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Price bargaining based on the Stackelberg game in two-tier orthogonal frequency division multiple access femtocell networks

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Abstract: This study presents a solution to the interference management scheme and resource allocation strategy for the two-tier femtocell networks, where the femtocell users (FUEs) share the same frequency band with the existing macrocell users. It is assumed that the FUEs compete for the available spectrum to fulfil their own communication. And the macrocell base station protects itself by pricing the interference from the FUEs, which formulates the Stackelberg game. In this study, two effective pricing schemes, uniform pricing scheme and non-uniform pricing scheme, combining with admission control are proposed to maximise the revenues and protect the quality of service requirements. The Stackelberg equilibriums for the proposed games are investigated. Besides, a novel distributed interference pricing algorithm is provided for the uniform pricing case. Numerical results show that, in two-tier femtocell networks with shared spectrum, the proposed pricing schemes are effective in resource allocation and performance protection.

1 Introduction

Femtocells have recently emerged as a promising technology to increase wireless network capacity, extend cellular coverage and introduce new services [1]. Femtocell is a small cellular base station (BS), typically designed for use in home or small business. It is connected through broadband IP, such as digital subscriber lines or cable modems, to the service provider's network. Femtocells consist of miniature personal BSs and stationary or low-mobility end users employed in indoor environment and are located within an existing cellular network [2]. However, end users' installation and unplanned deployment of femtocells in an existing macrocell network may cause severe interference between macrocells and femtocells, which is known as the cross-tier interference [3].

In recent years, schemes for interference coordination and mitigation in femtocell-deployed networks have been widely investigated. To cope with cross-tier interference, efficient allocation of the frequency, power and other resources in the system can be exploited [4]. Power control as a key technique for interference management which cannot be ignored in wireless resource management [5]. A great deal of scholarly work has recently appeared in the literature on the design of power control and interference mitigation strategies for spectrum sharing a two-tier femtocell networks. In [6], a power allocation strategy based on the received signal power level from the macrocell BS (MBS) is developed. The authors proposed the distributed power control algorithm to alleviate the cross-tier interference in [7, 8]. Interference mitigation strategy is studied in [9] by adjusting the femtocell users' maximum transmission powers.

Efficient pricing techniques not only increase the performance, but also improve network utilisation in light of the rapid growth and variety of network requirements. Different objectives for pricing communication networks have been investigated in the literature, such as social welfare maximisation and fairness guarantees and/or revenue maximisation. Under the cognitive radio scenario, the pricing scheme based on Stackelberg game is proposed in [10]. However, the interference threshold is not considered. To maximise the utility revenue of the primary, the dynamic adjustment of the interference threshold and the secondary users' transmission powers have been involved with pricing mechanism, such as in [11–13]. In two-tier femtocell networks, a dynamic optimised pricing scheme is proposed in [14], according to the signal-to-interference-plus-noise ratio (SINR) which alleviates the cross-tier interference and guarantees the fairness in the competition. However, assumptions that the femtocell has complete information about the network and that each FUE in the competition does not consider the corresponding interference to other co-channel FUEs lead to an unrealistic scenario. And, its' ability of directly control the cross-tier interference is poor. In [15], a price-based interference control scheme is proposed in two-tier femtocell networks. However, the inter-cell interference between the FUEs is ignored, and the FUEs' minimum performance requirements are not considered. Price-based resource allocation strategies are investigated in [16] based on the Stackelberg game for two-tier femtocell networks, but the interference price is adjusted by the experience which lacks dynamic adaptability. In [17], an effective distributed interference pricing scheme is proposed based on the Stackelberg game. The existence and uniqueness of the Stackelberg equilibrium (SE) point are established. In [18], the convergence and uniqueness of the hierarchical game with channel uncertainly is investigated. In [19], the spectrum leasing problem is studied for femtocells with hybrid access, and the decision-making process is modelled as a three-stage Stackelberg game. Achieving efficient Nash equilibrium (NE) point are studied using pricing schemes, such as [20–22]. The static pricing model and dynamic pricing model are investigated in [23] to analyse the NE price of macrocell and femtocell operators. In [24], the unique NE of the corresponding water-filling game is studied in the multiple-access channel capacity region. It is worth emphasising that the problem of the NE point for the non-uniform pricing scheme is less rigorously discussed in the above mentioned papers.

In our paper, both the inter-tier and cross-tier interference constraints are considered. Besides, an adaptive interference pricing scheme is proposed by considering the price constraint. All these conditions lead to a more realistic network scenario. In such a network scenario, we propose the price-based interference management schemes. They integrate the utility optimisation and restrict the cross-tier interference at the MBS below a given threshold which are quite different from the pricing schemes proposed in [10, 16]. Specially, a new method is proposed to obtain the uniform interference pricing scheme which is also different from the work [16, 19]. Then, distributed power allocation schemes are developed based on Stackelberg game, where the MBS acts as the leader and FUEs act as the followers. In contrast to [15], our work requires only a small amount of information and shows good control ability to the cross-tier interference. At last, admission control algorithm is adopted to robustly protect the performances of all active FUEs.

The rest of the paper is organised as follows. Section 2 introduces the system model. Section 3 describes the problem formulation. Two different pricing schemes of the MBS are given in Section 4, which follows the admission control algorithm in Section 5. Section 6 provides numerical simulations to validate the proposed studies. Finally, Section 7 concludes the paper.

2 System model

In this paper, we present a system model in a two-tier frequency division multiple access (FDMA) based femtocell network. The network consists of one central macrocell and N randomly distributed femtocells. Fig. 1 gives the description of the uplink transmission in two-tier femtocell networks. For simplicity, the macrocell user (MUE) is omitted. The solid and dashed lines represent the uplink signals and interference, respectively. We assume that in each femtocell there is one corresponding femtocell BS (FBS) providing the service. Besides, orthogonal uplink signalling is assumed in each slot (one scheduled active user per cell during each signalling slot), where a slot refers to a time resource.

Under the above described framework, all the terminals involved are assumed to be equipped with one signal antenna and all the channels involved are assumed to be block-fading. That is, the channels may change from one block to another but remain constant during each transmission block. Besides, the channel gains $g_{i,j}$ as in the IMT-2000 specification [25] are presented using simplified path loss model

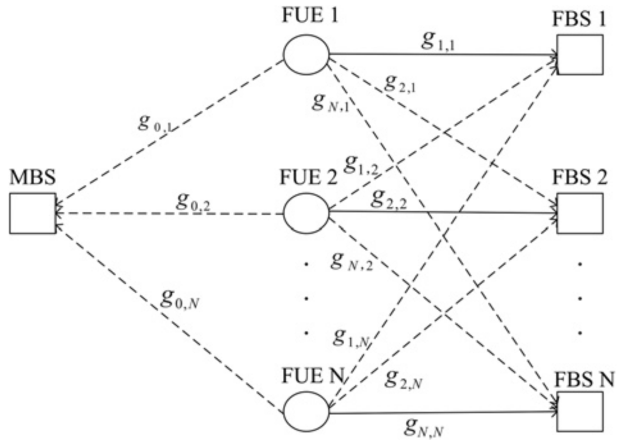

Fig. 1 *System model*

$$g_{i,j} = \begin{cases} K_c \min(D_{0,0}^{-\alpha_c}, 1), & i=j=0 \\ K_{fi} R_f^{-\alpha_\beta}, & i=j\neq 0 \\ K_{fo} W \min(D_{0,j}^{-\alpha_{fo}}, 1), & i=0, j>0 \\ K_c \min(D_{i,j}^{-\alpha_c}, 1), & i>0, j=0 \\ K_{fo} W^2 \min(D_{i,j}^{-\alpha_{fo}}, 1), & i\neq j\neq 0 \end{cases} \tag{1}$$

In (1), α_c, α_β and α_{fo} denote the macrocell, indoor and indoor-to-outdoor femtocell-path loss exponents, respectively. $D_{i,j}$ is the distance between user j and BS i. R_f denotes the coverage radius of the FBS. Noting that $i=0$ and $j=0$, respectively, denote the MBS and the MUE. Defining f_c as the carrier frequency in MHZ, the term $K_c = 30\log_{10} f_c - 71$ (dB) equals the fixed decibel propagation loss during the transmission from the MUE to the MBS. K_{fi} denotes the fixed loss between FUE i to its corresponding FBS. K_{fo} is defined as the fixed loss between FUE i to a different FBS and the assumption $K_{fo} = K_c$ is given. The term W explicitly models partition loss during indoor–outdoor transmission to the MBS.

Assumption 1: The outdoor path loss exponents from a MUE and a FUE to the MBS are equal. That is, $\alpha_c = \alpha_{fo} = \alpha$.

All the channel gains are assumed to be completely independent distributed random variables. Assume δ^2 is the Gaussian white noise and $\delta_i = \delta^2/g_{i,i}$ is the normalised noise. During a given time slot, let $i \in \{1, 2, \ldots, N\}$ denotes the scheduled user connected to its BS. The SINR of FUE i is expressed as

$$\gamma_i = \frac{p_i}{\sum_{j=1, j\neq i}^{N} \alpha_{i,j} p_j + \delta_i} \tag{2}$$

where p_i and p_j are, respectively, denote the transmission powers of the FUE i and the FUE j. And

$$\alpha_{i,j} = \begin{cases} \dfrac{g_{i,j}}{g_{i,i}}, & i \neq j \\ 1, & i = j \end{cases} \tag{3}$$

denotes the normalised channel gain or coefficient between the user and the corresponding BS.

3 Problem formulation

In this section, the Stackelberg game formulation for the price-based power allocation scheme is first presented. Then, we investigate the existence of the SE for the proposed schemes.

3.1 Stackelberg game formulation

In the Stackelberg game, the MBS as the leader wants to protect itself by pricing the interference introduced from FUEs' transmission powers. Thus we design a strategy to achieve revenue by selling the interference quota to FUEs. The revenue of the MBS is written as

$$U_{\text{MBS}}(\boldsymbol{c},\boldsymbol{p}) = \sum_{i=1}^{N} c_i p_i g_{0,i} \tag{4}$$

where c_i denotes the interference price of FUE i. It is worth emphasising that there exists some relationships between p_i and c_i, which presents the willing ness of FUE i to buy the interference quota in price set by the MBS. Taking the interference threshold Q into consideration, the optimisation problem at the MBS's side can be formulated as

$$\text{Problem 3.1:} \quad \max_{c_i>0} U_{\text{MBS}}(\boldsymbol{c},\boldsymbol{p}) = \sum_{i=1}^{N} c_i p_i g_{0,i} \text{s.t.} \quad \sum_{i=1}^{N} p_i g_{0,i} \leq Q \tag{5}$$

In this paper, we employ a profit function and a cost function to, respectively, represent the satisfaction degree of the FUE to the service quality and the cost incurred. Then we give the revenue of FUE i

$$U_i(p_i, \boldsymbol{p}_{-i}) = \lambda_i \log(1+\gamma_i) - c_i p_i g_{0,i} \tag{6}$$

where λ_i is the utility gain per unit transmission rate of FUE i, $\boldsymbol{p}_{-i}$ is the vector of power allocation for all FUEs except FUE i, that is, $\boldsymbol{p}_{-i} = [p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N]^{\text{T}}$. It is observed from (6) that if FUE i increases its transmission power, the transmission rate increases and so does the profit. However, with the increase of the transmission power, the FUE will definitely cause more interference to the MBS. As a result, it has to buy more interference quota from the MBS, which increases the cost. Therefore the FUEs need to select the optimal power allocation strategies to maximise their own utilities. Mathematically, the optimisation problem combining with the interference constraint can be formulated as

$$\begin{aligned} \text{Problem 3.2:} \quad & \max_{p_i>0} U_i(p_i, \boldsymbol{p}_{-i}) = \lambda_i \log(1+\gamma_i) - c_i p_i g_{0,i} \\ \text{s.t.} \quad & \sum_{i=1}^{N} p_i g_{0,i} \leq Q \\ & p_i \geq 0, \quad \forall i \in \{1, 2, \ldots, N\} \end{aligned} \tag{7}$$

Stackelberg game is a strategic game that consists a leader and several followers competing on certain resources. The leader moves first, then the followers adjust their corresponding actions. In this paper, the MBS prices the interference from the FUEs to achieve the maximum revenue. Then FUEs adjust their own transmission powers to maximise their individual utilities based on interference prices. Problem 3.1 and Problem 3.2 form the upper subgame and lower subgame, respectively. The two problems together formulate a Stackelberg game. Our target is to find the SE point, that is, the MBS and all FUEs have no motivations to deviate it.

The NE of the lower subgame can be obtained by optimising Problem 3.2. Resulting in the Karush–Kuhn–Tucker (KKT) conditions

$$\begin{aligned} & p_i \perp \left(-\frac{\lambda_i}{\delta_i + \sum_{j=1}^{N} \alpha_{ij} p_j} + c_i g_{0,i} \right) \\ \text{s.t.} \quad & p_i \geq 0 \\ & -\frac{\lambda_i}{\delta_i + \sum_{j=1}^{N} \alpha_{ij} p_j} + c_i g_{0,i} \geq 0 \end{aligned} \tag{8}$$

where the notation $a \perp b$ represents the complementarity condition of a and b, namely, $ab = 0$. Noting that $c_i > 0$ and $\delta_i > 0$, we can multiply formula (8) by a non-negative scalar

$$\frac{\delta_i + \sum_{j=1}^{N} \alpha_{ij} p_j}{c_i g_{0,i}}$$

Then, we have

$$p_i \perp \left(-\frac{\upsilon_i}{c_i} + \delta_i + \sum_{j=1}^{N} \alpha_{ij} p_j \right), \quad \forall i \in \{1, 2, \ldots, N\} \tag{9}$$

where $\upsilon_i = \lambda_i / g_{0,i}$. Considering the constraints mentioned in the above description, formula (9) is further equivalent to

$$p_i = \left[\frac{\upsilon_i}{c_i} - \delta_i - \sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j \right]^{+}, \quad \forall i \in \{1, 2, \ldots, N\} \tag{10}$$

where $[p_i]^+ = \max\{p_i, 0\}$.

3.2 Stackelberg equilibrium

Let $\boldsymbol{c}^*$ be a solution for Problem 3.1 and $\boldsymbol{p}^*$ be a solution for Problem 3.2 of FUE i. A strategy $(\boldsymbol{c}^*, \boldsymbol{p}^*)$ is called a SE strategy, if for any ($\boldsymbol{c}$, $\boldsymbol{p}$), the MBS achieves

$$U_{\text{MBS}}(\boldsymbol{c}^*, \boldsymbol{p}^*) \geq U_{\text{MBS}}(\boldsymbol{c}, \boldsymbol{p}) \tag{11}$$

Moreover, the FUE i achieves

$$U_i(p_i^*, \boldsymbol{p}_{-i}^*) \geq U_i(p_i, \boldsymbol{p}_{-i}), \quad \forall i \tag{12}$$

Generally, the SE for the Stackelberg game could be obtained by finding its subgame NE. In the game, FUEs compete in a strict non-cooperative game. NE is defined as the point that no player can unilaterally increase his own utility when any other players do not change their strategies, it is because that the adjustment of the strategy will sacrifice the corresponding FUE's utility. Therefore every rational FUE has no willingness to change the strategy individually in the NE point. In this paper, to obtain the optimal solutions, we have to obtain the best responses of both sides. An approach is provided to obtain the SE: for any given $\boldsymbol{c}$, we first solve Problem 3.2 for optimal transmission powers. Then, the optimal prices can be obtained by solving Problem 3.1.

3.3 Existence of SE

To ensure the existence of NE point, it is needed to select an appropriate utility function. Therefore, we give the following theorem.

Theorem 1: A NE exists in the lower subgame $G = \{S, \{P_i\} U_i\}\}$.

Proof:

(1) Since the convex set is a single point or a continuous line in the one-dimensional (1D) space. The strategy $\{p_i\}$ is a compact subset. Obviously, $\{p_i\}$ is a convex set.
(2) Quasi-concave function definition: function $f(x)$ is defined on a subset of the R^n, if and only if the function $f(x)$ satisfies the following property, $f(x)$ is quasi-concave

$$f(\lambda x_1 + (1-\lambda)x_2) \geq \min\big(f(x_1, f(x_2))\big), \quad \lambda \in [0, 1]$$

The derivation of $U_i(p_i, \boldsymbol{p}_{-i})$ is given as

$$\frac{\partial U_i}{\partial p_i} = \frac{\lambda_i}{1+\gamma_i(p_i, \boldsymbol{p}_{-i})} \cdot \frac{\gamma_i(p_i, \boldsymbol{p}_{-i})}{p_i} - c_i g_{0,i}$$

where $I_i(\boldsymbol{p}_{-i}) = \sum_{j \neq i} p_j \alpha_{i,j} + \delta_i$, $i, j \in S$. It is assumed that $\lambda_i \neq 0$, the second derivative is given as

$$\frac{\partial^2 U_i}{\partial p_i^2} = -\frac{\lambda_i}{(I_i(p_{-i}) + p_i)^2} < 0$$

Obviously, revenue function is quasi-concave. It is well known that if the above conditions are satisfied, the NE exists in the lower subgame [10]. Therefore the existence of the SE is also proved. □

4 Pricing mechanism of the MBS

In this section, two pricing schemes: uniform pricing scheme and non-uniform pricing scheme are proposed in the formulated Stackelberg game. For the uniform pricing scheme, a novel distributed interference price algorithm is proposed to manage the cross-tier interference at the MBS below a given threshold. For the non-uniform pricing scheme, the existence of SE point is investigated by considering the interference constraint. And, Lagrangian dual optimisation method is adopted to get the optimal solution.

4.1 Uniform pricing

Taking the transmission power and the interference threshold constraints into consideration, a more comprehensive description of Problem 3.1 can be formulated as

$$\begin{aligned} \text{Problem 4.1:} \quad & \max_{c_i > 0} U_{\text{MBS}}(\boldsymbol{c}, \boldsymbol{p}) = \sum_{i=1}^{N} c_i p_i g_{0,i} \\ \text{s.t.} \quad & p_i = \left[\frac{v_i}{c_i} - \delta_i - \sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j\right]^+, \quad \forall i \in S \\ & \sum_{i=1}^{N} p_i g_{0,i} \leq Q \end{aligned} \tag{13}$$

To simplify the problem, we can rewrite Problem 4.1 as

$$\begin{aligned} \text{Problem 4.2:} \quad & \min_{c_i > 0} U_{\text{MBS}}(\boldsymbol{c}, \boldsymbol{p}) = \sum_{i=1}^{N} c_i g_{0,i}\left[\delta_i + \sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j\right] \\ \text{s.t.} \quad & \left[\frac{v_i}{c_i} - \delta_i - \sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j\right] g_{0,i} \leq Q, \quad \forall i \in I \end{aligned} \tag{14}$$

where I is defined as the set of the active users for the given interference prices. It is easy to observe that Problem 4.2 is a convex optimisation problem. Thus, the iterative strategy of FUE i can be achieved based on (10)

$$p_i^{(t+1)} = \left\{\left[\frac{v_i}{c_i^{(t-1)}} - \delta_i - \sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j^{(t-1)}\right]^+, p_i^{\max}\right\} \quad \forall i \in S \tag{15}$$

where $p_i^{\max}$ is the maximum transmission power of FUE i. For the uniform pricing scheme, the MBS sets a unified interference price for all FUEs, that is, $c_i = c$, $\forall i \in S$. The SE and interference price bargaining scheme will be investigated in the following discussion.

Definition 1: Let us define $M(\cdot)$: $X \to X$ is a mapping and $\boldsymbol{x}^* \in X$ expresses the fixed point. M presents pseudo contractive mapping on the norm $|\cdot|$, if the following condition holds

$$\| M(\boldsymbol{x}) - \boldsymbol{x}^* \| \leq q \| \boldsymbol{x} - \boldsymbol{x}^* \|, \quad \forall \boldsymbol{x} \in X \tag{16}$$

Theorem 2: The uniform pricing scheme we proposed has a unique SE point.

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Proof: Let us define $\Delta p_i(t) = p_i(t) - p_i^*$, where $p_i(t)$ denotes the transmission power of FUE i in the step t in the power iteration. And the l_∞-norm of vector $\Delta \boldsymbol{p}$ over the player set S is denoted by $\|\Delta \boldsymbol{p}\|_S$, that is, $\|\Delta \boldsymbol{p}\|_S = \max_{i\in S}|\Delta p_i|$ which expresses the maximum deviation for all FUEs in a certain step of the iteration. Then we have

$$\begin{aligned}|\Delta p_i(t+1)| &= \left|\sum_{j\neq i,j=1}^{N} \alpha_{ij} p_j^* - \sum_{j\neq i,j=1}^{N} \alpha_{ij} p_j\right| \\ &= \left|\sum_{j\neq i,j=1}^{N} \alpha_{ij} \Delta p_j(t)\right| \\ &\le \max|\Delta p(t)| \sum_{j\neq i,j=1}^{N} \alpha_{ij} \\ &< \|\Delta \boldsymbol{p}\|_S \end{aligned} \tag{17}$$

Considering the direct channel gains dominate the cross-channel gains, α_{ij} typically takes small values in two-tier femtocell systems. And if the network is not very congested, we usually have the inequality $\sum_{j\neq i,j=1}^{N} \alpha_{ij} < 1$ [15]. Combining with Definition 1, we can verify the convergence by $\Delta \boldsymbol{p}(t+1) = M\ (p(t))$ which satisfies the above inequality. □

Remark 1: Note that the proposed power control strategy with the maximum power constraint still converges to a unique NE point. It is suggested that $A(p_i) = p_i^{\max}$ is a standard interference function with $p_i^{\max} \ge 0$, if it meets positivity, monotonicity and scalability. In the following, a brief proof is given.

Let $A(p_i) = p_i^{\max}$. (i) Positivity: since $p_i^{\max} > 0$, then $A(p_i) > 0$, (ii) monotonicity: when $p'_i > p_i$, we have $A(p'_i) = p_i^{\max} = A(p_i)$ and (iii) scalability: $\forall\theta > 1$, $\theta A(p_i) = \theta p_i^{\max} = \theta A(\theta p_i) > A(\theta p_i)$.

Next, a lemma is given to convert some problems into minimum value maximisation problems and finally obtain the optimal solution. Based on it, the equivalent form of Problem 4.1 will be proposed.

Lemma 1 [26, Razaviyayn]: Let $f(x)$ and $g(x)$ be strictly increasing function and decreasing function of the variable x, respectively. If x^* is the maximiser of the point-wise minimum function of $f(x)$ and $g(x)$, that is

$$x^* = \text{Arg}\max_x \min\{f(x), g(x)\}$$

we have $f(x^*) \ge g(x^*)$.

Let us define $U = \min\{U_1, U_2\}$, where

$$U_1 = cQ \tag{18}$$

$$U_2 = c\sum_{i=1}^{N} p_i g_{0,i} \tag{19}$$

U_2 can be rewritten as

$$U_2 = c\boldsymbol{G}^{\mathrm{T}}\boldsymbol{p} = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{B} - c\boldsymbol{D}] \tag{20}$$

where

$$\boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} g_{0,1} \\ g_{0,2} \\ \vdots \\ g_{0,N} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \\ \vdots \\ \upsilon_N \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_N \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,N} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N,1} & \alpha_{N,2} & \cdots & \alpha_{N,N} \end{bmatrix}$$

In order to transform Problem 4.1 into a minimum value maximisation problem, the monotonicity of U_2 should be first investigated. Therefore we will give the analysis of the monotonicity of $g(c)$ in the following discussion.

To show the monotonicity of U_2, we only need to prove $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{D} \ge 0$. Besides, we have

$$\boldsymbol{G}^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{D} = \boldsymbol{e}^{\mathrm{T}}\widetilde{\boldsymbol{H}}^{-1}\widetilde{\boldsymbol{D}} \tag{21}$$

where $\widetilde{\boldsymbol{D}} = \text{Diag}(\boldsymbol{G})\boldsymbol{D} > 0$. Therefore the problem can be simplified as

$$\boldsymbol{e}^{\mathrm{T}}\widetilde{\boldsymbol{H}}^{-1} \ge 0 \tag{22}$$

We define

$$\boldsymbol{e} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad \boldsymbol{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\boldsymbol{F} = \begin{bmatrix} 0 & \dfrac{g_{0,2}\alpha_{1,2}}{g_{0,1}} & \cdots & \dfrac{g_{0,N}\alpha_{1,N}}{g_{0,1}} \\ \dfrac{g_{0,1}\alpha_{2,1}}{g_{0,2}} & 0 & \cdots & \dfrac{g_{0,N}\alpha_{2,N}}{g_{0,2}} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{g_{0,1}\alpha_{1,N}}{g_{0,N}} & \dfrac{g_{0,2}\alpha_{2,N}}{g_{0,N}} & \cdots & 0 \end{bmatrix}$$

and

$$\widetilde{\boldsymbol{H}} = \text{Diag}(\boldsymbol{G})^{-1}\boldsymbol{H}\text{Diag}(\boldsymbol{G})$$

With above notations, we have

$$\begin{aligned}\boldsymbol{e}^{\mathrm{T}}\widetilde{\boldsymbol{H}}^{-1} &= \boldsymbol{e}^{\mathrm{T}}[\text{Diag}(\boldsymbol{G})^{-1}\boldsymbol{H}\text{Diag}(\boldsymbol{G})]^{-1} \\ &= \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{I}+\boldsymbol{F})^{-1} \\ &= \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{I}-\boldsymbol{F})(\boldsymbol{I}-\boldsymbol{F}^2)^{-1} \\ &= \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{I}-\boldsymbol{F})\left(\sum_{j=0}^{\infty}\boldsymbol{F}^{2j}\right)\end{aligned} \tag{23}$$

Let $\rho(\boldsymbol{X})$, which presents the maximum modulus eigenvalue of $\boldsymbol{X}$, denotes the spectral radius of matrix $\boldsymbol{X}$. In the above formulation, based on $\widetilde{\boldsymbol{H}} = \boldsymbol{I} + \boldsymbol{F}$ and the interference condition $\sum_{j\neq i,j=1}^{N} \alpha_{ij} < 1$, we have spectral radius $\rho(\boldsymbol{F}) < 1$. Therefore, $\boldsymbol{I} - \boldsymbol{F} > 0$. Obviously, $\sum_{j=0}^{\infty} \boldsymbol{F}^{2j} > 0$. Furthermore, it can be seen that U_2 is a linear function when the active FUEs' set I is fixed. And, the slope of U_2 may change with the vary of set I. We can get a conclusion that U_2 is a piecewise non-increasing function of c.

Based on the above analysis, Problem 4.1 with the unified price setting can be rewritten as

$$\text{Problem 4.3:} \quad \max_{(c,\boldsymbol{p})} \min\left\{cQ,\, c\sum_{i=1}^{N} p_i g_{0,i}\right\}$$
$$\text{s.t.} \quad p_i = \left[\frac{\upsilon_i}{c} - \delta_i - \sum_{j\neq i,j=1}^{N} \alpha_{ij} p_j\right]^+, \quad \forall i \in S \tag{24}$$

For the uniform pricing scheme, we have

$$p_i = \begin{cases} \dfrac{\upsilon_i}{c} - \delta_i - \displaystyle\sum_{j\neq i,j=1}^{N} \alpha_{ij} p_j, & \text{if } c < \dfrac{\upsilon_i}{\delta_i + \sum_{j\neq i,j=1}^{N} \alpha_{ij} p_j} \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

In Fig. 2, we define $c_{\max} = \max_{\forall i \in S}(\upsilon_i/\sigma_i)$. And thick dotted line describes Problem 4.3. It can be seen from the figure, the maximum revenue is obtained at the point $U_1 = U_2$. With

$$c^* = \operatorname{Arg}\max_{c} \min\left\{cQ,\, c\sum_{i=1}^{N} p_i g_{0,i}\right\} \tag{26}$$

holds, an analytical method is offered to get the optimal solution of Problem 4.3. For $U_1 < U_2$, the preset price c should be increased by $c + \Delta c$. And the preset price is decreased by $c - \Delta c$, if $U_1 > U_2$, until $|U_1 - U_2| \le \varepsilon$ (where ε is a positive constant that controls the accuracy of the algorithm). Based on the above analysis, a distributed interference price bargaining algorithm will be proposed.

Algorithm 1: Interference price bargaining algorithm based on price adaptation

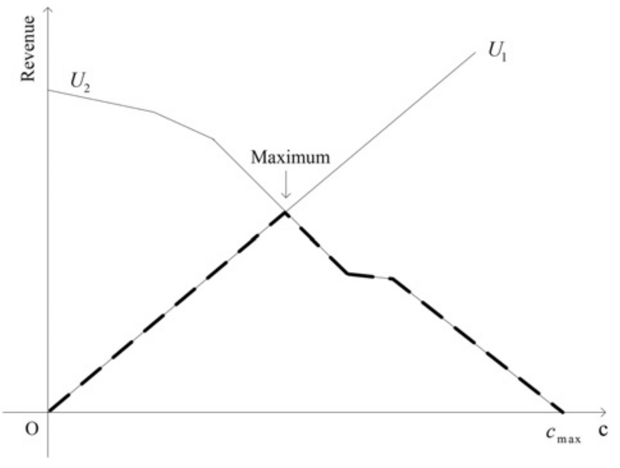

Fig. 2 *Monotonicity of two revenues*

1. Set the indicator $\xi = 1$, and the deviation allowable value $\varepsilon = 1$.
2. The MBS initialises the interference price $c = \max_{\forall i \in S}(\upsilon_i/2\delta_i)$ for each FUE and broadcasts it to all FUEs.
3. The MBS measures $g_{0,i}$ and each FBS i measures $g_{i,j}$, $\forall (i, j) \in S$ meanwhile broadcasts them to FUE i through the backhaul links.
4. **while** $\xi = 1$, **do**
5. Compute p_i.
6. Calculate U_1 and U_2.
7. Compare U_1 with U_2. **If** $U_1 < U_2 - \varepsilon$, the MBS increases the interference price by $c = c + \Delta c$.
8. **else if** $U_1 > U_2 - \varepsilon$, the MBS decreases the interference price by $c = c - \Delta c$.
9. **end if**.
10. After the adjustment of the interference price with step 7 or step 8, the MBS broadcasts the new interference price to all FUEs through the backhaul links.
11. Step 5 to step 10 are repeated until $|U_1 - U_2| \le \varepsilon$, then $\xi = 0$.
12. **end while**.

The optimal solution of this problem is given by the following proposition.

Proposition 1: We rewrite the best response

$$p_i^* = \frac{\upsilon_i}{c_i} - \delta_i - \sum_{j\neq i,j=1}^{N} \alpha_{ij} p_j^* \tag{27}$$

by the matrix form

$$\boldsymbol{p}^* = \boldsymbol{H}^{-1}\boldsymbol{A} - \boldsymbol{H}^{-1}\boldsymbol{D} \tag{28}$$

where

$$\boldsymbol{p}^* = \begin{bmatrix} p_1^* \\ p_2^* \\ \vdots \\ p_{N^*} \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} \dfrac{\upsilon_1}{c_1} \\ \dfrac{\upsilon_2}{c_2} \\ \vdots \\ \dfrac{\upsilon_N}{c_N} \end{bmatrix}$$

It is assumed that the normalised channel gain matrix $\boldsymbol{H}$ is an invertible matrix, the NE for the non-cooperative power selection game $\boldsymbol{p}^*$ can be derived as

$$\boldsymbol{p}^* = \frac{\boldsymbol{H}^{-1}\boldsymbol{B}}{c} - \boldsymbol{H}^{-1}\boldsymbol{D} \tag{29}$$

Besides, based on the conclusion that the optimal price is obtained at the point $U_1 = U_2$, we have

$$c^* = \frac{\boldsymbol{G}^T\boldsymbol{H}^{-1}\boldsymbol{B}}{Q + \boldsymbol{G}\boldsymbol{H}^{-1}\boldsymbol{D}} \tag{30}$$

Proof: The proof can be found in Appendix 1. □

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4.2 Non-uniform pricing

As the general case of the uniform pricing scheme, we first investigate the SE point of the Stackelberg game.

Proposition 2: The optimal interference prices of Problem 4.2 can be expressed as

$$c_i^* = \frac{N\upsilon_i g_{0,i}}{Q + \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^*\right)g_{0,i}}, \quad i \in S \tag{31}$$

when the system is designed to admit all N FUEs simultaneously, the interference threshold constraint should satisfy

$$Q > \frac{\sum_{i=1}^{N} g_{0,i}\sqrt{\upsilon_i(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^*)}}{\min_i \sqrt{\frac{\upsilon_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^*}}} - \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^*\right)g_{0,i} \tag{32}$$

Proof: The proof can be found in Appendix 2. □

Theorem 3: There exists a unique NE point in the lower subgame defined by Problem 3.2.

Proof: If the interference threshold is fixed and the worst case of the interference is considered, that is

$$\boldsymbol{G}\left(\boldsymbol{H}^{-1}\boldsymbol{A} - \boldsymbol{H}^{-1}\boldsymbol{D}\right) = Q \tag{33}$$

we can obtain a fixed set of optimal interference prices c_i*, $i \in S$. Next, the detailed proof will be given. Similar to the proof of Theorem 2, we have (see (34))

$$\begin{aligned} |\Delta p_i(t+1)| &= \left|\frac{\nu_i}{c_i} - \frac{\nu_i}{c_i^*} + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^* - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right| \\ &\leq \left|\frac{\nu_i}{c_i} - \frac{\nu_i}{c_i^*}\right| + \left|\sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^* - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right| \\ &\leq \left|\sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^* - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right| + \left|\sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^* - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right| \\ &< \|\Delta \boldsymbol{p}\|_S \end{aligned} \tag{34}$$

Remark 2: Similar to Remark 1, we can obtain that the proposed power control strategy still converges to a unique NE point, if the maximum power constraint is considered.

Therefore there is a unique NE point in the lower subgame. Obviously, the SE point is unique.

To guarantee the fairness of the FUEs' competition and eventually achieve the revenue maximisation, the reasonable pricing scheme should be given. Therefore, as the part of the pricing scheme, the boundary constraints of the interference prices are also necessary to be investigated.

Proposition 3: The following constraints should be held for the proposed pricing schemes

$$\frac{\sum_{i=1}^{N}\upsilon_i g_{0,i}}{Q + \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}} \leq c_i \leq \frac{\upsilon_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j} \tag{35}$$

Proof: The proof can be found in Appendix 3. □

In this subsection, the reasonable constraints are discussed to obtain the optimal solution of Problem 4.2. In Proposition 2, the constraint of the interference threshold is investigated as a prerequisite condition for getting the optimal solution. Then, the pricing constraints are proposed in Proposition 3 to achieve the positive FUEs' transmission powers and protect the MBS's SINR performance. Obviously, Propositions 2 and 3 together form the solvable conditions of Problem 4.2. Next, an algorithm to remove the unreasonable pricing FUEs will be discussed in detail.

In the algorithm

$$\phi_K = \frac{\sum_{i=1}^{K}\sqrt{\upsilon_i\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}}}{Q + \sum_{i=1}^{K}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}}$$

$$\varsigma_K = \frac{\sum_{i=1}^{N}\upsilon_i g_{0,i}}{Q + \sum_{i=1}^{N}(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j)g_{0,i}}$$

and

$$\psi_i = \sqrt{\frac{\upsilon_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j}}, \quad i \in S$$

are defined, where K is the number of the active FUEs. By comparison of the ϕ_K and ψ_i, the overpriced FUEs can be removed from the Stackelberg game. Besides, to ensure the SINR requirement of the MBS, the restriction of the lower bound interference price is also considered. The details of the algorithm are given as follows.

Algorithm 2: Removal based on the FUEs' unreasonable interference prices

1. Set $K = N$.
2. The MBS measures $g_{0,i}$ and the FBSs measure $g_{i,j}$, $\forall(i, j) \in S$ meanwhile broadcast them to FUE i through the backhaul links.

3. Compute δ_i and ψ_i.
4. Sort the K FUEs according to ψ_i.
5. Calculate ϕ_K and ς_K.
6. Compare ϕ_K with ψ_i. **If** $\phi_K > \psi_i$, remove the overpriced FUE i from the Stackelberg game, set $K = K - 1$ and go to step 5.
7. **end**.
8. Calculate $\phi_K \psi_i$.
9. Compare $\phi_K \psi_i$ with ψ_i. **If** $\phi_K \psi_i < \varsigma_K$, remove the underpriced FUE i from the Stackelberg game, set $K = K - 1$ and go to step 5.
10 **end**.
11. The interference prices with ϕ_K are given as follows

$$c_i = \begin{cases} \phi_K \psi_i, & \text{if } i \le K \\ \infty, & \text{otherwise} \end{cases}$$

12. Set $N = K$.

Now we discuss several practical issues regarding the implementation of the algorithm. It is assumed that $\sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j$ in the algorithm is a constant obtained in the previous iteration. Therefore, only a limited amount of information need to be exchanged among the FUEs and BSs (either the MBS or FBSs). In step 5 of the algorithm, we calculate ϕ_K and compare it with the ψ_i to guarantee the upper bound of the price constraint, that is, $\phi_K < \psi_i$ must be satisfied to ensure the corresponding $p_i > 0$. Moreover, in step 9, the lower bound based on Proposition 2 is also considered to ensure the MBS's SINR requirement. Obviously, all the unreasonable pricing FUEs have already been removed before the execution of step 11.

The pricing mechanisms of the MBS under the uniform pricing scheme and the non-uniform pricing scheme are presented in this section. For the uniform pricing scheme, Fig. 2 presents the view of getting the optimal price in the monotone case of U_2. Based on the analysis, a distributed interference price bargaining algorithm is proposed. For the non-uniform pricing scheme, the interference constraint is considered to ensure the uniqueness of the SE point. The removals of unreasonable pricing FUEs are also involved.

5 Admission control method for the proposed pricing schemes

In the above section, the pricing problem is mainly considered. However, the SINR requirements should also be satisfied to ensure the connection between FUEs and the corresponding FBSs. Thus

$$\gamma_i \ge \Gamma_{\min}, \quad \forall i \in S \tag{36}$$

is required, where $\Gamma_{\min}$ is the minimum SINR required for an FUE to communicate with the FBS. If FUEs SINR is below this threshold, it would simply not access the network. Under the uniform pricing scheme, general form of the admission control algorithm is considered. Therefore we have dedicated to study the admission control algorithm under the non-uniform pricing scheme which can simplify the process of the removal.

Assumption 2: In this paper, to facilitate the presentation and analysis, we assume that the aggregate interference at FBS i because of all other FUEs is bounded, i.e., $\sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j \le \varepsilon$, where ε denotes the upper bound. If the worst case is considered, that is, $\sum_{j \neq i, j=1}^{N} \alpha_{ij} p_j = \varepsilon$, we propose an admission control algorithm based on the following proposition to reduce the computation of information.

Proposition 4: The SINR requirements of all FUEs $\gamma_i \ge \Gamma_{\min}$, $\forall i \in S$ can be guaranteed, only when the following condition is satisfied

$$\frac{1}{\sqrt{\delta_i^+ + \varepsilon}\left(\sum_{i=1}^{N} \sqrt{v_i(\delta_i + \varepsilon) g_{0,i}}/Q + \sum_{i=1}^{N} (\delta_i + \varepsilon) g_{0,i}\right)} - 1 \ge \Gamma_{\min}$$

where $\delta_i^+ = \arg\max_{i \in S} \delta_i$.

Proof: The proof can be found in Appendix 4. □

Based on Assumption 2 and Proposition 4, the admission control algorithm under the non-uniform pricing is proposed.

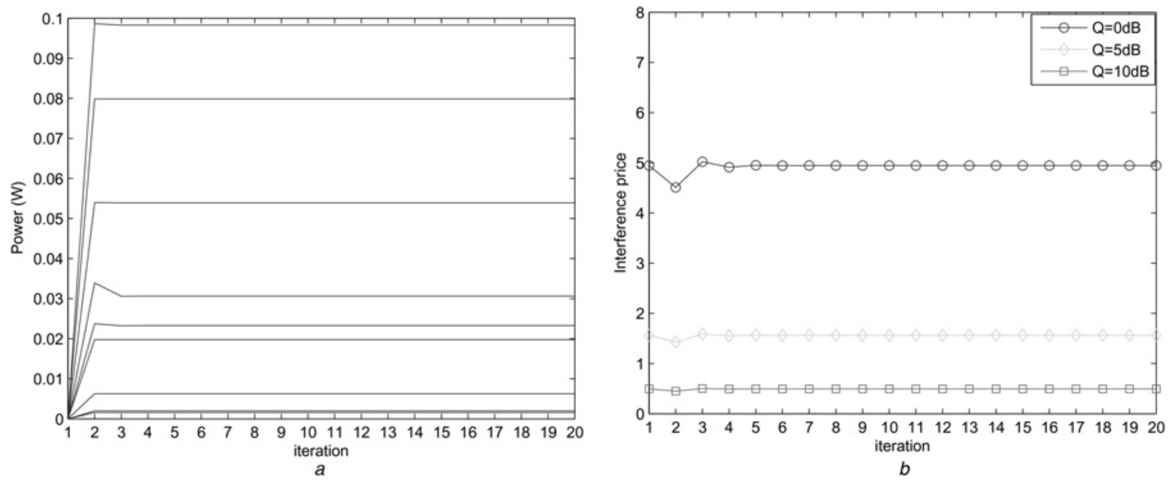

Fig. 3 *Convergence performance under the uniform pricing scheme (n = 10)*
a Power convergence of the uniform pricing scheme
b Distributed interference price algorithm under the uniform pricing scheme

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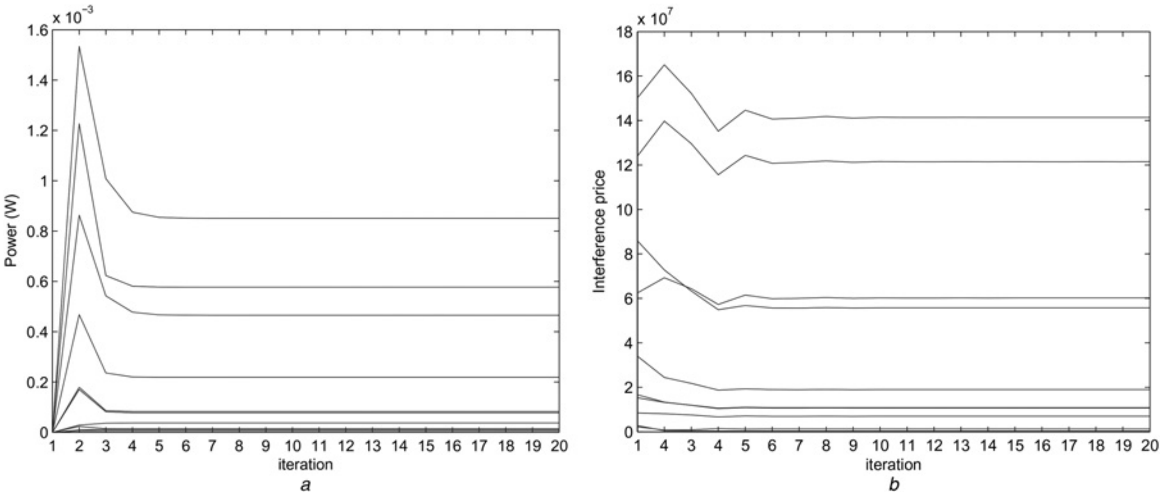

Fig. 4 *Convergence performance under the non-uniform pricing scheme (n = 10)*

a Power convergence of the non-uniform pricing scheme
b Price convergence of the non-uniform pricing scheme

Algorithm 3: Admission control algorithm under the non-uniform pricing

1. Set $K = N$, and the upper bound aggregate interference $\varepsilon = 0.1$.
2. The FUEs set the minimal performance requirement $\Gamma_{\min}$.
3. The MBS measures $g_{0,i}$ and each FBS measure $g_{i,j}$, $\forall (i, j) \in S$ meanwhile broadcast them to FUE i and the MBS through the backhaul links.
4. Compute δ_i.

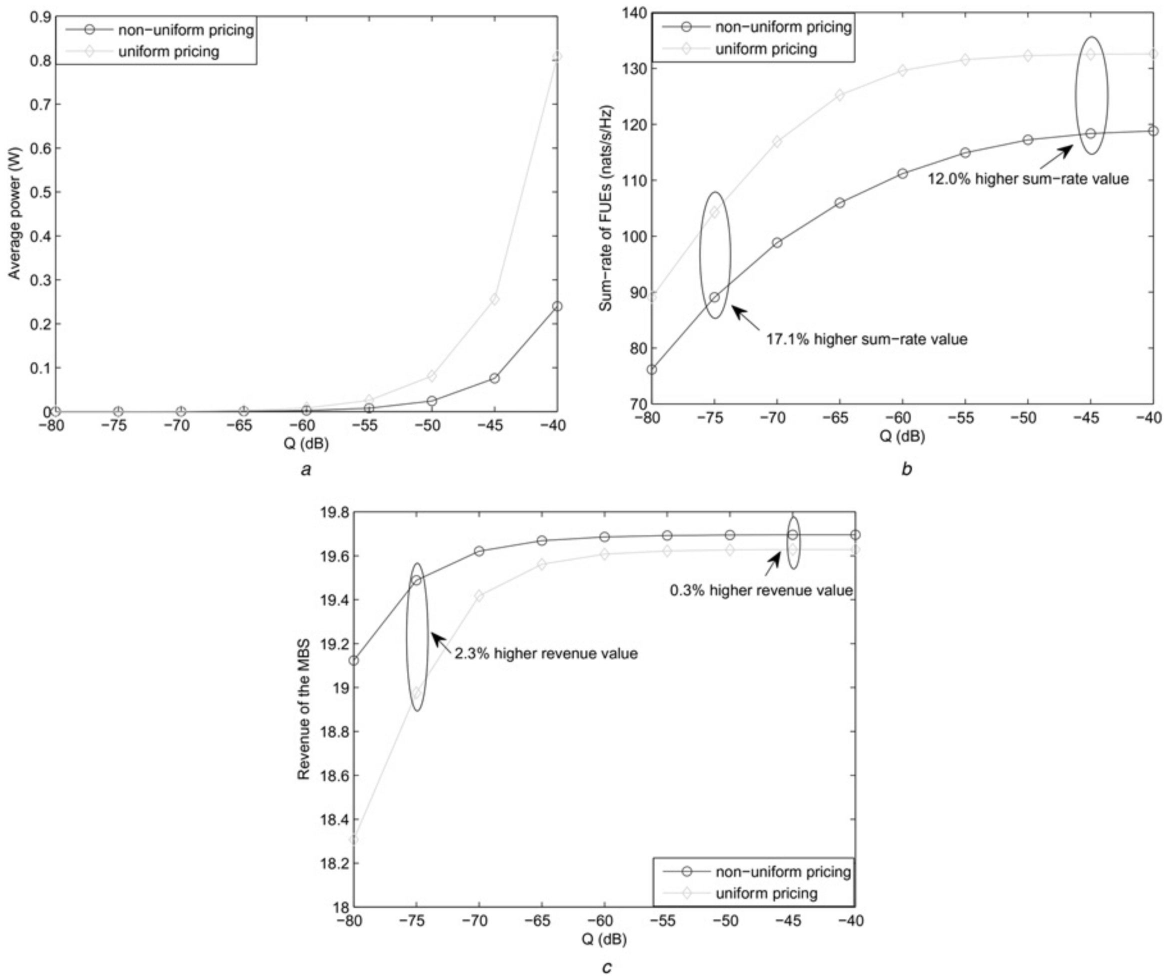

Fig. 5 *Performance comparison of FUEs and the MBS under two pricing schemes against Q*

a Average powers under two pricing schemes ($n = 20$)
b Sum-rate values of FUEs under two pricing schemes against Q ($n = 20$)
c Revenues of the MBS under two pricing schemes ($n = 20$)

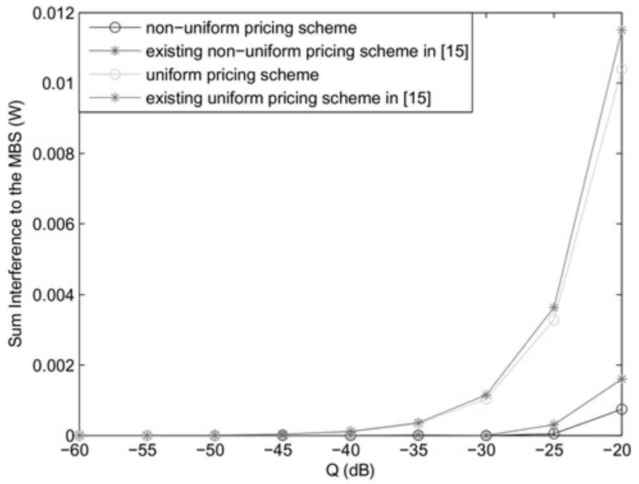

Fig. 6 *Sum interference to the MBS*

5. Sort the K FUEs according to $\delta_i = \delta^2/g_{i,i}$, that is, $\left(\frac{\delta^2}{g_{1,1}} < \frac{\delta^2}{g_{2,2}} < \cdots < \frac{\delta^2}{g_{N,N}}\right)$.
6. Calculate $\eta_k = \left(\sum_{i=1}^{K}\sqrt{\upsilon_i(\delta_i + \varepsilon)g_{0,i}}/Q + \sum_{i=1}^{K}(\delta_i + \varepsilon) g_{0,i}\right)$ and $\varphi_K = [1/\eta_k(\Gamma_{\min} + 1)]^2 - \varepsilon$.
7. Compare δ_N with φ_K. **If** $\delta_N > \varphi_K$, FUE K removes itself from the Stackelberg game, set $K = K - 1$ and go to step 5.
8. **else if** $\delta_N < \varphi_K$
9. **end**.
10. Set $N = K$.

It can be seen in step 7 that the comparison result of the factor δ_N and φ_K is provided. It shows that the minimum SINR requirement can be satisfied for each FUE, if $\delta_N < \varphi_K$, which can reduce the computation of the information. Besides, in any other case, it is also not needed to compare each δ_i with φ_K except that all FUEs cannot satisfy the minimum SINR requirement simultaneously. Noting that if the worst case is not suitable for the system, we consider the general form of the admission control, that is, remove the FUEs which cannot satisfy $\gamma_i \geq \Gamma_{\min}$, $i \in S$.

In this section, admission control algorithm is presented to improve the requirement of the active FUEs. Then it follows with numerical simulation results and analysis.

6 Simulations results and analysis

In this section, numerical results are provided to evaluate the performance of the proposed resource allocation schemes based on the approach of interference pricing mechanism. In the two-tier femtocell network, the MBS is located at the centre and the FBSs are randomly distributed within the circle with radius $R_c = 200$ m from the centre of the MBS. Then, the FUEs are randomly distributed in the centre of the corresponding FBSs with radius $R_f = 10$ m. Other specific parameter settings are presented as follows. The rate parameters $\lambda_i = 1$, $\forall i \in S$, carrier frequency $f_c = 2000$ MHz, $K_{fi} = 37$ dB. Besides, outdoor loss can be expressed as $K_{fo} = 30\log_{10} f_c - 71$(dB). The term $p_i^{\max} = 0.5$ W, $\forall i \in S$.

The convergence performance of the transmission powers and interference prices under the uniform pricing scheme will be given in Fig. 3.

In Figs. 3 and 4, we set a scenario beginning with ten FUEs in the two-tier femtocell network, all the initial powers are set to be 0 W. The maximum tolerable interference is chosen as $Q = 10^{-6}$. In Figs. 3 and 4, the powers and prices converge rapidly after a few iterations which prove the convergence and efficiency of our algorithms. Besides, it is observed from Fig. 3*b* that the proposed algorithm converges for all values of Q which also shows the reasonability of the algorithm.

Fig. 5 shows the different aspects performance of FUEs and the MBS against the interference threshold under two pricing schemes. The sum-rate of FUEs are defined as $\sum_{i=1}^{N} \lambda_i \log(1 + \gamma_i)$, $i \in S$. It can be seen from Figs. 5*a* and *b* that the average power and sum-rate value under the uniform pricing scheme are higher than that under the non-uniform pricing scheme. From Fig. 5*b*, we can see that the value of sum-rate under the uniform pricing scheme varies from 17.1% higher to 12.0% higher when Q changes from −75 to −45 dB. And the difference converges to a constant when Q is sufficiently large. Obviously, the uniform pricing scheme has an advantage in achieving larger sum-rate value and more satisfaction for FUEs. It is

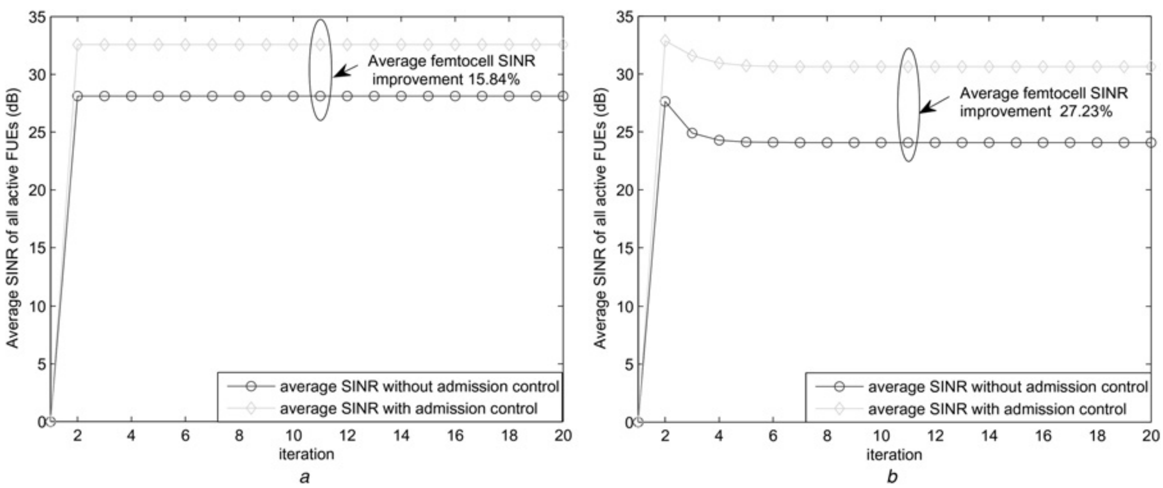

Fig. 7 *Average SINR comparison of active FUEs without and with admission control under the uniform pricing scheme and the non-uniform pricing scheme against Q (n = 20)*

a Uniform pricing scheme
b Non-uniform pricing scheme

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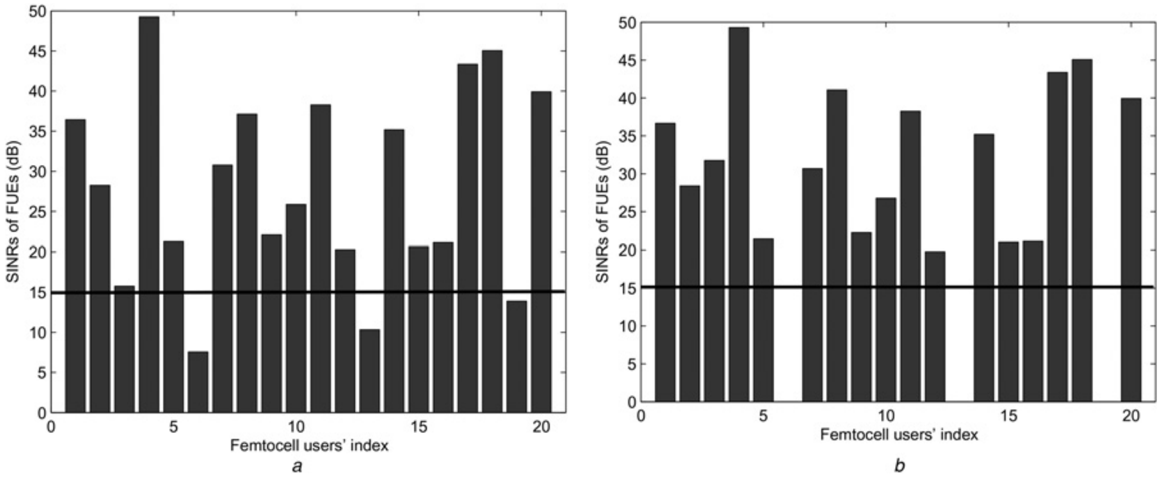

Fig. 8 *SINRs of all FUEs without and with admission control under the uniform pricing scheme (n = 20)*
a Non-admission control
b Admission control

observed from Fig. 5*c* that the revenue of the MBS under the non-uniform pricing scheme is higher than that under the uniform pricing scheme. This can be explained that the dynamic interference prices can adjust the power allocation timely to earn more MBSs revenue while the interference threshold is fixed. We can get conclusion that the non-uniform pricing scheme is superior to the uniform pricing scheme in protecting the performance of the MBS. Therefore, the uniform pricing scheme and the non-uniform pricing scheme, respectively, have their own superiorities in different aspects of the network performance. The description of the sum interference to the MBS is investigated in Fig. 6.

The example of Fig. 6 shows the sum interference to the MBS under the proposed schemes and the existing schemes in [15]. It is observed that the sum interference values to the MBS under the proposed pricing schemes are lower than that under the corresponding schemes in [15], respectively. This is because that the schemes in [15] does not fully consider the interference between the FUEs. It is not conducive to protect the quality of service (QoS) of the MBS. In addition, we can see that the sum interference values to the MBS under non-uniform pricing schemes are lower than that under uniform pricing schemes. And, the sum interference values increase very rapidly from $Q = -40$ dB under the uniform pricing scheme and $Q = -25$ dB under the non-uniform pricing scheme, respectively. Obviously, the results also show that the non-uniform pricing scheme has advantages in protecting the performance of the MBS. To illustrate the effectiveness of the admission control, the SINR performance comparison is shown in Fig. 7.

We set a scenario with 20 FUEs in Figs. 7 and 8. And the minimal SINR requirement is chosen as $\Gamma_{\min} = 15$. It can be observed that the average SINRs with the admission control, respectively, have 15.84 and 27.23% improvement under uniform pricing and non-uniform pricing. Besides, under the uniform scenario, the SINR values for FUEs have the improvement in various degrees with the removal of FUE6, FUE13 and FUE19. The explanation can be offered by the equation $\gamma_i = p_i / \sum_{j=1, j\neq i}^{N} \alpha_{i,j} p_j + \delta_i$. Obviously, the total interference reduces with the removal of the FUEs which cannot satisfy the SINR requirements. In summary, the results shown by Figs. 7 and 8 verify the effectiveness of the admission control algorithm.

7 Conclusion

In this paper, two price-based interference management schemes are proposed in two-tier femtocell networks. With utility optimisation integrated, the schemes not only restrict the cross-tier interference at the MBS below a given threshold but also ensure the optimisation of FUEs' power allocation. To obtain a more realistic network scenario, the boundary constraints of interference prices are also discussed. The Stackelberg game model is adopted to jointly study the utility maximisation of the MBS and FUEs. In the game, the problems of the SE are investigated. At last, the admission control algorithm is adopted, which is capable of robustly protecting the performances of all the active FUEs. It can be observed by the simulation results that the QoS requirements of all active FUEs have been greatly improved. The results of this paper are useful to practically obtain the proper pricing schemes for the different performance requirements in the realistic femtocell networks.

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10 Appendices

10.1 Appendix 1: Proof of Proposition 1

From (29), we can obtain its equivalent form

$$p_i^* + \Sigma_{j\neq i,j=1}^N \alpha_{ij} p_j^* = \frac{\upsilon_i}{c} - \delta, \quad \forall i \in S \tag{37}$$

Equation (37) can be changed into the matrix form

$$\boldsymbol{H}\boldsymbol{p}^* = \frac{\boldsymbol{B}}{c} - \boldsymbol{D}$$

Thus, the equality

$$\boldsymbol{p}^* = \frac{\boldsymbol{H}^{-1}\boldsymbol{B}}{c} - \boldsymbol{H}^{-1}\boldsymbol{D}$$

holds. If the condition

$$Q = \frac{\boldsymbol{G}\boldsymbol{H}^{-1}\boldsymbol{B}}{c} - \boldsymbol{G}\boldsymbol{H}^{-1}\boldsymbol{D}$$

is satisfied, the optimal price can be obtained, that is,

$$c^* = \frac{\boldsymbol{G}^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{B}}{Q + \boldsymbol{G}\boldsymbol{H}^{-1}\boldsymbol{D}}$$

10.2 Appendix 2: Proof of Proposition 2

Problem 4.1 is a convex optimisation problem, the duality gap does not exist between it and its dual optimisation problem. Therefore the best solution of Problem 4.1 can be obtained by solving Problem 4.2. The Lagrangian function can be written as

$$\begin{aligned}\mathcal{L}(\boldsymbol{c}, \alpha, \beta) &= \sum_{i=1}^N c_i g_{0,i}\left(\delta_i + \sum_{j\neq i,j=1}^N \alpha_{ij} p_j\right) \\ &+ \alpha\left[\sum_{i=1}^N\left(\frac{\upsilon_i}{c_i} - \delta_i - \sum_{j\neq i,j=1}^N \alpha_{ij} p_j\right) g_{0,i} - Q\right] - \sum_{i=1}^N \beta_i c_i\end{aligned} \tag{38}$$

Then, the KKT conditions can be expressed as follows

$$\begin{aligned}\frac{\partial \mathcal{L}(\boldsymbol{c}, \alpha, \beta)}{\partial c_i} &= g_{0,i}\left(\delta_i + \sum_{j\neq i,j=1}^N \alpha_{ij} p_j\right) - \alpha\frac{\upsilon_i}{c_i^2} - \beta_i \\ &= 0\end{aligned} \tag{39}$$

$$\alpha\left[\sum_{i=1}^N\left(\frac{\upsilon_i}{c_i} - \delta_i - \sum_{j\neq i,j=1}^N \alpha_{ij} p_j\right) g_{0,i} - Q\right] = 0 \tag{40}$$

$$\beta_i c_i = 0 \tag{41}$$

$$\alpha \geq 0 \tag{42}$$

$$\beta_i \geq 0 \tag{43}$$

$$c_i > 0 \tag{44}$$

With (39), we can obtain

$$c_i^2 = \alpha \frac{\upsilon_i}{g_{0,i}\left(\delta_i + \sum_{j\neq i,j=1}^N \alpha_{ij} p_j\right) - \beta_i} \tag{45}$$

Lemma 2: $\beta_i = 0$ and $\sum_{i=1}^N \left(\frac{\upsilon_i}{c_i} - \delta_i - \sum_{j\neq i,j=1}^N \alpha_{ij} p_j\right) g_{0,i} - Q = 0$.

Proof: If $\beta_i \neq 0$, we can obtain $c_i = 0$ which contradicts the assumption that $c_i > 0$. Besides, assuming that

$$\sum_{i=1}^{N}\left(\frac{v_i}{c_i} - \delta_i - \sum_{j\neq i, j=1}^{N}\alpha_{ij}p_j\right)g_{0,i} - Q \neq 0$$

from (40), we can obtain $\alpha = 0$. Substituting it into (45), the result $c_i = 0$ can be obtained which is opposite to the aforementioned assumption $c_i \neq 0$. Therefore

$$\sum_{i=1}^{N}\left(\frac{v_i}{c_i} - \delta_i - \sum_{j\neq i, j=1}^{N}\alpha_{ij}p_j\right)g_{0,i} - Q = 0$$

□

Besides, according to (45), we can obtain

$$c_i = \sqrt{\alpha\frac{v_i}{g_{0,i}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)}} \tag{46}$$

Substituting (46) into

$$\sum_{i=1}^{N}\left(\frac{v_i}{c_i} - \delta_i - \sum_{j\neq i, j=1}^{N}\alpha_{ij}p_j\right)g_{0,i} - Q = 0$$

we have

$$\sqrt{\alpha} = \frac{\sum_{i=1}^{N}\sqrt{v_i\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}}}{Q + \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}} \tag{47}$$

Then, it follows

$$c_i^* = \frac{N v_i g_{0,i}}{Q + \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j^*\right)g_{0,i}} \tag{48}$$

Besides, in order to guarantee that the transmission powers of FUEs are positive, the condition

$$c_i \leq \frac{v_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j} = \frac{v_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j}, \quad \forall i \in S$$

should be held. Substituting c_i^* into above inequality (see equation at the bottom of the page) we have

$$Q > \frac{\sum_{i=1}^{N} g_{0,i}\sqrt{v_i(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j)}}{\min_i\sqrt{\frac{v_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j}}} - \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}$$

Proposition 2 is proved.

10.3 Appendix 3: Proof of Proposition 3

To obtain non-negative transmission powers, we have

$$c_i \leq \frac{v_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j}, \quad i \in S \tag{49}$$

Besides, to meet the QoS requirement of the MBS, the total interference introduced by the FUEs' transmission powers must be restricted within the fixed interference threshold, that is

$$\sum_{i=1}^{N}\left(\frac{v_i}{c_i} - \delta_i - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i} \leq Q$$

We consider the worst case to meet the requirement of the interference threshold

$$\sum_{i=1}^{N}\left(\frac{v_i}{c_i} - \delta_i - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i} \leq \sum_{i=1}^{N}\left(\frac{v_i}{c_{\min}} - \delta_i - \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i} \leq Q$$

Therefore

$$c_i \geq \frac{\sum_{i=1}^{N} v_i g_{0,i}}{Q + \sum_{i=1}^{N}(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j)g_{0,i}} \tag{50}$$

Here, the proof is completed.

10.4 Appendix 4: Proof of Proposition 4

It can be seen from the following derivation that γ_i is a monotone decreasing function of δ_i, $\forall i \in S$.

$$\gamma_i = \frac{(v_i/c_i) - \delta_i - \varepsilon}{\delta_i + \varepsilon} = \frac{v_i}{c_i(\delta_i + \varepsilon)} - 1 = \frac{1}{\theta\sqrt{\delta_i + \varepsilon}} - 1 \geq \Gamma_{\min} \tag{51}$$

where $\theta = \left(\sum_{i=1}^{N}\sqrt{v_i(\delta_i + \varepsilon)g_{0,i}}/Q + \sum_{i=1}^{N}(\delta_i + \varepsilon)g_{0,i}\right)$. Thus, the proposition naturally follows.

$$\sqrt{\frac{v_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j}}\frac{\sum_{i=1}^{N} g_{0,i}\sqrt{v_i\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)}}{Q + \sum_{i=1}^{N}\left(\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j\right)g_{0,i}} < \frac{v_i}{\delta_i + \sum_{j\neq i,j=1}^{N}\alpha_{ij}p_j}$$