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Spectrally encoded code division multiple access-based cognitive relay networks

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Abstract: Cognitive radio and cooperative networks are increasingly regarded as revolutionary technologies that utilise the radio spectrum efficiently. In this study, spectrally encoded (spread-time) code division multiple access (SE-CDMA) scheme is proposed as a flexible and adaptable technique for physical layer of cognitive relay networks. All primary and secondary users transmit their signals to a common base station (BS) and then the BS selects one of the cognitive secondary users based on feedback information to relay the primary user's (PU's) data. To study the efficiency of the system, two scenarios are considered. In the first scenario, the secondary users transmit their information using the SE-CDMA technique and the PU is considered narrowband. In the second scenario, both primary and secondary users utilise the SE-CDMA technique. The performance of the system is evaluated in terms of the error probability of the PU, the outage probability of the primary and secondary users and comparing theoretical results with simulations. Moreover, the performance of the proposed method is compared with orthogonal frequency division multiple access-based cognitive relaying scheme. The results indicate the efficiency of the SE-CDMA technique.

1 Introduction

Owing to growing demand of various applications with higher data rates and increase in the number of users, spectrum scarcity has become a serious problem in wireless communications. On the other hand, according to the measurements conducted by the federal communications commission, it is shown that merely about 30% of spectrum allocated to different applications is used and most of the frequency bands remain unutilised [1]. To improve the spectrum utilisation, cognitive radio was proposed. Cognitive radio is an intelligent wireless system, which is aware of its environment and can adapt the network parameters to statistical variations [2].

Cooperative communication represents a new technology to combat fading in wireless channels. It enables single antenna users in a multiple user environment to share their antennas and create a virtual multiple antenna transmitter to achieve diversity in fading channel [3]. The combination of cognitive radio and cooperation improves the whole system reliability and efficiency [4].

Adaption of physical layer is one of the important issues of cognitive and cooperative systems, which is the focus of this paper. The parameters that affect the design of physical layer waveforms include power, frequency, modulation, data rate, pulse shaping and decoding. Orthogonal frequency division multiplexing (OFDM) technique has been considered as a promising candidate for physical layer of cognitive network because of its flexibility incorporate resource allocations. The performance of OFDM technique in cognitive and cooperative networks has been studied in several works such as [5–8] in recent years. In [5, 6, 8], resource and power allocation problems of OFDM for cognitive cooperative systems have been investigated. In [7], a cognitive relaying scheme based on the orthogonal frequency division multiple access (OFDMA) has been proposed which considers two different cases of relay assignment and power allocation. Golrezaei-Khuzani and Nasiri-Kenari [7] evaluated the error probability of the primary user (PU) and the outage probability of both primary and cognitive secondary users. The analyses show substantial improvement in the error probability and outage probability of the PU, whereas the outage probability of the cognitive secondary user does not change significantly when compared with the non-cooperative scheme. It is shown in [8] that in an OFDM-based cognitive radio system, even with near-perfect spectrum sensing, the secondary users can produce significant interference to the PU operating in the adjacent bands and vice versa, especially when the PU does not adopt OFDM technique.

Considering the sensitivity of OFDM to the environment impairments, for instance, timing offset, frequency offset, multipath issues, carrier phase noise, peak-to-average power ratio and Doppler effect, Najafi and Shayesteh [9] proposed the spectrally encoded (spread-time) code division multiple access (SE-CDMA) technique for cognitive networks. The abilities of SE-CDMA to satisfy cognitive network necessities have been demonstrated in [9] in detail where the overlay and underlay techniques were considered as two enabling cognitive methods. In [9], the performance of the SE-CDMA method for cognitive system has been appraised in additive white Gaussian noise (AWGN) and fading channels for the primary and secondary users. In addition, its performance has been compared with cognitive radio using soft decision spectrally modulated SE waveforms. The results demonstrated that SE-CDMA schemes are efficient.

The SE-CDMA technique is considered as the dual of direct sequence spread spectrum CDMA technique. In the SE-CDMA, the data pulse spectrum are multiplied by a pseudorandom code sequence in frequency domain. Thereupon, the inverse Fourier transform is applied. As a result, the transmitted pulse is spread in the time domain. At the receiver, the data are recovered by sampling the output of matched filter. Different codes are allocated to different users to attain multiple access scheme [9].

The major advantage of the SE-CDMA technique is its ability in adapting to the channel impairments such as discontinuous frequency bands. In addition, it is shown that the SE-CDMA method has better signal-to-interference-plus noise ratio (SINR) than the spread spectrum CDMA in AWGN and fading channels. The inherent flexibility of spectrum shaping and spectrum management in SE-CDMA efficiently contends against the narrowband interference, which degrades the performance of ultra-wideband (UWB) system [9–13]. In [14], temporal power level profile of a typical SE-CDMA signal was investigated. In

[15, 16], Mashhadi and Salehi studied the performance of UWB SE-CDMA method considering Gaussian interference for RAKE receiver and maximum-likelihood receiver, and proposed a three-level spectrally encoding technique. Design and implementation of SE-CDMA transceiver was presented in [17], which is suitable for multiuser and anti-jamming wireless communication systems. In [18], in order to eliminate multiple access interference (MAI), novel orthogonal spreading codes were designed for SE-CDMA. More details about the SE-CDMA technique, its characteristics and advantages have been explained in [9–18].

In this paper, we propose the SE-CDMA scheme for cognitive relay networks and study its performance in fading channel. Both primary and secondary users transmit their signals to a common base station (BS). We consider two schemes for evaluating the performance of the proposed system. At first, we use SE-CDMA technique for the physical layer of secondary users while the PU is assumed narrowband. In the second scenario, the SE-CDMA technique is considered for both of the primary and secondary users. The error probability of the secondary user has been investigated in [9], considering the PU as a narrowband interference. As a result, in this paper the performance of the proposed cognitive relay SE-CDMA method is assessed by computing the error probability of the PU and outage probability of both primary and secondary users. In addition, simulations are provided to validate analytical results. The results are compared with those of [7] for the same channel characteristics. It is shown that employing SE-CDMA technique for primary and secondary users is more efficient than the OFDMA technique.

For cooperative system, the incremental feedback decode and forward relaying protocol is used. It means that, at first, the secondary users must correctly decode the data of the PU and then the secondary user with better channel condition is chosen as the relay. We assume that the secondary users can access the channel perfectly. In this case, the probability of detection is $P_d = 1$ and the probability of false alarm is $P_f = 0$. This assumption has been considered similarly in [7, 9, 19].

The rest of this paper is organised as follows: in Section 2, the system model is explained. In Sections 3 and 4, we evaluate the performance of narrowband and SE-CDMA technique for PU in fading channel, respectively. Section 5 provides numerical results. Finally, Section 6 summarises the approach.

2 System description

2.1 Cognitive relay scheme

As shown in Fig. 1, the cooperation system is composed of one BS and several clusters. There are one PU and multiple cognitive secondary users in each cluster. Each user has its own data to transmit to the BS. Hence, the uplink transmission is considered. We assume that the channel state information is available at the BS. This model has been considered similarly for OFDM-based systems [7, 19].

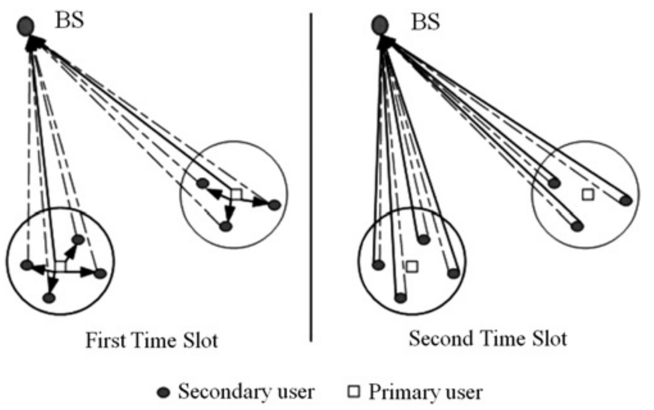

Fig. 1 *Cooperative system scheme*

Two time slots are considered for data transmission. In the first time slot, all users including primary and secondary send their data to the BS utilising spectrum access methods such as underlay or overlay. In this time slot, the secondary users receive the PU's data as well. In this case, the secondary users must have full duplex capability.

Using feedback information (channel gain between the secondary users and BS), the BS selects the secondary user that must relay the data of the PU which is unsuccessful in transmitting data. In other words, incremental relaying is used as cooperation protocol [7].

In the second time slot, the PU does not send data, the selected secondary user relays the PU's data and the other secondary users transmit their own data. Note that our goal is to detect the PU's data without error with the assistance of cognitive secondary users (cooperative scheme); besides, the secondary users will send their own data utilising the PU's frequency band (cognitive scheme).

2.2 SE-CDMA technique

In the SE-CDMA technique, the data modulated pulses are spectrally encoded by a pseudorandom code sequence which is uniquely assigned to each user, and as a result the transmitted pulses are spread in time. Fig. 2 demonstrates the block diagram of the SE-CDMA transmitter, typical data pulse spectrums of the SE-CDMA and narrowband users and a typical spreading code sequence. In SE-CDMA scheme, the spreading code sequence of each user encodes the data pulse spectrum $P(f)$ in the frequency domain. The spreading sequence spectrum of the kth SE-CDMA user $PN^k(f)$ in the baseband consists of $N/2$ distinct chips and is defined as

$$PN^k(f) = \sum_{i=-N/2}^{N/2-1} c_i^k \,\mathrm{rect}\left(\frac{f-(i+1/2)\Omega_c}{\Omega_c}\right) \tag{1}$$

where c_i^k is the ith component of the pseudorandom spreading code of the kth SE-CDMA user in the frequency domain. For the underlay method, c_i^k takes the values (+1, −1) and in the overlay case, it has three values, that is, (+1, 0, −1). rect(x) is the rectangle function in the interval (−1/2, 1/2) and the bandwidth of each frequency chip is $\Omega_c = (2W/N)$, where W is the bandwidth of the SE-CDMA baseband signal [9].

As shown in Fig. 2*a*, the data pulse spectrum $P(f)$ of SE-CDMA user is multiplied by $PN^k(f)$ in the frequency domain, and afterwards the inverse Fourier transform is applied. Thus, the transmitted pulse of the kth SE-CDMA user will be

$$q_s^k(t) = F^{-1}\{Q_s^k(f)\} = F^{-1}\{P(f)PN^k(f)\}$$

For the rectangular

$$P(f) = \begin{cases} 1/\sqrt{2W}, & |f| < W \\ 0, & \text{else} \end{cases}$$

and assuming (1), we obtain

$$q_s^k(t) = \sqrt{\frac{\Omega_c}{N}} e^{j\pi\Omega_c t} \sin c(\Omega_c t) \sum_{i=-N/2}^{N/2-1} c_i^k e^{j2\pi i\Omega_c t} \tag{2}$$

The transmitted signal of the kth SE-CDMA user considering 4-quadrature amplitude modulation (QAM) signalling can be written as

$$s_s^k(t) = \sqrt{E_{CR_k}} \sum_m d_m^k q_s^k(t-mT) \tag{3}$$

where $d_m^k = (\pm 1 \pm j)$ is the mth data symbol of the kth SE-CDMA

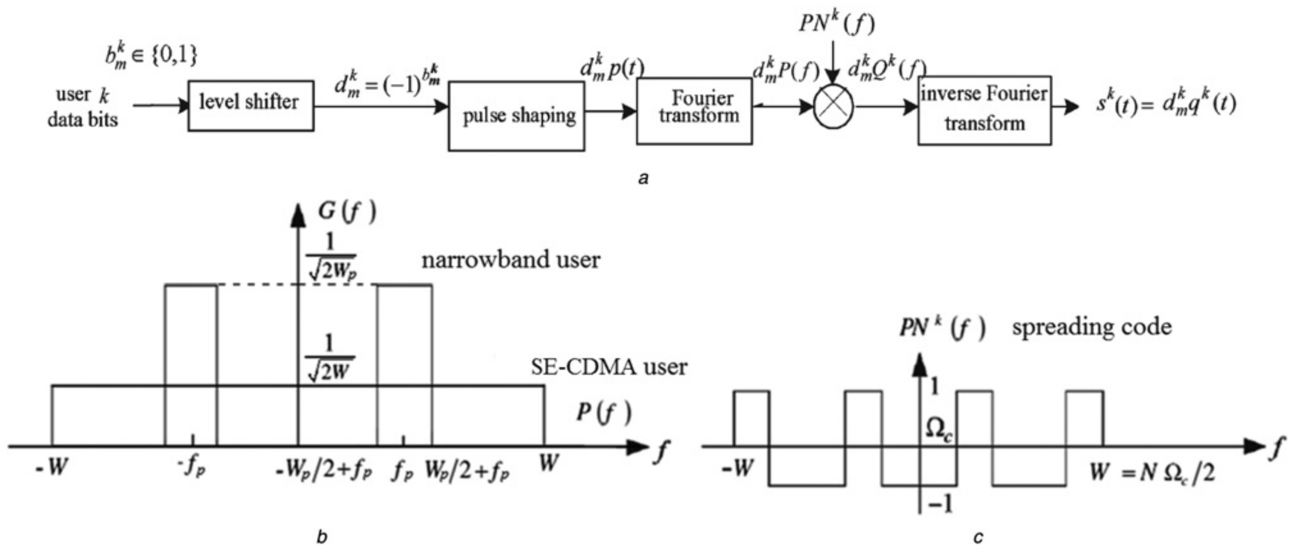

Fig. 2 *Block diagram of the SE-CDMA transmitter, typical data pulse spectrums of the SE-CDMA and narrowband users and a typical spreading code sequence*

a Block diagram of SE-CDMA transmitter for the *k*th user
b Data pulse spectrums of the SE-CDMA and narrowband users
c Typical *PN* sequence

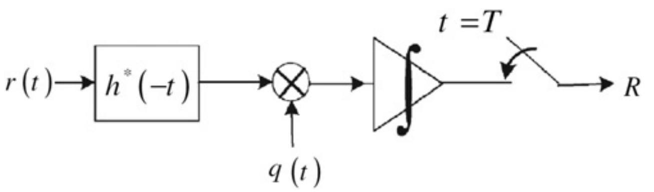

Fig. 3 *Block diagram of the receiver for the desired user*

user, E_{CR_k} is the transmitted energy per symbol and $1/T$ is the symbol rate.

Fig. 3 depicts the block diagram of the receiver, where $r(t)$ denotes the received signal, $h(t)$ is the impulse response of the channel and $q(t)$ indicates the transmitted pulse. At the receiver of SE-CDMA user, by applying the same spectral code to the time spread signal, the original short pulse will be recovered, whereas the signals of other users, which are spread by different pseudorandom codes, will spread more in time.

In the following two sections, two scenarios for PU transmission are considered and the performance is assessed.

3 Narrowband PU scheme

In this scenario, the PU is considered narrowband and the SE-CDMA is used for the secondary users. In the first time slot, the primary and secondary users transmit their data, where their data pulse spectrums are shown in Fig. 2*b*. In this time slot, in order to eliminate the interference between the primary and secondary users, we use overlay technique. It means that according to the spectrum information of the PU, at the transmitter three-level codes are utilised in the SE-CDMA technique for the secondary users. That is, zero code is assigned to the frequency chips in which the PU exists [9].

The model of channel for the narrowband PU (which is assumed to occupy one frequency chip) is assumed frequency flat slow fading. Hence, the channel can be written as

$$H_{\mathrm{p}}(f) = \alpha_{f_{\mathrm{p}}}\,\mathrm{rect}\left(\frac{f - f_{\mathrm{p}}}{\Omega_{\mathrm{c}}}\right), \quad \alpha_{f_{\mathrm{p}}} = \beta_{f_{\mathrm{p}}}\,\mathrm{e}^{\mathrm{j}\theta_{f_{\mathrm{p}}}} \tag{4}$$

where f_{p} is the central frequency of the PU's frequency chip, $\alpha_{f_{\mathrm{p}}}$ is the complex coefficient of channel in f_{p}, $\beta_{f_{\mathrm{p}}}$ is the amplitude of channel coefficient which has Rayleigh distribution with $E(\beta_{f_{\mathrm{p}}}^2) = \sigma^2$ and $\theta_{f_{\mathrm{p}}}$ is the random phase of $\alpha_{f_{\mathrm{p}}}$ having uniform distribution.

The channel of each SE-CDMA secondary user is considered frequency selective slow fading in the whole bandwidth W, but it is assumed frequency non-selective in each frequency chip [10, 20]. Accordingly, the channel of the kth secondary user can be expressed as

$$H_{\mathrm{s}}^k(f) = \sum_{n=-N/2}^{N/2-1} \alpha_n\,\mathrm{rect}\left(\frac{f - (n + 1/2)\Omega_{\mathrm{c}}}{\Omega_{\mathrm{c}}}\right), \quad \alpha_n = \beta_n\,\mathrm{e}^{\mathrm{j}\theta_n} \tag{5}$$

where α_n, β_n and θ_n have the same properties as explained for the channel of the PU. Note that $\beta_n = \beta_{-n-1}$. The correlation between β_n and $\beta_{n'}$ is not considered in order to attain closed-form expressions for eventuating equations [9]. It is shown that the channel model in (5) approximates the standard model 802.14.5.a for UWB communications [13].

The PU's transmitted signal using 4-QAM for data is

$$s_{\mathrm{p}}(t) = \sqrt{E_{\mathrm{p}}} \sum_m d_m^{\mathrm{p}} q_{\mathrm{p}}(t - mT) \tag{6}$$

where E_{p} is the transmitted energy per symbol of the PU and $d_m^{\mathrm{p}} = (\pm 1 \pm \mathrm{j})$ is the mth transmitted data symbol. $q_{\mathrm{p}}(t)$ is the transmitted pulse in the interval $(0, T)$ which has rectangular spectrum in the frequency domain as shown in Fig. 2*b*. Therefore (see (7)) where W_{p} is the bandwidth of the PU. It is assumed that the narrowband PU occupies one frequency chip of spreading code sequence, that is, $W_{\mathrm{p}} = \Omega_{\mathrm{c}}$.

$$Q_{\mathrm{p}}(f) = \begin{cases} \dfrac{1}{\sqrt{2W_{\mathrm{p}}}}, & -\dfrac{W_{\mathrm{p}}}{2} + f_{\mathrm{p}} < |f| < \dfrac{W_{\mathrm{p}}}{2} + f_{\mathrm{p}} \\ 0, & \text{else} \end{cases} \Rightarrow q_{\mathrm{p}}(t) = 2\sqrt{2W_{\mathrm{p}}}\cos(2\pi f_{\mathrm{p}} t)\sin c(W_{\mathrm{p}} t) \tag{7}$$

3.1 First time slot

The received signal at the BS in the first time slot is

$$r(t) = s_\mathrm{p}(t) \otimes h_\mathrm{p}(t) + \sum_{k=1}^{N_\mathrm{s}} s_\mathrm{s}^k(t) \otimes h_\mathrm{s}^k(t) + n(t) \tag{8}$$

where $\otimes$ denotes convolution, N_s is the number of secondary users, $s_\mathrm{s}^k(t)$ is the transmitted signal of the kth secondary user defined in (3) and $n(t)$ is the zero-mean AWGN with power spectral density $N_0/2$.

The receiver output signal owing to the PU is computed as [9]

$$\begin{aligned} S_{(\mathrm{PU,BS})} &= \int_0^\mathrm{T} \Big(\big(s_\mathrm{p}(t) \otimes h_\mathrm{p}(t) \big) \otimes h_\mathrm{p}^*(-t) \Big) q_\mathrm{p}(t)\, \mathrm{d}t \\ &= \sqrt{E_\mathrm{p} d_0^p} \beta^2_{f_\mathrm{p}(\mathrm{PU,BS})} \end{aligned} \tag{9}$$

where $\beta_{f_{\mathrm{p}(A,B)}}$ indicates the channel coefficient between A and B in the frequency f_p. The variance of noise conditioned on the channel coefficients at the receiver output will be calculated as

$$\begin{aligned} \mathrm{Var}\big(n | \beta_{f_\mathrm{p}(\mathrm{PU,BS})}\big) &= \frac{N_0}{2} \times 2 \int_{-W_\mathrm{p}/2+f_\mathrm{p}}^{W_\mathrm{p}/2+f_\mathrm{p}} \\ &\times \big|H_\mathrm{p}(f)\big|^2 \big|Q_\mathrm{p}(f)\big|^2 \,\mathrm{d}f = \frac{N_0}{2} \beta^2_{f_\mathrm{p}(\mathrm{PU,BS})} \end{aligned} \tag{10}$$

In the overlay scheme, the interference from the secondary users to the PU is zero because zero code is allocated to the frequency band occupied by the PU [9]. Therefore, considering (9) and (10), the SINR of the PU at the BS is computed as

$$\mathrm{SINR}_{(\mathrm{PU,BS})} = \frac{2E_\mathrm{p} \beta^2_{f_\mathrm{p}(\mathrm{PU,BS})}}{N_0} \tag{11}$$

Similarly, the effect of PU's signal at the receiver of the ith secondary user in the first time slot is obtained as

$$S_{(\mathrm{PU,CR}_i)} = \sqrt{E_\mathrm{p} d_0^p} \beta^2_{f_\mathrm{p}(\mathrm{PU,CR}_i)} \tag{12}$$

3.2 Second time slot

If the data symbol of the PU is not received successfully at the BS, one of the secondary users will transmit the PU's data as the relay in the second time slot using the SE-CDMA technique. The relay is selected by the BS based on the feedback information. There are two relay assignment methods for OFDMA method in [7]. In the first method, for each subcarrier of the PU the best cognitive user is selected as the relay, and in the second method for the entire subcarriers one secondary user is selected as the relay.

If the received SINR at the secondary user's receiver is lower than a pre-defined value, it is considered that this secondary user could not correctly decode the data of the PU. On the basis of the incremental feedback relaying protocol if the chosen relay can decode the data, it will retransmit it to the BS [7].

In this paper, among the secondary users that correctly decoded the PU's data denoted by $D(\mathrm{PU})$, the secondary user with better channel condition is selected as relay ($\mathrm{CR_r}$), that is

$$\mathrm{CR_r} = \max_{\mathrm{CR}_i \in D(\mathrm{PU})} \left(\sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR}_i,\mathrm{BS})} \right) \tag{13}$$

Therefore the output signal of the relay receiver at the BS in the second time slot is

$$R_{(\mathrm{CR_r,BS})} = S_{(\mathrm{CR_r,BS})} + \sum_{\substack{k=1 \\ k \neq r}}^{N_\mathrm{s}} I_{(\mathrm{CR}_k,\mathrm{BS})} + n \tag{14}$$

where $S_{(\mathrm{CR_r,BS})}$ is owing to the relay signal that is obtained as [9]

$$S_{(\mathrm{CR_r,BS})} = \frac{2}{N} \sqrt{E_{\mathrm{CR_r}} d_0^p} \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR_r,BS})} \tag{15}$$

where $E_{\mathrm{CR_r}}$ is the transmitted energy per symbol of the relay. In (14), $I_{(\mathrm{CR}_k,\mathrm{BS})}$ is the interference from the kth secondary user to the relay which is computed as [9]

$$\begin{aligned} I_{(\mathrm{CR}_k,\mathrm{BS})} &= \int_{-\infty}^{\infty} S_\mathrm{s}^k(f) H_\mathrm{s}^k(f) H_\mathrm{s}^{r*}(f) Q_\mathrm{s}^{r*}(f)\, \mathrm{d}f \\ &= \frac{2}{N} \sqrt{E_{\mathrm{CR}_k} d_0^k} \sum_{n=0}^{N/2-1} \alpha_{n(\mathrm{CR}_k,\mathrm{BS})} \alpha^*_{n(\mathrm{CR_r,BS})} c_n^k c_n^r \end{aligned} \tag{16}$$

The conditional variances of the interference and noise are derived as

$$\begin{aligned} \mathrm{Var}\Big(I_{(\mathrm{CR}_k,\mathrm{BS})} \big| \beta_{n(\mathrm{CR_r,BS})}\Big) &= \frac{4}{N^2} E_{\mathrm{CR}_k} \sigma^2 \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR_r,BS})}, \\ \mathrm{Var}\Big(n \big| \beta_{n(\mathrm{CR_r,BS})}\Big) &= \frac{N_0}{N} \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR_r,BS})} \end{aligned} \tag{17}$$

Therefore the SINR at the BS owing to the relay is obtained as

$$\mathrm{SINR}_{(\mathrm{CR_r,BS})} = \frac{4E_{\mathrm{CR_r}} \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR_r,BS})}}{4\sigma^2 \sum_{\substack{k=1 \\ k\neq r}}^{N_\mathrm{s}} E_{\mathrm{CR}_k} + N \cdot N_0} \tag{18}$$

3.3 Performance assessment of PU

The performance of narrowband PU is evaluated in terms of the outage probability and error probability.

3.3.1 Outage probability: The outage will occur when the average mutual information becomes smaller than a desired rate. For this reason, the data could not be detected correctly. The maximum of average mutual information between the PU and the BS according to the first theorem of Shannon for both non-cooperative (I) and cooperative (I_{CO}) schemes are computed as [7, 19]

$$\begin{aligned} I &= \frac{1}{2} \log_2(1 + \mathrm{SINR}_{(\mathrm{PU,BS})}), \\ I_{\mathrm{CO}} &= \frac{1}{2} \log_2\Big(1 + \mathrm{SINR}_{(\mathrm{PU,BS})} + \mathrm{SINR}_{(\mathrm{CR_r,BS})}\Big) \end{aligned} \tag{19}$$

By replacing (11) in (19), the PU's outage probability in the first time slot is given by

$$\begin{aligned} P^1_{\mathrm{p-out}} &= \Pr[I < R] \\ &= \Pr\left[\frac{1}{2} \log_2 \left(1 + \frac{2E_\mathrm{p} \beta^2_{f_\mathrm{p}(\mathrm{PU,BS})}}{N_0} \right) < R \right] \end{aligned} \tag{20}$$

where R is the desired data rate. Since we consider Rayleigh distribution for β_{f_p}, so $\beta^2_{f_\mathrm{p}}$ has exponential distribution [19]. After several computations, for the first time slot we obtain

$$P^1_{\mathrm{p-out}} = 1 - \mathrm{e}^{-\lambda_{(\mathrm{PU,BS})} Z} \tag{21}$$

where $Z = 2^{2R} - 1$ and $\lambda_{(\mathrm{PU,BS})} = (N_0/(2E_\mathrm{p}\sigma^2))$.

The total outage probability considering the first and second time slots is computed as [7]

$$P_{\text{p-out}}^{\text{co}} = P_{\text{ND}} + \sum_{D(\text{PU})} P_{\text{out}}(D(\text{PU}))\,\Pr[D(\text{PU})] \tag{22}$$

where $\Pr[D(\text{PU})]$ is the probability of the set $D(\text{PU})$, $P_{\text{out}}(D(\text{PU}))$ is the outage probability conditioned on set $D(\text{PU})$ and P_{ND} is the probability that none of the secondary users can decode the data of PU.

The parameters of (22) have been calculated in Appendix 1. By replacing (39)–(41) in (22), the total outage probability is attained.

3.3.2 Error probability: The error probability conditioned on occurring outage in the first time slot is defined as [7]

$$P_{\text{e}}^{\text{c}} \equiv \Pr[\text{error}|\text{outage in first time slot}] = \frac{P_{\text{e}}}{P_{p-\text{out}}^{1}} \tag{23}$$

where P_{e} is the error probability of cooperative scheme. When the PU is unsuccessful in data transmission in the first time slot, P_{e} is dependent on the set $D(\text{PU})$ and is computed as follows [7]

$$P_{\text{e}} = P_{\text{ND}}P_{\text{e}_{\text{NC}}} + \sum_{D(\text{PU})} \Pr[D(\text{PU})]\Big[P_{\text{e}_{\text{CO}}}(D(\text{PU}))\big(1 - P_{\text{e}_{\text{r}}}(D(\text{PU}))\big) + \big(1 - P_{\text{e}_{\text{CO}}}(D(\text{PU}))\big)P_{\text{e}_{\text{r}}}(D(\text{PU}))\Big] \tag{24}$$

where $P_{\text{e}_{\text{NC}}}$ is the error probability of PU when none of the secondary users can decode the data of PU, $P_{\text{e}_{\text{CO}}}(D(\text{PU}))$ is the error probability in cooperative scheme conditioned on the set $D(\text{PU})$ and $P_{\text{e}_{\text{r}}}(D(\text{PU}))$ is the error probability when error occurs in the relay conditioned on the set $D(\text{PU})$.

We have calculated the parameters of (24) in Appendix 1. By replacing (40)–(42), (46) and (49) in (24), the error probability of cooperative scheme is obtained. Consequently, by substituting P_{e} and $P_{\text{p-out}}^{1}$ in (23), P_{e}^{c} can be computed.

3.4 Performance evaluation of secondary users

The error probability of cognitive secondary users employing SE-CDMA was obtained in [9]. Hence, in this part the performance of the secondary user is investigated by computing the outage probability in two time slots. The average outage probability of the desired secondary user can be obtained as

$$P_{\text{s-out}} = \frac{P_{\text{s-out}}^{1} + P_{\text{s-out}}^{2}}{2} \tag{25}$$

where $P_{\text{s-out}}^{1}$ and $P_{\text{s-out}}^{2}$ are the outage probabilities in the first and second time slots, respectively.

3.4.1 Outage probability in the first time slot: In the first time slot, overlay scheme is applied to cognitive system, in which zero code is allocated to the frequency chip occupied by the PU. Therefore the interference from the PU is omitted. Consequently, the number of frequency chips should be considered $N/2-1$ instead of $N/2$.

The BS receiver outputs owing to the desired (first) secondary user and the kth secondary user's interference term are, respectively, derived as [9]

$$S_{(\text{CR}_1,\text{BS})} = \frac{2}{N}\sqrt{E_{\text{CR}_1}}d_0^1 \sum_{\substack{n=0\\ n\neq f_{\text{p}}}}^{N/2-1} \beta_{n(\text{CR}_1,\text{BS})}^2;$$
$$I_{(\text{CR}_k,\text{BS})} = \frac{2}{N}\sqrt{E_{\text{CR}_k}}d_0^k \sum_{\substack{n=0\\ n\neq f_{\text{p}}}}^{N/2-1} \alpha_{n(\text{CR}_k,\text{BS})}\alpha_{n(\text{CR}_1,\text{BS})}^{*}c_n^k c_n^1 \tag{26}$$

The conditional variances of the interference and noise are obtained as

$$\text{Var}\big(I_{(\text{CR}_k,\text{BS})}|\beta_{n(\text{CR}_1,\text{BS})}\big) = \frac{4}{N^2}E_{\text{CR}_k}\sigma^2 \sum_{\substack{n=0\\ n\neq f_{\text{p}}}}^{N/2-1} \beta_{n(\text{CR}_1,\text{BS})}^2;$$
$$\text{Var}\big(n|\beta_{n(\text{CR}_1,\text{BS})}\big) = \frac{N_0}{N}\sum_{\substack{n=0\\ n\neq f_{\text{p}}}}^{N/2-1} \beta_{n(\text{CR}_1,\text{BS})}^2 \tag{27}$$

Therefore the SINR in the first time slot is attained as

$$\text{SINR}_{(\text{CR}_1,\text{BS})}^{1} = \frac{4E_{\text{CR}_1}\sum_{n=0,\, n\neq f_{\text{p}}}^{N/2-1} \beta_{n(\text{CR}_1,\text{BS})}^2}{4\sigma^2\sum_{k=2}^{N_{\text{s}}} E_{\text{CR}_k} + N\cdot N_0} \tag{28}$$

In this way, the outage probability in the first time slot is computed as

$$P_{\text{s-out}}^{1} = \Pr\left[\frac{1}{2}\log_2\big(1 + \text{SINR}_{(\text{CR}_1,\text{BS})}^{1}\big) < R\right] = \frac{\gamma\big(N/2-1,\ \lambda_{(\text{CR}_1,\text{BS})}^{1}Z\big)}{(N/2-2)!} \tag{29}$$

where

$$\gamma(L,\ x) = \int_0^x t^{L-1}\,\text{e}^{-t}\,\text{d}t$$

is the lower incomplete Gamma function [21] and

$$\lambda_{(\text{CR}_1,\text{BS})}^{1} = \frac{4\sigma^2\sum_{k=2}^{N_{\text{s}}} E_{\text{CR}_k} + N\cdot N_0}{4E_{\text{CR}_1}\sigma^2(N/2-1)}$$

3.4.2 Outage probability in the second time slot: In this time slot, the PU does not send data and the SE-CDMA relay will send the data of PU in all frequency chips. The other secondary users transmit their own data to the BS. Hence, in the second time slot, the interference exists because of the other secondary users. As a result, the outage probabilities in the first and second time slots in this scheme are the same with a difference in the number of frequency chips, that is, $N/2$ instead of $N/2-1$ in (29), since the secondary users send in all frequency chips. Therefore the outage probability in the second time slot is obtained as

$$P_{\text{s-out}}^{2} = \frac{\gamma\big(N/2,\ \lambda_{(\text{CR}_1,\text{BS})}^{2}Z\big)}{(N/2-1)!} \tag{30}$$

where

$$\lambda_{(\text{CR}_1,\text{BS})}^{2} = \frac{4\sigma^2\sum_{k=2}^{N_{\text{s}}} E_{\text{CR}_k} + N\cdot N_0}{2E_{\text{CR}_1}\sigma^2 N}$$

By replacing (29) and (30) in (25), the average outage probability is achieved.

4 Spectrally encoded PU scheme

In this scheme, the primary and secondary users send their data utilising the SE-CDMA method. In the first time slot, the underlay technique is applied as cognitive radio approach that permits the secondary users to use the frequency band of the PU simultaneously [9]. In the second time slot, such as the previous part, the relay transmits PU's data and the other secondary users send their own data.

4.1 Performance assessment of PU

Now, we will attain expressions for the outage probability and error probability of the PU. The SINR of this case considering (15) and (17) for the SE-CDMA PU is obtained as

$$\mathrm{SINR}_{(\mathrm{PU,BS})} = \frac{4E_{\mathrm{p}} \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{PU,BS})}}{4\sigma^2 \sum_{k=1}^{N_{\mathrm{s}}} E_{\mathrm{CR}_k} + N \cdot N_0} \quad (31)$$

4.1.1 Outage probability: Since the $\mathrm{SINR}_{(\mathrm{PU,BS})}$ in (31) has Gamma distribution with the parameters $N/2$ and $\lambda_{(\mathrm{PU,BS})}$[20], hence the PU's outage probability in the first time slot is calculated as follows

$$\begin{aligned} P^1_{\text{p-out}} &= \Pr\left[\frac{1}{2}\log_2\left(1 + \mathrm{SINR}_{(\mathrm{PU,BS})}\right) < R\right] \\ &= \frac{\gamma\left(N/2,\ \lambda_{(\mathrm{PU,BS})} Z\right)}{(N/2-1)!} \end{aligned} \quad (32)$$

where

$$\lambda_{(\mathrm{PU,BS})} = \frac{4\sigma^2 \sum_{k=1}^{N_{\mathrm{s}}} E_{\mathrm{CR}_k} + N \cdot N_0}{2NE_{\mathrm{p}}\sigma^2}$$

The total outage probability in the cooperative scheme was defined in (22), where the parameters have been computed in Appendix 2. By substituting (51), (53) and (54) in (22), the outage probability of the PU is computed.

4.1.2 Error probability: To attain the error probability $P^{\mathrm{c}}_{\mathrm{e}}$ in (23), at first we must determine the error probability of cooperative scheme (P_{e}) given in (24) where its parameters are computed in Appendix 2. Therefore, by substituting (41), (42), (53), (56) and (58) in (24), P_{e} is obtained for the case that SE-CDMA technique is applied to all users. Finally, by replacing (24) and (32) in (23), $P^{\mathrm{c}}_{\mathrm{e}}$ will be computed.

4.2 Performance evaluation of secondary user

In the first time slot, in addition to the other secondary users, the PU makes interference to the desired secondary user as well. Although in the second time slot, there is interference just from the secondary users.

4.2.1 Outage probability in the first time slot: The SINR of the desired (first) secondary user in the first time slot noting (26) and (27) is

$$\mathrm{SINR}^1_{(\mathrm{CR}_1,\mathrm{BS})} = \frac{4E_{\mathrm{CR}_1} \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR}_1,\mathrm{BS})}}{\sum_{k=2}^{N_{\mathrm{s}}} 4E_{\mathrm{CR}_k}\sigma^2 + 4E_{\mathrm{p}}\sigma^2 + N \cdot N_0} \quad (33)$$

Therefore the outage probability is calculated as

$$\begin{aligned} P^1_{\text{s-out}} &= \Pr\left[\frac{1}{2}\log_2\left(1 + \mathrm{SINR}^1_{(\mathrm{CR}_1,\mathrm{BS})}\right) < R\right] \\ &= \frac{\gamma\left(N/2,\ \lambda^1_{(\mathrm{CR}_1,\mathrm{BS})} Z\right)}{(N/2-1)!} \end{aligned} \quad (34)$$

where

$$\lambda^1_{(\mathrm{CR}_1,\mathrm{BS})} = \frac{\sum_{k=2}^{N_{\mathrm{s}}} 4E_{\mathrm{CR}_k}\sigma^2 + 4E_{\mathrm{p}}\sigma^2 + N \cdot N_0}{2E_{\mathrm{CR}_1}\sigma^2 N}$$

4.2.2 Outage probability in the second time slot: As mentioned, in this time slot, the PU does not send data and the interference is just from the other secondary users. Therefore the SINR of the desired secondary user in the second time slot is

$$\mathrm{SINR}^2_{(\mathrm{CR}_1,\mathrm{BS})} = \frac{4E_{\mathrm{CR}_1} \sum_{n=0}^{N/2-1} \beta^2_{n(\mathrm{CR}_1,\mathrm{BS})}}{\sum_{k=2}^{N_{\mathrm{s}}} 4E_{\mathrm{CR}_k}\sigma^2 + N \cdot N_0} \quad (35)$$

Thereby, the outage probability is derived as

$$\begin{aligned} P^2_{\text{s-out}} &= \Pr\left[\frac{1}{2}\log_2\left(1 + \mathrm{SINR}^2_{(\mathrm{CR}_1,\mathrm{BS})}\right) < R\right] \\ &= \frac{\gamma\left(N/2,\ \lambda^2_{(\mathrm{CR}_1,\mathrm{BS})} Z\right)}{(N/2-1)!} \end{aligned} \quad (36)$$

where

$$\lambda^2_{(\mathrm{CR}_1,\mathrm{BS})} = \frac{\sum_{k=2}^{N_{\mathrm{s}}} 4E_{\mathrm{CR}_k}\sigma^2 + N \cdot N_0}{2E_{\mathrm{CR}_1}\sigma^2 N}$$

By replacing $P^1_{\text{s-out}}$ and $P^2_{\text{s-out}}$ in (25), the outage probability of the secondary user is achieved.

5 Numerical results

In this section, the performance of the proposed system is evaluated using analytical derivations and simulation results. The performance metrics are the error probability of the PU and the outage probabilities of both primary and secondary users. The transmitted energy per symbol for all users is assumed the same (E) in both scenarios (narrowband and SE-CDMA PU). The average signal-to-noise ratio (SNR) per symbol is defined as $((E\sigma^2)/(N_0/2))$. Monte Carlo simulation is used for error probability computation and 4-QAM symbols are generated randomly. The variance of Rayleigh fading channel is set to $\sigma^2=1$. The desired data rate is $R=2$ bit/s/Hz. The number of frequency chips in SE-CDMA method is $N/2=4$ and the code sequences for users in the SE-CDMA technique are generated randomly.

5.1 Performance of the PU

In Figs. 4*a* and *b*, the outage probability and the error probability of the PU are shown, respectively, where two schemes are considered: narrowband PU and SE-CDMA PU. The number of secondary users is $N_{\mathrm{s}}=3$. It is observed that, applying SE-CDMA method to the PU achieves better performance. Since in this scheme the SINR is more than the SINR of narrowband PU case and noting (19), increasing SINR increases the average mutual information and consequently the outage probability and error probability reduce. It is seen that simulation results verify the analytical derivations.

Now, the effect of the number of secondary users on the performance of the PU is evaluated. Figs. 5*a* and *b* depict the outage probability of both narrowband and SE-CDMA PU for different numbers of secondary users, respectively. As observed from Fig. 5*a*, increasing the number of secondary users reduces the outage probability of narrowband PU. The reason is that increasing the number of cooperative secondary users leads to high spatial diversity gain. In addition, more number of secondary users provide more information, and accordingly more reliable decisions about relay can be made. Even though one secondary user may fail to detect the signal of the PU, there are still many chances for other secondary users to detect it.

In the second scheme in which SE-CDMA is applied to the PU, the secondary users make interference to the PU. By increasing the number of secondary users, the interference to the PU increases which reduces the SINR and consequently the outage probability increases. On the other hand, increasing the number of

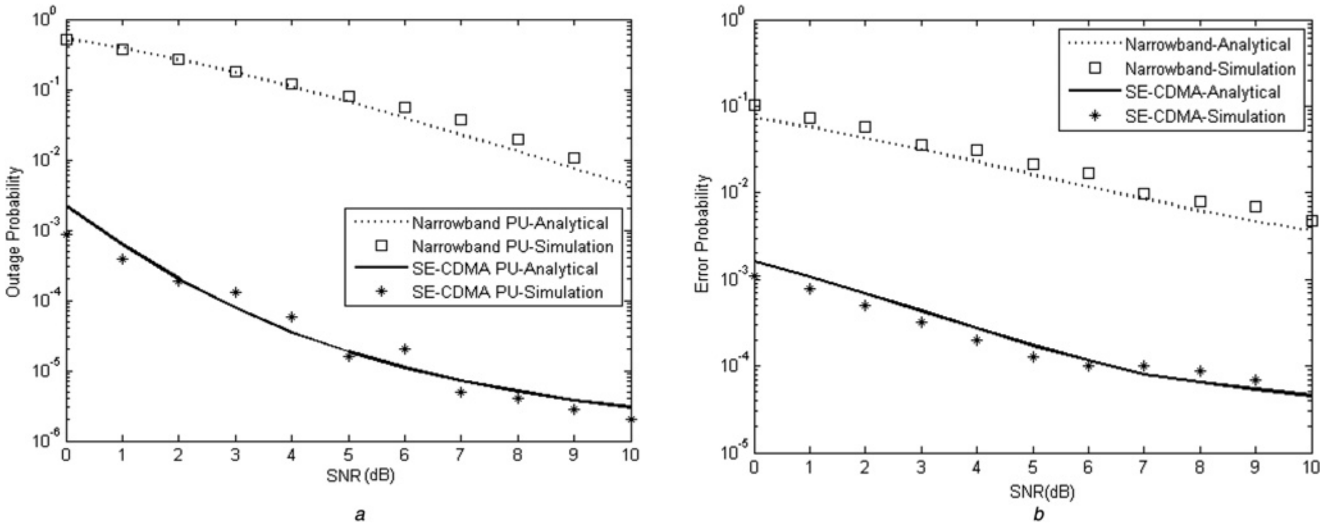

Fig. 4 *Performance of the PU in narrowband and SE-CDMA schemes, $N = 8$, $N_s = 3$*

a Outage probability
b Error probability

secondary users increases the spatial diversity which in turn improves the outage performance. Therefore, as Fig. 5*b* demonstrates, there is a trade-off between increasing the number of secondary users and SNR for the SE-CDMA PU. In lower SNRs, the outage performance for $N_s = 3$, 4 is better than for $N_s = 2$, but when SNR increases, increasing the number of secondary users degrades the outage performance.

Figs. 6*a* and *b* depict the error probability of the PU for different numbers of secondary users in the cases of narrowband and SE-CDMA schemes. Increasing the number of secondary users affects the performance of narrowband and SE-CDMA PU differently. When the PU is narrowband, increasing the number of secondary users increase the space diversity gain which improves the error performance. However, for the SE-CDMA PU, increase in the number of secondary users for SNRs higher than 5 dB will increase the error probability. The reason is the same as explained for Fig. 5.

In Figs. 7*a* and *b*, the outage probability and the error probability of the SE-CDMA PU are shown for different values of N, respectively. We observe that by increasing N, the performance improves. Since the coding gain increases and consequently SINR increases [10, 11, 13].

5.2 Performance of the secondary user

Fig. 8 demonstrates the outage probability of the secondary user. It is observed that the outage performance of the secondary user is better when the PU utilises the SE-CDMA scheme. Since in the narrowband PU scheme, we use overlay technique and assign zero code to the frequency band which is occupied by the PU. Therefore the number of frequency chips when SE-SDMA scheme is applied to the PU is greater than the case of narrowband PU. Increasing the number of frequency chips increases the code length and processing gain. This increases SINR and consequently the outage performance improves.

In Figs. 9*a* and *b*, the effect of the number of secondary users on the performance of the secondary user is illustrated. As expected, the increment in the number of secondary users degrades the outage performance of the secondary user. The reason is that by

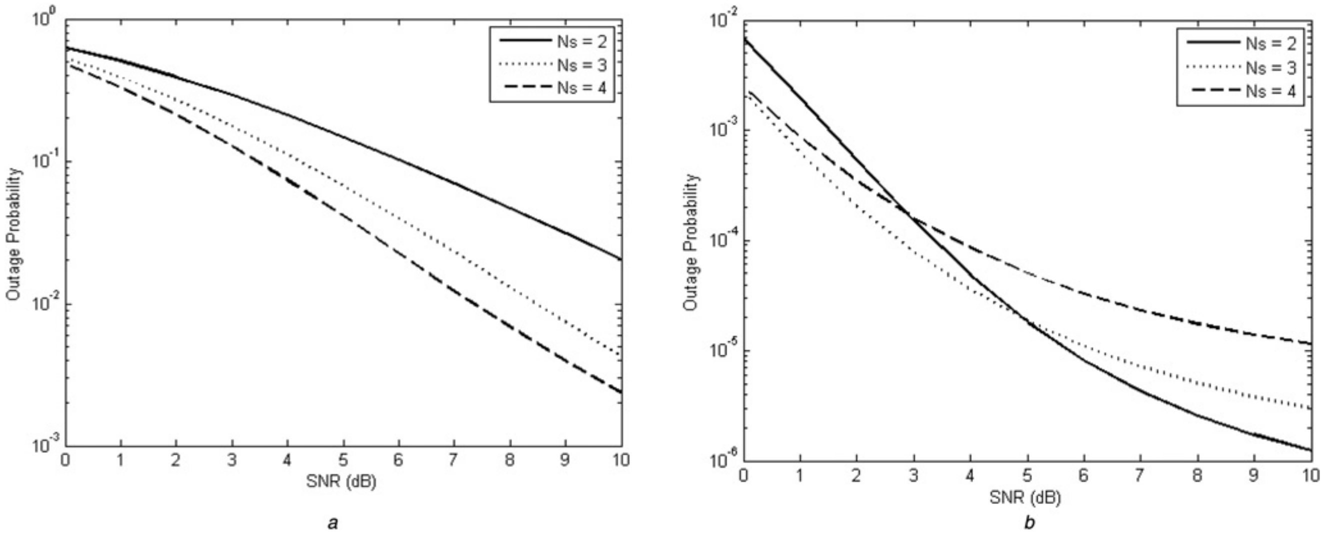

Fig. 5 *Outage probability of the PU for different numbers of secondary users, $N = 8$*

a Narrowband PU
b SE-CDMA PU

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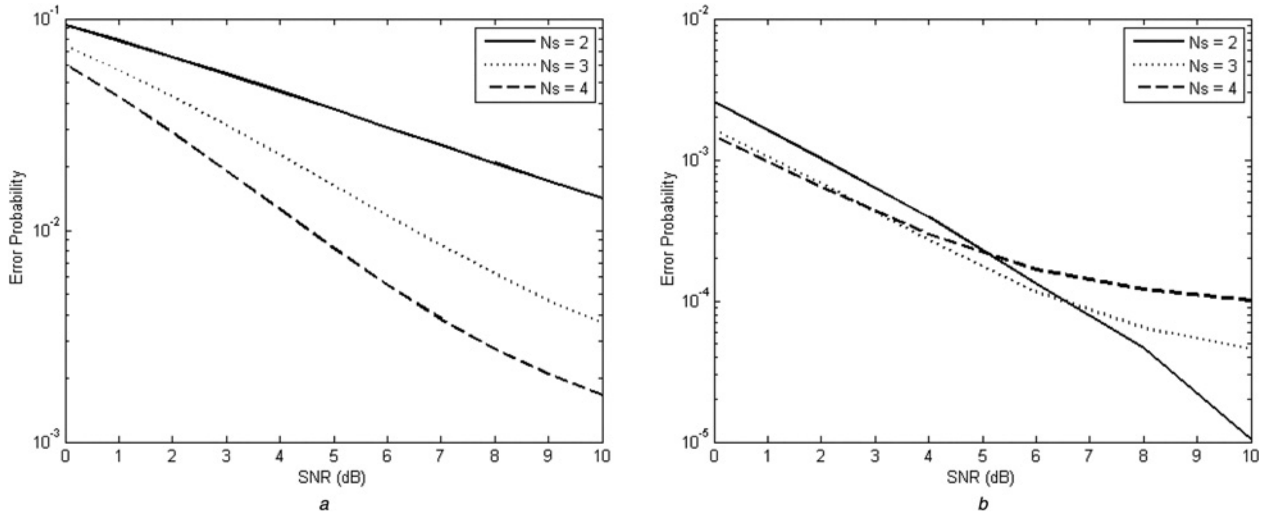

Fig. 6 *Error probability of the PU for different numbers of secondary users, N = 8*

a Narrowband PU
b SE-CDMA PU

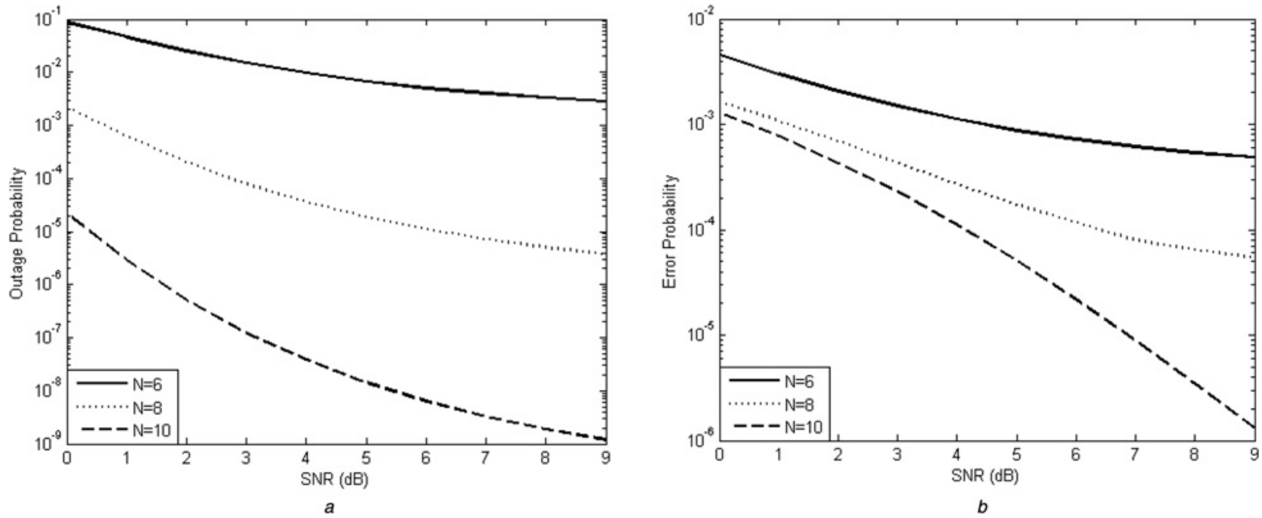

Fig. 7 *Performance of the SE-CDMA PU for different values of N, $N_s = 3$*

a Outage probability
b Error probability

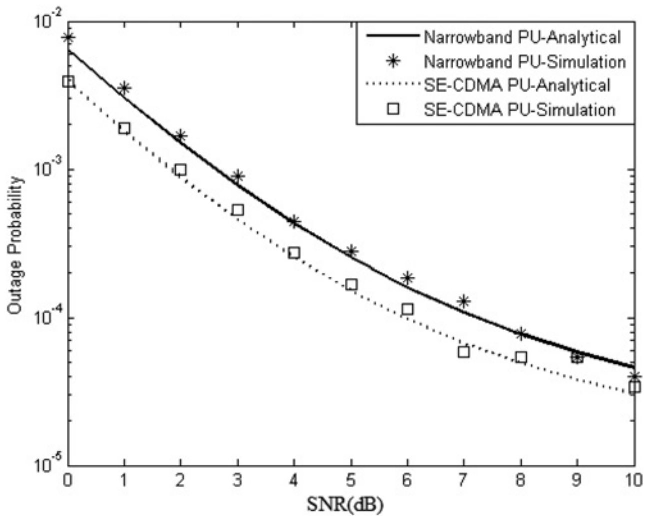

Fig. 8 *Outage probability of the secondary user for narrowband and SE-CDMA PU schemes, N = 8, $N_s = 3$*

increasing the number of secondary users MAI increases. This reduces the SINR and eventually the outage probability increases.

5.3 Performance comparison

Here, we compare the results of the proposed scheme with the case that all users utilise OFDMA method considered in [7] where we assume the same channel conditions for fair comparison. In [7], Golrezaei-Khuzani and Nasiri-Kenari have studied the OFDMA-based cognitive cooperative relaying scheme where the inter cluster interference have been neglected in the performance analysis. In Figs. 10*a* and *b*, the outage probability and error probability of the PU are demonstrated, respectively, considering OFDMA and the proposed two schemes. It should be noted that the channel condition, number of secondary users and transmitted energy per symbol are assumed the same in all scenarios. The total number of subcarriers and the number of subcarriers utilised by the PU in the OFDMA scheme are 12 and 10, respectively, as considered in [7]. The number of secondary users is $N_s = 3$ and in SE-CDMA scheme we consider $N = 8$. From the figures, it is obvious that employing SE-CDMA method for the primary and

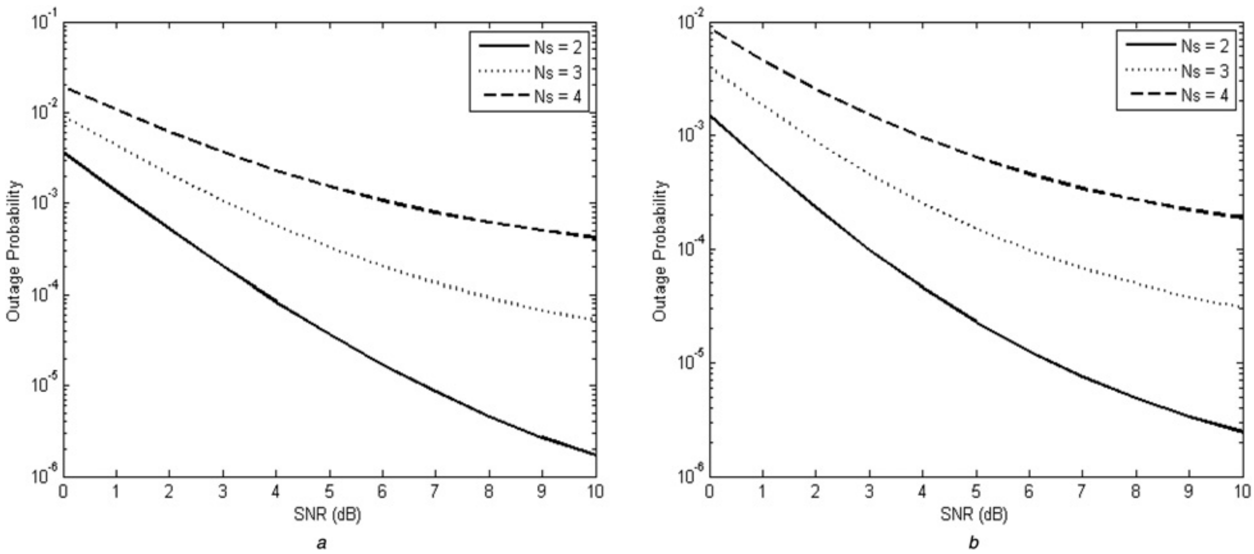

Fig. 9 *Outage probability of the secondary user for different numbers of secondary users, N = 8*
a Narrowband PU
b SE-CDMA PU

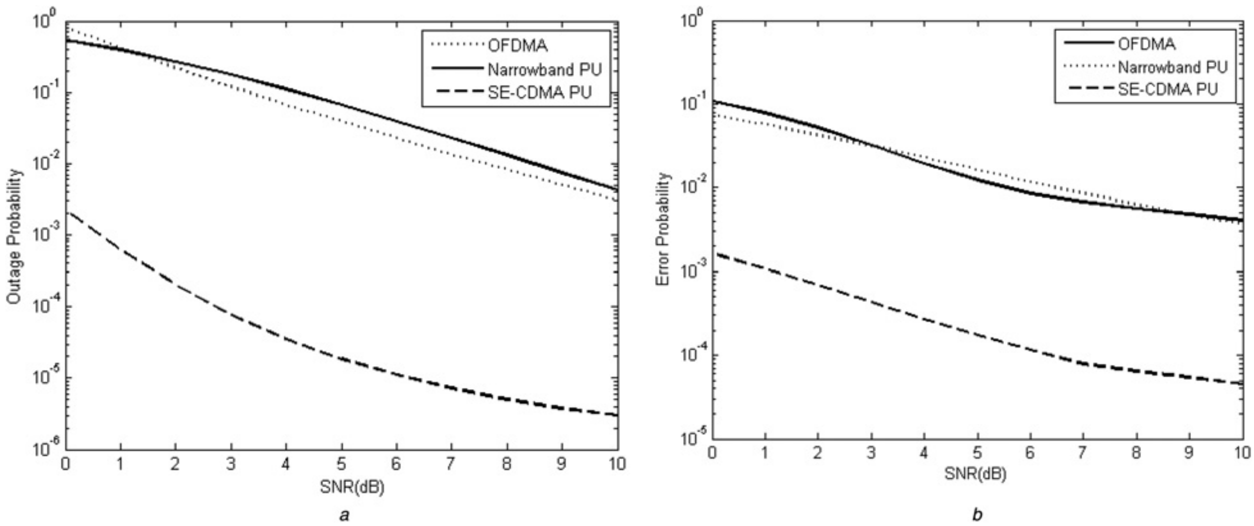

Fig. 10 *Performance comparison of OFDMA and proposed SE-CDMA schemes for the PU, $N_s = 3$*
a Outage probability of PU
b Error probability of PU

secondary users outperforms OFDMA and narrowband PU schemes. As explained before, this significant superiority is because of increasing SINR when SE-CDMA method is used.

6 Conclusions

In this paper, the performance of multiple access SE-CDMA in cognitive cooperative networks was investigated. In these networks, the secondary users relay the data of the PU that is not successful in transmitting data. For secondary users, we used the SE-CDMA method and for PU two scenarios were considered. In the first scheme, we assumed the PU narrowband occupying one frequency chip, whereas the secondary users utilised the overlay technique so that there is no interference to the PU. In the second scheme, both primary and secondary users used SE-CDMA for data transmission and the cognitive system employed underlay technique. The results of comparing two schemes indicated that using the SE-CDMA technique for the PU yields better performance for both primary and secondary users in terms of the outage probability and the error probability. The results also illustrated that increasing the number of secondary users enhances the performance of the narrowband PU. However, for the SE-CDMA PU, increasing the number of secondary users degrades the system performance in high SNRs. It was shown that using the SE-CDMA technique for the primary and secondary users achieves better results in comparison with OFDMA scheme in [7] under the same channel conditions. Performance evaluation of the system using other code sequences such as orthogonal codes [18] can be considered as a future work.

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8 Appendix

8.1 Appendix 1: Calculation of the outage and error probabilities for narrowband PU scheme

Here we will compute the parameters of (22) and (24). From (18) and (19), the outage probability conditioned on the set $D(\text{PU})$ after several calculations is obtained as (see (37)). Since β_n^2s are independent exponential random variables, $\sum_{n=0}^{N/2-1}\beta_n^2$ has Gamma distribution [21]. Consequently, U in (37) is a Gamma random variable with the parameters $N/2$ and

$$\lambda_{(\text{CR}_i,\text{BS})} = \frac{4\sigma^2\sum_{k=1,\,k\neq i}^{N_s} E_{\text{CR}_k} + N\cdot N_0}{2NE_{\text{CR}_i}\sigma^2}$$

Thus, the cumulative density function (CDF) of the variable X shown in (37) is computed as

$$F_X(x) = \prod_{\text{CR}_i\in D(\text{PU})}\left(\frac{\gamma\left(N/2,\ \lambda_{(\text{CR}_i,\text{BS})}x\right)}{(N/2-1)!}\right)$$

for $x \geq 0$ where $\gamma(L,x) = \int_0^x t^{L-1}\,\mathrm{e}^{-t}\,\mathrm{d}t$ is the lower incomplete Gamma function.

In addition, the conditional probability density function of the variable

$$Y = \frac{2E_\text{p}\beta^2_{f_\text{p}(\text{PU,BS})}}{N_0}$$

in (37) assuming that the outage occurs in the first time slot, is obtained as [21]

$$f_{Y|Y<Z}(y) = \begin{cases} \dfrac{\lambda_{(\text{PU,BS})}\,\mathrm{e}^{-\lambda_{(\text{PU,BS})}y}}{P^1_{\text{p-out}}}, & 0 < y \leq Z \\ 0, & y > Z \end{cases} \tag{38}$$

We compute the conditional outage probability in (22) as follows [21]

$$\begin{aligned} P_\text{out}\big(D(\text{PU})\big) &= \Pr[X < Z - Y] \\ &= \int_0^Z F_x(Z-y) f_{Y|Y<Z}(y)\,\mathrm{d}y \\ &= \int_0^Z \left(\frac{\lambda_{(\text{PU,BS})}\mathrm{e}^{-\lambda_{(\text{PU,BS})}y}}{P^1_{\text{p-out}}}\right) \\ &\quad\times \prod_{\text{CR}_i\in D(\text{PU})}\left(\frac{\gamma\left(N/2,\ \lambda_{(\text{CR}_i,\text{BS})}(Z-y)\right)}{(N/2-1)!}\right)\mathrm{d}y \end{aligned} \tag{39}$$

Next, we calculate the $\Pr[D(\text{PU})]$ in (22). It is considered that the secondary user can correctly decode the data of the PU, when the mutual information of the primary and secondary users is more than the desired rate. Hence, $\Pr[D(\text{PU})]$ is obtained as [7]

$$\Pr\big[D(\text{PU})\big] = \prod_{\text{CR}_i\in D(\text{PU})}\left(\mathrm{e}^{-\lambda_{(\text{PU,CR}_i)}Z}\right)\times\prod_{\text{CR}_i\notin D(\text{PU})}\left(1-\mathrm{e}^{-\lambda_{(\text{PU,CR}_i)}Z}\right) \tag{40}$$

where $\lambda_{(\text{PU,CR}_i)} = (N_0/(2E_\text{p}\sigma^2))$.

Finally, the probability that none of the secondary users can decode the data of the PU can be written as follows

$$P_\text{ND} = \prod_{\text{CR}_i}\left(1-\mathrm{e}^{-\lambda_{(\text{PU,CR}_i)}Z}\right) \tag{41}$$

Now, we compute the parameters of (24) in order to attain the error probability of the narrowband PU. The decoding error in the relay depends on which secondary user is chosen as the relay. Therefore $P_{\text{e}_\text{r}}(D(PU))$ can be expressed as [7]

$$P_{\text{e}_\text{r}}\big(D(\text{PU})\big) = \sum_{\text{CR}_i\in D(\text{PU})} P_{\text{CR}_i}\big(D(\text{PU})\big)P_{\text{e}_{\text{CR}_i}} \tag{42}$$

where $P_{\text{CR}_i}(D(\text{PU}))$ is the probability that the ith cognitive secondary

$$P_\text{out}(D(\text{PU})) = \Pr\left[\frac{1}{2}\log_2\left(1+\underbrace{\left(\left(2E_\text{p}\beta^2_{f_\text{p}(\text{PU,BS})}\right)/N_0\right)}_{Y}+\underbrace{\max_{\text{CR}_i\in D(\text{PU})}\underbrace{\left(\left(4E_{\text{CR}_i}\sum_{n=0}^{N/2-1}\beta^2_{n(\text{CR}_i,\text{BS})}\right)\Big/\left(4\sigma^2\sum_{k=1,\,k\neq i}^{N_s}E_{\text{CR}_k}+N\cdot N_0\right)\right)}_{U}}_{X}\right)^{\text{SINR}} < R\right] \tag{37}$$

user (CR_i) is selected as the relay and $P_{e_{CR_i}}$ is the decoding error of CR_i. From (13), CR_i is selected as the relay if it has the highest channel gain, that is

$$\underbrace{\left(\sum_{n=0}^{N/2-1} \beta^2_{n(CR_i,BS)}\right)}_{X_1} > \underbrace{\max_{CR_m \in D(PU), CR_m \neq CR_i}\left(\sum_{n=0}^{N/2-1} \beta^2_{n(CR_m,BS)}\right)}_{X_2} \tag{43}$$

Therefore $P_{CR_i}(D(PU))$ is obtained as

$$\begin{aligned} P_{CR_i}(D(PU)) &= \Pr[X_1 > X_2] = \int_0^\infty F_{X_1}(x) f_{X_2}(x)\,dx \\ &= \int_0^\infty \prod_{CR_m \in D(PU), CR_m \neq CR_i} \left(\frac{\gamma\left(N/2,\ \lambda_{(CR_m,BS)}x\right)}{(N/2-1)!}\right) \\ &\times \left(\frac{\lambda_{(CR_i,BS)}\,e^{-\lambda_{(CR_i,BS)}x}\left(\lambda_{(CR_i,BS)}x\right)^{N/2-1}}{(N/2-1)!}\right) dx \end{aligned} \tag{44}$$

To compute the conditional error probability, the CDF of SINR in (37) must be obtained. After several manipulations it is attained as (see (45)). Then, the conditional error probability $P_{e_{CO}}(D(PU))$ is computed as (see (46)). Now, we calculate $P_{e_r}(D(PU))$. To this end, $P_{e_{CR_i}}$ must be computed in (42). When the narrowband PU sends its data to the secondary users, the received SINR at the relay noting (12) is

$$SINR_{(PU,CR_i)} = \frac{2E_p \beta^2_{f_p(PU,CR_i)}}{N_0}$$

Thus, the CDF of the SINR is computed as

$$\begin{aligned} F_{SINR|SINR>Z} &= \Pr\left(SINR < y \middle| SINR > Z\right) \\ &= \begin{cases} 0, & 0 < y < Z \\ 1 - e^{-\lambda_{(PU,CR_i)}(y-Z)}, & y \geq Z \end{cases} \end{aligned} \tag{47}$$

Thereby, $P_{e_{CR_i}}$ is obtained as [7]

$$\begin{aligned} P_{e_{CR_i}} &= Q\left(\sqrt{2Z}\right) - \frac{1}{2\sqrt{\pi}} e^{\lambda_{(PU,CR_i)}Z} \\ &\times \sqrt{\frac{1}{1+\lambda_{(PU,CR_i)}}}\,\Gamma\left(\frac{1}{2},\ \left(1+\lambda_{(PU,CR_i)}\right)Z\right) \end{aligned} \tag{48}$$

where $\Gamma(L, x) = \int_x^\infty t^{L-1} e^{-t}\,dt$ is the upper incomplete Gamma function.

If none of the secondary users can decode the PU's data, the received SINR in the BS is the SINR of direct transmission link of (PU, BS) which is given in (11); consequently $P_{e_{NC}}$ is derived as follows

$$\begin{aligned} P_{e_{NC}} &= \frac{1}{P^1_{p-out}}\Bigg[0.5 - Q\left(\sqrt{2Z}\right) e^{-\lambda_{(PU,BS)}Z} \\ &- \frac{1}{2\sqrt{\pi}}\sqrt{\frac{1}{1+\lambda_{(PU,BS)}}}\,\gamma\left(\frac{1}{2},\ \left(1+\lambda_{(PU,BS)}\right)Z\right)\Bigg] \end{aligned} \tag{49}$$

8.2 *Appendix 2: Calculation of the outage and error probabilities for the SE-CDMA PU scheme*

To obtain (22), at first $P_{out}(D(PU))$ must be computed. The maximum average mutual information considering (19), (31) and (35) is (see (50)). Therefore, the conditional outage probability

$$F_{SINR}(w) = \begin{cases} \dfrac{1}{P^1_{p-out}} \displaystyle\int_0^Z e^{-\lambda_{(PU,BS)}y}\lambda_{(PU,BS)} \prod_{CR_i \in D(PU)} \left(\frac{\gamma\left(N/2,\ \lambda_{(CR_i,BS)}(w-y)\right)}{(N/2-1)!}\right) dy, & w \geq Z \\ \dfrac{1}{P^1_{p-out}} \displaystyle\int_0^w e^{-\lambda_{(PU,BS)}y}\lambda_{(PU,BS)} \prod_{CR_i \in D(PU)} \left(\frac{\gamma\left(N/2,\ \lambda_{(CR_i,BS)}(w-y)\right)}{(N/2-1)!}\right) dy, & 0 < w < Z \end{cases} \tag{45}$$

$$\begin{aligned} P_{e_{CO}}(D(PU)) &= \int_0^\infty Q\left(\sqrt{2y}\right) f_{SINR}(y)\,dy = \frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{e^{-w}}{\sqrt{w}} F_{SINR}(w)\,dy \\ &= \frac{\lambda_{(PU,BS)}}{2\sqrt{\pi}P^1_{p-out}}\Bigg[\int_0^Z \frac{e^{-w}}{\sqrt{w}}\left(\int_0^w e^{-\lambda_{(PU,BS)}y} \prod_{CR_i \in D(PU)} \left(\frac{\gamma\left(N/2,\ \lambda_{(CR_i,BS)}(w-y)\right)}{(N/2-1)!}\right) dy\right) dw \\ &+ \int_Z^\infty \frac{e^{-w}}{\sqrt{w}}\left(\int_0^Z e^{-\lambda_{(PU,BS)}y} \prod_{CR_i \in D(PU)} \left(\frac{\gamma\left(N/2,\ \lambda_{(CR_i,BS)}(w-y)\right)}{(N/2-1)!}\right) dy\right) dw\Bigg] \end{aligned} \tag{46}$$

$$I_{CO} = \frac{1}{2}\log_2\left(1 + \overbrace{\underbrace{\left(\left(4E_p \sum_{n=0}^{N/2-1} \beta^2_{n(PU,BS)}\right)\Big/\left(4\sigma^2 \sum_{k=1}^{N_s} E_{CR_k} + N\cdot N_0\right)\right)}_{Y} + \underbrace{\max_{CR_i \in D(PU)}\left(\left(4E_{CR_i}\sum_{n=0}^{N/2-1}\beta^2_{n(CR_i,BS)}\right)\Big/\left(4\sigma^2\sum_{\substack{k=1\\k\neq i}}^{N_s} E_{CR_k} + N\cdot N_0\right)\right)}_{X}}^{SINR}\right) \tag{50}$$

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$P_{out}(D(PU))$ is calculated as

$$P_{out}(D(PU)) = \int_0^Z \left(\frac{\lambda_{(PU,BS)} e^{-\lambda_{(PU,BS)}y} (\lambda_{(PU,BS)}y)^{N/2-1}}{(N/2-1)! P^1_{p-out}} \right) \times \prod_{CR_i \in D(PU)} \left(\frac{\gamma(N/2, \lambda_{(CR_i,BS)}(Z-y))}{(N/2-1)!} \right) dy \quad (51)$$

The probability that the secondary user CR_i can decode the data of the PU is

$$\Pr[CR_i \in D(PU)] = \Pr\left[\frac{4E_P \sum_{n=0}^{N/2-1} \beta^2_{n(PU,CR_i)}}{N \cdot N_0} > Z \right] = 1 - \frac{\gamma(N/2, \lambda_{(PU,CR_i)}Z)}{(N/2-1)!} \quad (52)$$

where

$$\lambda_{(PU,CR_i)} = \frac{N_0}{2E_p\sigma^2}$$

Therefore, the probability that only the secondary users of the set D (PU) can decode the data of the PU can be derived as

$$\Pr[D(PU)] = \prod_{CR_i \in D(PU)} \left(1 - \frac{\gamma(N/2, \lambda_{(PU,CR_i)}Z)}{(N/2-1)!} \right) \times \prod_{CR_i \notin D(PU)} \left(\frac{\gamma(N/2, \lambda_{(PU,CR_i)}Z)}{(N/2-1)!} \right) \quad (53)$$

Finally, the probability that none of the secondary users can decode the PU's data is

$$P_{ND} = \prod_{CR_i} \left(\frac{\gamma(N/2, \lambda_{(PU,CR_i)}Z)}{(N/2-1)!} \right) \quad (54)$$

Now, the parameters of (24) for SE-CDMA PU are calculated. As mentioned before, in order to calculate $P_{e_{CO}}(D(PU))$, we compute the CDF of SINR in (50) as follows (see (55)). Consequently, the conditional error probability $P_{e_{CO}}(D(PU))$ is calculated as (see (56)) where

$$A = \frac{(\lambda_{(PU,BS)})^{N/2}}{(N/2-1)! P^1_{p-out}}$$

Now, we derive $P_{e_r}(D(PU))$ defined in (42). According to (43, $P_{CR_i}(D(PU))$ is given in (44). Thus, $P_{e_{CR_i}}$ must be computed. To this end, we note that the SINR at the ith secondary user's receiver is

$$SINR_{(PU,CR_i)} = \frac{4E_p \sum_{n=0}^{N/2-1} \beta^2_{n(PU,CR_i)}}{N \cdot N_0}$$

By computing the CDF of the SINR and after some manipulations we obtain (see (57)). The error probability when none of the secondary users can decode the PU's data ($P_{e_{NC}}$) is computed as follows (see (58)).

$$F_{SINR}(w) = \begin{cases} \int_0^Z \left(\frac{\lambda_{(PU,BS)} e^{-\lambda_{(PU,BS)}y} (\lambda_{(PU,BS)}y)^{N/2-1}}{(N/2-1)! P^1_{p-out}} \right) \prod_{CR_i \in D(PU)} \left(\frac{\gamma(N/2, \lambda_{(CR_i,BS)}(w-y))}{(N/2-1)!} \right) dy & w > Z \\ \int_0^w \left(\frac{\lambda_{(PU,BS)} e^{-\lambda_{(PU,BS)}y} (\lambda_{(PU,BS)}y)^{N/2-1}}{(N/2-1)! P^1_{p-out}} \right) \prod_{CR_i \in D(PU)} \left(\frac{\gamma(N/2, \lambda_{(CR_i,BS)}(w-y))}{(N/2-1)!} \right) dy & 0 < w \le Z \end{cases} \quad (55)$$

$$P_{e_{CO}}(D(PU)) = \frac{A}{2\sqrt{\pi}} \left[\int_0^Z \frac{e^{-w}}{\sqrt{w}} \left(\int_0^w e^{-\lambda_{(PU,BS)}y} y^{N/2-1} \prod_{CR_i \in D(PU)} \left(\frac{\gamma(N/2, \lambda_{(CR_i,BS)}(w-y))}{(N/2-1)!} \right) dy \right) dw + \int_Z^\infty \frac{e^{-w}}{\sqrt{w}} \left(\int_0^Z e^{-\lambda_{(PU,BS)}y} y^{N/2-1} \prod_{CR_i \in D(PU)} \left(\frac{\gamma(N/2, \lambda_{(CR_i,BS)}(w-y))}{(N/2-1)!} \right) dy \right) dw \right] \quad (56)$$

$$P_{e_{CR_i}} = \frac{1}{2\sqrt{\pi}} \int_Z^\infty \frac{e^{-y}}{\sqrt{y}} \left(1 - e^{-\lambda_{(PU,CR_i)}(y-Z)} \frac{\sum_{n=0}^{N/2-1} \left(\left((\lambda_{(PU,CR_i)}y)^n \right) / n! \right)}{\sum_{n=0}^{N/2-1} \left(\left((\lambda_{(PU,CR_i)}Z)^n \right) / n! \right)} \right) dy$$
$$= Q(\sqrt{2Z}) - \frac{1}{2\sqrt{\pi}} \frac{e^{\lambda_{(PU,CR_i)}Z}}{\sum_{n=0}^{N/2-1} \left((\lambda_{(PU,CR_i)}Z)^n / n! \right)} \sum_{n=0}^{N/2-1} \frac{(\lambda_{(PU,CR_i)})^n}{n!} \left(\frac{1}{1+\lambda_{(PU,CR_i)}} \right)^{n+1/2} \Gamma\left(n+\frac{1}{2}, (1+\lambda_{(PU,CR_i)})Z \right) \quad (57)$$

$$P_{e_{NC}} = (P^1_{p-out})^{-1} \left[0.5 + (P^1_{p-out} - 1) Q(\sqrt{2Z}) - \frac{1}{2\sqrt{\pi}} \left(\sum_{n=0}^{N/2-1} \frac{(\lambda_{(PU,BS)})^n}{n!} \left(\frac{1}{1+\lambda_{(PU,BS)}} \right)^{n+(1/2)} \gamma\left(n+\frac{1}{2}, (1+\lambda_{(PU,BS)})Z \right) \right) \right] \quad (58)$$