the performance of the JZF equaliser is the worst, even when compared with a system without CFOs compensation.

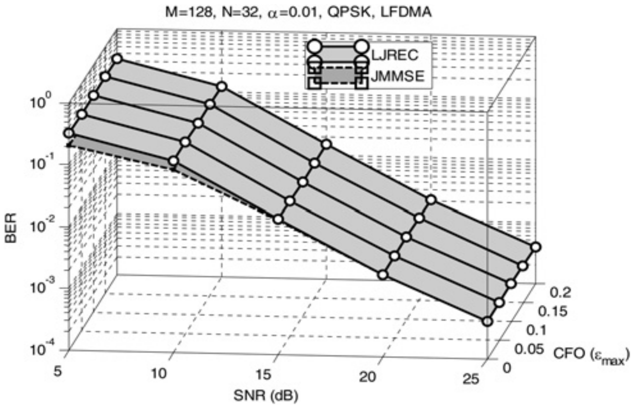

Fig. 6 *Variation of the BER with both SNR and ε_{max} for the LFDMA system with the LJREC and the JMMSE schemes*

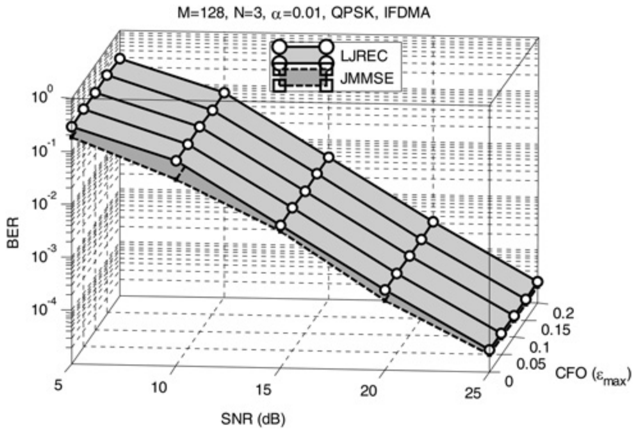

Fig. 7 *Variation of the BER with both SNR and ε_{max} for the IFDMA system with the LJREC and the JMMSE schemes*

Fig. 8 shows that when the SNR is large beyond 21 dB, the JMMSE and the proposed LJREC have the same BER performance. However, on comparing (12) (for JMMSE) and (13) (for LJREC), the two equations are not approximately identical when the SNR approaches infinity. To illustrate the difference between the proposed LJREC and the JMMSE, the uncoded case must be studied. Based on the obtained results in Fig. 8, since the proposed LJREC scheme is able to eliminate the effects of the CFOs for the localised systems, the interleaved systems are only considered in Fig. 9.

Fig. 9 illustrates the uncoded BER performance of the proposed LJREC and the JMMSE schemes. For a comparison purpose, the coded case is also simulated. $\alpha = 0.01$, $\varepsilon_{\max} = 0.15$ and ½ convolutional code are used. For the uncoded case, one can see that the JMMSE scheme provides better BER performance than the LJREC scheme at

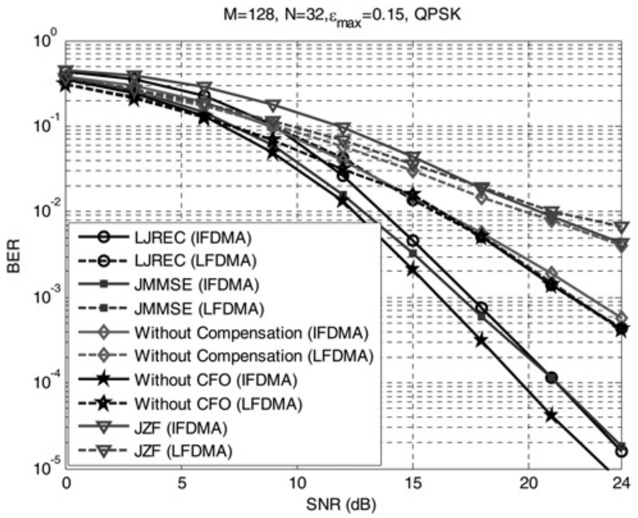

Fig. 8 *BER against the SNR for the LFDMA and the IFDMA systems with different CFOs compensation schemes*

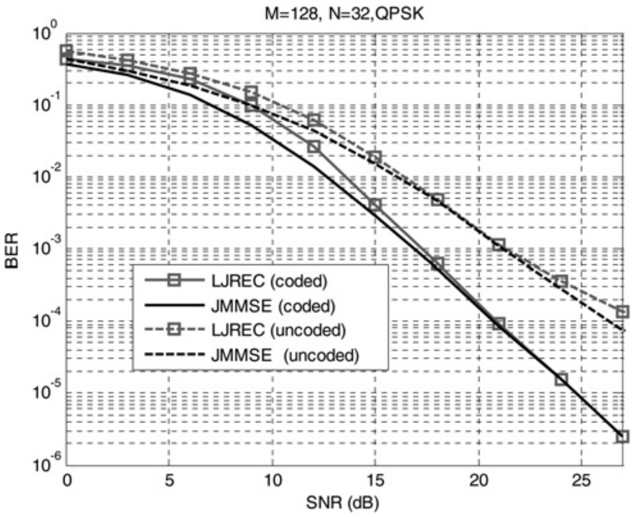

Fig. 9 *BER against the SNR for the IFDMA system with the LJREC and the JMMSE schemes for the uncoded case*

high SNR. Based on the coded and the uncoded results, it is found that the performance degradation at high SNR values can be compensated for by using an efficient coding scheme. On the other hand, the BER performance of the LJREC scheme in the presence of the coding scheme is the same as that of the JMMSE scheme, especially at moderate to high values of the SNR where the dominant is the interference.

5.4 Effect of the estimation errors

The effect of the estimation errors on the performance of the proposed LJREC scheme for the LFDMA and the IFDMA systems is studied and shown in Figs. 10 and 11. SNR = 15 dB, $\alpha = 0.01$, $\varepsilon_{max} = 0.15$ and ½ convolutional code are considered. The estimated CFOs are obtained by adding the true values of the CFOs to a zero-mean independent Gaussian random variable. The estimated channels are obtained by the same manner.

Fig. 10 shows that the performance of the LFDMA system with the proposed LJREC scheme starts to degrade as the standard deviation of the CFOs estimation error (δ_{CFO}) becomes larger than 0.1 and the standard deviation of the channel coefficients estimation error (δ_{Ch}) becomes larger than 0.01.

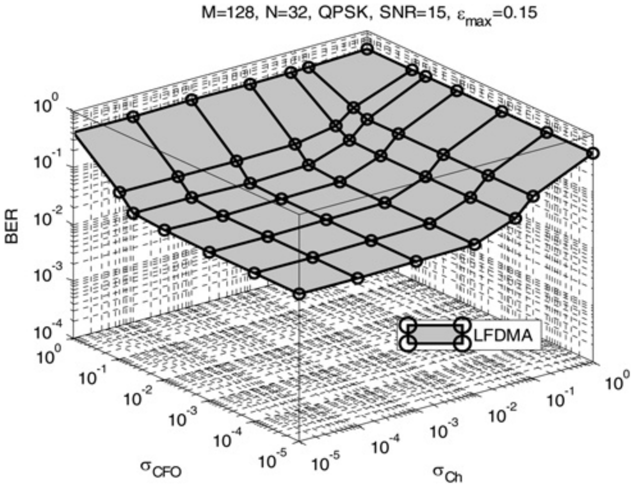

Fig. 10 *BER performance of the LFDMA system with the proposed LJREC scheme in the presence of the estimation errors at SNR = 15 dB and $\varepsilon_{max} = 0.15$*

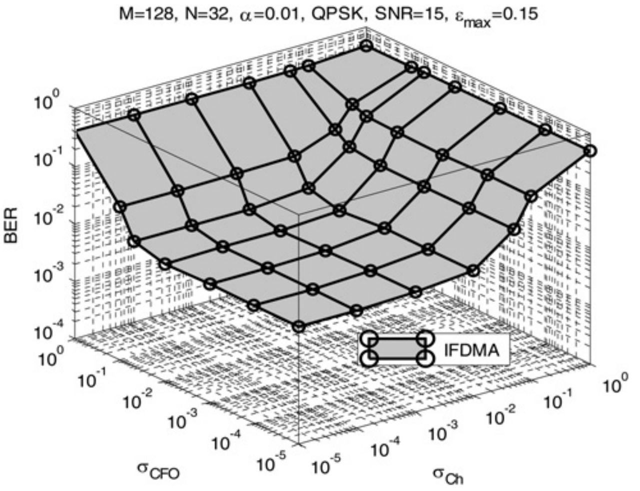

Fig. 11 *BER performance of the IFDMA system with the proposed LJREC scheme in the presence of the estimation errors at SNR = 15 dB and $\varepsilon_{max} = 0.15$*

Fig. 11 shows that the performance of the IFDMA system with the proposed LJREC schemes starts to degrade as δ_{CFO} becomes larger than 0.1 and δ_{Ch} becomes larger than 0.01. This indicates that the proposed LJREC scheme is robust to the estimation errors.

6 Conclusion

In this paper, we have introduced a new scheme to achieve the robust compensation of the multiple CFOs and the channels for the uplink SC-FDMA system. It has been shown that the complexity of the proposed scheme is of the order of $O(N)$, since it avoids the noise amplification problem and the estimation of the SNR and the interference matrices by adding a regularisation parameter. Also, it avoids the use of the direct matrix inversion method by taking advantage of the banded matrix approximation method. Simulation results have demonstrated that, for the uplink SC-FDMA systems with multiple CFOs, the proposed LJREC scheme achieves a good performance and it is more robust to the estimation errors. It has been found that the LJREC scheme provides the same performance as that of the JMMSE scheme and better performance than that of the JZF, the single-user and the circular-convolution schemes. Results have also shown that values between 0.1 and 0.01 are a suitable choice for the regularisation parameter, regardless of the SNR and the CFOs values.

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