Visualization of chaos

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Abstract – *Subject of this article is visualization of chaotic attractor in 3D space. 3D view of chaos enables create a demonstrative projection about relations of chaos generated by physical circuit - Chua's circuit. The program created in C++ program language enables view on chaotic attractor both in 2D and 3D space together with plane objects visualization – elements of singularities. The Chua's attractor visualisation represents visualization of large graphical data set.*

Keywords: chaos, Chua's circuit, singularity, 3D.

I. INTRODUCTION

The first work aimed on Chua's circuit [1] draws a publication explosion dealing with Chua's circuit, chaos and its utilization. The work [2] states nearly 800 scientific articles dealing with this subject by the year 2004. Based on works [3], [4] and [5] the work [6] was published about influence of elements of singularities on chaos relations. This work motivated an idea of singularity element 3D visualization together with chaotic attractor presented in this paper.
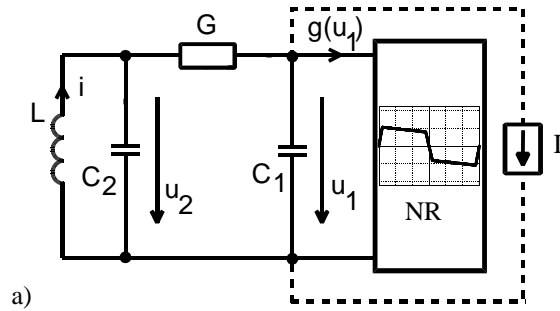
II. CHUA'S CIRCUIT

Chua's circuit is shown on Fig.1a, I-V characteristic of non-linear resistor (NR) is shown on Fig.1b as voltage dependence u_1 – $g(u_1)$. The circuit is described by equation system (1).

$$
\begin{aligned}
C_1\left(du_1/dt\right) &= G(u_2 - u_1) - g(u_1) - I &\equiv Q_1 \\
C_2\left(du_2/dt\right) &= G(u_1 - u_2) + i &\equiv Q_2 \\
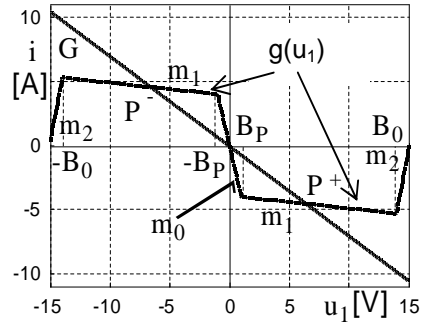L\left(di/dt\right) &= -u_2 - \rho i &\equiv Q_3
\end{aligned}
\qquad (1)
$$

where

$$
\begin{aligned}
g(u_1) &= m_2 u_1 + \tfrac{1}{2}(m_1 - m_0)\left(|u_1 - B_P| - |u_1 + B_P|\right) + \\
&+ \tfrac{1}{2}(m_2 - m_1)\left(|u_1 - B_O| - |u_1 + B_O|\right)
\end{aligned}
\qquad (2)
$$

Next we consider control pulse I=0, the resistance of the inductance ρ=0 and parameters (3)

a)

b)

Fig.1a) Chua's circuit

b) I - V characteristics of conductance G and NR

C_1=0,1, C_2=2, L=0,142857, G=0,7,
m_0=−4, m_1=−0,1, m_2=5, B_p=1, B_0=14   (3)

III. LINEARIZATION OF THE SYSTEM AND SINGULARITIES

The linearization of (1) can be written in the form

$$
d\hat{x}/dt = [\mathbf{A}]\,\hat{x}
\qquad (4)
$$

where $\hat{x} = [u_1, u_2, i]^t$ and [A] is the Jacobi matrix with eigenvectors $\lambda_1,\ \lambda_2,\ \lambda_3$, belonging to corresponding singularity $\mathbf{P^+}, \mathbf{P^-}, \mathbf{0}$ in Fig.1b. Since we are in the space R^3, $g(u_1)$ in Fig. 2a) is a surface coplanar with u_2. However, since the singularities are solutions to the system

$$
Q_1 = 0,\ Q_2 = 0\ \text{ a }\ Q_3 = 0
\qquad (5)
$$

it is obvious that they lie in the plane u_2=0 which is the plane of Monge projection in Fig. 2a). In it lies also the cross-section of the load plane Q_2=0, whose

cross-section at $Q_3=0$ is a straight line. Hence its common points define three singularities $\mathbf{P^+}$, $\mathbf{P^-}$, $\mathbf{0}$ in all three projection planes in Fig. 2a) b) c). Chua's circuit is the first physical circuit with all three singularities unstable at (2) and (3). Co-ordinates of the singularities are listed in Table 1.

TABLE 1. Co-ordinates of the singularities of Chua's circuit

Singularity	i [A]	u_1 [V]	u_2 [V]
P^-	4,55	-6,5	0
0	0	0	0
P^+	-4,55	6,5	0

It is necessary to point, that existence of three unstable singularities is necessary, but not sufficient condition for presence of chaos in Chua's circuit. Presence of 3 unstable singularities and consequently generation of chaos with help of structure similar as Chua's circuit is presented at work [7].

IV. GEOMETRIZATION OF THE SYSTEM (1) WITH PARAMETERS (3)

With the above assumptions the system exhibits two attractors. One of them is a limit cycle C_S and the other is a chaotic attractor C_{CH} in Fig. 2. The projections C_S are stunning in that despite the Monge projection C_S in Fig.2 a) and b) the redundant view c)

is perfectly elliptic. [6] The shape of this view is explained by Fig.1a since i and u_2 are quantities corresponding to the ideal LC_2 filter. From a technical viewpoint the task of twisting a wire so that two projections have the shape a) b) and the other projection the shape c) would be a difficult one if only one of the projections would be available. Let us remark that the chaotic attractor C_{CH} in Fig. 2 is purposely "sparse" for clarity. The arrows on LC indicate direction of representative point movement in state space along trajectory. The symbol "+" indicates time track t=0,18s on LC in the various projections.

V. ELEMENTS OF THE SINGULARITIES

There was, in work [6], defined singularity element by equation:

$$y_1 = \alpha_{11}\Delta u_1 + \alpha_{12}\Delta u_2 + \alpha_{13}\Delta i \qquad (6)$$

regardless of whether $\lambda_1>0$ or $\lambda_1<0$. When $\lambda_1>0$, the element would be a *REPULSOR*. At $\lambda_1<0$ the element would be an attractor. Eigenvalues and eigenvectors of singularities are listed in Table 2 Since the singularities and the characteristics $g(u_1)$ are symmetric, eigenvalues and eigenvectors (α_{11}, α_{12}, α_{13}) of $\mathbf{P^+}$ are identical to those of $\mathbf{P^-}$.
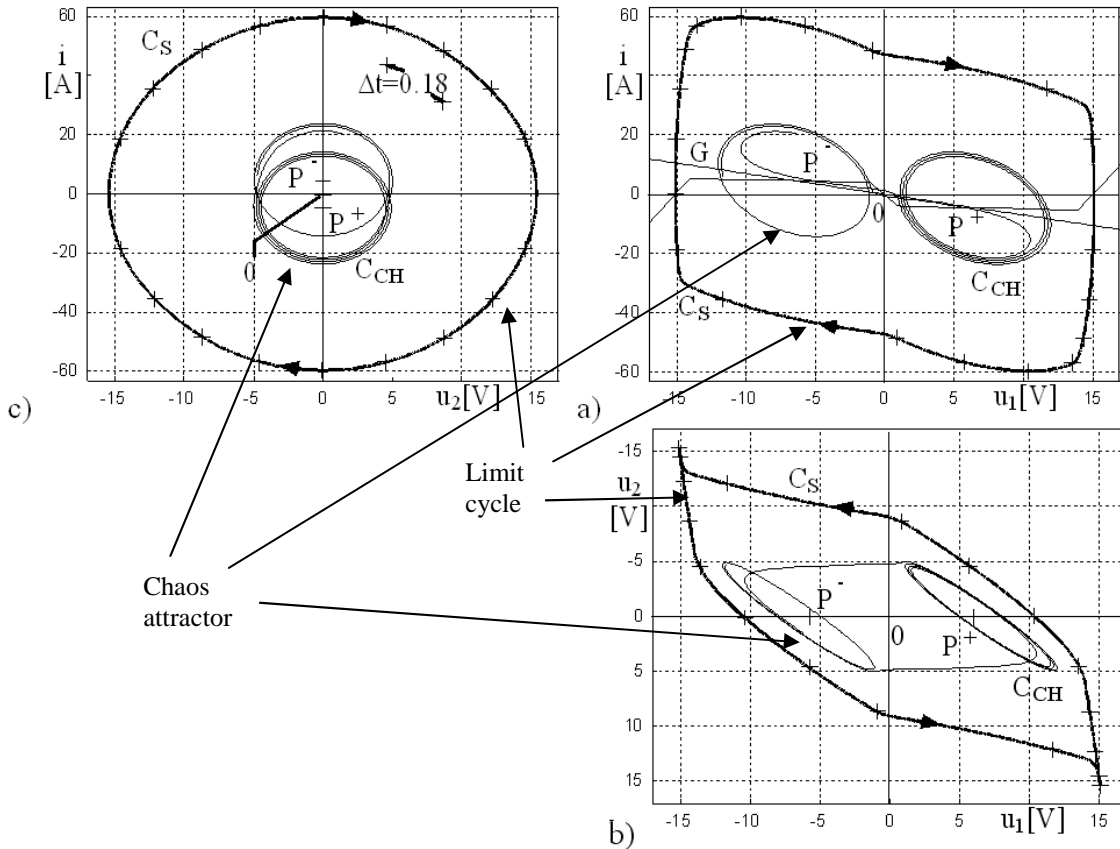

Fig.2 Projection of attractors of the system (1)

TABLE 2. The eigenvalues and eigenvectors of the singularities P (P^+, P^-) and 0

			α_{11}	α_{12}	α_{13}
P	λ_1	-6,372793	11,966291	-12,74558	1
	$\lambda_{2,3}$	0,0113967 ±1,815248i			
0	λ_1	33,073071	316,8296	66,1461426	1
	$\lambda_{2,3}$	-0,211535 ±1,856749i			

In Fig. 3a are denoted the elements containing all singularities. They are uniquely defined by main lines (ML) of elements in such a way that ML passing for example through the singularity **P**$^+$ is denoted in Fig.3a by the symbol **MLP**$^+$. The parameters *Δi=10A* and *Δi=−10A* suggest that the ML are parallel with the projection plane *i=0A* so that they are lower than the main line **MLP**$^+$, or higher by the value *Δi=10A*. The interpretation of symbols with the other singularities is analogous. More intricate attractor according [1] for parameters (7) is shown on Fig.3b. Denotes and descriptions of main lines are consistent with Fig.3a. This attractor will be subject of visualisation in next part of the contribution. 3D visualisation of this attractor including elements of singularities (**E0** – is element of singularity **0**, **EP**$^+$ and **EP**$^-$ - are elements of singularities **P**$^+$ and **P**$^-$) is shown in Fig.4, Fig.5.

$$C_1=1/9, C_2=1, L=0,142857, G=0,7,$$
$$m_0=−0,8, m_1=−0,5, m_2=5, B_p=1, B_0=14 \qquad (7)$$

Coordinates of singularities, its eigenvalues and eigenvectors are listed in Table 4 and Table 5.

TABLE 4. Co-ordinates of the singularities of Chua's circuit for parameters (7)

Singularity	i [A]	u_1 [V]	u_2 [V]
P^-	1,05	-1,5	0
0	0	0	0
P^+	-1,05	1,5	0

TABLE 5. The eigenvalues of the singularities for parameters (7)

			α_{11}	α_{12}	α_{13}
P	λ_1	-2,75949	2,0131973	-2,7594902	1
	$\lambda_{2,3}$	0,129745 ±2,13289i			
0	λ_1	1,552176	1,6659961	1,55217653	1
	$\lambda_{2,3}$	-0,6760882 ±1,8978201i			

V. VISUALISATION

During the past several years, high-performance and feature-rich PC graphics interfaces have become
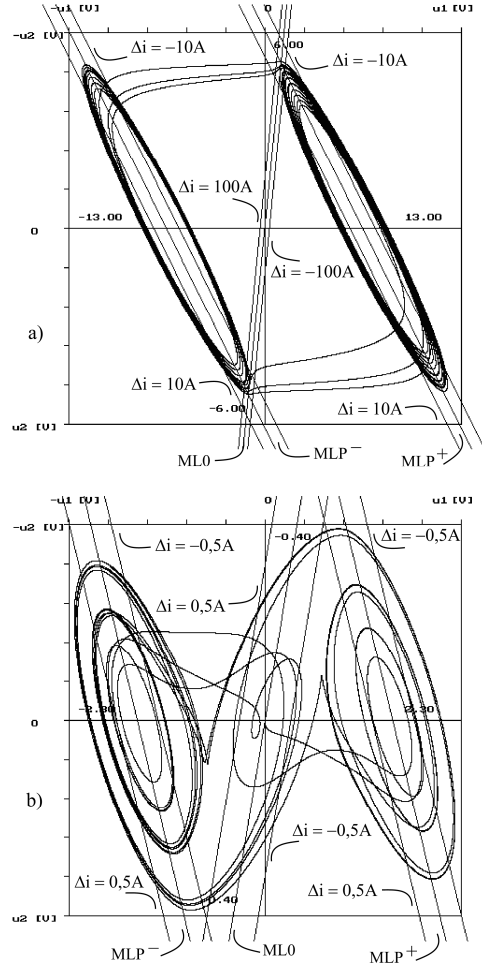

Fig.3 Projection of chaotic attractors and elements of singularities in plane u_2, u_1 of the system (1) for parameters: a) (3), b) (7)

available at low cost. High-performance graphics devices shorten computational visualising time. High-resolution display solutions increase quality and ability of immersion into 3D explored space (e.g. in virtual labs [8]). Graphical visualising systems are often based on serial using of information. On the input side is description of visualised space (e.g. using appropriate formulas) and on output side are correct coloured pixels on the screen. Between these two stages there are some other stages partially changed the input information. A modern visualisation brings new dimensions to the understanding of actions and events still hard understandable. Chua's circuit comes under this area.

Therefore visualising system was developed [9]. It visualizes Chua's attractor consist of appropriate curves or planes. The curves/planes can be single coloured with different colours. The visualisation of chaotic attractor in time and 3D space is graphical output of application. This visualisation depicts representative point movement and it creates chaotic attractor using two basic modes (*continuous* and *sequential*). The animation of representative point

movement in state space in *comet form* or in *continuous attractor form* is final result of this application.

Chaos visualising system provides three basic visualising modes: *continuous mode* (see Fig. 4a), *sequential mode* (see Fig. 4b) and *I-V characteristics visualisation mode*. System allows using four projection types ($3D\ u_1 – i - u_2$ projection (see Fig. 5), $2D\ i – u_2$ projection, $2D\ i – u_1$ projection and $2D\ u_2 – u_1$ projection) for better examined circuit understanding.

a   b
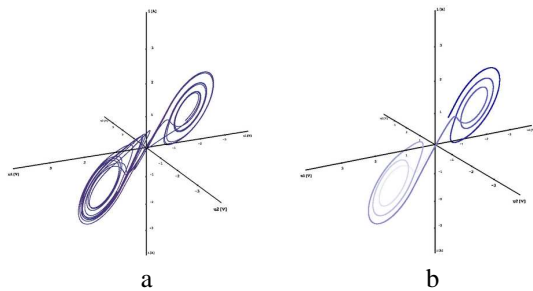
Fig.4 Examples of attractor displaying in continuous mode (a) and in sequential mode (b)
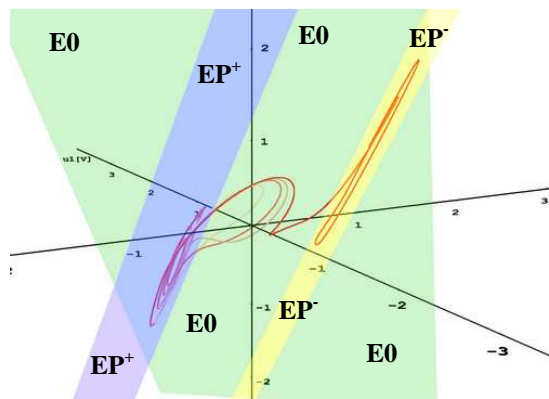

Fig.5 An elements of the singularities displaying as transparent planes in 3D space

Settable basic visualising parameters for chaotic attractor visualisation are: *drawing speed*, *points omission*, *chaotic attractor point size*, *comet length* and *attractor colour*. The combination of these parameters defines final visualisation form of chaotic attractor. In case of 3D graphic accelerator supported acceleration of graphic interface OpenGL is drawing of 3D primitives accelerated by graphic card. It enables increasing performance and using some graphical improvements can not be used without acceleration in real time.

Chaos visualising system is designed for visualization Chua's attractor in 3D space in real time and it is based on visualising kernel developed on DCI FEEI TU Košice [10]. An application is implemented in C++ language using OpenGL graphics library in Borland Developer Studio development environment. It enables visualisation for user defined Chua's attractor where user has in standard visualising mode (3D projection) 6 degrees of freedom for motion in explored attractor's space, which mean translation in 3 axis and rotating around them. It means that system is capable to visualise the Chua's attractor from any point and from any angle. The application works under MS Windows XP.

VI. CONCLUSIONS

Visualization of chaotic attractor provides better understanding of representative point movement in state space, what was still possible only with help of representation in projection planes.

ACKNOWLEDGMENTS

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