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Iterative receiver with soft-input-based-channel estimation for orthogonal frequency division multiplexing-interleave division multiple access systems

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Abstract: This article presents two sets of channel estimation methods that employ soft input from the decoder to enhance channel estimation process for orthogonal frequency division multiplexing -interleave division multiple access (OFDM-IDMA) system. The first channel estimation scheme exploits both time and frequency domains for channel estimation and prediction. The estimator exploits turbo principle while using iterative based sequential linear minimum mean square error (ISLMMSE) for channel transfer function (CTF) estimation, and regularised variable step size normalised least mean square (ℓ_1-VSSNLMS) algorithm for channel impulse response (CIR) prediction. The channel estimation scheme exchanges information with the Multi-User Detector (MUD), and also employs soft information feedback from the decoder for the enhancement of channel estimation. The second iterative channel estimation scheme, in time domain, is based on regularised noise power estimate-based variable forgetting factor recursive least square (ℓ_1-NPEVFF-RLS)-based CIR estimator. The performances of the proposed estimators are documented through computer simulation. Their comparative performances with other schemes in the literature are presented in this article. From the simulation results, the two proposed channel estimators exhibit better performance in comparison with other schemes in the literature, but with higher computational complexities. However, of the two proposed methods, the ℓ_1-NPEVFF-RLS-based CIR estimator that exhibits almost the same performance as the combined ISLMMSE-based CTF estimator and ℓ_1-VSSNLMS-based predictor exhibits lower computational complexity.

1 Introduction

Among the various multiple-access schemes based on orthogonal frequency division multiplexing (OFDM) transmission techniques, orthogonal frequency division multiplexing-interleave division multiple access (OFDM-IDMA) scheme has been confirmed to inherit most of the advantages of the multiple access schemes such as Orthogonal Frequency Division Multiple Access (OFDMA), Code Division Multiple Access (CDMA), OFDM-CDMA and IDMA schemes; and avoids their individual disadvantages [1]. Channel estimation is one of the many outstanding issues that demand investigation in this relatively new system as emphasised in [1].

There have been only few channel estimation methods that have been reported in the literature for OFDM-IDMA systems in the last five years. Least mean square (LMS) algorithm is employed in [2, 3] to estimate time-domain channel impulse response (CIR) of the OFDM-IDMA system in a soft decision directed channel estimation (DDCE) scheme. This is similar to the way the algorithm was employed to obtain CIR for a turbo equalisation-based wireless communication systems presented in [4]. In the reported scheme, the channel estimate employ feedback from decoder in order to enhance channel estimator performance. However, the limitations of the LMS algorithm have been emphasised in [4, 5]. The algorithm exhibits slow convergence and excess mean square error (MSE) at steady state. This by extension imposes a limitation to the performance of the proposed channel estimation method. In [6], the pilot-based channel estimation technique is reported. The reported technique also exploits the feedback from the decoder to improve the channel estimation quality. However, such schemes that employ pilot symbols inserted between data streams waste the scarce communication bandwidth. In addition, such schemes could also suffer from the problem of unresolved error introduced into the estimation process at the data streams location, especially in a fast fading channel scenario. In [6], channel estimation scheme that employs a combination of linear estimation of the frequency domain-channel transfer function (FD-CTF) based on linear minimum mean square error (MMSE) estimator, and the adaptive prediction of the time domain CIR is proposed. In the initial results presented, the channel estimation scheme exchanges information only with the multi-user detector (MUD). In [7] a review of iterative channel estimation algorithms based on pilot scheme and DDCE schemes including the one presented for OFDM-IDMA systems in [2, 3] are documented. Overall the authors concluded that decision directed method employing space alternating generalised expectation-maximisation maximum likelihood solution seems to be the most attractive schemes in terms of performance and computational complexity.

In this paper, two types of iterative channel estimation schemes are proposed. The first method exploits both time and frequency domains for both channel estimation and prediction for the OFDM-IDMA systems. The scheme does not only exchange information with the MUD as the case in [4] but also employs feedback from the decoder for channel estimation enhancement. On the other hand, the second method exploits only the underlying sparseness in the OFDM channel to develop time domain CIR estimator for OFDM-IDMA systems similar to the procedure presented in [3]. The results of the proposed schemes are documented in comparison with the scheme in [3] that exploits only time domain for DDCE based on the LMS algorithm and the scheme that exchanges information only with the MUD in [4]. The proposed schemes are also compared with DDCE employing other improved versions of the LMS algorithm-normalised least mean square (NLMS) and variable step size normalised least mean square (VSSNLMS) algorithms.

The contributions of this paper are as follows. Iterative principle is applied to the combine channel estimation and prediction exploiting both time domain and frequency domain for the channel estimation process. Linear MMSE is derived based on soft input information from the decoder to form the iterative-based sequential linear minimum mean square error (ISLMMSE) algorithm. This is employed for the implementation of the CTF module of the proposed channel estimation scheme. A novel time domain regularised noise power estimate-based variable forgetting factor recursive least square (ℓ_1-NPEVFF-RLS)-based CIR estimator is developed for OFDM-IDMA system. Comparative complexities of the proposed schemes are also presented along that of the channel estimation employing LMS, NLMS and VSSNLMS algorithms.

The rest of this paper is organised as follows. Section 2 describes the OFDM-IDMA system model. Section 3 present the elementary signal estimator (ESE) employed as MUD in this paper, whereas the channel estimator modules are detailed in Section 4. The proposed ISLMMSE algorithm is presented in Section 4. In Section 5, the regularised noise power estimate-based variable forgetting factor recursive least square (ℓ_1-NPEVFF-RLS)-based CIR estimator is developed. Computational complexities of the proposed estimators in comparison with LMS, NLMS and VSSNLMS algorithms-based estimators are presented in Section 6. Section 7 gives the comparative computer simulation results of the proposed schemes. Finally, in Section 8, the conclusions are drawn for the proposed schemes.

2 OFDM-IDMA system model

The OFDM-IDMA transmitter is shown in Fig. 1. At the transmitter, the uth user's message is first encoded as $c_{u,q}$ and then interleaved by user specific interleaver π_u to $x_{u,q}[n]$, where $q = 1, \ldots, Q$ (from the 2^Q-ary signal constellation). The interleaved signal from each user is converted to parallel, using a serial to parallel converter in order to have it modulated onto OFDM subcarriers using inverse fast Fourier transform (IFFT). After the cyclic prefix (CP) of length K_{cp} has been added, the time domain users' signals are transmitted over a wideband fading channel. A sample-spaced multipath channel model described by an M-tap complex random vector $\boldsymbol{h}_u$, for user u given as

$$\boldsymbol{h}_u[n] = \left[h_u[n, 1], h_u[n, 2], \ldots, h_u[n, M]\right]^{\mathrm{T}} \tag{1}$$

is considered.

The various users in the system are distinguished by their specific interleavers. The iterative receiver of the OFDM-IDMA system is shown in Fig. 2. At the receiver, the time domain nth sample received signal could be expressed as

$$\begin{aligned} z[n] &= \sum_{u=1}^{U}\sum_{m=1}^{M} h_u[n, m]x_u[n-m] + w[n], \\ n &= 0, 1, \ldots, K_g - 1 \end{aligned} \tag{2}$$

where $w[n]$ denotes additive white Gaussian noise (AWGN), $K_g = K + K_{cp}$ and K is the number of OFDM subcarriers. However, after the CP has been removed and FFT demodulation has taken place, the discrete frequency domain baseband model of the received signal on kth

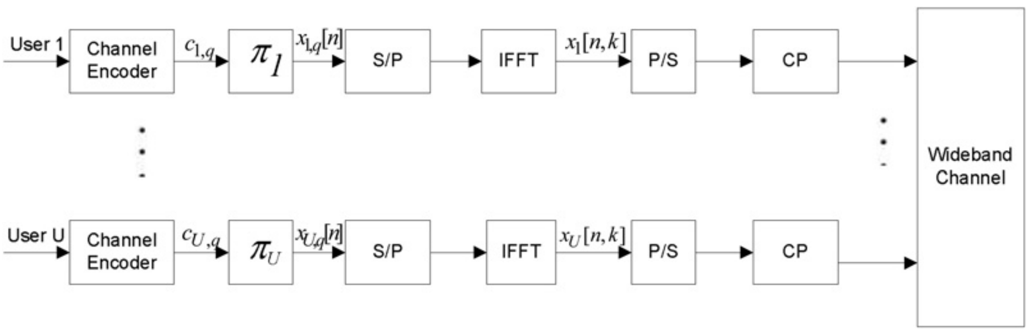

Fig. 1 *OFDM-IDMA transmitter*

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subcarrier at nth OFDM symbol period is given as

$$\boldsymbol{z}[n, k] = \sum_{u=1}^{U} H_u[n, k] x_u[n, k] + w[n, k] \tag{3}$$

By letting $\boldsymbol{H}[n, k] = [H_1[n, k], \ldots, H_u[n, k]]$ and $\boldsymbol{x}[n, k] = [x_1[n, k], \ldots, x_u[n, k]]^{\mathrm{T}}$, (3) can be written as

$$z[n, k] = \boldsymbol{H}[n, k]\boldsymbol{x}[n, k] + w[n, k] \tag{4}$$

where superscript T denotes the transpose; $x_u[n, k]$, $H_u[n, k]$ and $w[n, k]$ are the transmitted chip, FD-CTF coefficient for user u, and the samples of the AWGN with a zero mean and variance σ_w^2, respectively. The received chip on the kth subcarrier for user u can then be written as

$$z_u[n, k] = H_u[n, k] x_u[n, k] + \varsigma_u[n, k] \tag{5}$$

The symbol $\varsigma_u[n, k] = \sum_{u' \neq u}^{U} H_{u'}[n, k] x_{u'}[n, k] + w[n, k]$ is the net sum of interference from the other users imposed on the uth user plus AWGN. This distortion is the summation of the received signals from the other U-1 users (except user u) plus noise. From the central limit theorem, if these signals are assumed to be random and independent of each other, $\varsigma_u[n, k]$ can be approximated by a Gaussian random variable, when U is sufficiently large.

As shown in Fig. 2, the iterative receiver comprises the ESE, a posteriori probability decoders (APP decoders), and iterative channel estimator module. The ESE and the iterative channel estimator module are described in the following sections.

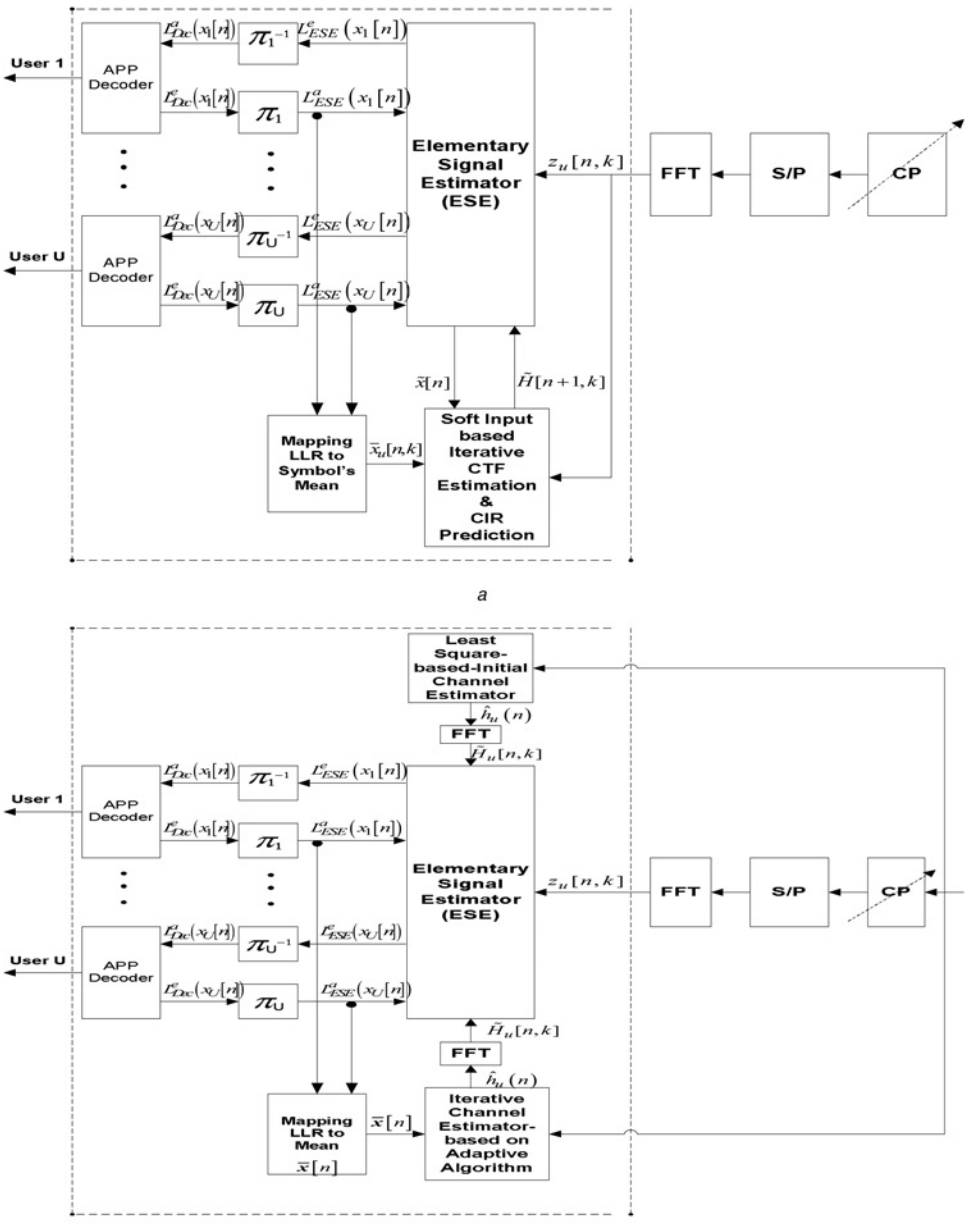

Fig. 2 *Iterative receiver of the OFDMIDMA system*

a Iterative receiver with soft input based iterative CTF estimator and CIR predictor
b Iterative receiver with CIR estimator based on adaptive algorithm

3 Elementary signal estimator

The MUD employed in this paper is the ESE. This follows after the one presented in [8–10]. By dropping the time index term 'k' for convenience sake, from the a priori information $L^{a}_{\mathrm{ESE}}(x_u[n])$ coming from the APP decoder, expectation (E) and variance (V) of $x_u[n, k]$ in (5) for M-phase shift keying (M-PSK) signal constellation can be, respectively, computed as [8–10]

$$E(x_u[n]) = \tanh\left(\frac{L^{\mathrm{a}}_{\mathrm{ESE}}(x_u[n])}{2}\right) \tag{6}$$

$$V(x_u[n]) = 1 - \left(E(x_u[n])\right)^2 \tag{7}$$

Using (6) and (7), the ESE computes the mean and variance of the interference imposed on the received signal as

$$E(\varsigma_u[n]) = E(z_u[n]) - \tilde{H}_u[n]E(x_u[n]) \tag{8}$$

$$V(\varsigma_u[n]) = V(z_u[n]) - |\tilde{H}_u[n]|^2 V(x_u[n]) \tag{9}$$

where $\tilde{H}_u[n]$ is the estimated channel state information provided by the proposed iterative channel estimator described in the subsequent section. The total received signals' mean and variance in (8) and (9) are described, respectively, as

$$E(z_u[n]) = \sum_{u=1}^{U} \tilde{H}_u[n]E(x_u[n]) \tag{10}$$

$$V(z_u[n]) = \sum_{u=1}^{U} |\tilde{H}_u[n]|^2 V(x_u[n]) + \sigma_w^2 \tag{11}$$

The extrinsic log likelihood ratio (LLR) from the ESE, $L^{\mathrm{e}}_{\mathrm{ESE}}(x_u[n])$ about $x_u[n, k]$ is then computed as

$$L^{\mathrm{e}}_{\mathrm{ESE}}(x_u[n]) = \ln\left(\frac{\Pr(z_u[n]|x_u[n] = +1)}{\Pr(z_u[n]|x_u[n] = -1)}\right) \tag{12}$$

However, from the central limit theorem point of view, if the number of users is sufficiently high, the conditional Gaussian probability density function used to characterise the received signal $\Pr(z_u[k]/x_u[k] = \pm 1)$ can be approximated as [8]

$$\Pr(z_u[n]|x_u[n] = \pm 1) = \frac{1}{\sqrt{2\pi V(\varsigma_u[n])}} \exp\left(\frac{\left(z_u[n] - \left(\pm \tilde{H}_u[n] + E(\varsigma_u[n])\right)\right)^2}{2V(\varsigma_u[n])}\right) \tag{13}$$

Substituting (13) into (12), the extrinsic LLR from the ESE becomes

$$L^{\mathrm{e}}_{\mathrm{ESE}}(x_u[n]) = 2\tilde{H}_u[n] \cdot \frac{z_u[n] - E(\varsigma_u[n])}{V(\varsigma_u[n])} \tag{14}$$

Equation (14) illustrates the soft interference cancellation employed by the ESE [8, 9].

The decoders in Fig. 2 employed the output of the ESE in order to generate the 'extrinsic' LLR, $L^{\mathrm{e}}_{\mathrm{Dec}}(x_u[n])$ and a priori LLR, $L^{\mathrm{a}}_{\mathrm{Dec}}(x_u[n])$ in turn. Interested readers are referred to [11] for detail operation of the APP decoders.

4 Iterative channel estimator module

Fig. 2 illustrates the major components of the proposed soft-input-based iterative channel estimator. The proposed iterative-based sequential linear MMSE-based-frequency domain (ISLMMSE-FD) CTF Estimator is described in the following.

In [12] MMSE estimator documented in [13] is employed for temporary CTF estimation of the proposed DDCE scheme for OFDM systems. However, it is also noted in [13] that the class of MMSE estimators in which MMSE estimator of [12] belongs are very difficult to determine in a closed from. In practice it is also too computationally intensive to implement. In order to circumvent these problems, we hereby propose an iterative-based sequential linear MMSE (ISLMMSE) estimator for the implementation of the FD-CTF estimation schemes for the multi-user-based OFDM-IDMA systems.

In order to derive the ISLMMSE-FD CTF estimator suitable for the multi-user OFDM-IDMA, we employ the sequential linear MMSE approach of [13] which has its input fed with soft symbols. In Fig. 2*a*, after the first iteration, the soft input (the mean), $\bar{x}_u[n, k]$ is fed into the first stage of the proposed soft input iterative channel estimator, iterative-based FD-CTF estimator instead of the received symbols for the subsequent channel estimation process. Following [14], the mean $\bar{x}_u[n, k]$ is obtained from the code bit $L^{\mathrm{a}}_{\mathrm{ESE}}(x_u[n])$ coming from the APP decoder as

$$\bar{x}_u[n, k] = \sum_{s \in \boldsymbol{S}} s \cdot \Pr(x_u[n] = s) \tag{15}$$

where $\boldsymbol{S}$ is a vector of symbols 's' in 2^Q-ary signal constellation such as M-PSK constellation and quadrature amplitude modulation (QAM) constellation. For the quadrature phase shift keying (QPSK) constellation, the symbol $Q = 2$, hence $q = 1, 2$. Using the QPSK modulation scheme with a mapping rule given as

$$\left\{ \begin{array}{ccc} (x_{u,1}[n], x_{u,2}[n]) & \Rightarrow & \left(\frac{1}{\sqrt{2}}(s)\right) \\ (0, 0) & \Rightarrow & (1 + i) \\ (1, 0) & \Rightarrow & (-1 + i) \\ (1, 1) & \Rightarrow & (-1 - i) \\ (0, 1) & \Rightarrow & (1 - i) \end{array} \right\} \tag{16}$$

the code bit probabilities for $\{x_u[n]\}^{q=1}$ and $\{x_u[n]\}^{q=2}$ of $x_u[n]$, given as $\Pr[x_u[n] = \pm 1]$, are described by the following input LLRs

$$\Pr\left[\{x_u[n]\}^{q=1} = +1\right] = \frac{1}{2}\left(1 + \tanh\frac{L^{\mathrm{a}}_{\mathrm{ESE}}\left(\{x_u[n]\}^{q=1}\right)}{2}\right) \tag{17}$$

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and

$$\Pr\left[\{x_u[n]\}^{q=2} = -1\right] = \frac{1}{2}\left(1 - \tanh\frac{L_{\mathrm{ESE}}^{\mathrm{a}}\left(\{x_u[n]\}^{q=2}\right)}{2}\right) \tag{18}$$

whereas $\{.\}^q$ indicates the position of the code bit.

By substituting (17), (18) and the values in (16) in (15), then we have the mean value of $\bar{x}_u[n, k]$ given as

$$\bar{x}_u[n, k] = \left(\frac{1}{\sqrt{2}}\right) \times \left(\tanh\frac{L_{\mathrm{ESE}}^{\mathrm{a}}\left(\{x_u[n]\}^{q=1}\right)}{2} + \tanh\frac{L_{\mathrm{ESE}}^{\mathrm{a}}\left(\{x_u[n]\}^{q=2}\right)}{2} i\right) \tag{19}$$

Extension of this to other signal constellations is straightforward.

The mean value calculated by (19) is then used by the ISLMMSE-FD CTF estimator in the first stage of the proposed soft input iterative channel estimator after the first iteration instead of the received symbol.

Following the Bayesian linear model theory of [13], the FD-CTF coefficients $\boldsymbol{H}[n, k]$ of the linear model portrayed by (4) can be sequentially estimated. If an MMSE estimator of $z[n, k]$ can be established based on $z[n-1, k]$, described as $\hat{z}[n|n-1, k]$the estimation error $\Im[n, k]$ will be defined as

$$\Im[n, k] = z[n, k] - \hat{z}[n|n-1, k] \tag{20}$$

Since $z[n, k] = \boldsymbol{H}[n, k]\boldsymbol{x}[n, k] + w[n, k]$, then by using the property of linear MMSE estimator that states that linear MMSE estimator commutes over linear transformations [13], consequent upon which

$$\hat{z}[n|n-1, k] = \hat{\boldsymbol{H}}[n-1, k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n, k] + \hat{w}[n|n-1, k] \tag{21}$$

Equation (21) indicates that the estimate of $z[n, k]$ based on $z[n-1, k]$ is equivalent to the product of past estimates of $\boldsymbol{H}[n, k]$ and the known vector $\boldsymbol{x}[n, k]$, the output of the detector, plus the estimate of $w[n, k]$ based on $w[n-1, k]$. At this instance, only immediate past estimate of $\boldsymbol{H}[n, k]$ and current estimate of the transmitted symbol are available.

Based on the Bayesian linear model assumed, $\boldsymbol{H}[n, k]$ and $w[n, k]$ are uncorrelated, and $\hat{\boldsymbol{H}}[n-1, k]$ and $w[n, k]$ are also uncorrelated such that $E\left\{\left(\boldsymbol{H}[n, k] - \hat{\boldsymbol{H}}[n-1, k]\right)w[n, k]\right\} = 0$. Furthermore, it is assumed that $w[n, k]$ is a sequence of uncorrelated random variables, hence, $w[n, k]$ is uncorrelated with the past noise samples [13]. Consequently, $\hat{w}[n|n-1, k]$, the LMMSE estimator of $w[n, k]$ based on the past samples $\{w[0, k], w[1, k], \ldots, w[n-1, k]\}$, according to the property of LMMSE [13], is equal to zero: $\hat{w}[n|n-1, k] = 0$, since $w[n, k]$ that is a sequence of uncorrelated random variables will be uncorrelated with the past data samples [13].

Hence,

$$\hat{z}[n|n-1, k] = \hat{\boldsymbol{H}}[n-1, k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n, k] \tag{22}$$

It is important to, however, note that the focus here is to estimate $\boldsymbol{H}[n, k]$ rather than $w[n, k]$.

The sequential estimator update for the entire $\hat{H}[n, k]$ using vector space approach [13] is then given as

$$\begin{aligned}\hat{\boldsymbol{H}}[n,k] &= \hat{\boldsymbol{H}}[n-1,k] + \mathrm{B}[n,k]\Im[n,k] \\ &= \hat{\boldsymbol{H}}[n-1,k] + \mathrm{B}[n,k]\left(z[n,k] - \hat{z}[n|n-1,k]\right) \\ &= \hat{\boldsymbol{H}}[n-1,k] + \mathrm{B}[n,k]\left(z[n,k] - \hat{\boldsymbol{H}}[n-1,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\right)\end{aligned} \tag{23}$$

Following [13], the gain $\boldsymbol{B}[n, k]$ can be obtained as

$$\mathrm{B}[n, k] = \frac{E\left\{\boldsymbol{H}[n, k]\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right\}}{E\left\{\left|\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right|^2\right\}} \tag{24}$$

where $E\{.\}$ is the expectation function.

The denominator of (24) can be evaluated, upon substitution of (4) and (22), as

$$\begin{aligned}&E\left\{\left|\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right|^2\right\} \\ &= E\left\{\left|\left(\boldsymbol{H}[n, k]\boldsymbol{x}^{\mathrm{T}}[n, k] + w[n, k] - \hat{\boldsymbol{H}}[n-1, k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n, k]\right)\right|^2\right\}\end{aligned} \tag{25}$$

In the case of error free signal detection, the symbol corresponding to the mean will be approximately equal to the transmitted symbol $\boldsymbol{x}[n, k]$. However, this is only possible at the instance where known symbol or pilot symbol is transmitted. Hence, its mean can be equated to the symbol $\boldsymbol{x}[n, k]$ as $\bar{\boldsymbol{x}}[n, k] = \boldsymbol{x}[n, k]$. Consequently, (25) becomes

$$\begin{aligned}&E\left\{\left|\left(z[n,k] - \hat{z}[n|n-1,k]\right)\right|^2\right\} \\ &= E\left\{\left|\left(\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\left(\boldsymbol{H}[n,k] - \hat{\boldsymbol{H}}[n-1,k]\right) + w[n,k]\right)\right|^2\right\} \\ &= \bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]E\left\{\left(\boldsymbol{H}[n,k] - \hat{\boldsymbol{H}}[n-1,k]\right)\left(\boldsymbol{H}[n,k] - \hat{\boldsymbol{H}}[n-1,k]\right)^{\mathrm{H}}\right\} \\ &\quad \times \bar{\boldsymbol{x}}[n,k] + E\left\{|w[n,k]|^2\right\} \\ &= \bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k] + \sigma_w^2\end{aligned} \tag{26}$$

where superscript H denotes the conjugate transpose. $E\left\{\bar{\boldsymbol{x}}[n, k]\bar{\boldsymbol{x}}^{\mathrm{H}}[n, k]\right\} = \|\bar{\boldsymbol{x}}[n, k]\|^2$, σ_w^2 is the variance of the AWGN, and parameter $\boldsymbol{D}[n]$ is the minimum MSE matrix.

By employing the properties of Bayesian linear model assumed in this derivation [13] where $\hat{z}[n|n-1, k]$ is the LMMSE estimator of $z[n, k]$ based on $\{z[0, k], z[1, k], \ldots, z[n-1, k]\}$, as such it could be assumed from (23) that $\hat{\boldsymbol{H}}[n-1, k]$ is a linear combination $\{z[0, k], z[1, k], \ldots, z[n-1, k]\}$, and $z[n, k] - \hat{z}[n|n-1, k]$ is uncorrelated with the past data samples. Hence it follows that $E\left\{\hat{\boldsymbol{H}}[n-1, k]\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right\} = 0$.

Based on these assumptions, the numerator of (24) can be expanded and evaluated as

$$\begin{aligned}&E\left\{\boldsymbol{H}[n, k]\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right\} \\ &= E\left\{\boldsymbol{H}[n, k]\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right. \\ &\quad \left. - \hat{\boldsymbol{H}}[n-1, k]\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right\} \\ &= E\left\{\left(\boldsymbol{H}[n, k] - \hat{\boldsymbol{H}}[n-1, k]\right)\left(z[n, k] - \hat{z}[n|n-1, k]\right)\right\}\end{aligned} \tag{27}$$

Upon substitution of (4) and (22), (27) becomes

$$\begin{aligned} &E\{\boldsymbol{H}[n,k](z[n,k]-\hat{z}[n|n-1,k])\} \\ &= E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k]) \\ &\quad \times(\boldsymbol{H}[n,k]\boldsymbol{x}^{\mathrm{T}}[n,k]+w[n,k]-\hat{\boldsymbol{H}}[n-1,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k])\} \end{aligned} \tag{28}$$

Since, $\boldsymbol{H}[n, k]$ and $w[n, k]$ are uncorrelated, $\hat{\boldsymbol{H}}[n-1, k]$ and $w[n, k]$ will also be uncorrelated. As a result

$$E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])w[n,k]\}=0$$

Therefore (28) becomes (see (29))

Substituting (26) and (29) in (24) gives

$$\mathrm{B}[n,k]=\frac{\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]}{\bar{\boldsymbol{x}}^{\mathrm{H}}[n,k]\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]+\sigma_w^2} \tag{30}$$

From (26) and (29), it is obvious that $\boldsymbol{D}[n, k]$ can be expressed as

$$\boldsymbol{D}[n,k]=E\left\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n,k])(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n,k])^{\mathrm{H}}\right\} \tag{31}$$

If (23) is substituted into (31), then $\boldsymbol{D}[n, k]$ becomes (see (32))

Upon further expansion of (32), we have (see (33))

From (24) and (30) we have

$$\begin{aligned} &\boldsymbol{B}[n,k]E\{|(z[n,k]-\hat{z}[n|n-1,k])|^2\} \\ &\quad =\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k] \end{aligned} \tag{34}$$

Also, knowing that $E\{\hat{\boldsymbol{H}}[n-1,k](z[n,k]-\hat{z}[n|n-1,k])\}=0$ then from (33)

$$\begin{aligned} &E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(z[n,k]-\hat{z}[n|n-1,k])\} \\ &= E\{(\boldsymbol{H}[n,k])(z[n,k]-\hat{z}[n|n-1,k])\} \\ &\quad - E\{(\hat{\boldsymbol{H}}[n-1,k])(z[n,k]-\hat{z}[n|n-1,k])\} \\ &= E\{(\boldsymbol{H}[n,k])(z[n,k]-\hat{z}[n|n-1,k])\}-0 \end{aligned} \tag{35}$$

From (29), (35) could be written as

$$\begin{aligned} &E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(z[n,k]-\hat{z}[n|n-1,k])\} \\ &= \boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k] \end{aligned} \tag{36}$$

Substituting (34) and (36) into (33), we have

$$\begin{aligned} \boldsymbol{D}[n,k] &= \boldsymbol{D}[n-1,k]-\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]\boldsymbol{B}^{\mathrm{H}}[n,k] \\ &\quad -\boldsymbol{B}[n,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\boldsymbol{D}[n-1,k]+\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]\boldsymbol{B}^{\mathrm{H}}[n,k] \\ &= \boldsymbol{D}[n-1,k]-\boldsymbol{B}[n,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\boldsymbol{D}[n-1,k] \\ &= (\mathrm{I}-\boldsymbol{B}[n,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k])\boldsymbol{D}[n-1,k] \end{aligned} \tag{37}$$

The iterative sequential linear MMSE estimator for OFDM-IDMA system is summarised in Table 1. The description of the algorithm in Table 1 is as follows. At the initial stage of the estimation, a known OFDM symbols which serves as the training symbol is employed to initialise the channel estimation process. Subsequently, the means of the detected signal obtained from the soft information feedback from the decoder are used by the ISLMMSE-FD CTF estimation. Based on the known OFDM symbol (at initial stage of the estimation) $\tilde{\boldsymbol{x}}[n, k]$, or the means of the detected symbol from the soft information feedback from decoder, $\bar{\boldsymbol{x}}[n, k]$ as shown in Fig. 2*a*, the ISLMME-FD CTF estimator

$$\begin{aligned} E\{\boldsymbol{H}[n,k](z[n,k]-\hat{z}[n|n-1,k])\} &= E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k](\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k]))\} \\ &= E\left\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])^{\mathrm{H}}\right\}\bar{\boldsymbol{x}}[n,k] \\ &= \boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k] \end{aligned} \tag{29}$$

$$\begin{aligned} \boldsymbol{D}[n,k]=E\{&(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k]-\boldsymbol{B}[n,k](z[n,k]-\hat{z}[n|n-1,k])) \\ &\cdot(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k]-\boldsymbol{B}[n,k](z[n,k]-\hat{z}[n|n-1,k]))^{\mathrm{H}}\} \end{aligned} \tag{32}$$

$$\begin{aligned} \boldsymbol{D}[n,k] &= E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])\} \\ &\quad - E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(z[n,k]-\hat{z}[n|n-1,k])\boldsymbol{B}^{\mathrm{H}}[n,k]\} \\ &\quad - \boldsymbol{B}[n,k]E\left\{(z[n,k]-\hat{z}[n|n-1,k])(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])^{\mathrm{H}}\right\} \\ &\quad + \boldsymbol{B}[n,k]E\{|(z[n,k]-\hat{z}[n|n-1,k])|^2\}\boldsymbol{B}^{\mathrm{H}}[n,k] \\ &= \boldsymbol{D}[n-1,k]-E\{(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])(z[n,k]-\hat{z}[n|n-1,k])\boldsymbol{B}^{\mathrm{H}}[n,k]\} \\ &\quad - \boldsymbol{B}[n,k]E\left\{(z[n,k]-\hat{z}[n|n-1,k])(\boldsymbol{H}[n,k]-\hat{\boldsymbol{H}}[n-1,k])^{\mathrm{H}}\right\} \\ &\quad + \boldsymbol{B}[n,k]E\{|(z[n,k]-\hat{z}[n|n-1,k])|^2\}\boldsymbol{B}^{\mathrm{H}}[n,k] \end{aligned} \tag{33}$$

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Table 1 Iterative-based sequential linear MMSE-based FD-CTF estimator for OFDM-IDMA system

$\hat{\boldsymbol{H}}[n-1,k]$ and $\boldsymbol{D}[n-1,k]$ are initialised to the mean and variance of $H[n,k]$, which are zero and σ_{H}^2 respectively.
$\mathrm{B}[n,k] = \dfrac{\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]}{\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]+\sigma_w^2}$
$\hat{\boldsymbol{H}}[n,k] = \hat{\boldsymbol{H}}[n-1,k] + \mathrm{B}[n,k]\left(z[n,k] - \hat{\boldsymbol{H}}[n-1,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\right)$
$\boldsymbol{D}[n,k] = \left(\mathrm{I} - \mathrm{B}[n,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\right)\boldsymbol{D}[n-1,k]$

makes the temporary CTF estimation of $\hat{\boldsymbol{H}}[n,k]$ based on Table 1 as follows. In Table 1, with $\hat{\boldsymbol{H}}[n-1,k]$ and $\boldsymbol{D}[n-1,k]$ initialised to zero and σ_{H}^2, the gain $\boldsymbol{B}[n,k]$ is obtained using (30) as $\boldsymbol{B}[n,k] = \boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k]/\left(\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\boldsymbol{D}[n-1,k]\bar{\boldsymbol{x}}[n,k] + \sigma_w^2\right)$. Thereafter, $\hat{\boldsymbol{H}}[n,k]$ is obtained using (23) as $\hat{\boldsymbol{H}}[n,k] = \hat{\boldsymbol{H}}[n-1,k] + \boldsymbol{B}[n,k]\left(z[n,k] - \hat{\boldsymbol{H}}[n-1,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\right)$, and finally at this stage, the parameter $\boldsymbol{D}[n,k]$ is computed using (37) as $\boldsymbol{D}[n,k] = \left(\boldsymbol{I} - \boldsymbol{B}[n,k]\bar{\boldsymbol{x}}^{\mathrm{T}}[n,k]\right)\boldsymbol{D}[n-1,k]$ for subsequent channel estimation at the next $(n+1)$th OFDM time index. This process is repeated until the whole channel has been estimated.

In order to obtain the CTF estimate to detect the information data subsequent to the pilot OFDM symbol, at symbol time $n+1$ and beyond, IFFT is employed to convert the FD-CTF coefficients at time n to time-domain CIR coefficients. The nth time domain CIR coefficients obtained from IFFT transformation can be expressed as

$$\hat{h}[n,m] = \begin{cases} \mathrm{IFFT}_K\{\hat{H}[n,k]\} & 0 \le m \le M-1 \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

where M is the number of sparse CIR coefficients.

Subsequently, regularised adaptive predictor is employed to make prediction for the next CIR coefficient at time $(n+1)$, as shown in Fig. 2*a*. In this paper, the regularised-VSSNLMS (ℓ_1-VSSNLMS) predictor of [15] is employed. The predicted CIR converted to frequency domain as $\hat{\boldsymbol{H}}[n,k]$ is used by ESE for signal detection as depicted in Fig. 2*a*.

5 Regularised adaptive algorithm based-time domain iterative channel estimator

In a wideband OFDM communication system, the time domain (discrete) channel is approximately sparse, in that there are many near-zero tap coefficients, with only few large ones [15, 16]. In such a system, the number of sparse CIR coefficients (D) are much less than the total wireless channel length (M), that is ($0 < D << M$). The estimator presented in this section exploits this sparsity in OFDM channel for channel estimation process. The whole estimator comprises of initialisation estimator and the regularised recursive least square (RLS) algorithm-based CIR estimator detailed in the following.

5.1 Least square(LS)-based initial channel estimator

The LS estimator is used for initial channel estimate at the stage where the preambles are pilot symbols. Using the time domain received signal $\boldsymbol{z}[n]$ of (2), the initial channel estimates employing LS estimator are obtained as follows.

By following similar approach in [17], the linear LS CIR estimator could be expressed as:

$$\hat{\boldsymbol{h}}_u[n] = \frac{z[n]}{\tilde{\boldsymbol{x}}_u[n]} = \boldsymbol{h}_u[n] \times \left(\frac{\boldsymbol{x}_u[n]}{\tilde{\boldsymbol{x}}_u[n]}\right) + \frac{w[n]}{\tilde{\boldsymbol{x}}_u[n]} \tag{39}$$

where $\hat{\boldsymbol{h}}_u[n] = \left(\hat{h}_u[n,1], \hat{h}_u[n,2], \ldots, \hat{h}_u[n,M]\right)^{\mathrm{T}}$ and $\tilde{\boldsymbol{x}}_u[n] = \left(\tilde{x}_u[n], \tilde{x}_u[n-1], \ldots, \tilde{x}_u[n-M]\right)^{\mathrm{T}}$ for all $u = 1, 2, \ldots. U$. At the instance of error-free estimation, $\tilde{\boldsymbol{x}}_u[n] = \bar{\boldsymbol{x}}_u[n]$, hence (39) becomes

$$\hat{\boldsymbol{h}}_u[n] = \boldsymbol{h}_u[n] + \frac{w[n]}{\tilde{\boldsymbol{x}}_u[n]} \tag{40}$$

Here $\bar{\boldsymbol{x}}_u[n]$ is the known preamble used as training signals for initialisation of the channel estimation process. The initial estimate of $H[n,k]$ is obtained from $\hat{h}_u[n]$ with the aid of FFT conversion as

$$\tilde{H}[n,k] = \sum_{m=0}^{M} \hat{h}_u[n,m]\mathrm{e}^{-\mathrm{j}2\pi km/K}$$

5.2 ℓ_1-NPEVFF-RLS-based CIR estimator

By employing the conventional RLS algorithm, the next CIR estimate, is obtained by minimising the cost function

$$\Omega_{\mathrm{RLS}}[n] = \sum_{j=1}^{n} \lambda^{n-j}\left|e[n]\right|^2 \tag{41}$$

The subsequent CIR estimates is expressed as

$$\hat{\boldsymbol{h}}_u[n] = \hat{\boldsymbol{h}}_u[n-1] + \boldsymbol{k}_u[n-1]\boldsymbol{e}_u^*[n] \tag{42}$$

where the estimation error $\boldsymbol{e}_u[n]$ is given as

$$\boldsymbol{e}_u[n] = z[n] - \hat{\boldsymbol{h}}_u^{\mathrm{H}}[n-1]\bar{\boldsymbol{x}}_u[n] \tag{43}$$

The parameter $\boldsymbol{k}_u[n]$, the gain vector, in (42) is expressed as

$$\boldsymbol{k}_u[n] = \frac{\boldsymbol{G}_u[n-1]\bar{\boldsymbol{x}}_u[n]}{\lambda + \hat{\boldsymbol{x}}_{\boldsymbol{u}}^{\mathrm{H}}[n]\boldsymbol{G}_u[n-1]\bar{\boldsymbol{x}}_u[n]} \tag{44}$$

where $\boldsymbol{G}_u[n]$ in (11) is given as

$$\boldsymbol{G}_u[n] = \frac{1}{\lambda}\left(I - \boldsymbol{k}[n]\bar{\boldsymbol{x}}_{\boldsymbol{u}}^{\mathrm{H}}[n]\right)\boldsymbol{G}_u[n-1] \tag{45}$$

and λ is a fixed forgetting factor for the conventional RLS algorithm which can assume values between $0 < \lambda < 1$, the symbol $\bar{x}_u[n]$ is the mean of the transmitted user symbol obtained from the feedback from decoder. The forgetting factor with smaller value will result in faster convergence and tracking speed; however; with worse MSE at steady state. On the other hand, forgetting factor of larger value will result in relatively small MSE at steady state but with very slow convergence and tracking speed. Hence for fixed forgetting fact RLS algorithm, it is very difficult to achieve high convergence and tracking speed and low MSE at the same time. Because of the fact that the forgetting factor in the RLS algorithm has great influence on the system performance of a time-varying wireless communication system such as OFDM-IDMA system, it is essential to make this parameter variable.

In order to enhance the performance of the conventional RLS algorithm, the forgetting factor is to be made variable as $\lambda[n]$. The adaptation of the forgetting factor is based on the noise power estimate derived in the following.

The noise power estimate-based VFF-RLS (NPEVFF-RLS) algorithm-based CIR estimator is obtained by ensuring that error signal does not go to zero, otherwise noise will be introduced into the estimation of $\hat{\boldsymbol{h}}_u[n]$.

The error signal in (42) is the a priori error, since it is computed from previous CIR estimate, $\hat{\boldsymbol{h}}_u[n-1]$. Consequently, the a posteriori error signal could be defined as

$$\boldsymbol{\varepsilon}_u[n] = z[n] - \hat{\boldsymbol{h}}_u^{\mathrm{H}}[n-1]\bar{\boldsymbol{x}}_u[n] \tag{46}$$

By substituting (42) and (43) (46) can be written as

$$\boldsymbol{\varepsilon}_u[n] = \boldsymbol{e}_u[n]\left(1 - \boldsymbol{k}[n]\bar{\boldsymbol{x}}_u^{\mathrm{H}}[n]\right) \tag{47}$$

In order to remove the noise from the error signal, the following condition can be imposed [18]

$$E\left\{\boldsymbol{\varepsilon}_u^2[n]\right\} = \sigma_w^2 \tag{48}$$

By making substitutions for $\varepsilon_u[n]$ using (44) and (47), (48) becomes

$$E\left\{\left(1 - \frac{\chi[n]}{\lambda[n] + \chi[n]}\right)\right\} = \frac{\sigma_w^2}{\sigma_e^2[n]} \tag{49}$$

where $\chi[n] = \bar{\boldsymbol{x}}_{\boldsymbol{u}}^{\mathrm{H}}[n]\boldsymbol{G}_u[n-1]\bar{\boldsymbol{x}}_u[n]$, and $E\left\{e_u^2[n]\right\} = \sigma_e^2[n]$ is the power of the a priori error signal.

By solving (49), we have

$$\lambda[n] = \frac{\sigma_\chi[n]\sigma_w}{\sigma_e[n] - \sigma_w} \tag{50}$$

where $E\left\{\chi^2[n]\right\} = \sigma_\chi^2[n]$.

The parameters $\sigma_\chi[n]$ and $\sigma_e[n]$ in (50) can be obtained from the power estimates given as follows.

$$\hat{\sigma}_e^2[n] = \vartheta\hat{\sigma}_e^2[n-1] + (1-\vartheta)e^2[n] \tag{51}$$

$$\hat{\sigma}_\chi^2[n] = \vartheta\hat{\sigma}_\chi^2[n-1] + (1-\vartheta)\chi^2[n] \tag{52}$$

where $\vartheta a = 1-(1/(K_\vartheta D))$ is a weighting factor, with $K_\vartheta \geq 2$, and D is the number of sparse CIR coefficients. The third parameter, the power of the noise can be estimated from the error signal as

$$\hat{\sigma}_w^2[n] = \alpha\hat{\sigma}_w^2[n-1] + (1-\alpha)e^2[n] \tag{53}$$

where $\alpha = 1-(1/(K_\alpha D))$ and $K_\alpha > K_\vartheta$.

Consequently, the variable forgetting factor (VFF) could be expressed as

$$\lambda[n] = \frac{\hat{\sigma}_\chi[n]\hat{\sigma}_w}{\varsigma + \left|\hat{\sigma}_e[n] - \hat{\sigma}_w\right|} \tag{54}$$

where ς is small positive constant that prevent division by zero.

In order to ensure steady state of the excess MSE and stability of the algorithm, the value of $\lambda[n]$ as obtained in (54) is forced within allowable range of $0 < \lambda \leq 1$ as

$$\lambda[n] = \begin{cases} \lambda_{\max} & \text{if } \lambda[n] > \lambda_{\max} \\ \lambda_{\min} & \text{if } \lambda[n] < \lambda_{\min} \\ \lambda[n] & \text{otherwise} \end{cases} \tag{55}$$

where $0 < \lambda_{\min} < \lambda_{\max} < 1$.

In [17, 19–21] the cost function of conventional RLS algorithm is regularised by addition of the weighted convex ℓ_1 norm. Employing this approach and by making the forgetting factor time varying as $\lambda[n]$,

In order to exploit the underlying sparseness in OFDM channels the ℓ_1 norm penalty term on the filter coefficients is incorporated to the cost function of the NPEVFF-RLS algorithms-based CIR estimator following similar approach in [15, 22–24] in order to obtain the ℓ_1 (regularised)-NPEVFF-RLS algorithms-based CIR estimator.

After the initial channel estimation by the LS estimator, in the iterative conditions by employing the mean of $\boldsymbol{x}_u[n]$ $\left(\bar{\boldsymbol{x}}_u[n]\right)$ obtained from the a priori information $L_{\mathrm{ESE}}^a\left(\boldsymbol{x}_u[n]\right)$ coming from the APP decoders, the ℓ_1-NPEVFF-RLS-based channel estimator computes the estimate of the CIR by minimising the cost function

$$\begin{aligned} \Omega_{\mathrm{RLS}_{\ell 1},u}[n] &= \frac{1}{2}\sum_{j=1}^{n}\lambda^{n-j}[n]\left|e_u[n]\right|^2 + \beta\left\|\hat{\boldsymbol{h}}_u[n]\right\|_1 \\ &= \frac{1}{2}\sum_{j=1}^{n}\lambda^{n-j}[n]\left|e_u[n]\right|^2 + \beta\sum_{m=0}^{M-1}\left|\hat{\boldsymbol{h}}_u[n,m]\right| \end{aligned} \tag{56}$$

where β is the parameter that denotes the tradeoff between sparsity of the estimator filter coefficients and the estimation error and $\lambda[n]$ is a variable forgetting factor (VFF). The CIR estimate is then obtained as

$$\begin{aligned} \hat{\boldsymbol{h}}_u[n] = {} & \hat{\boldsymbol{h}}_u[n-1] + \boldsymbol{k}_u[n-1]\boldsymbol{e}_u^*[n] + \beta\left(\frac{\lambda[n]-1}{\lambda[n]}\right) \\ & \times \left\{\boldsymbol{I}_M - \boldsymbol{k}_u[n-1]\hat{\boldsymbol{x}}_{\boldsymbol{u}}^H[n-1]\right\} \\ & \times \boldsymbol{G}_u[n-1]\mathrm{sgn}\left(\hat{\boldsymbol{h}}_u[n-1]\right) \end{aligned} \tag{57}$$

where sgn(.) denotes a component-wise sign function given as

$$\mathrm{sgn}(h) = \left\{\begin{array}{cc} \dfrac{h}{|h|} & h \neq 0 \\ 0 & \text{elsewhere} \end{array}\right\}$$

$\hat{\boldsymbol{h}}_u[n-1]$ is equivalent to the previously estimated CIR of (40).

6 Comparative computational complexity analysis of the channel estimator

In terms of complexity, the ISLMMSE-based CTF estimator's module of the proposed combined CTF estimator and CIR predictor will require $12K^2 + 10M$ multiplication/division operations and $4K^2 + 5M$ addition/subtraction operations. Its ℓ_1-VSSNLMS-based CIR predictor's module will require $M(9L_{\mathrm{prd}}+2)$ multiplication/division operations and $M(7L_{\mathrm{prd}}+2)$ addition/subtraction operations. However, each of the adaptive algorithms-based time domain CIR estimators will require M^2 operations for the LS-based

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Table 2 Computational complexity per iteration

Estimator	×/÷	+/-
combined CTF-estimator and CIR predictor	$12K^2+10M+M(9F+2)$	$4K^2+5M+M(7F+2)$
LS-based initialisation estimator	M^2	
LMS-based CIR estimator	$M(3F+1)$	$M(F+2)$
NLMS-based CIR estimator	$M(4F+2)$	$M(3F+2)$
VSSNLMS-based CIR estimator	$M(6F+2)$	$M(5F+2)$
ℓ_1-NPEVFF-RLS-based CIR estimator	$D(6F^2+7F+19)$	$D(6F^2+9F+14)$

initialisation estimator. After the initialisation by the LS algorithm, the second proposed estimator, the ℓ_1-NPEVFF-RLS algorithms-based CIR estimator requires $D(6F^2+7F+19)$ multiplication/division operations and $D(6F^2+9F+14)$ addition/subtraction operations.

The LMS-based iterative estimator will require $M(3F+2)$ multiplication/division operations and $M(F+2)$ addition/subtraction operations. For the NLMS-based iterative estimator, it will require $M(4F+2)$ multiplication/division operations and $M(3F+2)$ addition/subtraction operations. The VSSNLMS-based iterative estimator will require $M(6F+2)$ multiplication/division operations and $M(5F+2)$ addition/subtraction operations. These are all tabulated in Table 2. The symbol F is the length of the adaptive filter.

For $D=4$, $F=10$, $K=64$ and $M=16$, the overall computational complexity of the combined CTF-estimator and CIR predictor is 68 400. With the same values for the parameters, the overall computational complexity of the ℓ_1-NPEVFF-RLS algorithms-based CIR estimator is 5828. The overall computational complexities of the LMS-based CIR estimator, NLMS-based CIR estimator and VSSNLMS-based CIR estimator for OFDM-IDMA system are 960, 1440 and 2080, respectively. From these analyses the LMS-based CIR estimator exhibits the lowest computational complexity whereas combined CTF-estimator and CIR predictor exhibit the highest computational complexity.

7 Simulation results

This section presents results obtained from the computer simulation of the proposed channel estimation for OFDM-IDMA systems. In the simulation conducted, a QPSK modulated OFDM-IDMA system with $K=64$ subcarriers and operating carrier frequency of 2 GHz is assumed. The system bandwidth is also assumed to be 800 kHz with symbol duration of 80 µs. A six-path time-varying Rayleigh fading COST 207 Typical Urban (TU) channel model presented in [13] is assumed. In the simulation of the decision directed-based iterative channel estimation scheme employed at the OFDM system's receiver, sample spaced (SS)-CIR channel model with total length $M=16$ and the number of sparse CIR coefficients $D=4$ is assumed. Hence, there is introduction of a sparsity of 1/4 into the system.

In the first proposed estimator, the soft-input-based iterative channel estimator, the first component as shown in Fig. 4 is the sequential linear MMSE estimator of Section 4. This is employed to obtain the frequency domain CTF estimate. After the IFFT has been applied, the regularised adaptive predictor presented in [15] is employed to make prediction of the next $(n+1)$th CIR. This is used by the ESE of Section 3 for signal detection. The predicted time domain CIR is converted to the frequency domain CTF with the aid of the FFT module before being made available for the ESE. The ESE exchanges extrinsic information in an iterative mode with the a posteriori probability decoders (APP DECs), as shown in Fig. 1*b*. After the first iteration, perfect soft data are assumed. This is used by the proposed soft-input-based iterative channel estimator for subsequent channel estimation process instead of the received symbols. The second proposed iterative channel, the ℓ_1-NPEVFF-RLS algorithms-based CIR estimator and other time domain iterative CIR estimators: the NLMS-based iterative channel estimator, and the VSSNLMS-based iterative channel estimator, all used the receiver shown in Fig. 2*b*. The LS channel estimator [17] described in Section 5.1 is used to initialise all these time domain adaptive algorithms-based iterative channel

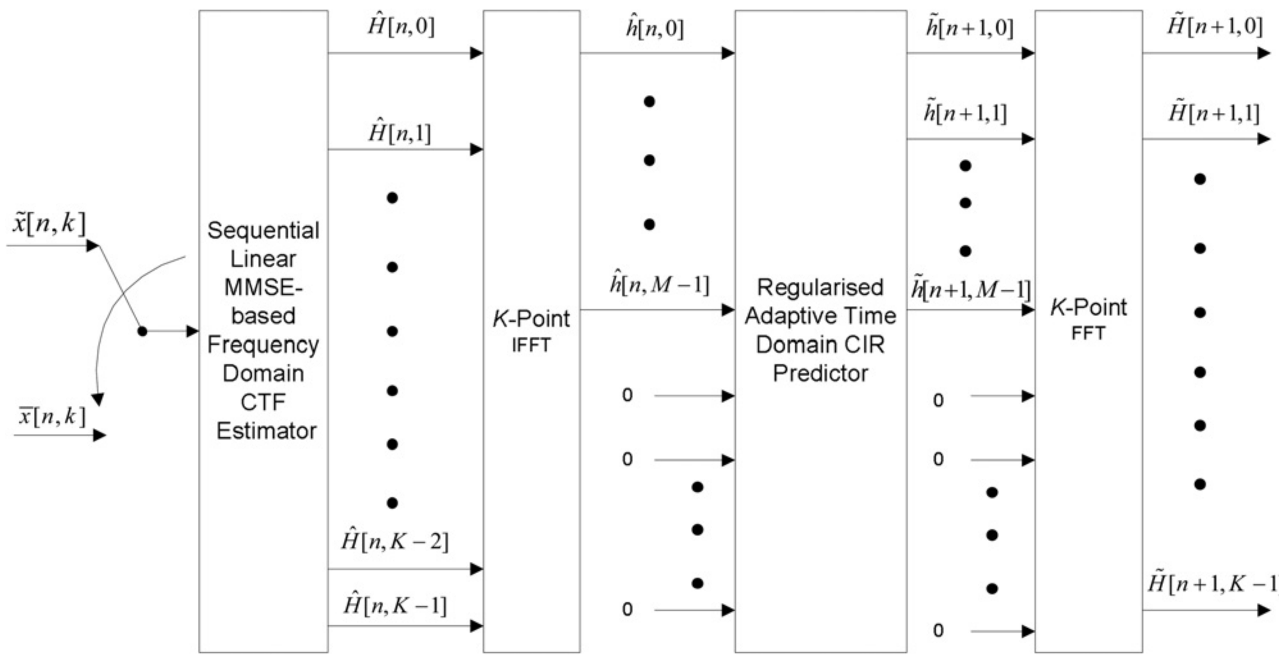

Fig. 3 *Soft-input-based iterative CTF estimator and CIR predictor for MC-IDMA system*

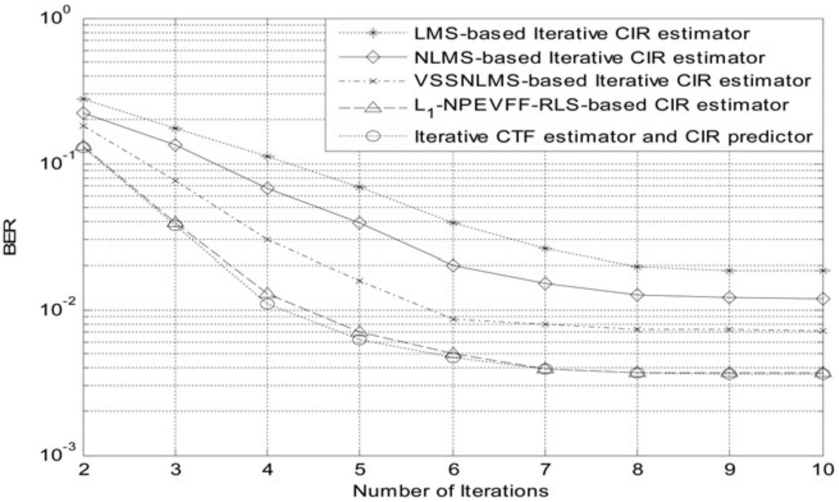

Fig. 4 *Average BER against number of iterations exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative CIR estimators, with SNR = 5 dB, number of user U = 2 and v = 5 Km/h*

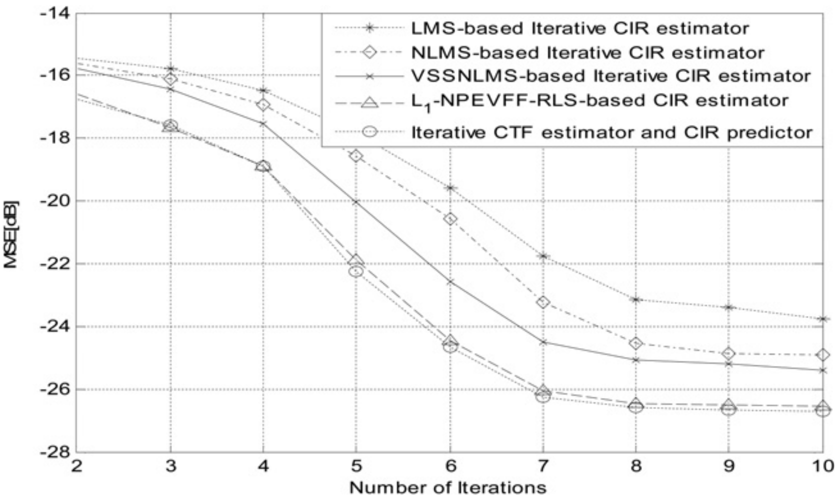

Fig. 5 *Average MSE against number of Iterations exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative CIR estimators, with SNR = 5 dB, number of user U = 4 and v = 5 Km/h*

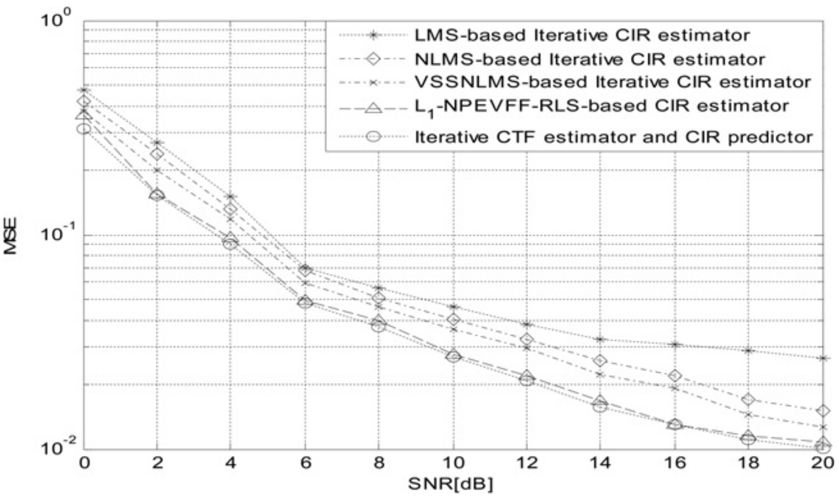

Fig. 6 *Average MSE against SNR exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative CIR estimators at seventh iteration, with fDn = 0.0045, v = 5 Km/h and number of user U = 4*

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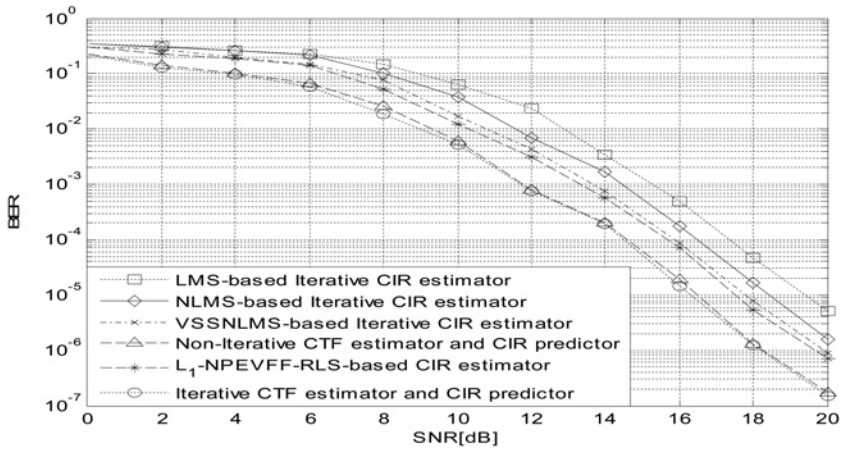

Fig. 7 *Average BER against SNR exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative CIR estimators at seventh iteration, and non-iterative CTF estimator and CIR prediction for OFDM-IDMA system, fDn = 0.0045, v = 5 Km/h, and number of user U = 4*

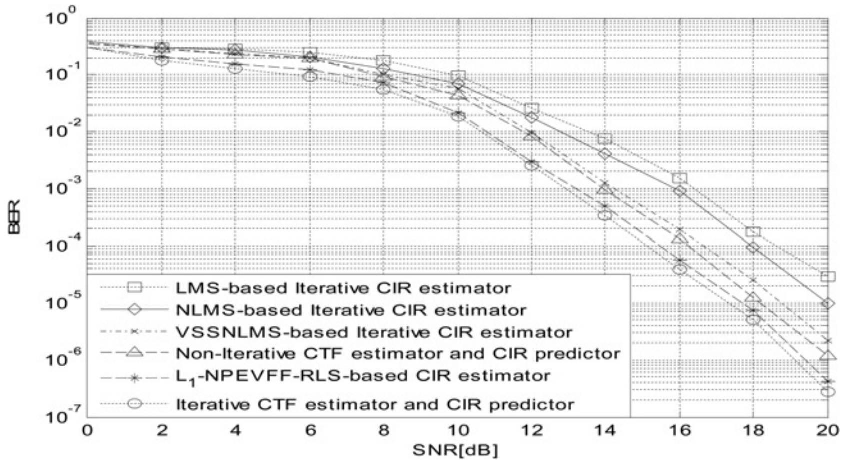

Fig. 8 *Average BER against SNR exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative CIR estimators at seventh iteration, and non-iterative CTF estimator and CIR prediction for OFDM-IDMA system, fDn = 0.108, v = 120 Km/h and number of user U = 4*

estimators. However, in [3] an un-named method was employed for channel estimate initialisation exploiting the autocorrelation of users message's preamble based on Zadoff–Chu sequence [25]. The estimated CIRs employing the adaptive algorithms-based iterative channel estimators are converted to CTF with the aid of FFT. The symbols K_ϑ, K_α, ς, β are set for the ℓ_1-NPEVFF-RLS algorithms-based CIR estimator as $K_\vartheta = 2$; $K_\alpha = 5K_\vartheta$ and $\varsigma = 10^{-8}$ as employed for white Gaussian noise input in [18]; and β is set to 3 according to [15]. The parameters of the ℓ_1-VSSNLMS predictor are set, following the optimum values indicated in [15], as $\mu[0] = 0.5$, $\rho = 0.002$[3] and $\kappa = 5 \times 10^{-4}$. The step size, $\mu = \mu[0]$ is also used for the LMS-based iterative channel estimator [3], the NLMS-based iterative channel estimator, and to initialise the VSSNLMS-based iterative channel estimator. For the VSSNLMS-based iterative channel estimator, ρ is set to 0.002.

The first sets of simulations are to determine the optimum number of iterations that give the best results in the scenario considered in this paper. Figs. 5 and 6 illustrate average bit error rate (BER) and MSE against number of iteration exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative channel estimators. Seven iterations seem to be the optimum number of iterations as shown in the figures. The results after the seventh iteration do not improve significantly. Fig. 7 demonstrates the achievable average MSE against signal-to-noise ratio (SNR) of the proposed estimators in comparison with other adaptive algorithms-based iterative channel estimators after seven iterations. The results indicate that the proposed iterative channel estimators outperform all the other adaptive algorithms-based iterative channel estimators with LMS-based iterative channel estimator exhibiting the poorest performance, whereas the ℓ_1-NPEVFF-RLS algorithms-based CIR estimator's performance is very close to that of the iterative channel estimator and predictor.

Furthermore, in Figs. 8 and 9 the proposed iterative channel estimators and the other adaptive algorithms-based iterative channel estimators are compared with non-iterative channel estimator and predictor presented in [6] in terms of BER against SNR at both slow and fast mobile speed scenarios. The effects of iteration on the proposed iterative schemes are quite significant. Fig. 10 shows the comparative computational complexity of the proposed

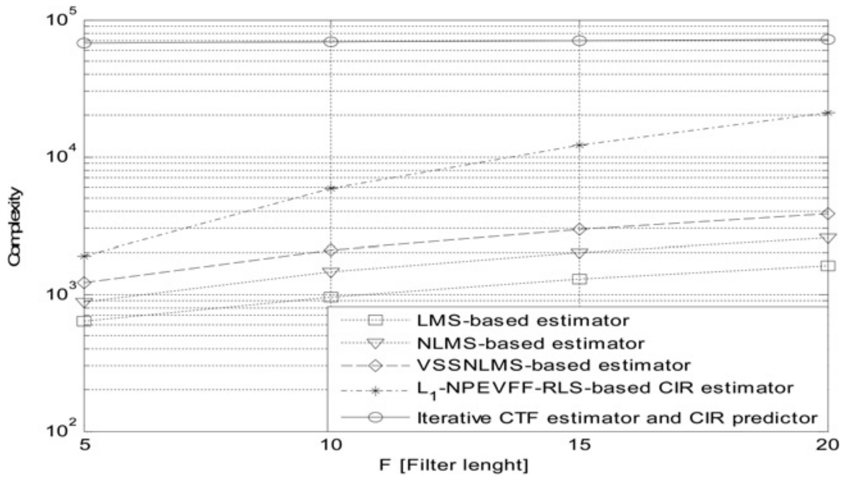

Fig. 9 *Computational complexity against F at fixed values of M = 16, D = 4 and K = 64 exhibited by the proposed estimators in comparison with other adaptive algorithms-based iterative CIR estimators*

estimators against adaptive filter length whereas other parameters are fixed as $D = 4$, $K = 64$ and $M = 16$. From the results, it could be seen that the LMS-based CIR estimator exhibits the lowest computational complexity, whereas the proposed CTF-estimator and CIR predictor for OFDM-IDMA system exhibits the largest computational complexity. The computational complexity of the ℓ_1-NPEVFF-RLS algorithms-based CIR estimator is close to the other adaptive algorithms-based iterative channel estimators at lower filter length. It could be seen that the computational complexity of the CTF-estimator and CIR predictor for OFDM-IDMA system seems to be linear as the filter length increase. The reason for this observation is that the computational complexity of CTF-estimator and CIR predictor for OFDM-IDMA system is a linear function of F unlike computational complexities of the ℓ_1-NPEVFF-RLS algorithms-based CIR and other adaptive algorithms-based iterative channel estimators that are of function of F^2. However, the computational complexity of CTF-estimator and CIR predictor for OFDM-IDMA system is a function of K^2. Hence, if the number of carrier is increased, its computational complexity will grow rapidly contrary to its linearity characteristic as displayed in Fig. 10.

8 Conclusion

In this paper, channel estimation for OFDM-IDMA communication systems is studied. Iterative channel estimation based on combination of FD-CFT estimator and time domain CIR predictor and another estimator based on regularised noise power estimate-based VFF RLS are presented. The ISLMMSE algorithm is derived for implementation of the CTF estimator module of the first channel estimator. Regularised adaptive VSSNLMS algorithm is employed for the CIR predictor module of the scheme. The iterative channel estimation scheme exchanges information with the MUD, and also employs soft information feedback from decoder for enhancement of channel estimation. Furthermore, regularised noise power estimate-based variable forgetting factor recursive least square (ℓ_1-NPEVFF-RLS) algorithm is developed to implement time domain CIR estimator for OFDM-IDMA systems. The achievable performances of the proposed iterative channel estimation schemes in comparison with other adaptive algorithms-based iterative channel estimators and another non-iterative channel estimator and predictor have been documented in the context of the multi-users-based OFDM-IDMA system operating under various conditions. The proposed schemes outperform all the other adaptive algorithms-based iterative channel estimators and the non-iterative channel estimator and predictor scheme, but with higher computational complexity in comparison with them. However, in terms of the computational complexity and performance of channel estimation scheme, the ℓ_1-NPEVFF-RLS)-based CIR estimator whose performance is close to that of the combined ISLMMSE-based CTF estimator and ℓ_1-VSSNLMS-based predictor and with lower computational complexity will be the best channel estimator for the multi-user-based OFDM-IDMA wireless communication systems.

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