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Performance analysis of non-orthogonal multiple access in downlink cooperative network

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Abstract: Non-orthogonal multiple access (NOMA), an emerging technology to improve system capacity and spectrum efficiency, has attracted significant attention. In this study, the authors propose a NOMA-based downlink cooperative cellular system, where the base station communicates with two paired mobile users simultaneously through the help of a half-duplex amplify-and-forward relay. The outage performance of the system is investigated and closed-form expressions for their respective exact and asymptotic outage scheme are derived. Furthermore, they study the ergodic sum rate of the two paired users and the upper bound of the ergodic sum rate is obtained. By comparing the NOMA with conventional multiple access (MA) via numerical simulations, they have shown that NOMA can obtain the same diversity order with conventional MA, and achieve nearly the same sum rate with conventional MA. Furthermore, NOMA can offer better spectral efficiency and user fairness since more users are served at the same time, frequency, and spreading code.

1 Introduction

To achieve significant gains in capacity and quality of user experience, the research defining the fifth generation (5G) networks have received considerable attention [1]. Non-orthogonal multiple access (NOMA) is considered to be a potential candidate for 5G multiple access (MA) [2–4]. NOMA is fundamentally different from conventional orthogonal MA schemes, where multiple users are multiplexed in the same time domain, frequency domain, and code domain, but in different power domain. Multi-user signal separation on the receiver side is conducted based on successive interference cancellation (SIC) where the users with better channel conditions need to decode the messages for the others before decoding their own [5]. The system-level performance of cellular downlink NOMA was investigated in [6], and a weighted proportional fair-based multi-user scheduling scheme was proposed to achieve a good tradeoff between the total user throughput and cell-edge user throughput. In [4], the outage behaviour and the ergodic sum rate were studied for a cellular downlink NOMA scenario with randomly deployed users. In [7], the author studied the impact of user pairing on the performance of NOMA systems. In [8], a cooperative NOMA transmission scheme was proposed. Since the users with better channel conditions can decode the messages for the others, and therefore these users can be used as relays to enhance the transmit reliability for the users with poor channel conditions.

In addition, cooperative relaying has attracted lots of attention, owning to its ability to improve the throughput and transmit reliability of wireless network. Therefore, it is considerable to design NOMA for cooperative relaying and this motivates our work.

In this paper, we consider a downlink cooperative cellular scenario, where the base station communicates with multiple mobile users simultaneously through the help of a half-duplex amplify-and-forward (AF) relay. We assume that the mobile users in the system are ordered according to the channel quality of direct links. In NOMA system, multiple users are admitted at the same time, frequency, and spreading code, hence co-channel interference will be strong. Consequently, asking all the users in the system to perform NOMA jointly may not be realistic and can incur a high complexity. A promising solution is to build a hybrid MA system, in which NOMA is combined with conventional MA. In particular, the users in the system can be divided into multiple groups, where NOMA will be implemented among the users within each group and conventional orthogonal multiple access (OMA) can be used for inter-group MA. Without loss of generality, we consider two users are paired together in a group to perform NOMA, and the user with better (worst) channel quality of direct link, that is, the strong user (the weak user), will be allocated less (more) power.

The main contributions of this paper are summarised as follows:

(i) We combine NOMA with cooperative relaying system to improve the system performance. The outage behaviour of the users in a group is investigated and closed-form expressions for the outage probability of the users are derived. Furthermore, the asymptotic outage behaviour in high signal-to-noise ratio (SNR) regions is studied and the diversity order of the user is obtained. Simulation results show that NOMA can obtain the same diversity order with conventional MA.
(ii) We analyse the ergodic sum rates of the two users in a group and the upper bound of the ergodic sum rate is attained. Simulation results demonstrate that NOMA can achieve nearly the same sum rate with conventional MA.

The remainder of this paper is organised as follows: in Section 2, the system model is presented. In Section 3, we investigate the outage behaviour of the system. In Section 4, the ergodic sum rate of the users in a group is analysed. In Section 5, numerical results verify the superiority of NOMA. Finally, Section 6 concludes this paper.

2 System model

Consider a downlink cooperative cellular scenario as shown in Fig. 1. One base station S intends to transmit information to M mobile users with the help of a relay R and the direct links between base station and mobile users exist. We assume that all the nodes are single-antenna devices and operate in a half-duplex mode. For mathematical tractability, we restrict our attention primarily to a homogeneous network topology, where all wireless

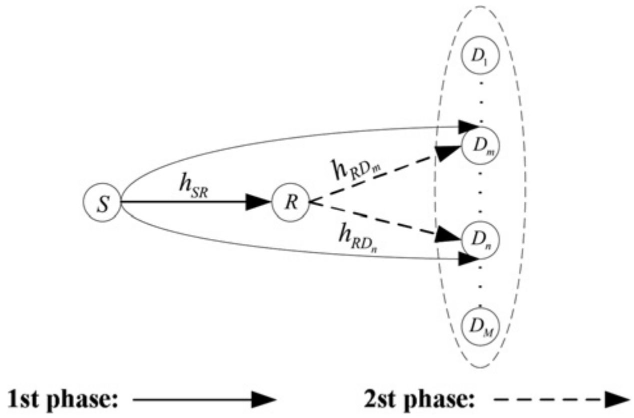

Fig. 1 *NOMA downlink cooperative system*

links exhibit frequency non-selective Rayleigh block fading and additive white Gaussian noise (AWGN). The complex channel coefficient between S and R is denoted by $h_{\rm SR} \sim \mathcal{CN}(0,\ \Omega_{\rm SR})$, the complex channel coefficient between R and D_k is denoted by $h_{{\rm RD}_k} \sim \mathcal{CN}(0,\ \Omega_{RD})$, and the complex channel coefficient between S and D_k is denoted by $h_{{\rm SD}_k} \sim \mathcal{CN}(0,\ \Omega_{SD})$. Without loss of generality, assume that the users' channels have been ordered as $|h_{{\rm SD}_1}|^2 \le |h_{{\rm SD}_2}|^2 \le \cdots \le |h_{{\rm SD}_{\rm M}}|^2$. Consider that the mth user and the nth user, $m < n$, are paired together to perform NOMA. The transmit power at S and R are denoted by $P_{\rm S}$ and $P_{\rm R}$, respectively.

In this paper, we use the conventional AF scheme for NOMA downlink cooperative system. Two consecutive phases are involved to complete the information exchange.

In the first phase, the base station S broadcasts $x_{\rm S}$, whereas R, D_m, and D_n listen. $x_{\rm S}$ is a summation of the coded modulation symbol of the two users, and can be expressed as $x_{\rm S} = \sqrt{a_m P_{\rm S}} x_m + \sqrt{a_n P_{\rm S}} x_n$ ($a_m > a_n$), where a_m and a_n are the power allocation coefficient and ${a_m}^2 + {a_n}^2 = 1$. The received signals at R, D_m, and D_n are given by

$$y_{\rm R} = h_{\rm SR} x_{\rm S} + n_{\rm R} = h_{\rm SR}\left(\sqrt{a_m P_{\rm S}} x_m + \sqrt{a_n P_{\rm S}} x_n\right) + n_{\rm R}, \quad (1)$$

$$y_{D_m} = h_{{\rm SD}_m} x_{\rm S} + n_{D_m} = h_{{\rm SD}_m}\left(\sqrt{a_m P_{\rm S}} x_m + \sqrt{a_n P_{\rm S}} x_n\right) + n_{D_m}, \quad (2)$$

$$y_{D_n} = h_{{\rm SD}_n} x_{\rm S} + n_{D_n} = h_{{\rm SD}_n}\left(\sqrt{a_m P_{\rm S}} x_m + \sqrt{a_n P_{\rm S}} x_n\right) + n_{D_n}, \quad (3)$$

where $n_{\rm R} \sim \mathcal{CN}(0,\ {\sigma_{\rm R}}^2)$ and $n_{D_i} \sim \mathcal{CN}(0,\ {\sigma_D}^2)$, $i = m,\ n$ denote the AWGN.

In the second phase, R broadcasts $y_{\rm R}$ after multiplying with an amplifying gain $G = \sqrt{P_{\rm R}/(P_{\rm S}|h_{\rm SR}|^2 + {\sigma_{\rm R}}^2)}$. Therefore, the received signals at D_m and D_n can be expressed as

$$y_{{\rm RD}_m} = G h_{{\rm RD}_m} h_{\rm SR}\left(\sqrt{a_m P_{\rm S}} x_m + \sqrt{a_n P_{\rm S}} x_n\right) + G h_{{\rm RD}_m} n_{\rm R} + n_{{\rm RD}_m}, \quad (4)$$

$$y_{{\rm RD}_n} = G h_{{\rm RD}_n} h_{\rm SR}\left(\sqrt{a_m P_{\rm S}} x_m + \sqrt{a_n P_{\rm S}} x_n\right) + G h_{{\rm RD}_n} n_{\rm R} + n_{{\rm RD}_n}, \quad (5)$$

where $n_{{\rm RD}_i} \sim \mathcal{CN}(0,\ {\sigma_{\rm RD}}^2)$, $i = m,\ n$, denote the AWGN. With loss of generality, we assume ${\sigma_{\rm R}}^2 = {\sigma_D}^2 = {\sigma_{\rm RD}}^2 = \sigma^2$, $P_{\rm S} = P_{\rm R} = P$, and define $\gamma \triangleq P/\sigma^2$ to represent the average system SNR.

Therefore, in the first phase the instantaneous signal-to-interference-and-noise ratio (SINR) at D_m can be given as

$$\gamma_{{\rm SD}_m} = \frac{a_m \gamma |h_{{\rm SD}_m}|^2}{a_n \gamma |h_{{\rm SD}_m}|^2 + 1}. \quad (6)$$

SIC will be carried out at D_n. The nth user will detect the mth user's message and then remove the message from its observation. The instantaneous SINR for the nth user to detect the mth user's message is given by

$$\gamma_{{\rm SD}_{m\to n}} = \frac{a_m \gamma |h_{{\rm SD}_n}|^2}{a_n \gamma |h_{{\rm SD}_n}|^2 + 1}, \quad (7)$$

and the instantaneous SINR at D_n can be given as

$$\gamma_{{\rm SD}_n} = a_n \gamma |h_{{\rm SD}_n}|^2. \quad (8)$$

In the second phase, the instantaneous SINR of the mth user related to link $R \to D_m$ can be expressed as

$$\gamma_{{\rm RD}_m} = \frac{a_m \gamma^2 |h_{{\rm RD}_m}|^2 |h_{\rm SR}|^2}{a_n \gamma^2 |h_{{\rm RD}_m}|^2 |h_{\rm SR}|^2 + \gamma\left(|h_{{\rm RD}_m}|^2 + |h_{\rm SR}|^2\right) + 1}. \quad (9)$$

The instantaneous SINR at D_n for the nth user to detect the mth user's message and the instantaneous SINR of the nth user related to link $R \to D_n$ can be expressed as

$$\gamma_{{\rm RD}_{m\to n}} = \frac{a_m \gamma^2 |h_{{\rm RD}_n}|^2 |h_{\rm SR}|^2}{a_n \gamma^2 |h_{{\rm RD}_n}|^2 |h_{\rm SR}|^2 + \gamma\left(|h_{{\rm RD}_n}|^2 + |h_{\rm SR}|^2\right) + 1}, \quad (10)$$

and

$$\gamma_{{\rm RD}_n} = \frac{a_n \gamma^2 |h_{{\rm RD}_n}|^2 |h_{\rm SR}|^2}{\gamma\left(|h_{{\rm RD}_n}|^2 + |h_{\rm SR}|^2\right) + 1}, \quad (11)$$

respectively. Finally, the user terminals process the received signals during the two-phase transmission by using selection combining.

3 Outage behaviour

In this section, we consider the target SINRs of the two users are determined by the users' quality of service (QoS) requirements, that is, each user has a preset target SINR, $\gamma{\rm th}_i$, $i = m,\ n$. In this case, it is important to examine the probability of the two paired users.

3.1 *Exact outage behaviour*

From the scheme descriptions in the last section, an outage event occurs if neither the direct transmission nor the relaying transmission succeeds. Therefore, the outage probability of the mth user can be expressed as

$$\begin{aligned}\mathcal{P}_{\rm out}^m &= \Pr\left(\max\left[\gamma_{{\rm SD}_m},\ \gamma_{{\rm RD}_m}\right] < \gamma{\rm th}_m\right) \\ &= \Pr(\gamma_{{\rm SD}_m} < \gamma{\rm th}_m)\Pr(\gamma_{RD_m} < \gamma{\rm th}_m).\end{aligned} \quad (12)$$

With the aid of order statistics, the pdf of $|h_{{\rm SD}_m}|^2$ is given by [9]

$$f_{|h_{{\rm SD}_m}|^2}(x) = \frac{M!}{(M-m)!(m-1)!} f_{|h_{SD}|^2}(x) \times \left[F_{|h_{SD}|^2}(x)\right]^{m-1}\left[1 - F_{|h_{SD}|^2}(x)\right]^{M-m}$$

$$
= \frac{M!}{(M-m)!(m-1)!}\frac{1}{\Omega_{\rm SD}}\sum_{k=0}^{m-1}\binom{m-1}{k} \times (-1)^k \mathrm{e}^{-x(M-m+k+1)/\Omega_{\rm SD}}. \tag{13}
$$

In what follows, both $\Pr(\gamma_{\mathrm{SD}_m} < \gamma\mathrm{th}_m)$ and $\Pr(\gamma_{\mathrm{RD}_m} < \gamma\mathrm{th}_m)$ will be attained

$$
\begin{aligned}
\Pr(\gamma_{\mathrm{SD}_m} < \gamma\mathrm{th}_m) &= \Pr\left(\frac{a_m\gamma\left|h_{\mathrm{SD}_m}\right|^2}{a_n\gamma\left|h_{\mathrm{SD}_m}\right|^2+1} < \gamma\mathrm{th}_m\right) \\
&\underset{(a)}{=} \Pr\left(\left|h_{\mathrm{SD}_m}\right|^2 < \frac{\gamma\mathrm{th}_m}{(a_m - a_n\gamma\mathrm{th}_m)\gamma} \triangleq \tau\right) \\
&= \frac{M!}{(M-m)!(m-1)!}\frac{1}{\Omega_{\rm SD}}\sum_{k=0}^{m-1}\binom{m-1}{k} \times (-1)^k\int_0^{\tau}\mathrm{e}^{-x(M-m+k+1)/\Omega_{\rm SD}}\,\mathrm{d}x \\
&= \frac{M!}{(M-m)!(m-1)!}\sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^k \times \frac{1}{M-m+k+1}\left(1-\mathrm{e}^{-\tau(M-m+k+1)/\Omega_{SD}}\right).
\end{aligned} \tag{14}
$$

Note that step (a) is obtained by assuming the following condition holds

$$
\gamma\mathrm{th}_m < \frac{a_m}{a_n}. \tag{15}
$$

When such a condition is not satisfied, the users outage probability is always one

$$
\begin{aligned}
&\Pr(\gamma_{\mathrm{RD}_m} < \gamma\mathrm{th}_m) \\
&= \Pr\left(\frac{a_m\gamma^2\left|h_{\mathrm{RD}_m}\right|^2|h_{\rm SR}|^2}{a_n\gamma^2\left|h_{\mathrm{RD}_m}\right|^2|h_{\rm SR}|^2 + \gamma\left(\left|h_{\mathrm{RD}_m}\right|^2 + |h_{\rm SR}|^2\right)+1} < \gamma\mathrm{th}_m\right) \\
&= \Pr\left(\left|h_{\mathrm{RD}_m}\right|^2 < \tau\right) \\
&\quad + \Pr\left(|h_{\rm SR}|^2 < \frac{\tau(1+\gamma\left|h_{\mathrm{RD}_m}\right|^2)}{\gamma(\left|h_{\mathrm{RD}_m}\right|^2 - \tau)},\ \left|h_{\mathrm{RD}_m}\right|^2 > \tau\right) \\
&= \Pr\left(\left|h_{\mathrm{RD}_m}\right|^2 < \tau\right) + \int_{\tau}^{\infty} f_{\left|h_{\mathrm{RD}_m}\right|^2}(x) \int_0^{\frac{\tau(1+\gamma y)}{\gamma(y-\tau)}} \frac{1}{\Omega_{\rm SR}}\mathrm{e}^{-x/\Omega_{\rm SR}}\,\mathrm{d}x\,\mathrm{d}y \\
&= 1 - \int_{\tau}^{\infty}\frac{1}{\Omega_{\rm RD}}\mathrm{e}^{-y/\Omega_{\rm RD}}\mathrm{e}^{-\tau(1+\gamma y)/\gamma(y-\tau)\Omega_{\rm SR}}\,\mathrm{d}y \\
&\underset{(b)}{=} 1 - \mathrm{e}^{-\tau(1/\Omega_{\rm SR}+1/\Omega_{RD})}\sqrt{\frac{4\tau(1+\gamma\tau)}{\gamma\Omega_{\rm SR}\Omega_{RD}}}K_1\left(\sqrt{\frac{4\tau(1+\gamma\tau)}{\gamma\Omega_{\rm SR}\Omega_{RD}}}\right),
\end{aligned} \tag{16}
$$

where step (b) is obtained with the aid of [10] [Eq. (3.324.1)] and $K_1(\cdot)$ is the first-order modified Bessel function of the second kind.

By substituting (14) and (16) into (12), a closed-form expression for the outage probability of the mth user can be written as

$$
\begin{aligned}
\mathcal{P}_{\rm out}^m = &\left[\frac{M!}{(M-m)!(m-1)!}\sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^k \frac{1}{M-m+k+1}\left(1-\mathrm{e}^{-\tau(M-m+k+1)/\Omega_{SD}}\right)\right] \\
&\times\left[1-\mathrm{e}^{-\tau(1/\Omega_{\rm SR}+1/\Omega_{RD})}\sqrt{\frac{4\tau(1+\gamma\tau)}{\gamma\Omega_{\rm SR}\Omega_{RD}}}K_1\left(\sqrt{\frac{4\tau(1+\gamma\tau)}{\gamma\Omega_{\rm SR}\Omega_{RD}}}\right)\right].
\end{aligned} \tag{17}
$$

Next, we analyse the outage probability of the nth user. As the nth user needs to decode the signal of the mth user first, the probability to characterise such an event can be formulated as follows

$$
\begin{aligned}
\mathcal{P}_{\rm out}^n = &\underbrace{\left[1-\Pr\left(\gamma_{\mathrm{SD}_{m\to n}} \ge \gamma\mathrm{th}_m,\ \gamma_{\mathrm{SD}_n} \ge \gamma\mathrm{th}_n\right)\right]}_{J_1} \\
&\times\underbrace{\left[1-\Pr\left(\gamma_{\mathrm{RD}_{m\to n}} \ge \gamma\mathrm{th}_m,\ \gamma_{\mathrm{RD}_n} \ge \gamma\mathrm{th}_n\right)\right]}_{J_2},
\end{aligned} \tag{18}
$$

where J_1 can be solved as

$$
\begin{aligned}
J_1 &= 1 - \Pr\left(\gamma_{\mathrm{SD}_{m\to n}} \ge \gamma\mathrm{th}_m\right)\Pr\left(\gamma_{\mathrm{SD}_n} \ge \gamma\mathrm{th}_n\right) \\
&= 1 - \Pr\left(\frac{a_m\gamma\left|h_{\mathrm{SD}_n}\right|^2}{a_n\gamma\left|h_{\mathrm{SD}_n}\right|^2+1} \ge \gamma\mathrm{th}_m\right)\Pr\left(a_n\gamma\left|h_{\mathrm{SD}_n}\right|^2 \ge \gamma\mathrm{th}_n\right) \\
&= 1 - \Pr\left(\left|h_{\mathrm{SD}_n}\right|^2 \ge \tau\right)Pr(\left|h_{\mathrm{SD}_n}\right|^2 \ge \frac{\gamma\mathrm{th}_n}{a_n\gamma} \triangleq \beta\right) \\
&= 1 - \Pr\left(\left|h_{\mathrm{SD}_n}\right|^2 \ge \max(\tau,\ \beta) \triangleq \theta\right) \\
&= \Pr\left(\left|h_{\mathrm{SD}_n}\right|^2 \le \theta\right).
\end{aligned} \tag{19}
$$

According to (14), J_1 can be expressed as

$$
J_1 = \frac{M!}{(M-n)!(n-1)!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k \times \frac{1}{M-n+k+1}\left(1-\mathrm{e}^{-\theta(M-n+k+1)/\Omega_{SD}}\right). \tag{20}
$$

Now, J_2 can be solved as

$$
\begin{aligned}
J_2 &= 1 - \Pr\left(\gamma_{RD_{m\to n}} \ge \gamma\mathrm{th}_m\right)\Pr\left(\gamma_{\mathrm{RD}_n} \ge \gamma\mathrm{th}_n\right) \\
&= 1 - \Pr\left(\frac{a_m\gamma^2\left|h_{\mathrm{RD}_n}\right|^2|h_{\rm SR}|^2}{a_n\gamma^2\left|h_{\mathrm{RD}_n}\right|^2|h_{\rm SR}|^2+\gamma\left(\left|h_{\mathrm{RD}_n}\right|^2+|h_{\rm SR}|^2\right)+1} \ge \gamma\mathrm{th}_m\right) \\
&\quad\times\Pr\left(\frac{a_n\gamma^2\left|h_{\mathrm{RD}_n}\right|^2|h_{\rm SR}|^2}{\gamma\left(\left|h_{\mathrm{RD}_n}\right|^2+|h_{\rm SR}|^2\right)+1} \ge \gamma\mathrm{th}_n\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 1 - \Pr\left(|h_{\mathrm{SR}}|^2 \geq \frac{\tau\left(1+\gamma|h_{\mathrm{RD}_n}|^2\right)}{\gamma\left(|h_{\mathrm{RD}_n}|^2-\tau\right)}, \left|h_{\mathrm{RD}_n}\right|^2 \geq \tau\right) \\
&\quad \times \Pr\left(|h_{\mathrm{SR}}|^2 \geq \frac{\beta\left(1+\gamma|h_{\mathrm{RD}_n}|^2\right)}{\gamma\left(|h_{\mathrm{RD}_n}|^2-\beta\right)}, \left|h_{\mathrm{RD}_n}\right|^2 \geq \beta\right) \\
&= 1 - \Pr\left(|h_{\mathrm{SR}}|^2 \geq \frac{\theta\left(1+\gamma|h_{\mathrm{RD}_n}|^2\right)}{\gamma\left(|h_{\mathrm{RD}_n}|^2-\theta\right)}, \left|h_{\mathrm{RD}_n}\right|^2 \geq \theta\right) \\
&= 1 - \int_\theta^\infty \frac{1}{\Omega_{\mathrm{RD}}} \mathrm{e}^{-y/\Omega_{\mathrm{RD}}} \int_{\frac{\theta(1+\gamma y)}{\gamma(y-\theta)\Omega_{\mathrm{SR}}}}^\infty \frac{1}{\Omega_{\mathrm{SR}}} \mathrm{e}^{-x/\Omega_{\mathrm{SR}}}\, \mathrm{d}x\, \mathrm{d}y \\
&= 1 - \mathrm{e}^{-\theta(1/\Omega_{\mathrm{SR}}+1/\Omega_{\mathrm{RD}})} \sqrt{\frac{4\theta(1+\gamma\theta)}{\gamma\Omega_{\mathrm{SR}}\Omega_{\mathrm{RD}}}} K_1\left(\sqrt{\frac{4\theta(1+\gamma\theta)}{\gamma\Omega_{\mathrm{SR}}\Omega_{\mathrm{RD}}}}\right).
\end{aligned} \tag{21}
$$

By substituting (20) and (21) into (18), a closed-form expression for the outage probability of the nth user can be written as

$$
\begin{aligned}
\mathcal{P}_{\mathrm{out}}^n = &\left[\frac{M!}{(M-n)!(n-1)!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k \frac{1}{M-n+k+1}\left(1-\mathrm{e}^{-\tau(M-n+k+1)/\Omega_{\mathrm{SD}}}\right)\right] \\
&\times \left[1-\mathrm{e}^{-\theta(1/\Omega_{\mathrm{SR}}+1/\Omega_{\mathrm{RD}})}\sqrt{\frac{4\theta(1+\gamma\theta)}{\gamma\Omega_{\mathrm{SR}}\Omega_{\mathrm{RD}}}}K_1\left(\sqrt{\frac{4\theta(1+\gamma\theta)}{\gamma\Omega_{\mathrm{SR}}\Omega_{\mathrm{RD}}}}\right)\right].
\end{aligned} \tag{22}
$$

3.2 *Asymptotic outage behaviour*

On the basis of the preceding results, we perform asymptotic analysis for the outage probability of the mth user and nth user in high SNR regime.

Applying the series expansion of the exponential functions in (14), we have

$$
\begin{aligned}
&\Pr(\gamma_{\mathrm{SD}_m} < \gamma\mathrm{th}_m) \\
&\quad = \frac{M!}{(M-m)!(m-1)!}\sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^{k+1} \\
&\quad \times \frac{1}{M-m+k+1}\sum_{i=1}^{\infty}\frac{(-1)^i(M-m+k+1)^i\tau^i}{i!(\Omega_{SD})^i} \\
&= \sum_{i=1}^{\infty}\frac{(-1)^i\tau^i M!}{(M-m)!(m-1)!i!(\Omega_{SD})^i} \\
&\quad \times \sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^{k+1}(M-m+k+1)^{i-1} \\
&= \sum_{i=1}^{\infty}\frac{(-1)^{i+1}\tau^i M!}{(M-m)!(m-1)!i!(\Omega_{SD})^i} \\
&\quad \times \sum_{l=0}^{i-1}\binom{i-1}{l}(M-m+1)^{i-1-l}\sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^k k^l.
\end{aligned} \tag{23}
$$

Recall the following sums of the binomial coefficients [Eq. (0.154.3) in [10]]

$$
\sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^k k^l = 0, \tag{24}
$$

for $m-2 \geq l \geq 1$ and

$$
\sum_{k=0}^{m-1}\binom{m-1}{k}(-1)^k k^{m-1} = (-1)^{m-1}(m-1)!. \tag{25}
$$

Therefore, $\Pr(\gamma_{\mathrm{SD}_m} < \gamma\mathrm{th}_m)$ can be asymptotically written as

$$
\Pr(\gamma_{\mathrm{SD}_m} < \gamma\mathrm{th}_m) \simeq \frac{\tau^m M!}{(M-m)!m!(\Omega_{SD})^m} \propto \frac{1}{\gamma^m}. \tag{26}
$$

By using the fact that $\mathrm{e}^x \simeq 1+x$ and $K_1(x) \simeq 1/x$ when $x \to 0$, we have

$$
\Pr(\gamma_{\mathrm{RD}_m} < \gamma\mathrm{th}_m) \simeq 1-\mathrm{e}^{-\tau(1/\Omega_{RD}+1/\Omega_{\mathrm{SR}})} \simeq \tau\left(\frac{1}{\Omega_{RD}}+\frac{1}{\Omega_{\mathrm{SR}}}\right) \propto \frac{1}{\gamma}. \tag{27}
$$

Thus, the outage probability of the mth user can be asymptotically written as

$$
\mathcal{P}_{\mathrm{out}}^m \simeq \frac{\tau^{m+1}M!}{(M-m)!m!(\Omega_{SD})^m}\left(\frac{1}{\Omega_{\mathrm{RD}}}+\frac{1}{\Omega_{\mathrm{SR}}}\right) \propto \frac{1}{\gamma^{m+1}}. \tag{28}
$$

Similarly, we can obtain

$$
\mathcal{P}_{\mathrm{out}}^n \simeq \frac{\theta^{n+1}M!}{(M-n)!n!(\Omega_{SD})^n}\left(\frac{1}{\Omega_{\mathrm{RD}}}+\frac{1}{\Omega_{\mathrm{SR}}}\right) \propto \frac{1}{\gamma^{n+1}}. \tag{29}
$$

From (28) and (29), note that the mth user and the nth user will experience a diversity order of $m+1$ and $n+1$, respectively.

4 Ergodic sum rate

In this section, we consider the target SINRs of the two users are determined by the users' channel condition, that is, $\gamma\mathrm{th}_m = \min\left[\gamma_{\mathrm{SD}_m}, \gamma_{\mathrm{RD}_m}\right]$. Note that $\gamma_{\mathrm{SD}_{m\to n}} \geq \gamma_{\mathrm{SD}_m}$ always holds since $\left|h_{\mathrm{SD}_m}\right|^2 \leq \left|h_{\mathrm{SD}_n}\right|^2$. However, $\gamma_{\mathrm{RD}_{m\to n}} \geq \gamma_{\mathrm{RD}_m}$ may not be correct when $\left|h_{\mathrm{RD}_n}\right|^2 < \left|h_{\mathrm{RD}_m}\right|^2$. Therefore, after selection combining at the receivers, the rates achievable to the two users can be expressed as

$$
R_m = \frac{1}{2}\log_2\left(1+\max\left[\gamma_{\mathrm{SD}_m}, \gamma_{\mathrm{RD}_m}\right]\right), \tag{30}
$$

$$
R_n = \begin{cases} \frac{1}{2}\log_2\left(1+\max\left[\gamma_{\mathrm{SD}_n}, \gamma_{\mathrm{RD}_n}\right]\right), & \text{if} \left|h_{\mathrm{RD}_n}\right|^2 > \left|h_{\mathrm{RD}_m}\right|^2 \\ \frac{1}{2}\log_2\left(1+\gamma_{\mathrm{SD}_n}\right), & \text{if} \left|h_{\mathrm{RD}_n}\right|^2 < \left|h_{\mathrm{RD}_m}\right|^2. \end{cases} \tag{31}
$$

The ergodic rate of the two users can be written as

$$
R_{\mathrm{ave}}^m = E\left[\frac{1}{2}\log_2\left(1+\max\left[\gamma_{\mathrm{SD}_m}, \gamma_{\mathrm{RD}_m}\right]\right)\right], \tag{32}
$$

$$
\begin{aligned}
R_{\mathrm{ave}}^n = &\Pr\left(\left|h_{\mathrm{RD}_n}\right|^2 > \left|h_{\mathrm{RD}_m}\right|^2\right) \\
&\times \underbrace{E\left[\frac{1}{2}\log_2\left(1+\max\left[\gamma_{\mathrm{SD}_n}, \gamma_{\mathrm{RD}_n}\right]\right)\right]}_{R_{ave}^{n,1}} \\
&+ \Pr\left(\left|h_{\mathrm{RD}_n}\right|^2 < \left|h_{\mathrm{RD}_m}\right|^2\right)\underbrace{E\left[\frac{1}{2}\log_2\left(1+\gamma_{\mathrm{SD}_n}\right)\right]}_{R_{\mathrm{ave}}^{n,2}}.
\end{aligned} \tag{33}
$$

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It is difficult to obtain the exact expression for the ergodic sum rate, so in this paper we focus on the high SNR approximation.

When $\gamma \to \infty$, R_{ave}^{m} can be expressed as (see (34))

Since h_{RD_n} and h_{RD_m} are independent identically distributed random variables, we have $\Pr\left(\left|h_{\text{RD}_n}\right|^2 > \left|h_{\text{RD}_m}\right|^2\right) = \frac{1}{2}$. In what follows, R_{ave}^{n} will be calculated. First, $R_{\text{ave}}^{n,1}$ can be expressed as (see (35))

Denoting $X = \left|h_{\text{SD}_n}\right|^2$, $Y = |h_{\text{SR}}|^2$, and $Z = \left|h_{\text{RD}_n}\right|^2$, we can rewrite $R_{\text{ave}}^{n,1}$ as

$$\begin{aligned} R_{\text{ave}}^{n,1} &= E\left\{\frac{1}{2\ln 2}\ln\left(1+\max\left[a_n\gamma X,\ \frac{a_n\gamma^2 YZ}{\gamma(Y+Z)+1}\right]\right)\right\} \\ &\underset{(c)}{<} E\left\{\frac{1}{2\ln 2}\ln\left(1+a_n\gamma\max[X,\ \min[Y,\ Z]]\right)\right\}, \end{aligned} \tag{36}$$

where step (c) is obtained by the inequality [11]

$$\frac{\gamma^2 YZ}{\gamma(Y+Z)+1} < \min[\gamma Y,\ \gamma Z]. \tag{37}$$

Denote $W = \max[X,\ \min[Y,\ Z]]$. After some involved manipulations we have

$$\begin{aligned} F_W(w) &= \Pr(\max[X,\ \min[Y,\ Z]] \le w) \\ &= \Pr(X \le w)\left[1-\Pr(Y>w)\Pr(Z>w)\right] \\ &= \frac{M!}{(M-n)!(n-1)!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k \\ &\quad \times \frac{1}{M-n+k+1}\left(1-\mathrm{e}^{-w(M-n+k+1)/\Omega_{\text{SD}}}\right) \\ &\quad \times \left(1-\mathrm{e}^{-w\left(1/\Omega_{\text{SR}}+1/\Omega_{\text{RD}}\right)}\right). \end{aligned} \tag{38}$$

Therefore, a upper bound of $R_{\text{ave}}^{n,1}$ can be calculated as

$$\begin{aligned} R_{\text{ave}}^{n,1} &< \mathrm{E}\left[\frac{1}{2\ln 2}\ln(1+a_n\gamma W)\right] \\ &= \frac{1}{2\ln 2}\int_0^\infty f_W(w)\ln(1+a_n\gamma w)\mathrm{d}w \\ &= \frac{1}{2\ln 2}\int_0^\infty \ln(1+a_n\gamma w)\mathrm{d}F_W(w) \\ &= \frac{1}{2\ln 2}\left[\ln(1+a_n\gamma w)F_W(w)\Big|_0^\infty - a_n\gamma\int_0^\infty \frac{F_W(w)}{1+a_n\gamma w}\mathrm{d}w\right] \\ &= \frac{a_n\gamma}{2\ln 2}\int_0^\infty \frac{1-F_W(w)}{1+a_n\gamma w}\mathrm{d}w. \end{aligned} \tag{39}$$

Substituting (38) into (39) and performing some algebraic simplifications, (39) can be rewritten as

$$\begin{aligned} R_{\text{ave}}^{n,1} &< \frac{1}{2\ln 2}\frac{M!}{(M-n)!(n-1)!} \\ &\quad \times \sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k\frac{a_n\gamma}{M-n+k+1}\left[\int_0^\infty \frac{\mathrm{e}^{-w\left(M-n+k+1/\Omega_{SD}\right)}}{1+a_n\gamma w}\mathrm{d}w\right. \\ &\quad + \int_0^\infty \frac{\mathrm{e}^{-w\left(1/\Omega_{\text{SR}}+1/\Omega_{\text{RD}}\right)}}{1+a_n\gamma w}\mathrm{d}w \\ &\quad \left. - \int_0^\infty \frac{\mathrm{e}^{-w\left(M-n+k+1/\Omega_{\text{SD}}+1/\Omega_{\text{SR}}+1/\Omega_{\text{RD}}\right)}}{1+a_n\gamma w}\mathrm{d}w\right]. \end{aligned} \tag{40}$$

Denote $v = a_n\gamma w$, $\mu_1 = \frac{1}{a_n\gamma}\left(\frac{M-n+k+1}{\Omega_{\text{SD}}}\right)$, $\mu_2 = \frac{1}{a_n\gamma}\left(\frac{1}{\Omega_{\text{SR}}}+\frac{1}{\Omega_{RD}}\right)$, and $\mu_3 = \frac{1}{a_n\gamma}\left(\frac{M-n+k+1}{\Omega_{SD}}+\frac{1}{\Omega_{\text{SR}}}+\frac{1}{\Omega_{RD}}\right)$, then we have

$$\begin{aligned} R_{\text{ave}}^{n,1} &< \frac{1}{2\ln 2}\frac{M!}{(M-n)!(n-1)!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k\frac{1}{M-n+k+1} \\ &\quad \times \left[\underbrace{\int_0^\infty \frac{\mathrm{e}^{-\mu_1 v}}{1+v}\mathrm{d}v + \int_0^\infty \frac{\mathrm{e}^{-\mu_2 v}}{1+v}\mathrm{d}v - \int_0^\infty \frac{\mathrm{e}^{-\mu_3 v}}{1+v}\mathrm{d}v}_{\Phi}\right]. \end{aligned} \tag{41}$$

With the aid of [10] [Eq. (3.352.4)], Φ can be expressed as

$$\Phi = \mathrm{e}^{\mu_3}E_i(-\mu_3) - \mathrm{e}^{\mu_2}E_i(-\mu_2) - \mathrm{e}^{\mu_1}E_i(-\mu_1), \tag{42}$$

where $E_i(\cdot)$ is the exponential integral functions. Now, by substituting (42) into (41), one can obtain the upper bound of $R_{\text{ave}}^{n,1}$ as

$$\begin{aligned} R_{\text{ave}}^{n,1} &< \frac{1}{2\ln 2}\frac{M!}{(M-n)!(n-1)!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k \\ &\frac{1}{M-n+k+1}\times\left[\mathrm{e}^{\mu_3}E_i(-\mu_3) - \mathrm{e}^{\mu_2}E_i(-\mu_2) - \mathrm{e}^{\mu_1}E_i(-\mu_1)\right]. \end{aligned} \tag{43}$$

Next, $R_{\text{ave}}^{n,2}$ can be calculated as follows:

$$\begin{aligned} R_{\text{ave}}^{n,2} &= E\left[\frac{1}{2}\log_2\left(1+a_n\gamma\left|h_{\text{SD}_n}\right|^2\right)\right] \\ &= \frac{a_n\gamma}{2\ln 2}\int_0^\infty \frac{1-F_{\left|h_{\text{SD}_n}\right|^2}(x)}{1+a_n\gamma x}\mathrm{d}x \\ &= \frac{a_n\gamma M!}{2\ln 2(M-n)!(n-1)!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k \\ &\quad \times \frac{1}{M-n+k+1}\int_0^\infty \frac{\mathrm{e}^{-x(M-n+k+1)/\Omega_{SD}}}{1+a_n\gamma x}\mathrm{d}x. \end{aligned} \tag{44}$$

$$R_{\text{ave}}^{m} = E\left\{\frac{1}{2}\log_2\left(1+\max\left[\frac{a_m\gamma\left|h_{\text{SD}_m}\right|^2}{a_n\gamma\left|h_{\text{SD}_m}\right|^2+1},\ \frac{a_m\gamma^2\left|h_{\text{RD}_m}\right|^2|h_{\text{SR}}|^2}{a_n\gamma^2\left|h_{\text{RD}_m}\right|^2|h_{\text{SR}}|^2+\gamma\left(\left|h_{\text{RD}_m}\right|^2+|h_{\text{SR}}|^2\right)+1}\right]\right)\right\} \simeq \frac{1}{2}\log_2\left(1+\frac{a_m}{a_n}\right). \tag{34}$$

$$R_{\text{ave}}^{n,1} = E\left\{\frac{1}{2}\log_2\left(1+\max\left[a_n\gamma\left|h_{\text{SD}_n}\right|^2,\ \frac{a_n\gamma^2\left|h_{\text{RD}_n}\right|^2|h_{\text{SR}}|^2}{\gamma\left(\left|h_{\text{RD}_n}\right|^2+|h_{\text{SR}}|^2\right)+1}\right]\right)\right\}. \tag{35}$$

Similarly, with the aid of [10] [Eq. (3.352.4)], we have

$$R_{\text{ave}}^{n,2} = -\frac{M!}{2\ln 2(M-n)!(n-1)!} \times \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{e^{\mu_1} E_i(-\mu_1)}{M-n+k+1}. \tag{45}$$

By plugging (43) and (45) into (33), the upper bound of R_{ave}^n can be expressed as

$$R_{\text{ave}}^n < \frac{1}{4\ln 2} \frac{M!}{(M-n)!(n-1)!} \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \times \frac{1}{M-n+k+1} \times \left[e^{\mu_3} E_i(-\mu_3) - e^{\mu_2} E_i(-\mu_2) - 2e^{\mu_1} E_i(-\mu_1) \right]. \tag{46}$$

Finally, the upper bound of the ergodic sum rate can be expressed as

$$R_{\text{ave}}^{\text{sum}} < \frac{1}{2} \log_2 \left(1 + \frac{a_m}{a_n} \right) + \frac{1}{4\ln 2} \frac{M!}{(M-n)!(n-1)!} \times \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{1}{M-n+k+1} \times \left[e^{\mu_3} E_i(-\mu_3) - e^{\mu_2} E_i(-\mu_2) - 2e^{\mu_1} E_i(-\mu_1) \right]. \tag{47}$$

From (34) we can obtain that R_{ave}^m is a constant and independent of the channel condition of the mth user. However, R_{ave}^n depends on the channel condition of the nth user and increases as the system SNR increases. Therefore, a user with a better channel condition is more willing to perform NOMA, which is not true for a user with a poor channel condition. Thus, m should be as small as possible and n should be as large as possible.

5 Numerical results and discussion

In this section, we demonstrate the performance of the NOMA cooperative system in terms of outage probability and sum rate. We consider that the base station, the relay node, and mobile users are located on a straight line with the users clustered together. For illustration purposes and without loss of generality, we normalise the distance between S and D to unity and let d denote the normalised distance between S and R. As a result, $\Omega_{SD} = 1$, $\Omega_{SR} = d^{-\alpha}$, and $\Omega_{RD} = (1-d)^{-\alpha}$, where α is the path loss exponent. In the following simulations, we assume $M = 4$, $a_m = 4/5$, $a_n = 1/5$, $\gamma th_m = 2$ dB, $\gamma th_n = 4$ dB, $d = 0.5$, and $\alpha = 4$. The exact analysis curves for the outage probability of the two users are plotted according to (17) and (22), and the asymptotic curves for the outage probability of the two users are plotted according to (28) and (29). Monte Carlo simulations are performed to validate the derived exact as well as the asymptotic analytical results. Meanwhile, the ergodic sum rate of the system is investigated. The analysis upper bound of the ergodic sum rate is conducted according to (47), and the simulation result for the ergodic sum rate is averaged over 3000 channel realisations.

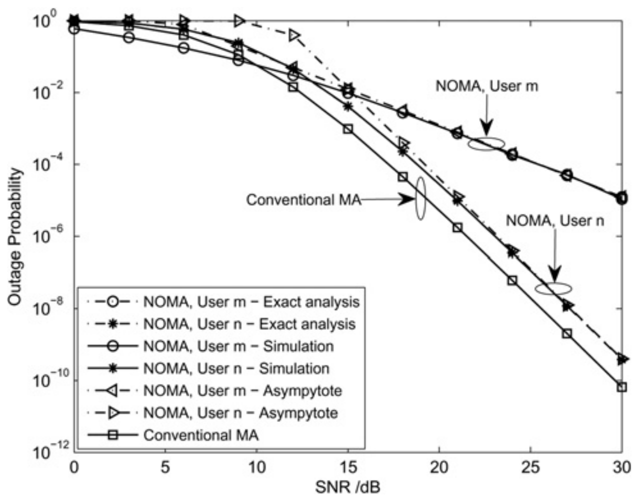

Fig. 2 *Outage probability against system SNR with m = 1 and n = 4*

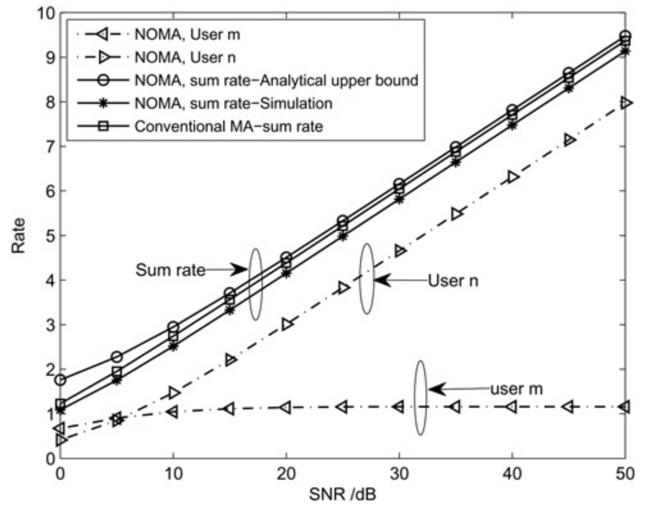

Fig. 3 *Rate against system SNR with m = 1 and n = 4*

Fig. 2 depicts the outage probability of the two paired users versus system SNR with $m = 1$ and $n = 4$. Note that an excellent agreement between the exact analytical results and simulations is observed, and the asymptotic curves match well with them in high SNR regions. In addition, we compared the proposed scheme with conventional cooperative system. Here, we assume that an opportunistic MA approach is adopted for conventional MA, where the user with the best channel condition is scheduled. The target SINR γth for conventional scheme satisfies $\frac{1}{2}\left[\log_2(1+\gamma th_m) + \log_2(1+\gamma th_n)\right] = 1/2\log_2(1+\gamma th)$. Simulation results show that conventional

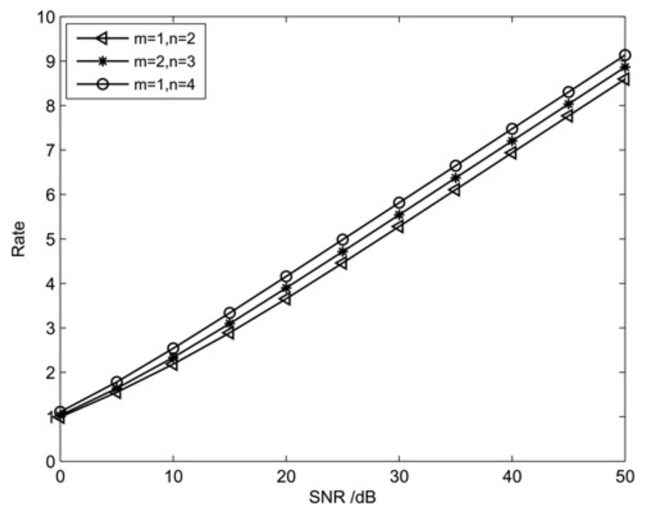

Fig. 4 *Ergodic sum rate against system SNR with different user pairings {m, n}: {1, 2}, {2, 3} and {1, 4}*

MA can achieve the same diversity gain with user n. Conventional MA obtains better outage performance than NOMA, but NOMA can offer better fairness since more users are served simultaneously.

The sum rate achieved by NOMA is shown in Fig. 3 with $m = 1$ and $n = 4$. We can observe that the analysis upper bound of the ergodic sum rate is close to the simulation results. Note that NOMA can achieve nearly the same sum rate with conventional MA, but NOMA can offer better spectral efficiency and user fairness since more users are served at the same time, frequency, and spreading code. In addition, it is observed that the rate of user m is a constant in the medium and high SNR regions, which is consistent to (34).

In Fig. 4, we show the ergodic sum rate with different user pairings $\{m, n\}$: $\{1, 2\}$ $\{2, 3\}$, and $\{1, 4\}$. It is shown that the user pairing $\{1, 4\}$ achieves the highest ergodic sum rate, which verifies the user selection criterion that m should be as small as possible and n should be as large as possible.

6 Conclusion

In this paper, we combine NOMA with downlink cooperative system. We first study the outage behaviour of the two paired users and derive closed-form expressions for their respective exact and asymptotic outage scheme. In addition, we discuss the ergodic sum rate of the two paired users in the system, and obtain the upper bound of the ergodic sum rate. Simulation results show that NOMA can obtain the same diversity order with conventional MA where the user with the best channel condition is scheduled, and achieve nearly the same sum rate with conventional MA, but NOMA can offer better spectral efficiency and user fairness.

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