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IMPLEMENTING THE ECONOMICAL $1 \to M$ PHASE-COVARIANT QUANTUM CLONING VIA THE DETECTION OF CAVITY DECAY

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We propose a scheme to realize the economical $1 \to M$ phase-covariant quantum cloning in 2-dimension with M Λ-type three-level atoms in an optical cavity. In our scheme, we do not require addressing and exactly manipulating on every atoms, never so much as controlling the time of interaction between atoms and photon. The success or failure of cloning can be determined simply by detecting the polarization of the photon leaking out of the cavity. With the use of an automatic feedback, the success probability of the scheme can be made to approach unity. And the desired output state is a superposition of different combination of two grounds states and thus is free from decoherence caused by spontaneous emission.

Keywords: Phase-covariant quantum cloning, optical cavity.

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1. Introduction

It is impossible to accurately copy an arbitrary quantum state because of the no-cloning theorem[1] and other limitations, which are the basic rules in quantum physics, specifically, the no-signaling condition[2] and the uncertainty relations. This feature is a consequence of the linearity of quantum mechanics. Since exact copy cannot be obtained, much more attention has been turned to approximate quantum cloning. In recent years, many works about quantum cloning transformations have been reported. For example, the optimal symmetric universal quantum cloning (UQC) to copy an input state with its phase factors and amplitudes completely unknown,[3] and the optimal symmetric phase-covariant quantum cloning (PCC)[4,5] to clone an input state with its phase factors unknown, have been studied theoretically. Asymmetric quantum cloning first proposed by N. J. Cerf in Ref. 6 locally produces two unnecessarily identical output qubits with the best distribution of the

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fidelity of clones. Recently, the explicit transformation of the optimal asymmetric $1 \to 1+1+1$ UQC[7] in d-dimension is presented. Very recently, the optimal asymmetric $1 \to 2$ PCC and the optimal real state cloning (RSC) to copy an input state with its amplitudes being real and unknown in d-dimension are presented.[8,9] These three types of cloning mentioned above, including UQC, PCC and RSC, can construct a generic cloning of discrete variables. However, it is a tremendous challenge to obtain the optimal copies during quantum cloning process based on practical physics systems. Very recently, many schemes of quantum cloning have been proposed in quantum optics[10] and cavity QED,[11–14] and experimental realizations have also been implemented in optical method,[15,16] NMR technique.[17–19]

In this letter, we propose a simple scheme to realize the economical $1 \to M$ phase-covariant quantum cloning via the detection of cavity decay. In our scheme, M Λ-type three-level atoms are trapped in an optical cavity. As we know, photons have long coherent time and are easy to implement single-qubit logical gate by using linear optics elements, but are inconvenient to be stored. Fortunately atoms trapped in the cavity are adopted as storing qubits because of their long-lived internal state. So in our scheme, a photonic polarization state with its phase unknown is cloned into the quantum state of M atoms in optical cavity. It is so important to copy and store unknown information for the quantum network of future quantum computer and quantum communication, whereas quantum copying is approximate and is also necessary. In the whole process of cloning, we do not require separate operations and localization on every atom. We only need detecting the polarization of the photon leaking out of the cavity to determine the success or failure of cloning. With the use of an automatic feedback, the success probability of the scheme can be made to approach unity.

This letter is organized as follows. In the next section, we briefly review the basic theory about the economical $1 \to M$ PCC in 2-dimension. In Sec. 3, we briefly introduce the model of n Λ-type three-level atoms coupled to a two-mode cavity field. Then, in Sec. 4, we present a scheme to realize the economical $1 \to M$ PCC with M Λ-type three-level atoms in an optical cavity via the detection of cavity decay. The letter ends with a brief discussion and a summary.

2. The Economical $1 \to M$ PCC in 2-Dimension

We first briefly review the economical $1 \to M$ PCC in 2-dimension.[4,5] In the standard parameterization by Euler angles θ and φ, in the case of $\theta = \pi/4$ (also called qubits on the x–y plane of the Bloch sphere), the input state is $|\psi\rangle^{(\mathrm{in})} = (|0\rangle + e^{i\varphi}|1\rangle)/\sqrt{2}$ with $\varphi \in [0, 2\pi)$ being unknown. This state has just one arbitrary phase parameter instead of two parameters for arbitrary qubit. So, we have known partial information of the input state. The explicit transformation of the economical $1 \to M$ PCC in 2-dimension can be defined as

$$|0\rangle_1 |00\cdots 0\rangle_{2,3,\ldots,M-1} \to |00\cdots 0\rangle_{1,2,\ldots,M} \qquad (1)$$

$$|1\rangle_1|00\cdots 0\rangle_{2,3,\ldots,M-1} \to \frac{1}{\sqrt{M}}(|100\cdots 0\rangle_{1,2,\ldots,M} + |010\cdots 0\rangle_{1,2,\ldots,M} + \cdots + |010\cdots 0\rangle_{1,2,\ldots,M}), \quad (2)$$

where qubit 1 of the left-hand side represents the input state and the others are the blank copies.

Furthermore, the fidelities of M copies do not depend on φ and are equal to average cloning fidelity

$$F = \langle\psi|\rho|\psi\rangle = \frac{1}{2} + \frac{1}{2\sqrt{M}} \quad (3)$$

where we note that the cloner is symmetric with respect to the M copies, whose reduced density matrices are identical as $\rho = \rho_1 = \rho_2 = \cdots = \rho_M$. It has been shown that the optimal fidelity for $1 \to M$ phase covariant of the qubits $F_{\text{even}} = \frac{1}{2} + \frac{\sqrt{M(M+2)}}{4M}$ is for even M, and $F_{\text{odd}} = \frac{1}{2} + \frac{M+1}{4M}$ for odd M. Therefore, for $M = 2$, Eq. (3) implements the optimal $1 \to 2$ phase-covariant cloning machine. For $M > 2$, Eq. (3) is always lower than the optimal fidelity.

3. The Interaction Between n Λ-type Three-Level Atoms and Two-Mode Cavity

The schematic set-up is shown in Fig. 1(a), with the relevant atomic levels depicted in Fig. 1(b). The states $|0\rangle$ and $|1\rangle$, which represent basis vectors of a qubit, correspond to two degenerate ground states, and $|e\rangle$ corresponds to an excited state. Without loss of generality, we assume that the transitions $|e\rangle \leftrightarrow |0\rangle$ and $|e\rangle \leftrightarrow |1\rangle$ are coupled to two degenerate cavity modes a_0 and a_1 with left- and right-circular polarizations, respectively. n identical Λ-type atoms are trapped in a resonant optical cavity, separated from each other by more than one optical wavelength of the $|e\rangle \leftrightarrow |0\rangle$ or $|e\rangle \leftrightarrow |1\rangle$ transition. So the dipole–dipole interaction can be neglected.

We first consider the ideal interaction of n Λ-type three-level atoms and the two-mode cavity without cavity decay and atomic spontaneous decay. The Hamiltonian

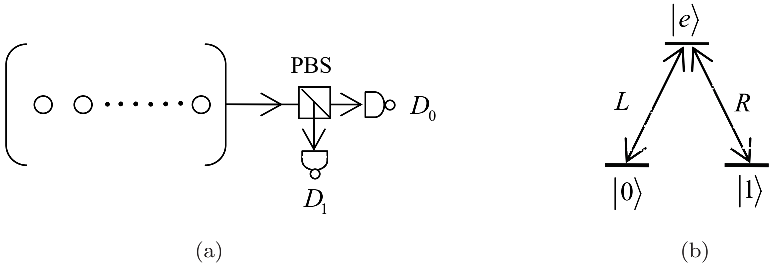

Fig. 1. (a) The schematic set-up to implement the optimal economical $1 \to M$ PCC. PBS denotes a polarization beam splitter. If a photon incident on PBS is left-(right-) circularly polarized, then it is detected by $D_0(D_1)$. (b) The relevant level structure of Λ-type three level atoms.

in the interaction picture is given by (assuming $\hbar = 1$)[20–22]

$$H_I = i\sum_{i=1}^{n}\sum_{\lambda=0,1}(g_\lambda a_\lambda|e\rangle_i\langle\lambda| - g_\lambda a_\lambda^+|\lambda\rangle_i\langle e|)\,, \tag{4}$$

where g_0 and g_1 are the corresponding real coupling strengths. a_λ and a_λ^+ ($\lambda = 0, 1$) denote the annihilation and creation operators for the left- or right-circularity polarized cavity field. $|\lambda\rangle_i$ ($\lambda = 0, 1$; $i = 0, 1, \ldots, n$) represents the two ground state of the atom i, and $|e\rangle_i$ represents the excited state of the atom i.

4. Scheme for Implementing the Economical 1 → M PCC

In this section, we turn to describe the detail of implementing the $1 \to M$ PCC. Generally speaking, the whole process of cloning can be divided into two steps, which are preparation and copying, respectively.

In the first step, we assume that the M atoms are prepared in the state $|00\cdots 0\rangle_{1,2,\ldots,M}$ and we inject the photon, which has the cloned state $|\psi\rangle^{(\mathrm{in})} = (|R\rangle + e^{i\varphi}|L\rangle)/\sqrt{2}$, into the cavity. Here we choose $|R\rangle$ and $|L\rangle$ to act as the logical bit $|0\rangle$ and $|1\rangle$, respectively.

So the initial state of the whole system can be written as

$$|\psi\rangle^{(1)}_{1,2,\ldots,M,c} = \frac{1}{\sqrt{2}}|00,\ldots,0\rangle_{1,2,\ldots,M}|R\rangle_c + \frac{1}{\sqrt{2}}e^{i\varphi}|00,\ldots,0\rangle_{1,2,\ldots,M}|L\rangle_c\,. \tag{5}$$

At a time t, if still no photon is detected, the system evolves to

$$\begin{aligned}|\psi\rangle^{(2)}_{1,2,\ldots,M,c} = {}& \frac{1}{\sqrt{2}}|00,\ldots,0\rangle_{1,2,\ldots,M}|R\rangle_c \\ & + \frac{1}{\sqrt{2}}e^{i\varphi}\left[\frac{g_1^2 + \sqrt{M}g_0^2\cos\alpha_1 t}{\alpha^2}|0\rangle_1|0\rangle_2|0\rangle_3|L\rangle_c\right. \\ & + \frac{g_0\sin\alpha t}{\alpha}(|e,0,\ldots,0\rangle_{1,2,\ldots,M} + |0,e,\ldots,0\rangle_{1,2,\ldots,M} \\ & + \cdots + |00,\ldots,e\rangle_{1,2,\ldots,M})|0\rangle_c \\ & + \frac{g_0 g_1(\cos\alpha t - 1)}{\alpha^2}(|10,\ldots,0\rangle_{1,2,\ldots,M} + |01,\ldots,0\rangle_{1,2,\ldots,M} \\ & \left. + \cdots + |00,\ldots,1\rangle_{1,2,\ldots,M})|R\rangle_c\right]\end{aligned} \tag{6}$$

where $\alpha = \sqrt{Mg_0^2 + g_1^2}$. If the detector D_0 is triggered at t_D, the cloning is failure. When the detector D_1 is only triggered at t_D, the system will relapse to the following state after normalization:

$$\begin{aligned}|\psi\rangle^{(\mathrm{out})}_{1,2,\ldots,M,c} = {}& \frac{1}{\sqrt{2}}|00,\ldots,0\rangle_{1,2,\ldots,M} + \frac{1}{\sqrt{2M}}e^{i\varphi}(|10,\ldots,0\rangle_{1,2,\ldots,M} \\ & + |01,\ldots,0\rangle_{1,2,\ldots,M} + \cdots + |00,\ldots,1\rangle_{1,2,\ldots,M})\,.\end{aligned} \tag{7}$$

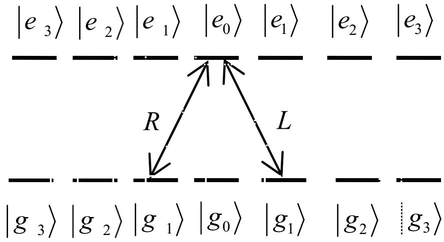

Fig. 2. Hyperfine levels of cesium ($F = 3$). R and L indicate that the two levels connected by the arrow are coupled by right- and left-circularly polarized light, respectively.

So we have realized the desired PCC including M identical clones shown in Eqs. (1) and (2). Here we note that a photonic state with phase unknown is cloned to the output state of M atoms in the optical cavity. Equation (6) indicates that the probability at time t of obtaining Eq. (7) is given by

$$P(t) = |{}^{(\mathrm{out})}_{1,2,\ldots,M}\langle\psi|\psi\rangle^{(2)}_{1,2,\ldots,M}|^2 = \frac{1}{4} + \frac{M\beta^2 \sin^4 \frac{\alpha t}{2}}{(M+\beta^2)^2} + \frac{\sqrt{M}\beta \sin^2 \frac{\alpha t}{2}}{M+\beta^2} \qquad (8)$$

where $\beta = g_1/g_0$. At $t = (2n+1)\pi/\alpha$ ($n = 0, 1, 2 \ldots$), the probability has the maximum value $P_{\max} = \frac{1}{4} + \frac{M\beta^2}{(M+\beta^2)^2} + \frac{\sqrt{M}\beta}{M+\beta^2}$. When $\beta = 1$, $P_{\max} = \left(\frac{1}{2} + \frac{\sqrt{M}}{M+1}\right)^2$. In particular, when $\beta = \sqrt{M}$, $P_{\max}$ reaches 1.

5. Discussion and Conclusion

Finally, let us briefly discuss the feasibility and some practical issues of the scheme. First, we consider hyperfine levels of cesium (nuclear spin $I = 7/2$) considered by Lange and Kimble.[22] The Zeeman sublevels of the states ($6S_{1/2}$, $F = 3$) and ($6P_{1/2}$, $F = 3$) are drawn in Fig. 2, where $|g_{mF}\rangle$ and $|e_{mF}\rangle$ denote the sublevels ($6S_{1/2}$, $F = 3$) and ($6P_{1/2}$ $F = 3$), respectively, with mF running from -3 through 3. The wavelength of the $|e\rangle \to |g\rangle$ transition is 852.36 nm. The transition between $|e_{mF}\rangle$ and $|g_{mF-1}\rangle$ is mediated by right-circularly polarized light and that between $|e_{mF}\rangle$ and $|g_{mF+1}\rangle$ by left-circularly polarized light.

Second, the requirement of a single photonic polarization state with its phase unknown is also difficult to meet, as a true single photon source does not exist.[21,22] One possible way of obtaining single-photon injection is to use hyperfine levels of cesium, as shown in Fig. 3. The idea is to prepare the atom on the two grounds state $(|g_1\rangle + e^{i\varphi}|g_{-1}\rangle)/\sqrt{2}$ and adiabatically drive it with classical field to the state $(|e_1\rangle + e^{i\varphi}|e_{-1}\rangle)/\sqrt{2}$. The atom will then decay spontaneously to $|g_0\rangle$ emitting a single photon, which has the required state $(|R\rangle + e^{i\varphi}|L\rangle)/\sqrt{2}$. By using the method of adiabatically drive and decay spontaneously, we can obtain a single polarized-photon injection easily.

We also note that the desired output state is a superposition of different combination of two grounds states (or metastable states) and thus is free from decoherence caused by spontaneous emission.[21,22]

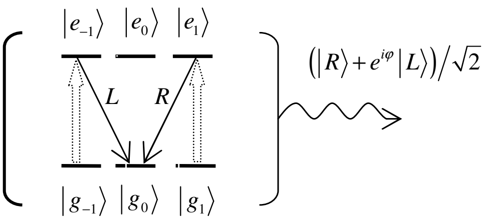

Fig. 3. The schematic set-up to generate a single photon.

In summary, we have presented here a simple scheme to implement the economical $1 \rightarrow M$ PCC with M Λ-type three-level atoms in an optical cavity via the detection of cavity decay. As we know, photons have long coherent time and are easy to implement single-qubit logical gate by using linear optics elements, but are inconvenient to be stored. Fortunately atoms trapped in the cavity are adopted as storing qubits because of their long-lived internal state. So in our scheme, a photonic polarization state with its phase unknown is cloned into the quantum state of three atoms in optical cavity. It is so important to copy and store unknown information for the quantum network of future quantum computer and quantum communication, whereas quantum copying is approximate and is also necessary. Importantly, according to feedback of detecting the polarization of the photon leaking out of the cavity, we can determine the success or failure of cloning.[20]

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