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Analysis of carrier sense multiple access protocols for channels supporting multi-packet reception

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Abstract: Exact throughput analysis of unslotted carrier sense multiple access (CSMA) protocols for channels supporting multi-packet reception (MPR) is carried out. The protocols considered include (i) non-persistent and (ii) 1-persistent CSMA. An adaptive non-persistent CSMA protocol for channels supporting MPR has been proposed and evaluated. A Markov chain model with states representing the transmission period of the 1-persistent CSMA is used in the analysis. Analytical results are validated using simulation. The authors show that for CSMA protocols supporting MPR the throughput is crucially dependent on the carrier sensing delay.

1 Introduction

Traditional medium access control (MAC) protocols assume the 'collision channel model', wherein the packet reception is successful only when there is exactly one transmission. Newer physical (PHY) layer technologies such as multi-user multiple-input multiple-output (MU-MIMO) supports multi-packet reception (MPR), wherein more than one simultaneous transmission can be successfully decoded at the receiver [1]. The support for MU-MIMO and hence MPR in WLAN is now available, through the recent WLAN standard IEEE 802.11ac. The IEEE 802.11ac aims to provide throughputs of the order of gigabit per second [2] supporting applications such as wireless display, domestic HDTV distribution and large file transfer.

Traditional MAC protocols when used in channels supporting MPR grossly underestimate the channel capacity leading to inefficiency. Therefore MAC protocols need to be redesigned to leverage MPR capability. For a survey of MAC protocols supporting MPR please refer [3] and references therein. Recently modifications to IEEE 802.11 DCF protocol to support MPR have been proposed which include threshold based protocol [4], Ack-aware protocol [5] and adaptive backoff scheme [6].

It is of fundamental importance to rederive the performance metrics of the basic MAC protocols under MPR. Ghez *et al.* [7] have introduced a MPR channel model and studied the stability issues of slotted Aloha under MPR. Naware *et al.* [8] have studied the stability and delay of finite user slotted aloha under a symmetric MPR channel. Nagaraj *et al.* [9] derived an exact formula for the throughput of Aloha for channels having MPR capability $K = 2$ and approximate expression for the case of $K > 2$. An exact expression for the throughput of pure Aloha valid for all K has been derived in [10] using an order statistics based analysis. Chan *et al.* [11] have analysed the performance of carrier sense multiple access (CSMA) under MPR and has evaluated through simulation the early specification of IEEE 802.11ac standard for MPR enabled MU-MIMO capable PHY layer. Further enhancements to IEEE 802.11ac protocols to support QoS through adaptive backoff mechanism leveraging the MPR nature of channel have been proposed [12].

In [13] Chan and Berger proposed a cross-layer solution for MPR known as cross layer CSMA (XL-CSMA). Other previous studies on CSMA over MPR channel either focused on closed feedback CSMA [11, 14] or on slotted version of CSMA [15]. However the performance analysis of classic CSMA protocols for channels supporting MPR and an analytical expression for throughput valid for arbitrary K (MPR capability) is still an open problem. The purpose of the present paper is to fill this gap.

The paper is organised as follows. In Section 2 we provide the system and the channel model adopted by us. Analysis of the non-persistent CSMA (NP-CSMA) is taken up in Section 3. In Section 3 an expression for the throughput of NP-CSMA is derived and theoretical results are compared against simulation. This is followed by a discussion on the maximum achievable throughput of NP-CSMA. Subsequently an adaptive MPR NP-CSMA protocol is proposed. The Section 4 is devoted to the analysis of the 1-persistent CSMA (1P-CSMA). A Markov chain whose states capture the transmission periods (TPs) of the 1P-CSMA is proposed. Throughput of the 1P-CSMA is obtained by solving for the stationary state occupancy probability of the Markov chain. Throughput is then computed and compared against simulation results. Our conclusions are drawn in Section 5.

2 System and channel model

To proceed with the analysis we need to assume a channel model. We consider the case of a multiple access system wherein all nodes are in the reception range of each other. In the K-MPR channel model [3], a node will be able to receive all packets successfully as long as the number of packets transmitted in the channel is not greater than the MPR capability 'K'. In case the number of packets transmitted in the channel exceeds K, collision occurs and none of the transmitted packets can be received by the node. In the generalised MPR channel model, the channel is specified using a matrix whose elements indicate the probability of reception for different cases [7]. In our work we follow the K-MPR channel model with MPR capability K. The present work does not consider the effect of 'Capture'.

We assume the infinite user model. Let $t = \tau$ denote the time of arrival of the packet in consideration. Let the transmission time of a packet be 1 unit. Let a be the carrier sensing delay, assumed to be same for all nodes. That is, all other nodes will be able to sense the start of a transmission by a given node only after a seconds has elapsed. Let Λ be the aggregate arrival rate to the channel (new as well as rescheduled). In accordance with the infinite user model, the aggregate arrival process is assumed to be Poisson.

We now give an overview of the CSMA protocols before analysing them. The basic idea behind carrier sensing protocols is to 'listen before transmit'. Variants of the CSMA protocols include the non-persistent, p-persistent and 1-persistent [16]. Each of these protocols can operate in either slotted or unslotted mode. We confine our analysis to then unslotted versions of CSMA protocols. In carrier sensing protocols, collisions happen because of the delay in sensing the channel. The carrier sensing delay is a manifestation of the finite propagation delay of the signal. In general, throughput of carrier sensing protocol will be functions of the carrier sensing delay.

3 Non-persistent CSMA

In this section, we derive the throughput of NP-CSMA under MPR. In NP-CSMA, a node senses the channel and if the channel is free, the packet is transmitted immediately. If the channel is not free, the node reschedules the packet to a later random time.

3.1 Analysis of NP-CSMA with MPR

We extend the analysis of CSMA (for non-MPR) by Kleinrock and Tobagi [17] to the MPR channel. In this renewal theoretic analysis, first we identify the renewal intervals consisting of an idle period followed by a busy period. In an idle period, no transmission goes on in the channel. A busy period of NP-CSMA is defined as the time in which the channel is occupied with one or more transmissions followed by a duration a, wherein the idleness of the channel is not yet sensed by the nodes. A typical way of occurrence of the idle and busy periods is shown in Fig. 1. Since a number of transmissions can begin immediately after τ (the start time of the first transmission) but within a (the duration of the carrier sensing time), we can infer that there can be more than one transmission in a single busy period. In such a case, a busy period ends, when the last of such packets (initiated during the first a seconds) finishes its transmission.

In a K-MPR channel model, a transmission is successful if the number of additional transmissions initiated in the first a seconds of the busy period (vulnerable interval), is less than or equal to $K-1$ where K is the MPR capability of the channel. Therefore given a transmission, the probability that it becomes successful is given by

$$\Pr(\text{success}) = \sum_{i=0}^{K-1} \frac{(\Lambda a)^i \mathrm{e}^{-\Lambda a}}{i!} \tag{1}$$

We note that the average duration of the idle time is $(1/\Lambda)$, which follows from the memory less property of inter arrival times. Let Y be the random variable denoting the time interval between the beginning of the first transmission (τ) and the last arrival in the interval (τ, $\tau + a$).

The cumulative distribution function of Y, $F_Y(y) = \Pr(Y \le y)$ is the probability of no arrivals in time duration $\tau + y$ to $\tau + a$. It is given by

$$F_Y(y) = \exp[-\Lambda(a-y)] \tag{2}$$

Also, the mean of Y, $\bar{Y}$ is given by

$$\bar{Y} = a - \frac{1}{\Lambda}(1 - \mathrm{e}^{-a\Lambda})$$

The average duration of the busy interval is $1 + \bar{Y} + a$ (refer Fig. 1) and is equal to

$$1 + 2a - \frac{1}{\Lambda}(1 - \mathrm{e}^{-a\Lambda})$$

A cycle (renewal interval) consists of consecutive idle and busy times. The mean renewal cycle length is therefore

$$1 + 2a - \frac{1}{\Lambda}(1 - \mathrm{e}^{-a\Lambda}) + \frac{1}{\Lambda}$$

Let the reward during the renewal cycle equal the number of successfully transmitted packets in that cycle. The long run expected reward equals the normalised throughput. The throughput is normalised since the packet transmission time is unity. Then, using renewal reward theorem [18], the normalised throughput is given by

$$S = \frac{\sum_{i=0}^{K-1} (i+1)\Big(\big((\Lambda a)^i e^{-\Lambda a}\big)/(i!)\Big)}{(1/\Lambda) + 1 + 2a - (1/\Lambda)(1 - \mathrm{e}^{-a\Lambda})} \tag{3}$$

3.2 NP-CSMA results and discussions

The throughput of NP-CSMA given by (3) is plotted in Fig. 2 for different values of a with the MPR limit $K = 4$. The theoretical computation was carried out in Mathematica [19]. To validate the theoretical results, simulation of NP-CSMA was carried out using Python [20]. Theoretical results are compared with that of simulation results in Fig. 2.

We have used an ideal PHY layer model for the simulation. We have simulated the aggregated arrival process as a Poisson process following infinite user model. A packet transmission is successful, whenever the number of transmissions overlapping with the current transmission is less than or equal to the MPR capability K. In an ideal PHY, all packets which do not encounter a collision will be successfully received. In our simulation, we assume that all nodes will be able to sense the channel after a delay of a. Finally, the throughput is calculated as the number of successful packet transmissions per unit time. All packets are of same length taking 1 unit of time for transmission. All these assumptions are in line with the assumptions used in our theoretical derivations. The simulation were done for 10^5 packet transmissions.

The simulation results are found to be in good agreement with the theoretical results (Fig. 2). The results indicate that the throughput is very sensitive to the value of a and exhibits a peak. As a is decreased

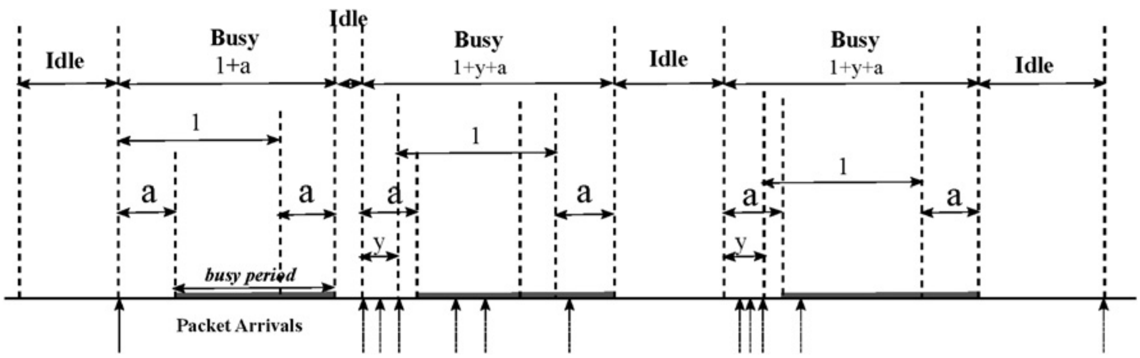

Fig. 1 *Illustrating the state of the channel against time of NP-CSMA*
Arrivals to a busy period are scheduled for transmission after a 'random' time

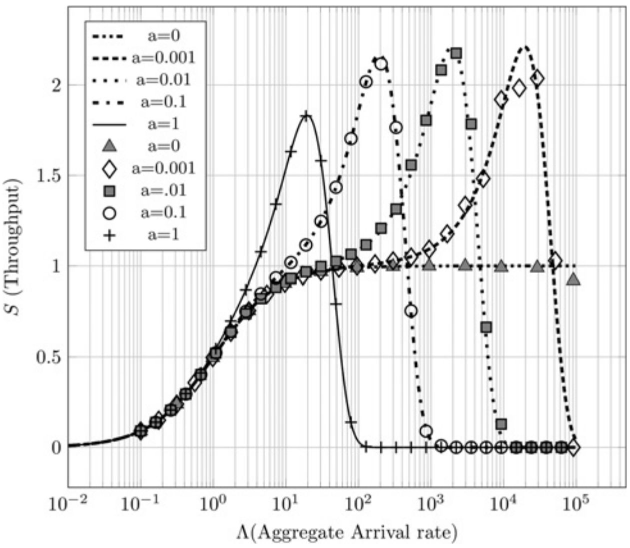

Fig. 2 *Normalised throughput of NP-CSMA against normalised aggregate arrival rate in packets per unit time on log scale for different values of carrier sensing delay (a)*

MPR limit is fixed at $K = 4$
Theory (lines) against simulation (symbols)

from 1, the peak value of the throughput increases. Further the peak occurs at higher values of arrival rate and flattens out for a tending to zero. Following is the explanation for the behaviour of the throughput of NP-CSMA for a MPR channel.

Assume for the sake of understanding that the carrier sensing delay a can be varied. First consider the case of traditional collision channel model with MPR capability K as 1. The throughput of the NP-CSMA for $K = 1$ is maximised under the following condition. As soon as a transmission is started in an idle channel, all other nodes should differ their attempt to avoid a collision. This would happen if the carrier sensing delay a is 0 (for all arrival rates). However for $K > 1$, the channel can support concurrent transmissions. Hence throughput increases as a increases from 0 for a given arrival rate. It reaches the peak when the expected number of transmission in the interval (τ to $\tau + a$) is around K. Further for smaller values of a, the peak in throughput should occur at higher arrival rates in order to make the expected number of transmissions to be the fixed value K.

The exact value at which the peak in throughput occurs depends upon the trade-off between the time wasted in idle period ($1/\Lambda$) and the time wasted because of collision (mean packet duration). However for a practical system a depends upon the nature of the multi access communication system. Therefore for a given a and MPR limit K the throughput curve exhibits a peak as the arrival rate is varied. We can also note from the curves corresponding to $a = 1$, 0.1, 0.01, 0.001 and 0.0001, that the peaks in the throughput occur at higher and higher arrival rates for systems with lower a.

3.3 Maximum achievable throughput of NP-CSMA

The expression for throughput given by (3) can be rewritten using the incomplete Gamma function [21] as below

$$S = \frac{\Lambda\, e^{a\Lambda}(a\Lambda + 1)\Gamma(K, a\Lambda) - \Lambda(a\Lambda)^K}{(2a + 1)\Lambda\, e^{a\Lambda}\Gamma(K) + \Gamma(K)} \tag{4}$$

For K, the MPR capability being an integer the Gamma function is given by $\Gamma(K) = (K - 1)!$, and the incomplete Gamma function is given by

$$\Gamma(K, a\Lambda) = (K - 1)!\, e^{-a\Lambda} \sum_{i=0}^{K-1} \frac{(a\Lambda)^i}{i!} \tag{5}$$

We are interested in finding the value of 'a' that maximises the throughput for a given arrival rate and K. For the same, the differential of S (given by (4)) with respect to a should be set to zero. Since the differential is complex, finding the maximum of the expression analytically and deriving a closed form expression is difficult. Therefore we have used Mathematica [19] to compute the maximum throughput which is plotted in Fig. 3.

Fig. 3*a* shows that for a given MPR limit the maximum achievable throughput decreases with a, which is also exhibited as flattening of the throughput curve as a tends to zero in Fig. 2. Fig. 3*b* shows that the arrival rates at which the maximum throughput occurs decreases exponentially with a. This means that if the carrier sensing delay is small, the arrival rates need to be increased to obtain higher throughput.

3.4 Adaptive MPR CSMA protocol

The insight obtained through the above discussions prompts us to propose a CSMA protocol for MPR channels described as follows. If an estimate of the arrival rate to the system is known the system

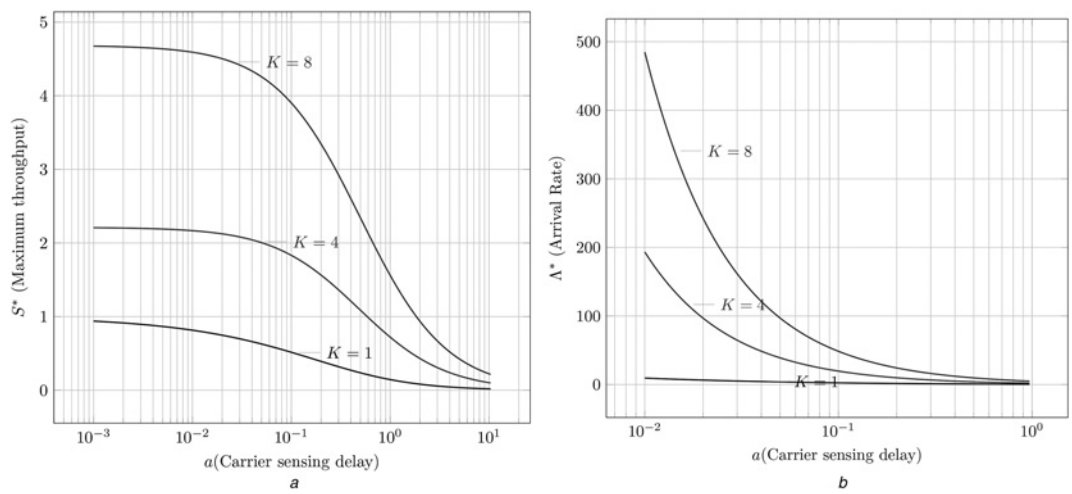

Fig. 3 *Effect of carrier sensing delay on*

a Maximum throughput
b Arrival rate at which throughput is maximum

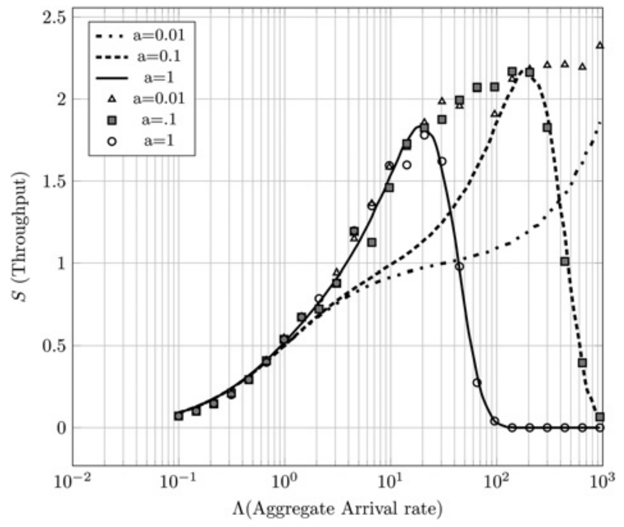

Fig. 4 *Normalised throughput of adaptive CSMA with MPR: NP-CSMA (lines) against Adaptive CSMA (symbols) for $K = 4$*

can calculate the value of a at which the throughput is maximised. Call this as $\hat{a}$. If this desirable value of vulnerable period $\hat{a}$ is more than the physical carrier sensing delay a then we can artificially extend the vulnerable period by allowing the nodes to continue transmitting packets even after sensing the channel to be busy for an additional duration of $\hat{a} - a$.

The adaptive MPR CSMA protocol can be implemented as follows. Let $a = f_k(\Lambda)$ denote a polynomial function which is the best fit for the curve Λ against a (for a given K) shown in Fig. 3*b*. The protocol estimates the arrival rate Λ at regular intervals and calculates the desired value of vulnerable period $\hat{a}$ as $f_k(\Lambda)$. If $\hat{a} < a$, then the transmission attempts are stopped as soon the carrier is sensed to be busy, else the transmission is continued for an additional duration of $\hat{a} - a$. Our numerical computation has shown that the function $a = f_k(\Lambda)$ is a one to one function, that allows us to compute a unique value of $\hat{a}$ for the estimated arrival rate Λ.

The throughput of adaptive CSMA and NP-CSMA are compared in Fig. 4. Throughput is plotted against arrival rates for three different values of carrier sensing delays. Adaptive CSMA shows throughput improvements over NP-CSMA for small values of a. The throughput improvements is more and is over a larger range for the case of $a = 0.01$ than that of for the case of $a = 0.1$. For the case of $a = 1$, with the given simulation parameters adaptive CSMA performs same as NP CSMA as adaptation is rarely invoked. This is because of the fact that the physical carrier sensing delay a is more as compared with the desired vulnerable period $\hat{a}$ over the range of arrival rate considered.

4 1-persistent CSMA with MPR

In 1-persistent CSMA, nodes sense the channel before transmission. If the channel is idle, the packet is transmitted right away. 1-persistent CSMA differs from NP-CSMA when the channel is sensed to be busy. When the channel is sensed to be busy, in a 1P-CSMA, the node waits for the channel to be idle and then transmit the packet as soon as the channel becomes idle.

4.1 *Markov chain based analysis*

We now analyse the 1P-CSMA under MPR conditions and derive its throughput using a Markov chain model. The analysis of the 1P-CSMA proceeds by extending the analysis of Sohraby *et al.* [22] for the non-MPR case of CSMA. The analysis proceeds by identifying three kinds of TP as the states of a Markov chain. The transition probabilities of the chain are computed, using which the stationary probabilities as well as the throughput is computed. The three kinds of TPs are

(i) An idle TP called Type 0.
(ii) A type 1 TP which follows the type-0 TP and starts with the transmission of a single packet.
(iii) A type 2 TP follows an arrival into a busy channel. A type-2 transmission may begin with more than one packet transmission.

The types of TPs are illustrated in Fig. 5, which is explained in the following. In Fig. 5 the busy period indicates the duration that is sensed to be busy by all nodes. Initially we have an idle (type 0) period. An arrival of a packet in an idle channel marks the beginning of type 1 period which extends for a duration $1 + a$, after which the channel is perceived as idle by all nodes. Note that the busy period (sensed as busy by all nodes) starts a time unit after the beginning of the first transmission and ends a time after the end of the last transmission. It so happens that after the first arrival, there are no further arrival in the vulnerable duration a. Further there are no arrivals in the first busy period. Therefore a type 0 or Idle period starts after the first busy period.

The second arrival takes place when the channel is idle and it starts transmitting. Since the third and fourth arrivals that occur in the vulnerable period have not yet sensed the transmission of 2, they also starts transmitting. The busy period starts from $t_2 + a$. The time of arrival of the last packet in the vulnerable period is denoted as y and it is $t_4 - t_2$ in the above case. The busy period extends for a duration of $1 + a$ from the time of last arrival t_4. Arrivals numbered 5–8 occurs during the second busy period. Therefore the second busy period is followed by a type 2 period. The duration of type 1 and type 2 periods are $1 + Y + a$.

Packets 5–8 will start their transmission as soon as they sense the channel to be idle, that is at the end of the second busy period. Packets 9–11 arriving during the vulnerable period would not sense the transmissions made by 5–8 and hence they also start their transmissions as soon as they arrive.

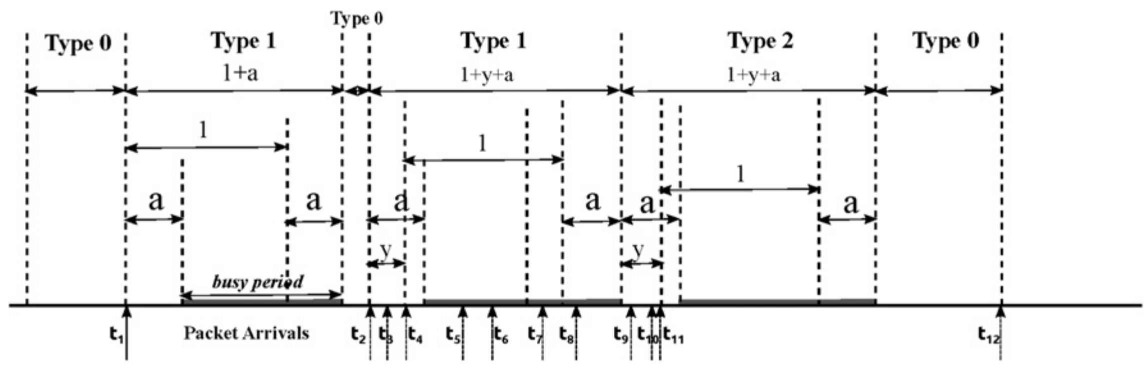

Fig. 5 *Illustrating the channel and time: 1P-CSMA*

Arrivals to a busy period are scheduled for transmission at the end of the current TP

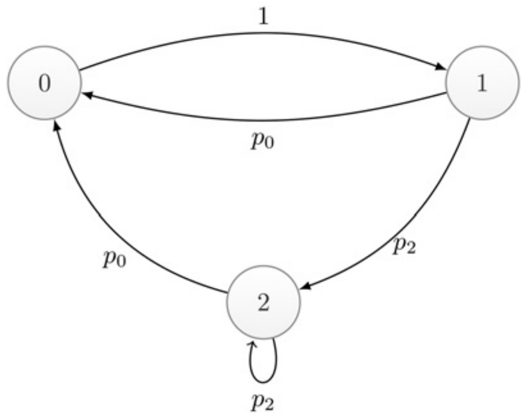

Fig. 6 *Markov chain for the TP of 1-persistent CSMA Protocol with MPR*

The state transition diagram with the TPs as states is shown in Fig. 6. We will shortly prove that Fig. 6 is a Markov chain. We derive the transitions probabilities and the dwell time for each state in the subsequent subsections. The results for non-MPR 1P-CSMA [22] is valid here because apart from the effect on the probability of collision, MPR and non-MPR CSMA protocols behave identically.

4.2 State transition probabilities of the Markov chain

The transition probabilities of the chain are indicated in Fig. 6. From the nature of the 1P-CSMA under MPR, the transition probabilities can be derived as follows. Every idle period (type 0) is always followed by a type 1 TP (ie) $P_{01} = 1$. It is also true that type 1 TPs can occur only at the end of an idle period. Now during a type 1 or type 2 TP, if no arrival takes place it is followed by a type 0 TP. Let the probability of occurrence of this event be p_0. If in TP1 or TP2 one or more packet(s) arrive (with a probability say p_2), they will find the channel to be busy and wait for the channel to become idle and then transmit together. Thus the next TP may start with the transmission of one or more packets (type 2).

Type 2, occurs whenever a packet arrives to a busy channel. In other words it can happen only at the end of a busy period (Type 1 or 2). Also, for both type 1 and type 2 the next state depends only on the number of arrivals in the current TP, which in turn depends only on the length of the interval (Poisson process). Since the length T_1 and T_2 are identically distributed we see that the transition probabilities to any state does not depend on the current state (for states 1 and 2). This justifies the Markovian nature of the chain shown in Fig. 6. This observation greatly reduces the number of transition probabilities to be found. Let us denote P_{j0} as p_0 for $(j = 1, 2)$ and P_{j2} as p_2 for $(j = 1, 2)$, and use it in further calculations of steady state occupancy probability.

4.3 Calculation of the steady state occupancy probability

Let us denote the steady state occupancy probabilities of states 0, 1 and 2 as π_0, π_1 and π_2, respectively. From local balance equation of the Markov chain, it immediately follows that

$$\begin{aligned} \pi_1 &= \pi_0 \\ \pi_0 &= \pi_1 p_0 + \pi_2 p_0 \\ \Rightarrow \pi_2 &= \frac{1 - p_0}{p_0}\pi_0 \end{aligned}$$

Normalising condition yields

$$\pi_0 = \pi_1 = \frac{p_0}{1 + p_0} \tag{6}$$

$$\pi_2 = \frac{1 - p_0}{1 + p_0} \tag{7}$$

Also for finding the transition probabilities, Sohraby *et al.* [22] identifies busy states to be success or collision states. However for MPR, even multiple transmissions does not necessarily result in collision. Therefore we do not consider the success/collision separately. Instead we condition on the value of the random variable Y, without requiring the success/collision information.

From the cdf of Y given in (2), it follows that the pdf of Y is given by

$$\begin{aligned} f_Y(y) &= \delta(y)\Pr(Y = 0) + \Lambda\,\mathrm{e}^{-\Lambda(a-y)} \\ &= \delta(y)\,\mathrm{e}^{-a\Lambda} + \Lambda\,e^{-\Lambda(a-y)}; \quad 0 \le y < a \end{aligned} \tag{8}$$

The random variable Y has a mixture distribution since it has non-zero probability at $Y = 0$. p_0 is given by

$$\begin{aligned} p_0 &= E_Y[\Pr\{\text{no arrival in}(1 + y)\}] \\ &= \int_{y=0}^{a} \mathrm{e}^{-\Lambda(1+y)} f_Y(y)\,\mathrm{d}y \\ &= \int_{y=0}^{a} \mathrm{e}^{-\Lambda(1+y)}(\delta(y)\,\mathrm{e}^{-a\Lambda} + \Lambda\,\mathrm{e}^{-\Lambda(a-y)})\,\mathrm{d}y \end{aligned}$$

We obtain

$$p_0 = (1 + a\Lambda)\,\mathrm{e}^{-\Lambda(1+a)} \tag{9}$$

Finally the equations of π_0, π_1 and π_2 can be written as

$$\pi_0 = \pi_1 = \frac{(1 + a\Lambda)\,\mathrm{e}^{-\Lambda(1+a)}}{1 + (1 + a\Lambda)\,\mathrm{e}^{-\Lambda(1+a)}} \tag{10}$$

$$\pi_2 = \frac{1 - (1 + a\Lambda)\,\mathrm{e}^{-\Lambda(1+a)}}{1 + (1 + a\Lambda)\,\mathrm{e}^{-\Lambda(1+a)}} \tag{11}$$

4.4 Dwell time in different states of the Markov chain

Let $E[T_i]$, be the average dwell time in the state i. First we note that, $E[T_0]$, the average idle duration is $(1/\Lambda)$. Now the duration of type 1 and type 2 periods ($E[T_1]$ and $E[T_2]$) are identically distributed as it depends only on Y. Thus we have

$$E[T_1] = E[T_2]$$

To find dwell times at each slot

$$\begin{aligned} E[T_1 | Y = y] &= 1 + a + y \\ \Rightarrow E[T_1] &= 1 + a + E[Y] \end{aligned}$$

Thus, we have, using value of E[Y] derived in Section 3.1

$$E[T_1] = E[T_2] = 1 + 2a - \frac{1 - \mathrm{e}^{-a\Lambda}}{\Lambda} \tag{12}$$

4.5 Throughput during different TPs

Let $\hat{S}_i$ indicate the throughput in the TP numbered i. In a type 0 TP, during which the channel is idle, the throughput will be 0. That is $\hat{S}_0 = 0$.

In a type 1 period, the throughput depends on the number of successful packet transmissions during that period. Thus the throughput is equal to the number of packets arriving in the vulnerable period (first a seconds) in case of success and zero for the case of collision. If we denote the expected value of the number of successful packets during a type 1 TP by $\hat{S}_1$, then we have

$$\hat{S}_1 = \sum_{i=0}^{K-1} (i+1)(a\Lambda)^i \frac{e^{-a\Lambda}}{i!} \tag{13}$$

The case of a type 2 TP is different. Here, the number of transmissions initiated is dependent on the number of arrivals that has taken place during the previous TP. The length of the TP is dependent on the value of the random variable Y (time of arrival of the last packet in the vulnerable period). Now, if we condition on the value of random variable $Y = y$ for the preceding TP, it will have a length of $a + 1 + y$. Any packet arriving in the interval of length $1 + y$, will be transmitted in the next TP (at the end of the current TP).

Further the arrivals during the first a seconds of the type-2 transmission will also be transmitted in the same TP. Let i denote the number of arrivals in the interval $1 + y$, and let j denote the number of arrivals in the vulnerable period a. In the example illustrated in Fig. 5 the value of $i = 4$ (t_5 to t_8) and $j = 3$ (t_9 to t_{11}). Note that $i + j$ packets will be successful if $i + j \le K$. Type 2 period is conditioned by the fact that there has to be at least one arrival in the interval $1 + y$. The throughput in a type-2 period, $\hat{S}_2$ conditioned by $Y = y$ is given by

$$E[\hat{S}_2 | Y = y] = \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{\Pr(i \text{ arrivals in } 1+y, \quad j \text{ arrivals in } a)}{\Pr(\text{at least one arrival in } 1+y)} \tag{14}$$

Since Poisson arrivals in non-overlapping intervals $(1 + y)$ and a are independent and having the same arrival rate Λ, we have

$$\begin{aligned} E[\hat{S}_2 | Y = y] &= \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{[(1+y)\Lambda]^i \left(\left(e^{-(1+y)\Lambda} \right) / (i!) \right) (a\Lambda)^j \left((e^{-a\Lambda}) / (j!) \right)}{1 - e^{-(1+y)\Lambda}} \\ &= \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{a^j \Lambda^{i+j} (1+y)^i \left((e^{-(1+y)\Lambda}) / (i!) \right) \left((e^{-a\Lambda}) / (j!) \right)}{1 - e^{-(1+y)\Lambda}} \end{aligned} \tag{15}$$

Now, unconditioning on Y, and noting that Y has mixture distribution with a non-zero probability at $Y = 0$, we obtain

$$\begin{aligned} E[\hat{S}_2] &= \Pr(Y=0) \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{a^j \Lambda^{i+j} \left(e^{-\Lambda}/i! \right) \left(e^{-a\Lambda}/j! \right)}{1 - e^{-\Lambda}} \\ &+ \int_{y=0}^{a} \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{a^j \Lambda^{i+j} (1+y)^i \left((e^{-(1+y)\Lambda}) / (i!) \right) \left(e^{-a\Lambda}/j! \right)}{1 - e^{-(1+y)\Lambda}} f_Y(y) \mathrm{d}y \\ &= e^{-a\Lambda} \sum_{i=1}^{K} \sum_{j=0}^{K-i} (i+j) \frac{a^j \Lambda^{i+j} \left(e^{-\Lambda}/i! \right) \left(e^{-a\Lambda}/j! \right)}{1 - e^{-\Lambda}} \\ &+ \sum_{i=1}^{K} \sum_{j=0}^{K-i} \frac{(i+j) a^j \Lambda^{i+j} e^{-(1+2a)\Lambda}}{i! j!} \int_{y=0}^{a} \frac{(1+y)^i}{1 - e^{-(1+y)\Lambda}} \mathrm{d}y \end{aligned} \tag{16}$$

The throughput can be written as (17). The (17) follows from the renewal-reward theorem. Identify the beginning of the TPs as renewal points. The expected duration of renewal cycle is obtained by the sum of product of expected dwell time in each state (transition periods) multiplied by the steady state occupancy probability. The expected reward in a renewal cycle is the weighted average of the throughput in each state

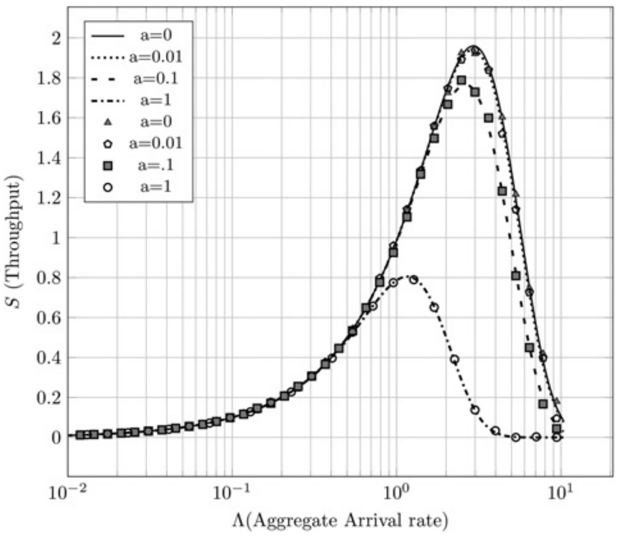

Fig. 7 *Normalised throughput of 1P-CSMA with MPR: Analysis (lines) against simulation (symbols) for K = 4*

$$S = \frac{\sum_{i=0}^{2} \pi_i \hat{S}_i}{\sum_{i=0}^{2} \pi_i E[T_i]} \tag{17}$$

Finally, the throughput is obtained by, substituting the stationary probabilities (10) to (11) in (17). For a general K (17) has to be computed numerically because of the complex nature of various terms involved and because of the presence of an integral in $\hat{S}_2$. However for the simpler case of $K = 1$, one can obtain an approximate but closed form expression for S. We carry out this exercise in Appendix.

1P-CSMA protocol is simulated with infinite user model and the throughput performance is plotted in Fig. 7. The Fig. 7 shows that the performance characteristics of 1P-CSMA are different from that of NP-CSMA protocol. It can be noted that for the range of a from 0 to 0.1 ($a \ll 1$) the 1P-CSMA throughput is not very sensitive to variation in the sensing delay. When the arrival rates of relatively larger, Type 2 TPs are more probable. $E[T_2] = 1 + a + E[y]$ and hence the throughput is not very sensitive to the value of a. The case of $a = 1$ is an extreme case where the packet transmission time is equal to the propagation delay. Although the case of $a = 1$ is uncommon in WLANs, it may be encountered in satellite network.

The persistent CSMA performance is inferior to the NP-CSMA for moderate to high values of arrival rates. This is expected since the persistent transmissions in the case of 1P-CSMA protocol increases the probability of collisions. The maximum throughput of 1P-CSMA remains however, close to the maximum throughput of NP-CSMA. Compared with the NP-CSMA, in the low arrival rate region, 1P-CSMA has higher throughput (see 0.1 to 1 range of arrival rate in Figs. 2 and 7) since the number of transmission opportunities wasted will be lesser in the case of 1P CSMA as compared with NP CSMA.

5 Conclusion

A clear understanding of the role of MPR on the performance of basic MAC protocols (Aloha, CSMA etc.) is fundamental to the design of MAC protocols for next generation WLANs. In this paper, we have analysed the carrier sensing protocols for MPR channels. The throughput performance of NP-CSMA and

1P-CSMA were computed for an infinite user model. This study sheds light on the general nature of performance characteristics of carrier sensing protocols in MPR channels. One difference from traditional channels is that the throughput does not monotonically decrease with the increase in sensing delay. In fact throughput is critically dependent upon arrival rates, MPR capability and the carrier sensing delay.

6 References

1 Gesbert, D., Kountouris, M., Heath, R.W., Chae, C.-B., Salzer, T.: 'Shifting the mimo paradigm', *IEEE Signal Process. Mag.*, 2007, **24**, (5), pp. 36–46
2 Bejarano, O., Knightly, E.W., Park, M.: 'IEEE 802.11 Ac: From channelization to multi-user MIMO', *IEEE Commun. Mag.*, 2013, **51**, (10), p. 85
3 Lu, J.-L., Shu, W., Wu, M.-Y.: 'A survey on multipacket reception for wireless random access networks', *J. Comput. Netw. Commun.*, 2012, **2012**, pp. 1–14, doi:10.1155/2012/246359
4 Babich, F., Comisso, M.: 'Theoretical analysis of asynchronous multi-packet reception in 802.11 networks', *IEEE Trans. Commun.*, 2010, **58**, (6), pp. 1782–1794
5 Mukhopadhyay, A., Mehta, N.B., Srinivasan, V.: 'Acknowledgement-aware MPR MAC protocol for distributed WLANS: Design and analysis'. IEEE GLOBECOM, 2012, pp. 5087–5092
6 Arun, I.B., Venkatesh, T.G.: 'Adaptive backoff algorithm for IEEE 802.11 DCF under MPR wireless channels'. Int. Conf. on Computer Communications and Networks (ICCCN), Bahamas (978-1-4673-5773-3/13), 2013
7 Ghez, S., Verdu, S., Schwartz, S.: 'Stability properties of slotted aloha with multipacket reception capability', *IEEE Trans. Autom. Control*, 1988, **33**, (7), pp. 640–649
8 Naware, V., Mergen, G., Tong, L.: 'Stability and delay of finite-user slotted aloha with multipacket reception', *IEEE Trans. Inf. Theory*, 2005, **51**, (7), pp. 2636–2656
9 Nagaraj, S., Truhachev, D., Schlegel, C.: 'Analysis of a random channel access scheme with multi-packet reception'. IEEE GLOBECOM, 2008, pp. 1–5
10 Arun, I., Venkatesh, T.: 'Order statistics based analysis of pure aloha in channels with multipacket reception', *IEEE Commun. Lett.*, 2013, **17**, (10), pp. 2012–2015
11 Chan, D., Berger, T., Tong, L.: 'Carrier sense multiple access communications on multipacket reception channels: Theory and applications to IEEE 802.11 wireless networks', *IEEE Trans. Commun.*, 2013, **61**, (1), pp. 266–278
12 Arun, I.B., Venkatesh, T.G., Bhasker, D.: 'QoS provisioning with adaptive backoff algorithm for IEEE 802.11ac under multipacket reception'. IEEE/IET Int. Symp. on Communications Systems Networks and Digital Signal Processing. (CSNDSP), 2014
13 Chan, D.S., Berger, T.: 'Performance and cross-layer design of CSMA for wireless networks with multipacket reception'. Conf. Record of the 38th Asilomar Conf. on Signals, Systems and Computers, 2004. vol. 2, pp. 1917–1921
14 Chan, D.S., Berger, T., Tong, L.: 'On the stability and optimal decentralized throughput of CSMA with multipacket reception capability'. Proc. of Allerton Conf. on Communication, Control, and Computing, 2004
15 Gau, R.-H.: 'Modeling the slotted nonpersistent CSMA protocol for wireless access networks with multiple packet reception', *IEEE Commun. Lett.*, 2009, **13**, (10), pp. 797–799
16 Bertsekas, D.P., Gallager, R.G., Humblet, P.: 'Data networks' (Prentice-Hall International, 1992), vol. 2
17 Kleinrock, L., Tobagi, F.: 'Packet switching in radio channels: Part I–carrier sense multiple-access modes and their throughput-delay characteristics', *IEEE Trans. Commun.*, 1975, **23**, (12), pp. 1400–1416
18 Sheldon, M.R.: 'Introduction to probability models' (Academic Press, 2010)
19 Wolfram, S.: 'The MATHEMATICA® Book, Version 4' (Cambridge University Press, 1999)
20 'Python Language Reference, version 2.7'. http://www.python.org, 2014
21 Arfken, G.B., Weber, H.J., Harris, F.E.: 'Mathematical methods for physicists: a comprehensive guide' (Academic press, 2011)
22 Sohraby, K., Molle, M., Venetsanopoulos, A.: 'Comments on 'throughput analysis for persistent CSMA systems'', *IEEE Trans. Commun.*, 1987, **35**, (2), pp. 240–243

7 Appendix

7.1 Throughput of 1-P CSMA for collision channel

In this appendix we derive a closed form expression for the throughput of 1P-CSMA for the case of classical collision channel model. The classical collision channel model is equivalent to the k-MPR channel model with $K = 1$.

Equation (17) for S holds good for $K = 1$ with appropriate values for the terms π_i, $\hat{S}_i$ and $E[T_i]$ for $i = 1$, 2 and 3. The equations for π_0, π_1, π_2 are given by (10) and (11). $E[T_1]$ and $E[T_2]$ are given by (12) while $E[T_0] = (1/\Lambda)$. Further, $\hat{S}_0 = 0$. Only $\hat{S}_1$ and $\hat{S}_2$ remains to be

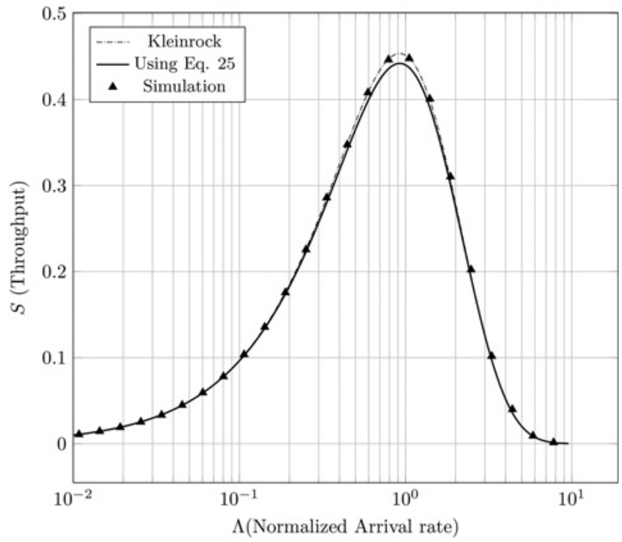

Fig. 8 *Normalised arrival rate against normalised throughput of 1P-CSMA with K = 1 and a = 0.1*

Our analysis, results of [17] and simulation

derived for $K = 1$. From (13), for $K = 1$ we obtain

$$\hat{S}_1 = \sum_{i=0}^{K-1} (i+1)(a\Lambda)^i \frac{\mathrm{e}^{-a\Lambda}}{i!} = \mathrm{e}^{-a\Lambda} \tag{18}$$

To obtain a closed form expression for $\hat{S}_2$, instead of using (16), we use an alternate derivation. Recall that $\hat{S}_2$ is the throughput in type-2 TP, which occurs as a result of one or more arrivals in a busy period.

$$\Pr(i \text{ transmission are made at the beginning of type}-2\text{ TP}|Y = y) = \frac{[(1+y)\Lambda]^i\,\mathrm{e}^{-(1+y)\Lambda}}{i!(1-\mathrm{e}^{-(1+y)\Lambda})} \tag{19}$$

Equation (19) follows from the fact that type-2 TP implies that there is atleast one arrival during the preceding TP. Unconditioning on Y in (19) we obtain

$$\Pr(i \text{ transmissions are made at the beginning of type}-2\text{ TP}) = \int_0^a \frac{[(1+y)\Lambda]^i\,\mathrm{e}^{-(1+y)\Lambda}}{i!(1-\mathrm{e}^{-(1+y)\Lambda})} f_Y(y)\,\mathrm{d}y \tag{20}$$

Let there be i arrivals in the duration $1 + y$, and j arrivals during the vulnerable duration a. Then

$$E[\hat{S}_2] = \sum_{i=1}^{K}\sum_{j=0}^{K-i} \Pr(i \text{ in } 1+y)\Pr(j \text{ arrivals in duration } a) \tag{21}$$

$$= \sum_{i=1}^{K}\sum_{j=0}^{K-i} \int_0^a \frac{[(1+y)\Lambda]^i\,\mathrm{e}^{-(1+y)\Lambda}}{i!(1-\mathrm{e}^{-(1+y)\Lambda})} \frac{\mathrm{e}^{-a\Lambda}(a\Lambda)^j}{j!} f_Y(y)\,\mathrm{d}y \tag{22}$$

For $K = 1$, substituting $i = 1$ and $j = 0$

$$E[\hat{S}_2] = \left\{ \frac{a(a+2)\Lambda^2}{2\left(\mathrm{e}^{(a+1)\Lambda}-1\right)} + \frac{\Lambda\,\mathrm{e}^{-(a+1)\Lambda}}{1-\mathrm{e}^{-(a+1)\Lambda}} \right\} \mathrm{e}^{-a\Lambda} \tag{23}$$

To compute the integral in (22), we have made an approximation $y = a$ in the denominator of the integral. Substituting (18), (23) and other known values in (17), the value of S after approximation and simplification with the help of Mathematica yields (see (25))

The form of throughput given by (24) differs in the function from the formula given in [17]. However the correctness of the analytical results is verified by its close match with both the formulas and the simulation as shown in Fig. 8. The slight deviation of our result (25) in Fig. 8 is because of the approximation invoked in the calculation of the integral.

$$S_{K=1} = \frac{\mathrm{e}^{-a\Lambda}\left(\Lambda\,\mathrm{e}^{(a+1)\Lambda}(\Lambda(a((a+2)\Lambda+2)+2)+2) - \Lambda(a\Lambda+1)(\Lambda(a(a+2)\Lambda+2)+2)\right)}{2\left(\mathrm{e}^{(a+1)\Lambda}-1\right)\left(a\Lambda+\mathrm{e}^{(a+1)\Lambda}(2a\Lambda+\Lambda-1)+\mathrm{e}^{\Lambda}+1\right)} \tag{24}$$