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Computationally efficient adaptive algorithm for resource allocation in orthogonal frequency-division multiple-access-based cognitive radio networks

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Abstract: In this study, the authors examine resource allocation in an orthogonal frequency-division multiple-access-based cognitive radio (CR) network which dynamically senses primary users (PUs) spectrum and opportunistically uses available channels. The aim is resource allocation such that the CR network throughput is maximised under the PUs maximum interference constraint and cognitive users (CUs) transmission power budget. This problem is formulated as a mixed-integer non-linear programming problem which is $\mathcal{NP}$-hard in general and infeasible to solve in real-time. To reduce the computational complexity, the authors decouple the problem into two separate steps. After initial power allocation, in the first step, an adaptive algorithm is employed to assign subcarriers to the CUs toward throughput maximisation by using these initial powers. In the second step, power is allocated optimally to the assigned subcarriers. Simulation results show that the proposed method nearly achieves the optimal solution in a small number of iterations meaning significant reduction in the computational complexity.

1 Introduction

By the significant growth of wireless applications, radio spectrum scarcity becomes more obvious. To support distinct services and applications without interference experience for users, traditionally fixed spectrum access (FSA) policy has been developed. In FSA, each portion of frequency spectrum is licensed to a subscriber. In this method, only the licensed subscriber is allowed to use the allocated spectrum regardless of whether the licensed subscriber is using it. On the other hand recent measurements by Federal Communications Commission have shown that large portions of the allocated spectrum are underutilised. To enhance spectral efficiency, dynamic spectrum access (DSA) is introduced as a new policy [1–3]. According to DSA, a portion of spectrum is allocated to a subscriber, known as a primary user (PU), although he is not the individual user of the allocated spectrum. Other users can also use the allocated spectrum with lower priority since it is not temporally in-use by PU. Cognitive radio (CR) technology is a solution for opportunistic use of the underutilised spectrum. In a CR system, cognitive users (CUs) sense the spectrum of the licensed users and smartly adapt their transmission characteristics according to the instantaneous behaviour of the licensed users. Although this concept can improve the overall spectral efficiency, it may result in harmful interference to the PUs of the spectrum. Hence, the goal is maximising the throughput of the whole network while keeping the interference power of the CUs into the PUs below a predefined threshold which is known as the 'interference temperature' [4].

Orthogonal frequency-division multiple-access (OFDMA) presents high flexibility in resource allocation and considerable reduction in the complexity of resource allocation algorithms. A resource allocation problem is usually formulated as an optimisation problem, where the objective is maximising sum bit rate or spectral efficiency or minimising the power consumption and interference. In [5–10] resource allocation in OFDMA-based networks has been studied extensively. A complete summary of this subject can be found in [11]. In CR networks because of the presence of unlicensed users, we have the PUs interference constraints that make the problem more complicated.

There have been recently numerous studies on radio resource allocation in CR networks, most of which are based on OFDMA. Zhang and Leung in [12], formulated subcarrier, bit and power allocation problem in OFDMA-based CR networks as a multidimensional knapsack problem and employed a greedy algorithm to solve it. Although the authors encountered multiple PUs but they considered only a single CU in the problem formulation. In [13], the average rate of the CR link is maximised using a dynamic programming scheme under the CUs power budget and maximum interference introduced to PUs constraints. In [14] after investigating an optimal power allocation in an OFDM-based CR system, two suboptimal schemes with less complexity have been proposed for power allocation and their performance have been compared with the classical schemes. In [15] the authors have designed an iterative waterfilling algorithm to control the transmission power dynamically in a non-cooperative game scheme and used some control theories to analyse performance of the network under time varying conditions. In [16], the resource allocation problem in CR network is formulated as the Nash bargaining (NB) cooperative game. It has been shown that the NB resource allocation game with spectrum mask and total power constraints is non-convex optimisation problem with high complexity. To find the NB solution, a classification of two-user systems is proposed and a suboptimal algorithm with low complexity is employed. Hoang et al formulated power control and channel allocation in CR networks with the purpose of maximising CR network throughput in [17]. A mixed distributed/centralised two-phase control algorithm consisting of two phases has been proposed to solve the problem by assuming cooperation between CUs and PUs. The authors of [18] have considered the resource allocation problem in an OFDMA-based network under PUs interference constraints with imperfect spectrum sensing assumption. In their proposed method, at first, initial powers and subbands are allocated to CUs and then power enhancement step is taken to maximise the total capacity. In [19], dynamic resource allocation problem in a CR network with imperfect channel sensing is studied. In the first step, the problem is solved under the known channel assumption and in the second step joint power and channel allocation is carried out by using a discrete stochastic optimisation method.

In this paper, we propose a computationally efficient adaptive algorithm for resource allocation in OFDMA-based CR networks. First, we model the system in which we bring the mutual interference between CUs and PUs into account. Then, we formulate the uplink resource allocation problem to maximise the CR network sum bit rate (throughput) under the CUs transmission power budget and PUs maximum interference constraints. This optimisation problem is non-linear, non-convex, includes binary and real variables and falls within the category of mixed-integer non-linear programming (MINLP) problems which are $\mathcal{NP}$-hard in general. To reduce the computational complexity, we treat the problem in two separate steps. After initial power allocation, in the first step, we introduce the proposed subcarrier assignment scheme based on an adaptive algorithm to assign the subcarriers to the CUs by using these initial powers such that the CR network throughput is maximised. In this step, by fixing CUs transmission powers to their initial values, we reformulate the subcarrier assignment problem into an equivalent problem defined on a conceptual system in such a way that a least square based adaptive algorithm can find the solution. In the second step, optimal power allocation is carried out for the subscriber assignments calculated in Step one, which is a convex problem and can be solved using interior-point against.

The rest of this paper is organised as follows. In Section 2, the system model and the interference formulation are presented. In Section 3, we formulate the resource allocation problem and propose our resource allocation algorithm. In Section 4, we evaluate the proposed method through simulations and finally the paper is concluded in Section 6.

2 System model and the interference formulation

Assume a system including a primary network and a CR network, as shown in Fig. 1. The primary network has L PUs and we also assume that two networks use OFDMA scheme. The CR network consists of K CUs and an access point (AP) which allocates the spectrum opportunistically to the CUs. We assume that the whole spectrum is divided into N_t subcarriers, each with bandwidth B.

We assume perfect knowledge of the channel-state information (CSI) where the AP knows all the uplink channel gains depicted in Fig. 1. However in practice having full CSI in both transmitter and receiver is not realistic. To see the effect of imperfect CSI in a network throughput, the interested readers are referred to [20, 21].

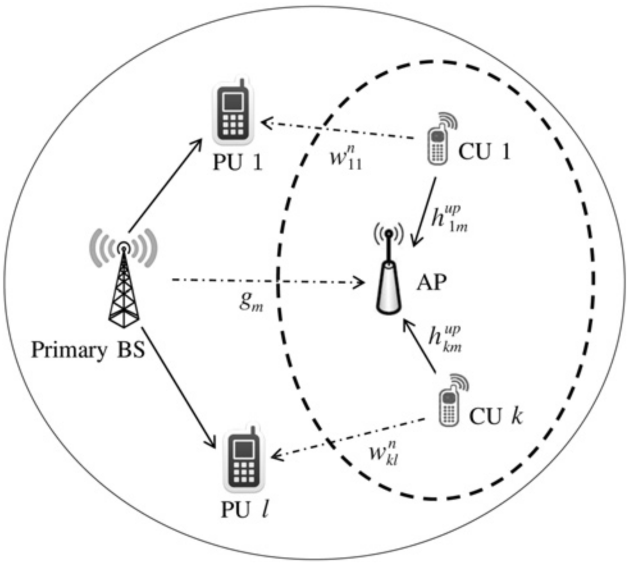

Fig. 1 *System model*

In this paper because of the perfect CSI assumption, the resultant throughput of the CR network can be seen as an upper bound for the case with the imperfect CSI.

There are lots of spectrum sensing techniques in the literature [22]. Here, we hire the cooperative sensing scenario in which each CU periodically senses N_t subcarriers and sends their sensing information to the AP via dedicated control channel (choosing the sensing method [e.g. cyclostationary features-based, energy-based etc.] is mostly depended on system characteristics, computational complexity, network requirements and required accuracy.). Then the AP decides whether the subcarrier is available or in-use. If $\mathcal{Y}_m$ represents the global test static calculated for subcarrier m by the AP, then the state of the subcarrier m is determined by comparing global test static $\mathcal{Y}_m$ with the decision threshold $\mathcal{T}_{\text{th}}$ as follows. When the given $\mathcal{Y}_m$ for the subcarrier m exceeds $\mathcal{T}_{\text{th}}$, the subcarrier m is declared in-use, otherwise the subcarrier is available. Hence the sets of available subcarriers $\mathcal{M}_{\text{av}}$ with N_{av} elements and the sets of in-use subcarriers belonging to the ℓth PU, $\mathcal{M}^{\ell}_{\text{us}}$ are determined. Since this paper mainly focus on proposing a computationally efficient adaptive algorithm to allocate subcarriers and power to the CUs, by perfect spectrum sensing assumption we do not encounter sensing error effects in the CR network performance, which can be found in [18, 19, 23].

Since both networks use OFDMA, inter-carrier interference for each network is negligible and can be ignored because of the orthogonality of the subcarriers. However, the rectangularly pulsed OFDMA signals represent relatively large power spectral sidelobes to the adjacent channels [18, 24]. So, we consider cross-channel interference to the PUs subcarriers because of the CUs transmission and vice versa, respectively, as follows. The amount of power spectral sidelobes introduced to the ℓth PU's subcarriers because of the kth CU transmission on subcarrier m is

$$\begin{aligned} I^{\ell}_{km} &= \sum_{n\in\mathcal{M}^{\ell}_{\text{us}}} \int_{f_n-B/2}^{f_n+B/2} w^n_{k\ell} p_{km} \phi(f-f_m)\,\mathrm{d}f \\ &= p_{km} q^{\ell}_{km} \end{aligned} \tag{1}$$

where $q^{\ell}_{km} = \sum_{n\in\mathcal{M}^{\ell}_{\text{us}}} \int_{f_n-B/2}^{f_n+B/2} w^n_{k\ell}\phi(f-f_m)\,\mathrm{d}f$ is the interference factor of PU ℓ, and $\phi(f) = T_s \text{sinc}^2(fT_s)$ is the power spectral density (PSD) of the CR network subcarrier OFDM signal. The parameter p_{km} is the kth CU transmission power to the AP on subcarrier m, $w^n_{k\ell}$ is the channel gain between the kth CU and the ℓth PU on subcarrier n and f_m and f_n are the central frequencies of the CU on subcarrier m and the PU on subcarrier n, respectively.

Moreover the sum interference power introduced to the AP on subcarrier m by the primary BS is

$$I^m_{\text{PU}} = \sum_{\ell=1}^{L} a^m_{\ell} \tag{2}$$

where

$$a^m_{\ell} = \sum_{i\in\mathcal{M}^{\ell}_{\text{us}}} \int_{f_m-B/2}^{f_m+B/2} g_m p^{\ell i}_{\text{PU}} \phi(f-f_i)\,\mathrm{d}f \tag{3}$$

is the interference power introduced by the primary BS transmitting to the ℓth PU on subcarrier m, g_m is the channel gain from primary BS to the AP on subcarrier m, and $p^{\ell i}_{\text{PU}}$ is the primary BS transmission power on subcarrier i belonging to the ℓth PU.

The bit rate of the kth CU on subcarrier m is

$$r_{km} = \log_2(1 + p_{km}\gamma_{km}) \tag{4}$$

where $\gamma_{km} = h^{\text{up}}_{km}/(N_0 B + I^m_{\text{PU}})$ is the signal to interference plus noise ratio (SINR). Parameter h^{up}_{km} is the channel gain between the kth CU and the AP on subcarrier m, N_0 is the noise PSD and I^m_{PU} is the interference from PUs introduced to the kth CU on subcarrier m.

3 Uplink resource allocation problem and proposed algorithm

The aim is subcarrier and power allocation to the K CUs to maximise the CR network throughput under the CUs transmission power budget and PUs maximum interference constraints. The optimisation problem is as follows

$$\text{maximise} \sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\text{av}}} y_{km} r_{km} \tag{5a}$$

subject to

$$\sum_{m \in \mathcal{M}_{\text{av}}} y_{km} p_{km} \le P_t^k, \quad \forall k \tag{5b}$$

$$\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\text{av}}} y_{km} p_{km} q_{km}^{\ell} \le I_{\text{th}}^{\ell}, \quad \text{for } \ell = 1, \ldots, L \tag{5c}$$

$$\sum_{k=1}^{K} y_{km} \le 1, \quad \forall m \in \mathcal{M}_{\text{av}} \tag{5d}$$

$$y_{km} \in \{0, 1\}, \quad \forall m \in \mathcal{M}_{\text{av}}, \ k = 1, \ldots, K \tag{5e}$$

$$p_{km} \ge 0, \quad \forall m \in \mathcal{M}_{\text{av}}, \ k = 1, \ldots, K \tag{5f}$$

where y_{km} is the subcarrier assignment variable such that $y_{km} = 1$ if subcarrier m is assigned to the kth CU and $y_{km} = 0$ otherwise. In (5b), P_t^k is the kth CU transmission power budget. The inequality (5c) indicates that the maximum interference power introduced to the ℓth PU should be less than the interference threshold I_{th}^{ℓ}. It should be noted that the fairness among CUs is not considered in the present resource allocation scenario in which maximising CR network throughput may be unfair to CUs with poor average channel gains. The proportional rate schemes in [5, 19] provides a certain level of fairness between CUs.

The optimisation problem in (5) is non-linear, non-convex and includes both binary y_{km} and real p_{km} variables. It is a subclass of MINLP problem, which is $\mathcal{NP}$-hard in general [25]. To solve, we decouple the problem into two steps. Before taking the steps, the CUs are allocated initial powers on all the subcarriers. In the first step, a subcarrier assignment scheme based on an adaptive algorithm is executed to assign the subcarriers to the CUs by using these initial powers such that the CR network throughput is maximised. In this step the subcarrier assignment problem is reformulated into an equivalent problem defined on a conceptual system in such a way that a least square based adaptive algorithm can find the solution. The adaptive algorithm modifies the subcarrier assignments iteratively to increase the CR network throughput until no modification leads to further gain. In the next step, optimal power allocation is carried out on the resulting subcarrier assignments.

3.1 Initial power allocation

To initialise power allocation, we use the following strategy where the transmission power of the kth CU on the available subcarrier $m \in \mathcal{M}_{\text{av}}$ is proportional to the subcarrier's SINR (γ_{km}) and inversely to the sum of weighted interference factors ($Q_{km} = \sum_{\ell} q_{km}^{\ell} / I_{\text{th}}^{\ell}$). The reason of normalising interference factors (q_{km}^{ℓ}) by their thresholds (I_{th}^{ℓ}) before summation is that each PU has a different threshold to tolerate interfering power. In this approach subcarriers with higher SINR and lower interference factors are allowed to transmit with higher power levels. That is, we have

$$\bar{p}_{km} = \frac{P_t^k (\gamma_{km} / Q_{km})}{\sum_{m \in \mathcal{M}_{\text{av}}} \gamma_{km} / Q_{km}}, \quad \forall m \in \mathcal{M}_{\text{av}}, \ \forall k \tag{6}$$

It is obvious that $\sum_{m \in \mathcal{M}_{\text{av}}} \bar{p}_{km} = P_t^k$ for all k which ensures CUs transmission power budget constraint (5b).

3.2 Step one: Subcarrier assignment scheme

After initial power allocation, in this step, we look for subcarrier assignment which maximises the CR network throughput under the PUs maximum interference constraints calculated using the initial allocated powers. We rewrite the uplink resource allocation (5) with CUs transmission power p_{km} fixed to their initial values according to (6) as follows

$$\text{maximise} \sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\text{av}}} y_{km} r_{km} \tag{7a}$$

subject to

$$\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\text{av}}} y_{km} I_{km}^{\ell} \le I_{\text{th}}^{\ell}, \quad \text{for } \ell = 1, \ldots, L \tag{7b}$$

$$\sum_{k=1}^{K} y_{km} \le 1, \quad \forall m \in \mathcal{M}_{\text{av}} \tag{7c}$$

$$y_{km} \in \{0, 1\}, \quad \forall m \in \mathcal{M}_{\text{av}}, \ k = 1, \ldots, K \tag{7d}$$

where

$$r_{km} = \log_2 (1 + \bar{p}_{km} \gamma_{km})$$

and

$$I_{km}^{\ell} = \bar{p}_{km} q_{km}^{\ell}$$

The problem in (7) is a version of the multiple-choice knapsack problem (MCKP) [26]. In the original MCKP, 'C classes of items are to be packed in a knapsack of given capacity. Each item has a profit and a weight. The aim is to maximise the sum profit by choosing one item from each class under a knapsack weight constraint' [27]. If we consider each subcarrier $m \in \mathcal{M}_{\text{av}}$ as a class of size K with the member items being the assignment of the mth subcarrier to CU 1 to K, then r_{km} plays the role of the profit in assigning subcarrier m to the kth CU and I_{km}^{ℓ} for $\ell = 1, \ldots, L$ play the role of the weights associated with each item. As can be seen, our problem differs from MCKP in that there are L weights associated with each item instead of one. Similary instead of one weight constraint, there are L knapsack weight constraints as defined by (7b).

The MCKP problem is $\mathcal{NP}$-hard in general. However, a variety of pseudo-polynomial time algorithms are available which find the solution. These algorithms usually belong to the dynamic programming, greedy or branch-and-bound algorithm categories [26]. While some of them are based on strict mathematics and get the exact solution, some others are heuristic and sometimes fail to find the optimal solution. Here, we introduce a computationally efficient adaptive algorithm which to increase the CR network throughput, modifies the subcarrier assignment iteratively until no modification leads to further gain. In this paper, we employ the normalised least mean square (NLMS) algorithm [28], sequentially having the chain of input-output pairs ($\boldsymbol{v}(1)$, $d(1)$), ($\boldsymbol{v}(2)$, $d(2)$), ... where the vector $\boldsymbol{v}(n)$ and the scalar $d(n)$ are the input and output of the conceptual system related to each other by

$$d(n) = \boldsymbol{y} \boldsymbol{v}(n)^{\text{T}} + \eta(n), \quad \text{for } n = 1, 2, \ldots \tag{8}$$

where $\eta(n)$ is a disturbance (measurement or additive noise) and T is the operator of transposition. The NLMS algorithm tries to iteratively

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estimate the unknown vector $\boldsymbol{y}$ by updating the recursion

$$\boldsymbol{y}(n) = \boldsymbol{y}(n-1) + \frac{\mu(n)}{\| \boldsymbol{v}(n) \|^2} \boldsymbol{v}(n) e(n) \tag{9}$$

where

$$e(n) = d(n) - \boldsymbol{y}(n-1)\boldsymbol{v}(n)^{\mathrm{T}} \tag{10}$$

and $\mu(n) > 0$ is the step-size of the algorithm at time instant n. Regularly the step-size is time-invariant ($\mu(n) = \mu = cte$). It is well-known that the NLMS algorithm (with fixed step-size), converges only in the mean and $\mu \in (0, 2)$ guarantees the mean convergence [28].

Assume $y_{km}^{(j)}$ is the subcarrier assignment indicator at the jth throughput enhancement iteration. The aim is to increase the throughput by modifying the subcarrier assignments. Hence, $y_{km}^{(j)}$ should satisfy the following constraints

$$\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\mathrm{av}}} y_{km}^{(j)} r_{km} \geq R_{\mathrm{tot}}^{(j-1)} \tag{11a}$$

$$\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\mathrm{av}}} y_{km}^{(j)} I_{km}^{\ell} \leq I_{\mathrm{th}}^{\ell}, \quad \text{for } \ell = 1, \ldots, L \tag{11b}$$

$$\sum_{k=1}^{K} y_{km}^{(j)} \leq 1, \quad \forall m \in \mathcal{M}_{\mathrm{av}} \tag{11c}$$

$$y_{km}^{(j)} \in \{0, 1\}, \quad \forall m \in \mathcal{M}_{\mathrm{av}}, \ k = 1, \ldots, K \tag{11d}$$

where

$$R_{\mathrm{tot}}^{(j-1)} = \sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\mathrm{av}}} y_{km}^{(j-1)} r_{km}$$

Constraint (11a) ensures that the new subcarrier assignments increase throughput in comparison with the previous iteration and constraints (11b)–(11d) are the same as the constraints (7b)–(7d). Here, we explain the steps needed to reformulate (11) into an equivalent problem defined on a conceptual system in such a way that a least square based adaptive algorithm can find the solution. For a better expression and ease of formulation, we define matrices $\boldsymbol{Y}^{(j)} = [y_{km}^{(j)}]$, $\boldsymbol{R} = [r_{km}]$ and $\boldsymbol{I}^{\ell} = [I_{km}^{\ell}]$ with the dimension $K \times N_{\mathrm{av}}$. Now, we rewrite (11a) and (11b), respectively, as follows

$$\sum_{k} \boldsymbol{y}_k^{(j)} \boldsymbol{r}_k^{\mathrm{T}} \geq R_{\mathrm{tot}}^{(j-1)} \tag{12}$$

and

$$\sum_{k} \boldsymbol{y}_k^{(j)} \boldsymbol{\iota}_k^{\ell \mathrm{T}} \leq I_{\mathrm{th}}^{\ell}, \quad \forall \ell \tag{13}$$

where $\boldsymbol{y}_k^{(j)}$, $\boldsymbol{r}_k$ and $\boldsymbol{\iota}_k^{\ell}$ indicate the kth row of $\boldsymbol{Y}^{(j)}$, $\boldsymbol{R}$ and $\boldsymbol{I}^{\ell}$. To rearrange the problem into the form required for executing an adaptive algorithm to estimate unknown vectors $\boldsymbol{y}_k^{(j)}$, we start with the inequalities (12) and (13) and we have

$$R_{\mathrm{tot}}^{(j-1)} = \sum_{k} \boldsymbol{y}_k^{(j)} \boldsymbol{r}_k^{\mathrm{T}} + u_R \tag{14}$$

$$I_{\mathrm{th}}^{\ell} = \sum_{k} \boldsymbol{y}_k^{(j)} \boldsymbol{\iota}_k^{\ell \mathrm{T}} + u_{I^{\ell}}, \quad \text{for } \ell = 1, \ldots, L \tag{15}$$

The non-negative parameter u_R and the non-positive parameters $u_{I^{\ell}}$, $\ell = 1, \ldots, L$ are unknown. Now, we define the sequences $x(n)$, $\boldsymbol{X}(n)$ and $z(n)$ according to the $L+1$ equalities (14) and (15) as follows

$$x(n) = \begin{cases} R_{\mathrm{tot}}^{(j-1)}, & \text{if } n = 0 \mod (L+1) \\ I_{\mathrm{th}}^{1}, & \text{if } n = 1 \mod (L+1) \\ \vdots & \\ I_{\mathrm{th}}^{L}, & \text{if } n = L \mod (L+1) \end{cases} \tag{16}$$

$$\boldsymbol{X}(n) = \begin{cases} \boldsymbol{R}, & \text{if } n = 0 \mod (L+1) \\ \boldsymbol{I}^{1}, & \text{if } n = 1 \mod (L+1) \\ \vdots & \\ \boldsymbol{I}^{L}, & \text{if } n = L \mod (L+1) \end{cases} \tag{17}$$

$$z(n) = \begin{cases} u_R, & \text{if } n = 0 \mod (L+1) \\ u_{I^1}, & \text{if } n = 1 \mod (L+1) \\ \vdots & \\ u_{I^L}, & \text{if } n = L \mod (L+1) \end{cases} \tag{18}$$

Now from (14) to (18), we have

$$x(n) = \sum_{k} \boldsymbol{y}_k^{(j)} \boldsymbol{x}_k(n)^{\mathrm{T}} + z(n) \tag{19}$$

where $\boldsymbol{x}_k(n)$ indicates the kth row of $\boldsymbol{X}(n)$. In (19), $x(n)$ is considered as the output of a conceptual system consisting of K subsystems with $\boldsymbol{y}_k^{(j)}$ and $\boldsymbol{x}_k(n)$ being the coefficient and input vectors of the kth subsystem at time instant n, respectively, and $z(n)$ being the system noise. The recursion (19) already has the form required for executing K NLMS algorithms, simultaneously to estimate K unknown vectors, $\boldsymbol{y}_k^{(j)}$. However, no adaptive algorithm could practically have a good estimate of the unknown vectors $\boldsymbol{y}_k^{(j)}$ for $k = 1, \ldots, K$, because of the high correlation between the matrices (note that $\boldsymbol{X}(n-L-1) = \boldsymbol{X}(n)$ for all n). A high correlation between the input vectors $\boldsymbol{x}_k(n)$, reduces the convergence speed of the algorithms. A simple trick helps to overcome this problem. Consider a white random matrix $\boldsymbol{V}(n)$ at time instant n and define

$$\widehat{\boldsymbol{X}}(n) = \boldsymbol{X}(n) - \boldsymbol{V}(n) \tag{20}$$

This uncertainty results in an internal noise in the conceptual system which helps the adaptive algorithm to converge. Assume $\boldsymbol{Y}^{(j)}(n-1)$ is an estimate of $\boldsymbol{Y}^{(j)}$ at time instant $n-1$. Defining

$$d(n) \equiv x(n) - \sum_{k} \boldsymbol{y}_k^{(j)}(n-1) \widehat{\boldsymbol{x}}_k(n)^{\mathrm{T}} \tag{21}$$

and

$$\eta(n) \equiv \sum_{k} (\boldsymbol{y}_k^{(j)} - \boldsymbol{y}_k^{(j)}(n-1)) \widehat{\boldsymbol{x}}_k(n)^{\mathrm{T}} + z(n) \tag{22}$$

where $\widehat{\boldsymbol{x}}_k(n)$ indicates the kth row of $\widehat{\boldsymbol{X}}(n)$. From (19) to (22), we have the following recursion

$$d(n) = \sum_{k} \boldsymbol{y}_k^{(j)} \boldsymbol{v}_k(n)^{\mathrm{T}} + \eta(n) \tag{23}$$

The unknown vectors $\boldsymbol{y}_k^{(j)}$ can be estimated by the K NLMS algorithms, executed simultaneously. The kth algorithm estimates $\boldsymbol{y}_k^{(j)}$ according to the following recursion, iteratively

$$\boldsymbol{y}_k^{(j)}(n) = \boldsymbol{y}_k^{(j)}(n-1) + \frac{\mu(n)}{\| \boldsymbol{v}_k(n) \|^2} \boldsymbol{v}_k(n) e(n), \quad \text{for } k = 1, \ldots, K \tag{24}$$

Algorithm 1

Input: $P_t^k, I_{\text{th}}^\ell, \gamma_{km}, \mathcal{M}_{av}, q_{km}^\ell,\ \forall m \in \mathcal{M}_{av}$;
Output: $y_{km}, p_{km},\ \forall k, m$;
Initial power allocation:
  $\bar{p}_{km} \leftarrow$ Initial power allocation according to (6);
Step 1: subcarrier assignment
  $\mu_n \leftarrow \mu \in (0, 2)$;
  $\boldsymbol{Y}^{(j)}\big|_{j=0} \leftarrow \big[0\big]_{K\times N_{av}}$;
  $R_{\text{tot}}^{(j)}\big|_{j=0} \leftarrow 0$;
  $j \leftarrow 0$;
  repeat
    $j \leftarrow j+1$;
    $\boldsymbol{Y}^{(j)}(n)\big|_{n=0} \leftarrow \boldsymbol{Y}^{(j-1)}$;
    for $n : 1$ to N **do**
      $z(n), x(n), \boldsymbol{X}(n) \leftarrow$ according to (16),(17),(18);
      $\boldsymbol{V}(n) \leftarrow$ random matrix;
      $\hat{\boldsymbol{X}}(n) \leftarrow \boldsymbol{X}(n) - \boldsymbol{V}(n)$;
      $d(n) \leftarrow x(n) - \sum_k \boldsymbol{y}_k^{(j)}(n-1)\hat{\boldsymbol{x}}_k(n)^{\mathrm{T}}$;
      $e(n) \leftarrow d(n) - \sum_k \boldsymbol{y}_k^{(j)}(n-1)\boldsymbol{v}_k(n)^{\mathrm{T}}$;
      for $k = 1$ to K **do**
        $\boldsymbol{y}_k^{(j)}(n) \leftarrow \boldsymbol{y}_k^{(j)}(n-1) + \frac{\mu}{\|\boldsymbol{v}_k(n)\|^2}\boldsymbol{v}_k(n)e(n)$;
      end for
      $\boldsymbol{Y}_Q(n) \leftarrow F_Q(\boldsymbol{Y}^{(j)}(n))$;
      if $(\sum_{k=1}^K \boldsymbol{y}_k^Q(n)\boldsymbol{r}_k^{\mathrm{T}} \geq R_{\text{tot}}^{(j-1)} \& \sum_{k=1}^K \boldsymbol{y}_k^Q(n)\boldsymbol{\iota}_k^{\ell\,\mathrm{T}} \leq I_{\text{th}}^\ell, \forall l)$ **then**
        $\boldsymbol{Y}^{(j)} \leftarrow \boldsymbol{Y}_Q(n)$;
        break
      end if
    end for
    $R_{\text{tot}}^{(j)} \leftarrow \sum_{k=1}^K \boldsymbol{y}_k^{(j)}\boldsymbol{r}_k$;
  until $(\| R_{\text{tot}}^{(j)} - R_{\text{tot}}^{(j-1)} \| / R_{\text{tot}}^{(j)} \leq \epsilon)$
Step 2: power allocation
  $p_{km} \leftarrow$ Solution of (27) using an interior point method;

Fig. 2 *Proposed resource allocation algorithm*

where $\boldsymbol{v}_k(n)$ is the kth row of $\boldsymbol{V}(n)$ and

$$e(n) \equiv d(n) - \sum_k \boldsymbol{y}_k^{(j)}(n-1)\boldsymbol{v}_k(n)^{\mathrm{T}} \tag{25}$$

Note that the sequence of the vectors $\boldsymbol{v}_k(n)$ is an uncorrelated sequence and the NLMS algorithm (9), (10) can be used to find the unknown vectors $\boldsymbol{y}_k^{(j)}$. A fixed step-size $\mu \in (0, 2)$ is considered and at each time instant, we quantise the elements of the estimated matrix $\boldsymbol{Y}^{(j)}(n)$ consisting of $\boldsymbol{y}_k^{(j)}(n)$ as its rows for $k = 1, \ldots, K$ to satisfy (11c) and (11d). To do so, define the function $F_Q(.)$

$$\boldsymbol{Y}_Q(n) = F_Q(\boldsymbol{Y}^{(j)}(n)) \tag{26}$$

takes $\boldsymbol{Y}^{(j)}(n)$ and assigns ones to the output elements corresponding to the largest element of each column and zeros to the rest of the elements. In Algorithm 1 (see Fig. 2), we consider the condition $(\| R_{\text{tot}}^{(j)} - R_{\text{tot}}^{(j-1)} \| / R_{\text{tot}}^{(j)} \leq \epsilon)$ as a stop criterium which is true only when the subcarrier assignment modification leads to no further gain.

3.3 Step two: Power allocation under given subcarrier assignments

When the subcarrier assignments y_{km} are given, the resource allocation problem (5) is rewritten as

$$\text{maximise} \sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\text{av}}} y_{km} r_{km} \tag{27a}$$

subject to

$$\sum_{m \in \mathcal{M}_{\text{av}}} y_{km} p_{km} \leq P_t^k, \quad \forall k \tag{27b}$$

$$p_{km} \geq 0, \quad \forall m \in \mathcal{M}_{\text{av}} \tag{27c}$$

$$\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{\text{av}}} y_{km} I_{km}^\ell \leq I_{\text{th}}^\ell, \quad \forall \ell \tag{27d}$$

where

$$r_{km} = \log_2(1 + p_{km}\gamma_{km})$$

$$I_{km}^\ell = p_{km} q_{km}^\ell$$

and the optimisation parameters are p_{km} for $k = 1, \ldots, K$ and $m \in \mathcal{M}_{\text{av}}$. The optimisation problem in (27) is convex and can be solved optimally using interior-point methods [29].

3.4 Computational complexity

The optimal resource allocation in OFDMA-based networks needs exhaustive search to find out the optimal assignment of N_{av} subcarriers to K users among $K^{N_{\text{av}}}$ assignment possibilities and jointly doing power allocation for each assignment possibility. Thus, the optimal resource allocation has a computational complexity of the order of $\mathcal{O}(K^{N_{\text{av}}})$ which grows exponentially with the number of subcarriers N_{av} and is computationally infeasible for large K and N_{av}. To determine the computational complexity of the proposed algorithm, we consider the first step, chooses one of the subcarrier assignment possibilities iteratively by quantising the estimated elements of matrix $\boldsymbol{Y}^{(j)}$ which requires $\mathcal{O}(KN_{\text{av}} \log_2(K))$ computational complexity and updating the **if**-condition until no modification leads to no further gain. Therefore the computational complexity of the proposed algorithm is $\mathcal{O}(J_{\max} NKN_{\text{av}} \log_2(K))$ where N is the maximum number of 'for–do' loop iterations and $J_{\max}$ is the maximum iteration numbers of 'loop–until' loop. In the next step, optimal power allocation is done for the resulting subcarrier assignments.

4 Simulation results

In this section, we evaluate the performance of our proposed method. We assume a PU network with $L = 2$ PUs and a CR network consists of K CUs. The whole spectrum licensed to the PU network is divided into $N_t = 15$ subcarriers each with bandwidth 40 kHz. We consider the noise PSD $N_0 = 0$ dBm/Hz. All the channel gains g_m, h_{km}^{up} and $w_{k\ell}^m$ have an exponential distribution with mean 0 dB and the primary BS transmission power $p_{\text{PU}}^{\ell i}$ are considered 0 dBm. Moreover we assume in each time slot, the AP identifies the following sets $\mathcal{M}_{\text{us}}^1 = \{3, 4, 5, 6\}$ and $\mathcal{M}_{\text{us}}^2 = \{10, 11, 12, 13\}$ as the subcarriers belonging to the L PUs based on the CUs sensing results. Hence the remaining $N_{\text{av}} = 7$ subcarriers are assigned to the K CUs such that the CR network throughput is maximised under the PUs maximum interference constraints and CUs transmission power budget. In the simulation results, 'exhaustive-search scheme' is used to represent exhaustive-search method to find out optimal subcarrier assignment, 'random scheme' to represent the random subcarrier assignment method following optimal power allocation, 'existing scheme' to represent subcarrier assignment scheme based on an item efficiency which is used commonly in knapsack problems. In this approach subcarriers with highest efficiency are assigned to the CUs while the overall interferences introduced to PUs do not exceed their threshold. In [18], the authors also proposed a subcarrier assignment scheme based on an item efficiency where they consider the ratio of the CU capacity on a subcarrier to the interference introduced to the PUs as the subcarrier efficiency. Since here, each PU has a different

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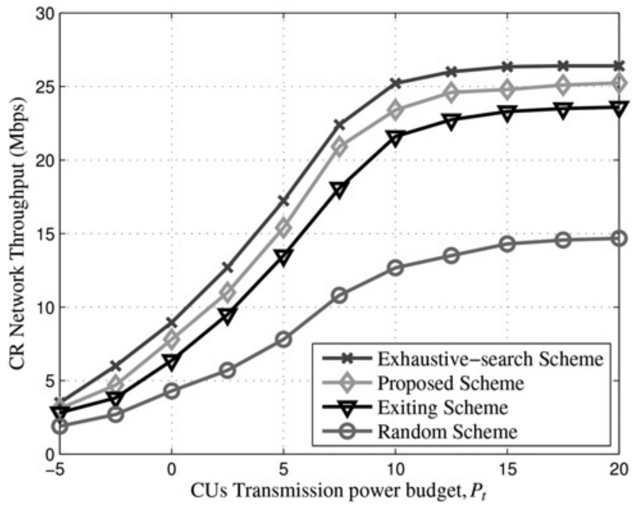

Fig. 3 *CR network throughput as a function of CUs transmission power budget P_t, for the 2 PUs interference thresholds, $I_{th}^1 = 0\,\text{dBm}$, $I_{th}^2 = 5\,\text{dBm}$*

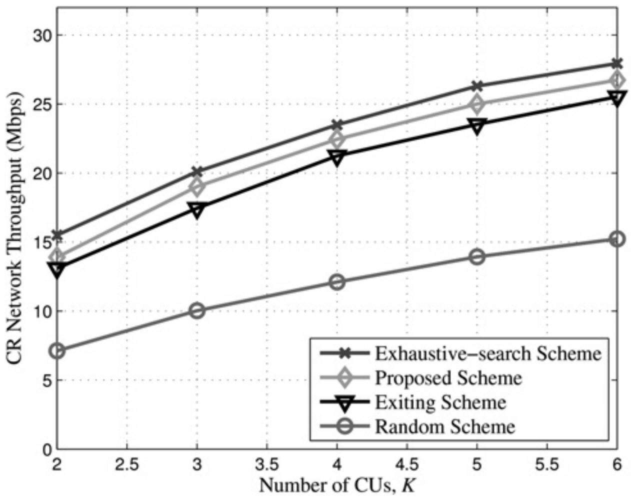

Fig. 4 *CR network throughput against the number of CUs*

interference threshold which is more practical, we consider the ratio of the kth CU capacity on the subcarrier m, r_{km} to the overall interference introduced to the PUs because of the CU transmission on that subcarrier weighted by their interference threshold, $\sum_\ell I_{km}^\ell / I_{\text{th}}^\ell$ as the efficiency of assigning subcarrier m to kth CU. The existing scheme is more efficient in comparison with the available algorithms based on dynamic programming and branch-and-bound, need large amount of computations and memory especially for large scale however there will be a considerable gap between the existing scheme results and optimal ones. And finally the 'proposed scheme' is used to represent resource allocation according to Algorithm 1 (Fig. 2).

In (20), the white random matrix $\boldsymbol{V}(n)$ should remove the correlation between the matrices $\boldsymbol{X}(n)$. Here, we produce the white random matrix as $\boldsymbol{V}(n) = \eta.\boldsymbol{X}(n) \circ \text{randn}(K, N_{\text{av}})$ where randn(K, N_{av}) is a Matlab command generates ($K \times N_{\text{av}}$) matrix of normally distributed pseudorandom elements with variance 1 and symbol $\circ$ indicates Hadamard product, also known as the element-wise product. We choose η from the interval (1/10, 1/5). All the results were obtained by averaging over 1 000 channel realisation.

Fig. 3 shows the CR network throughput as a function of the CUs transmission power budget for the PUs interference thresholds, $I_{\text{th}}^1 = 0$ dBm, $I_{\text{th}}^2 = 5$ dBm. We assume $K = 3$ CUs have an equal transmission power budgets P_t. The proposed scheme performance is compared with the three others. It can be seen that the increase in the CUs transmission power budget P_t leads to increase in the CR network throughput till the PUs interference thresholds become a limiting factor. Moreover, as shown the proposed scheme behaves near exhaustive-search scheme over a wide range of the CUs transmission power budget P_t. However, at the same time, the proposed scheme can reduce the computational complexity significantly.

In Fig. 4, we show the CR network throughput against the number of CUs for the PUs interference thresholds $I_{\text{th}}^1 = 0$ dBm, $I_{\text{th}}^2 = 3$ dBm and CUs transmission power budget $P_t^k = 7$ dBm for all k.

In Fig. 5, we compare the performance of the proposed scheme to the exhaustive-search scheme and existing scheme for the specific channel state of the CUs on the available subcarriers given in Tables 1–3 for the following parameters, CUs sum bit rate, CUs total transmission power and CUs overall interference introduced to the PU 1 and PU 2. We assume two PUs with interference thresholds $I_{\text{th}}^1 = 0\,\text{dBm}$ and $I_{\text{th}}^2 = 3\,\text{dBm}$ and $K = 3$ CUs with transmission power budgets $P_t^1 = 8\,\text{dBm}$, $P_t^2 = 8\,\text{dBm}$ and $P_t^3 = 10\,\text{dBm}$. In Fig. 5 the optimal parameter values are obtained

Table 1 CU1's SINR and interference factor

Subcarrier index, m	1	2	3	4	5	6	7
q_{1m}^1	0.1686	0.7150	0.6662	0.1620	0.0773	0.0143	0.0117
q_{1m}^2	0.0113	0.0140	0.0722	0.1686	0.7150	0.6662	0.1620
γ_{1m}	1.2217	2.4600	0.7334	0.5099	3.8243	0.6058	0.4726

Table 2 CU2's SINR and interference factors

Subcarrier index, m	1	2	3	4	5	6	7
q_{2m}^1	0.1087	0.5141	0.0816	0.0381	0.0229	0.0056	0.0046
q_{2m}^2	0.0060	0.0075	0.0451	0.1087	0.5141	0.0816	0.0381
γ_{2m}	1.6610	1.2297	1.0760	0.6410	1.6299	1.2823	0.0455

Table 3 CU3's SINR and interference factors

Subcarrier index, m	1	2	3	4	5	6	7
q_{3m}^1	0.3731	1.9853	0.7199	0.1696	0.0771	0.0157	0.0128
q_{3m}^2	0.0159	0.0200	0.1261	0.3731	1.9853	0.7199	0.1696
γ_{3m}	2.6053	3.3750	0.5179	4.8482	0.9048	0.2836	0.9036.

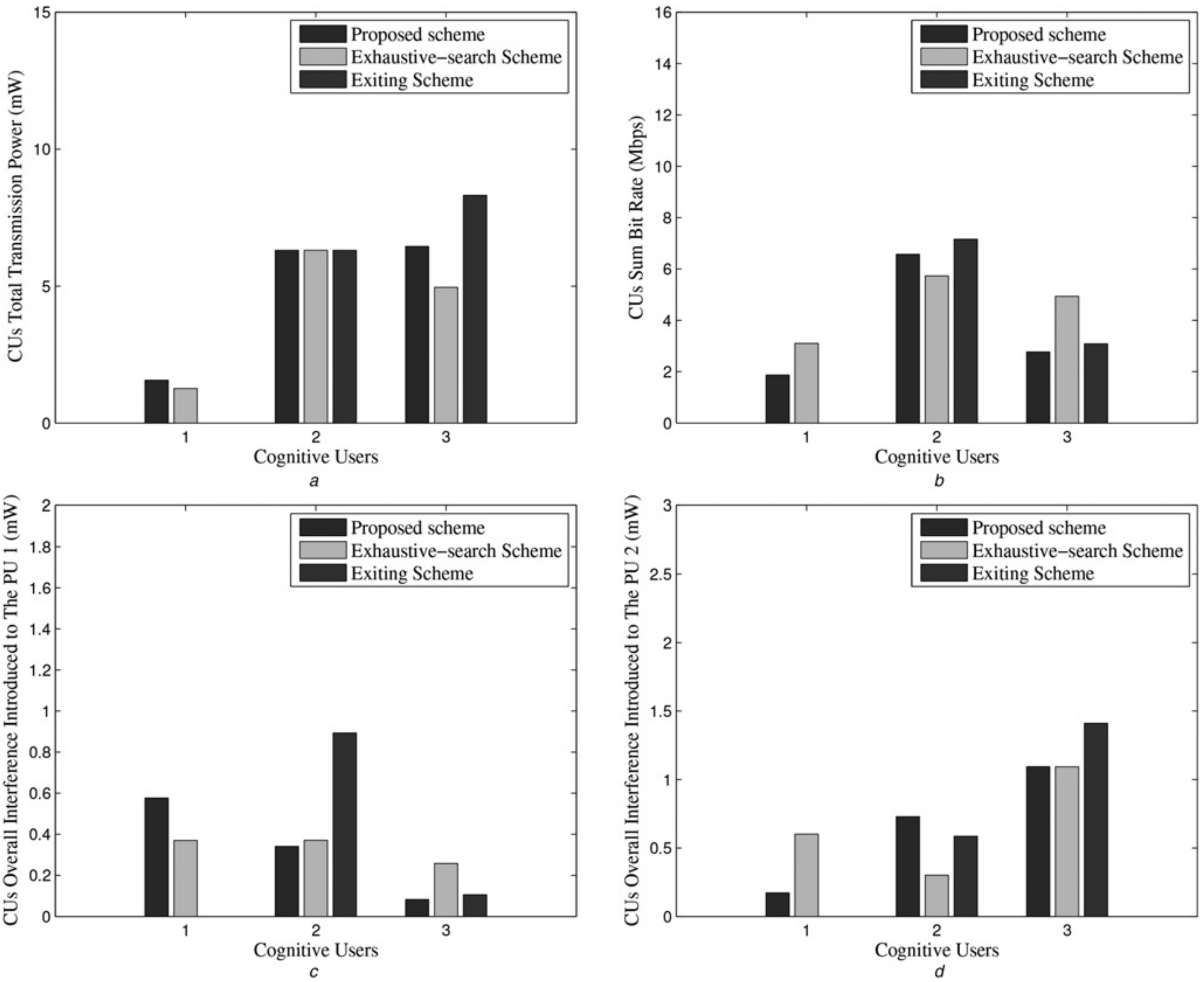

Fig. 5 *Uplink resource allocation for a specific channel state of the CUs with $P_t^1 = 8\,\text{dBm}$, $P_t^2 = 8\,\text{dBm}$ and $P_t^3 = 10\,\text{dBm}$ and PUs interference thresholds $I_{th}^1 = 0\,\text{dBm}$ and $I_{th}^2 = 3\,\text{dBm}$*

a CUs sum bit rate
b CUs total transmission power
c CUs overall interference introduced to the PU 1
d CUs overall interference introduced to the PU 2

by exhaustive search through $K^{N_{av}}$ subcarrier assignment possibilities following by optimal power allocation done by cvx

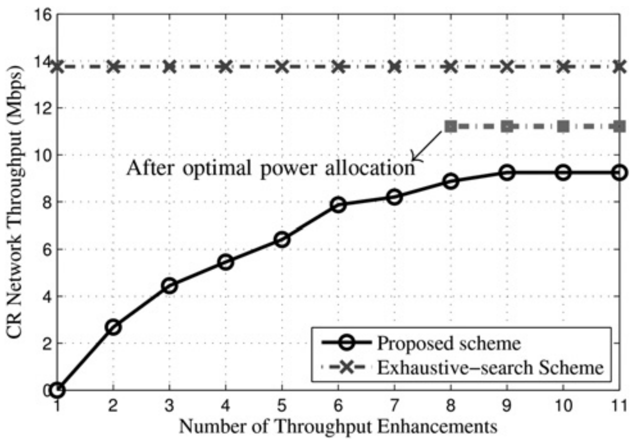

Fig. 6 *CR network throughput against the number of throughput enhancements [the parameter j in Algorithm 1 (Fig.)]*

version 1.21 [30] on the core i5 intel processor. This figure shows that the resource allocation done by the proposed scheme results in the CUs sum bit rate and of course, some interference power introduced to the PUs which are close to their optimal values determined by the exhaustive-search scheme. However, as shown the existing scheme fails to assign any subcarrier to the CU 1.

The convergence of the subcarrier assignment step for the previous example is depicted in Fig. 6, where the number of throughput enhancements [the parameter j in Algorithm 1 (Fig. 2)] is shown. The adaptive algorithm starts with $R_{\text{tot}}^{(0)} = 0$ as an throughput initial value and proceeds to modify the subcarrier assignment iteratively until no modification leads to further gain. The subcarrier assignment step converges through small number of iterations. Note that the throughput achieved in Fig. 6 is without the optimal power allocation step. Moreover, the red line determines the throughput that the proposed algorithm can achieve through optimal power allocation.

5 Summary and conclusion

In this paper, we studied uplink resource allocation in OFDMA-based CR networks considering mutual interference between PUs and CUs to maximise the throughput of the CR network. We discussed that this problem is $\mathcal{NP}$-hard in general. To resolve, we proposed a resource allocation algorithm consisting of two main steps. After initial power allocation, in the first step, we introduced the proposed subcarrier assignment scheme based on an adaptive algorithm to assign the subcarriers to the CUs by using these initial powers such that the CR network throughput is maximised. We explained how to reformulate the subcarrier assignment problem into an equivalent problem defined on a conceptual system in such a way that a least square based adaptive algorithm can find the solution. The proposed scheme proceeds to modify subcarrier assignment iteratively to increase the CR network throughput until no modification leads to further gain. Finally, in the next step, optimal power allocation is carried out under the assumption of known subcarrier assignment. The computational complexity of the proposed algorithm is significantly smaller than that of the optimal algorithm. Moreover, our simulation results show that the achievable bite rates for various CUs power budget and interference thresholds are very close to the maximum achievable bit rates.

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