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# Performance of a cooperative downlink multiplexing scheme using dynamic spectrum access principles

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**Abstract:** In this study, the authors propose a new cooperative multiplexing scheme for downlink transmission using dynamic spectrum access (DSA) principles. In this scheme, the base-station (BS) transmits to its distant user using a selected relay node in two hops. In the second hop, while the relay forwards information to the distant user, the BS transmits information to another selected user in its vicinity using DSA principles. Assuming peak power and peak interference constraints, the authors derive outage and ergodic rate performance of the user served by DSA assuming various user selection schemes. The authors consider gain-based selection, SINR based selection (SS), interference cancellation (IC), as well as a hybrid scheme that employs both SS and IC. The authors demonstrate that apportioning of time between the first and second hop of relaying can improve outage performance of the user served by DSA while ensuring certain guaranteed performance for the distant user. The presented analytical expressions are compared with computer simulation results.

## 1 Introduction

It has been established in recent years that spectrum scarcity can be alleviated by systems that utilise a dynamic spectrum access (DSA) framework. In these systems, secondary (unlicensed) users share the spectrum of the primary (licensed) users (PUs) [1, 2]. Such spectrum sharing clearly results in interference. The challenge in spectrum sharing systems is to deal with interference at both the primary and secondary user (SU) receivers, and maximise performance of the secondary network, while guaranteeing certain acceptable performance in the primary network. To this end, the secondary transmit power is constrained to maintain interference at the PU below an interference tolerance limit (ITL) [3, 4]. In such interference-limited scenarios, use of relays is clearly beneficial [5–10] since reduced transmit powers can decrease interference. Incorporating relays in the primary network raises the ITL, thereby increasing the secondary transmit powers and improving the primary signal-to-noise ratio (SNR) [5, 7]. Use of node selection schemes in the secondary link boosts the signal-to-interference plus noise ratio (SINR) at the secondary receiver [6, 8–10]. In DSA systems, the ITL can be either statistically determined or assumed to vary according to channel state information (CSI) of the primary link and the link between the secondary transmitter and the PU receiver. In the former, ITL can be chosen to limit degradation in outage [3] or ergodic rate [11] performance of the primary link. Else, it can be chosen based on the average interference to PU [8] or the probability that the interference exceeds a threshold [9].

Performance gains accrued by cooperative relaying and/or user selection have motivated researchers to investigate their use in the DSA framework. In [6], decode-and-forward (DF) cooperative relaying protocol [12, 13] has been used in the secondary network to improve its outage performance over Nakagami-$m$ fading channels. In this work, ITL was statistically determined. Exact expressions for secondary outage over Rayleigh fading channels have been presented in [8]. In this work, primary to secondary interference was also taken into account (and not ignored like in other DSA works). Exact outage performance of DSA with incremental regenerative relaying was discussed in [9], which used CSI based ITL. In [5, 7], relay selection was employed in the primary network to boost the ITL. An SU was selected from the set of users satisfying the ITL constraint to access the spectrum in the second hop of primary network. The outage performance of the secondary was analysed assuming DF relaying. The system model was generalised in [7]. In all these works, duration of the first and second phase of relaying was assumed to be the same. In this paper, we demonstrate that secondary outage performance can be improved by optimally choosing these durations.

Cooperative multiplexing (CM) has been suggested as a technique to improve spectral efficiency and improve frequency reuse [14]. In such systems, the base-station (BS) transmits information to a distant user using a relay. In the second phase, the relay forwards information to the remote user while the BS transmits data to another user in its vicinity. However, in such protocols, outage performance of the distant user is not guaranteed since it encounters interference because of transmission by the BS in the second phase of relaying. In this paper, we use DSA principles in the CM framework to guarantee performance of the distant user. We also employ relay selection and user selection in the second phase. We analyse outage and ergodic rate performance of the user served by DSA. We show that optimally choosing the first and second phase durations can improve outage performance of the user served by DSA while guaranteeing certain performance to the distant user. We analyse the performance of user selection schemes in this context. We assume CSI assisted ITL, which makes power used by the BS in the second phase a random variable dependent on the interference caused to the distant user.

The remaining paper is organised as follows. The novel CM system using DSA principles is described in Section 2, where the problem formulation is presented. Outage and ergodic rate analysis are presented in Section 3. In Section 4, numerical results are presented with discussions. Conclusions are drawn in Section 5.

*Notations:* Subscript 'p' and 's' denote primary and secondary network, respectively. '$\sim$' denotes 'distributed as'. Exponential integral $E_n(x) = \int_1^\infty (\mathrm{e}^{-xt}/t^n)\,\mathrm{d}t$ where $n$ is an integer. Pochhammer symbol $(a)_n = \Gamma(a+n)/\Gamma(a)$ where $\Gamma(a) = (a-1)!$. $|\mathcal{A}|$ denotes the cardinality of set $\mathcal{A}$. $\mathcal{CN}(c, \Omega)$ denotes the complex Normal distribution of mean $c$ and variance $\Omega$. $\mathbb{E}[.]$ denotes the expectation operator.

## 2 System model description and problem formulation

We consider a cellular cooperative downlink (DL) multiplexing scheme using DSA framework where a BS serves a distant user (termed the PU) as well as another user in two hops with the help of a half-duplex regenerative node selected from a closely-spaced cluster $\mathcal{J}$ of $J$ nodes. After transmitting the data of PU for relaying using the licensed spectrum, BS dynamically accesses the same frequency band in the second hop for DL data transmission to an opportunistically selected SU from a cluster $\mathcal{K}$ of $K$ SUs. As depicted in Fig. 1, the cellular network comprises of a total of $M = J + K$ SUs in two clusters. The user selection mechanisms are discussed later in the section.

We assume quasi-static Rayleigh fading channel model, and channels from nodes in each cluster are independent and identically distributed. The channel coefficients $h_{1j} \sim \mathcal{CN}(0, \Omega_{h1})$, $h_{2j} \sim \mathcal{CN}(0, \Omega_{h2})$ and $g_{1k} \sim \mathcal{CN}(0, \Omega_{g1})$ denote the channel between BS to $j$th node, $j$th node to PU and BS to $k$th SU, respectively, where $j \in \mathcal{J}$ and $k \in \mathcal{K}$. The cross-channel coefficients $g_2 \sim \mathcal{CN}(0, \Omega_{g2})$ and $h_{3jk} \equiv h_{3k} \sim \mathcal{CN}(0, \Omega_{h3})$ denote the channel between BS to PU, and $j$th node in $\mathcal{J}$ to $k$th SU in $\mathcal{K}$, respectively. *Note:* the index $j$ is dropped from $h_{3jk}$ to simplify notation. We also assume zero mean complex additive white Gaussian noise (AWGN) of variance $\sigma^2$ at each node.

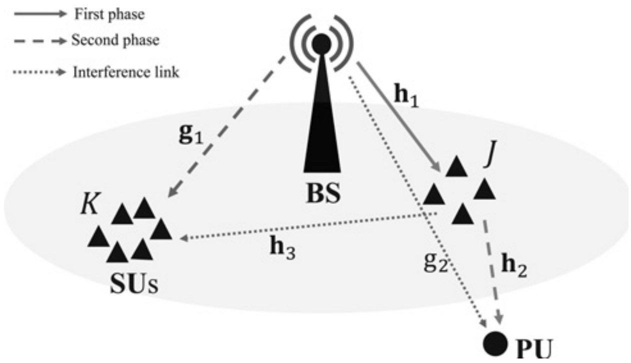


**Fig. 1** *System model for cooperative DL multiplexing scheme using DSA framework. In this $\boldsymbol{h}_x = [h_{x1}, h_{x2}, \ldots, h_{xJ}]^T$ where x = 1, 2 and $\boldsymbol{g}_1 = [g_{11}, g_{12}, \ldots, g_{1K}]^T$. Similarly, $\boldsymbol{h}_3 = [h_{31}, h_{32}, \ldots, h_{3K}]^T$ for jth node of cluster $\mathcal{J}$*

We denote the overall rate requirement of the primary link by $R_p$. Transmission from BS to PU takes place in time $T$ in two phases. We note that secondary DL transmissions take place only in the second phase. For this reason, use of equal duration time slots for the two phases is not well motivated. Unlike other papers related to cognitive radio, we assume unequal duration of phases, with the first and the second phases of duration $\alpha T$ and $(1-\alpha)T$, where $\alpha$ is phase duration allocation (PDA) factor taking values in the range (0, 1). We show later in this paper that $\alpha$ can be chosen to minimise the outage probability of the secondary system.

### 2.1 First phase

In the first phase of $\alpha T$ seconds, BS broadcasts unit energy symbols $x_p$ with power $P_p \le P$ and rate $R_{p1} = R_p/\alpha$ (a higher rate is used to make up for the smaller time duration $\alpha T$) where $P$ is peak transmit power. The $j$th node in set $\mathcal{J}$ receives signal $y_j = \sqrt{P_p} h_{1j} x_p + n_j$ (where $n_j \sim \mathcal{CN}(0, \sigma^2)$ is the AWGN) with SNR $\Gamma_{p,j} = \gamma_p |h_{1j}|^2$ where $\gamma_p = P_p/\sigma^2$. A subset $\mathcal{J}_d$ of nodes in $\mathcal{J}$ that successfully decode information from BS is defined as

$$\mathcal{J}_d = \left\{ j \in \mathcal{J} |\log_2(1 + \Gamma_{p,j}) > \frac{R_p}{\alpha} = R_{p1} \right\}$$

It can be seen that the probability of the cardinality of $|\mathcal{J}_d|$ being $\ell$ is given by

$$Pr(|\mathcal{J}_d| = \ell) = \binom{J}{\ell} p_{\mathrm{dec},p}^{\ell} (1 - p_{\mathrm{dec},p})^{J-\ell} \quad (1)$$

where decoding probability of each node $p_{\mathrm{dec},p}$ is readily written as $exp(-1/(\overline{\gamma}_p \Omega_{h1}))$ where $\overline{\gamma}_p = \gamma_p/\gamma_{p1}^{\mathrm{th}}$ and $\gamma_{p1}^{\mathrm{th}} = 2^{R_{p1}} - 1$. The $j$th node acquires the channel gain $|h_{2j}|^2$ by observing the reverse channel from PU. Relay selection is performed using the rule

$$j^\star = \arg\max_{j \in \mathcal{J}_d} |h_{2j}|^2$$

The best relay $j^\star \in \mathcal{J}_d$ is opportunistically selected in a distributed manner by using the timer-based scheme described in [15].

### 2.2 Second phase

In the second phase of duration $(1-\alpha)T$ seconds, $j^\star$ forwards the decoded symbol $x_p$ using power $P_r$. Clearly, the target primary second hop rate $R_{p2} = R_p/(1-\alpha)$. The BS also simultaneously transmits unit-energy symbols $x_s$ with power $P_s$ to selected user $k^\dagger$ by sharing the same frequency band. We discuss SU selection later in this section. PU and SU $k^\dagger$ receive the signals

$$y_{\mathrm{pu}} = \sqrt{P_r} h_{2j^\star} x_p + \sqrt{P_s} g_2 x_s + n_{\mathrm{pu}}$$

$$y_{k^\dagger} = \sqrt{P_s} g_{1k^\dagger} x_s + \sqrt{P_r} h_{3k^\dagger} x_p + n_{k^\dagger}$$

where $n_{\text{pu}} \sim \mathcal{CN}(0, \sigma^2)$ and $n_{k^\dagger} \sim \mathcal{CN}(0, \sigma^2)$ are AWGN samples at nodes PU and SU. Clearly, $P_s$ needs to be controlled to limit interference to PU.

The SINR at PU is given by

$$\Gamma_{\text{pu}} = \frac{\gamma_r |h_{2j^\star}|^2}{\gamma_s |g_2|^2 + 1} \tag{2}$$

where $\gamma_r = P_r/\sigma^2$ and interference power is $\gamma_s|g_2|^2$ with $\gamma_s = P_s/\sigma^2$. From (2), we deduce the peak interference power (denoted by $\mathcal{I}_{\text{pu}}$) using inequality $\Gamma_{\text{pu}} \geq \gamma_{p2}^{\text{th}} = 2^{R_{p2}} - 1$ that ensures rate $R_{p2}$ for the second hop. We obtain

$$\mathcal{I}_{\text{pu}} \leq \overline{\gamma}_r |h_{2j^\star}|^2 - 1 \tag{3}$$

where $\overline{\gamma}_r = \gamma_r/\gamma_{p2}^{\text{th}}$. Note that $\mathcal{I}_{\text{pu}}$ in (3) can be negative *iff* $j^\star \to$ PU link in outage (in which case the BS uses its peak transmit power $P$ to transmit in the second phase). Else, the BS transmit power $P_s$ can be constrained so that $P_s < \mathcal{I}_{\text{pu}}\sigma^2/|g_2|^2$. In summary, the transmit power of BS $P_s$ is allocated as (see (4)) where $P$ is peak transmit power of BS.

Broadly, there exist two common schemes for selecting the user $k^\dagger$ from the set $\mathcal{K}$

1. *Gain based selection (GS) scheme:* We assume that each SU monitors the forward link signals and acquires knowledge of $|g_{1k}|^2$ (a training can optionally be used too). The user with maximum $|g_{1k}|^2$ is chosen by using a timer based mechanism [15]. BS is then informed through control channels.
2. *SINR based selection (SS) scheme:* We assume that each SU acquires knowledge of $|g_{1k}|^2$ by monitoring the secondary forward channels, and knowledge of $|h_{3k}|^2$ by using a short training sequence transmitted by the selected node $k$ just prior to the second phase. The SINR of the secondary reverse channels $\Gamma_{s,k} = \gamma_s|g_{1k}|^2/(\gamma_r|h_{3k}|^2 + 1)$ is calculated at each SU, and the best user $k^\dagger$ is chosen as per $k^\dagger = \arg\max_{k \in \mathcal{K}} \Gamma_{s,k}$ using a timer based mechanism. Once again, control signals can be used to inform the BS.

The SINRs $\Gamma_{s,k^\dagger}^{\text{gs}}$ and $\Gamma_{s,k^\dagger}^{\text{ss}}$ of the schemes (with *gs* and *ss* in the superscript denoting GS and SNR-based selection, respectively) can be written as

$$\Gamma_{s,k^\dagger}^{\text{gs}} = \frac{\max_{k \in \mathcal{K}} \gamma_s |g_{1k}|^2}{\gamma_r |h_{3k^\dagger}|^2 + 1}, \qquad k^\dagger = \arg\max_{k \in \mathcal{K}} \gamma_s |g_{1k}|^2$$
$$\Gamma_{s,k^\dagger}^{\text{ss}} = \max_{k \in \mathcal{K}} \frac{\gamma_s |g_{1k}|^2}{\gamma_r |h_{3k}|^2 + 1}, \qquad k^\dagger = \arg\max_{k \in \mathcal{K}} \frac{\gamma_s |g_{1k}|^2}{\gamma_r |h_{3k}|^2 + 1} \tag{5}$$

From (5), we can observe that $\Gamma_{s,k^\dagger}^{\text{gs}}$ and $\Gamma_{s,k^\dagger}^{\text{ss}}$ are same for $K = 1$. It is also intuitive that the performance of both the schemes are almost same if interference from $\mathcal{J}$ is very weak. However the SS scheme that considers the interference power in the selection process outperforms the GS scheme [16].

In addition to the above classic schemes, we also consider two others. The first is based on interference cancellation (IC), and the second is a hybrid scheme (HS) based on a combination of IC and the SS scheme.

3. *IC scheme:* The IC scheme is based on the fact that IC can be applied by exploiting the prior knowledge of interfering symbols [14]. In such a scheme, SUs receive signals $y_k^I = \sqrt{P_p} g_{1k} x_p + n_k$ in the first transmission phase of the primary, and attempt to decode $x_p$ ($n_k \sim \mathcal{CN}(0, \sigma^2)$ is the additive Gaussian noise sample at SU $k$). *Note:* a superscript $I$ is used to denote the secondary signals and quantities relevant to the first phase of the primary transmission. It is also emphasised that secondary transmission only takes place in the second phase of the primary transmission. A subset $\mathcal{K}_d$ of SUs that successfully decode in the first phase is formed as follows

$$\mathcal{K}_d = \left\{ k \in \mathcal{K} | \log_2(1 + \Gamma_{s,k}^I) > R_{p1} \right\} \tag{6}$$

where $\Gamma_{s,k}^I = \gamma_p |g_{1k}|^2$ is SNR at $k$th SU. One of these nodes participates in the secondary reverse channel transmission. Hence the SNR at $k^\dagger \in \mathcal{K}_d$ after IC can be written as

$$\Gamma_{s,k^\dagger}^{\text{ic}} = \max_{k \in \mathcal{K}_d} \gamma_s |g_{1k}|^2 \tag{7}$$

It is intuitive that the performance of IC scheme is limited by the subset $\mathcal{K}_d$. The probability that cardinality of $|\mathcal{K}_d|$ equals $\zeta$ is given by

$$Pr(|\mathcal{K}_d| = \zeta) = \binom{\mathcal{K}}{\zeta} p_{\text{dec},s}^{\zeta} (1 - p_{\text{dec},s})^{K-\zeta} \tag{8}$$

where $p_{\text{dec},s} = \exp(-\gamma_{p1}^{\text{th}}/(\gamma_p \Omega_{g1}))$ is decoding probability of $k$th SU and $\gamma_p = P_p/\sigma^2$.

4. *Hybrid selection (HS) scheme:* It is emphasised that the IC scheme is highly dependent on cardinality of the set $\mathcal{K}_d$. If $|\mathcal{K}_d|$ is very small, most of the SUs are not considered for the transmission process. Consider a set $\chi$ of users who are not in $\mathcal{K}_d$. We consider a HS that allows the remaining $|\chi| = |\mathcal{K}| - |\mathcal{K}_d|$ SUs to participate in SU selection. Once again, the user with the best overall SINR is picked. Hence the SINR at $k^\dagger$ for the HS scheme is given by

$$\Gamma_{s,k^\dagger}^{\text{hs}} = \max\left( \max_{k_1 \in \mathcal{K}_d} \gamma_s |g_{1k_1}|^2, \ \max_{k_2 \in \chi} \frac{\gamma_s |g_{1k_2}|^2}{\gamma_r |h_{3k_2}|^2 + 1} \right) \tag{9}$$

Clearly, $\mathcal{K}_d$ and $\chi$ belong to disjoint sets. We denote by $k_1^\dagger$ and $k_2^\dagger$ the users with best SNRs in each set. The best SINR/SNR from each set is therefore given by $\Gamma_{s,k_1^\dagger|_{\mathcal{K}_d}}^{\text{ic}} = \max_{k_1 \in \mathcal{K}_d} \gamma_s |g_{1k_1}|^2$ and $\Gamma_{s,k_2^\dagger|_\chi}^{\text{ss}} = \max_{k_2 \in \chi} \left( \gamma_s |g_{1k_2}|^2 / (\gamma_r |h_{3k_2}|^2 + 1) \right)$.

It is emphasised that if the primary network is in outage (PO case), the BS selects $k^\dagger$ from set $\mathcal{K}$ with highest

$$P_s = \begin{cases} P & \text{for primary in outage (PO) case} \\ \min\left( \dfrac{(\overline{\gamma}_r |h_{2j^\star}|^2 - 1)\sigma^2}{|g_2|^2}, P \right) & \text{for primary not in outage (PNO) case} \end{cases} \tag{4}$$



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channel gain to BS. The SNR for PO case is given by

$$\Gamma^{\rm po}_{s,k^\dagger} = \gamma |g_{1k^\dagger}|^2 \quad (10)$$

where $\gamma = P/\sigma^2$.

In the next section we discuss the outage and ergodic rate performance of the selection schemes.

## 3 Performance analysis

In the following, we first analyse the outage performance of the secondary network for a target primary outage.

### 3.1 Outage probability

The outage probability for the secondary network can be written as

$$p_{o,s} \triangleq p_{o,p} p^{\rm po}_{o,s} + (1 - p_{o,p}) p^{\rm pno}_{o,s} \quad (11)$$

where $p_{o,p}$ is primary outage probability, and $p^{\rm po}_{o,s}$ and $p^{\rm pno}_{o,s}$ are secondary outage probabilities for PO and PNO cases, respectively. Clearly, the primary link is in outage when either $\{|\mathcal{J}_d| = |\emptyset|\}$ or $\{|\mathcal{J}_d| \neq |\emptyset|, |h_{2j^\star}|^2 < (1/\overline{\gamma}_r)\}$ where $\overline{\gamma}_r = \gamma_r/\gamma^{\rm th}_{p2}$ as defined earlier. Hence the outage probability $p_{o,p}$ is readily obtained as

$$p_{o,p} = \left[1 - \exp\left(-\frac{1}{\overline{\gamma}_p\,\Omega_{h1}} - \frac{1}{\overline{\gamma}_r\,\Omega_{h2}}\right)\right]^J \quad (12)$$

We obtain the primary outage probability function of PDA factor $\alpha$ as

$$p_{o,p}(\alpha) = \left[1 - \exp\left(-\frac{2^{R_p/\alpha} - 1}{\gamma_p \Omega_{h1}} - \frac{2^{R_p/(1-\alpha)} - 1}{\gamma_r \Omega_{h2}}\right)\right]^J \quad (13)$$

From (13), we can observe that the probability function of $\alpha$ is non-linear, and closed form expression for the range of $\alpha$ for which $p_{o,p}$ is below a certain threshold (to guarantee certain outage performance) is difficult to obtain. Numerical techniques are used in Section 4 to compute this range.

*3.1.1 Secondary outage probability for PO case:* Using SNR $\Gamma^{\rm po}_{s,k^\dagger} = \gamma|g_{1k^\dagger}|^2$ from (10), and the i.i.d. nature of the exponential random variables $|g_{1k^\dagger}|^2$, the outage probability $p^{\rm po}_{o,s}$ of the secondary link for PO case is given by

$$p^{\rm po}_{o,s} \triangleq Pr(\Gamma^{\rm po}_{s,k^\dagger} < \gamma^{\rm th}_s) = Pr\left(|g_{1k^\dagger}|^2 < \frac{1}{\overline{\gamma}}\right)$$
$$= \left[Pr\left(|g_{1k}|^2 < \frac{1}{\overline{\gamma}}\right)\right]^K = \left[1 - \exp\left(-\frac{1}{\overline{\gamma}\Omega_{g1}}\right)\right]^K$$

where $\gamma^{\rm th}_s = 2^{R_s/(1-\alpha)} - 1$ for secondary target rate $R_s$ and $\bar{\gamma} = \gamma/\gamma^{\rm th}_s$. We derive the outage probabilities for various selection schemes when primary is not in outage scenario in the following section.

*3.1.2 Secondary outage probability for PNO case:* Since SINRs for various schemes in PNO case are function of random variable $\gamma_s$, we derive its cumulative distribution function (CDF) first. From (4), for PNO case $\gamma_s = P_s/\sigma^2 = \min\left((\overline{\gamma}_r\,|h_{2j\star}|^2 - 1)/|g_2|^2, \gamma\right)$. With $Z = (\overline{\gamma}_r\,|h_{2j\star}|^2 - 1)/|g_2|^2$, $\gamma_s = \min(Z, \gamma)$. We note that $Z$ is dependent on set $\mathcal{J}_d$. The conditional probability density function (PDF) of $\gamma_s = \min(Z, \gamma)$ for PNO case (conditioned on $|\mathcal{J}_d|$) is given by

$$f_{\gamma_s||\mathcal{J}_d|}(x) = f_{Z||\mathcal{J}_d|}(x)(\mathcal{U}(x) - \mathcal{U}(x-\gamma)) + \overline{F}_{Z||\mathcal{J}_d|}(\gamma)\delta(x-\gamma) \quad (14)$$

where $\overline{F}_{Z||\mathcal{J}_d|}(\gamma) = 1 - F_{Z||\mathcal{J}_d|}(\gamma)$, $\mathcal{U}(x)$ is unit step function and $\delta(x)$ is Delta function. It is shown in Appendix 1 that the CDF and PDF of $Z$ are given by

$$F_{Z||\mathcal{J}_d|}(z) = 1 - \sum_{j=1}^{|\mathcal{J}_d|} \mathbb{A}_1(j)\frac{\exp\left(-j/(\overline{\gamma}_r\,\Omega_{h2})\right)}{1 + j(z/\overline{\gamma}_r\,\xi_{h2/g2})} \quad (15)$$

$$f_{Z||\mathcal{J}_d|}(z) = \sum_{j=1}^{|\mathcal{J}_d|} \frac{j\mathbb{A}_1(j)}{\overline{\gamma}_r\,\xi_{h2/g2}} \frac{\exp\left(-j/(\overline{\gamma}_r\,\Omega_{h2})\right)}{\left(1 + (jz/\overline{\gamma}_r\,\xi_{h2/g2})\right)^2} \quad (16)$$

where $\mathbb{A}_1(j) = \dfrac{\binom{|\mathcal{J}_d|}{j}(-1)^{j-1}}{1 - \left[1 - \exp\left(-1/(\overline{\gamma}_r\,\Omega_{h2})\right)\right]^{|\mathcal{J}_d|}}$ and $\xi_{h2/g2} = \Omega_{h2}/\Omega_{g2}$. The outage probability $p^{\rm pno}_{o,s}$ of secondary node for each user selection scheme can now be derived.

1. *Gain based selection scheme (GS):* The outage probability $p^{\rm pno}_{o,s}$ for GS scheme $p^{\rm gs}_{o,s}$ is given using (5) by

$$p^{\rm gs}_{o,s} \triangleq Pr(\Gamma^{\rm gs}_{s,k^\dagger} < \gamma^{\rm th}_s) = Pr\left(\frac{\max_{k\in\mathcal{K}} \gamma_s |g_{1k}|^2}{\gamma_r |h_{3k^\dagger}|^2 + 1} < \gamma^{\rm th}_s\right) \quad (17)$$

The PDF of random variable $\gamma_s = \min(Z, \gamma)$ is as in (14). Clearly, the CDF of channel gain $|g_{1k^\dagger}|^2 = \max_{k\in\mathcal{K}} |g_{1k}|^2$ is $F_{|g_{1k^\dagger}|^2}(x) = [1 - \exp(-x/\Omega_{g_1})]^K$. Since user selection does not use knowledge of the interference, it is clear that the distribution of interference channel gain $|h_{3k^\dagger}|^2$ is $F_{|h_{3k^\dagger}|^2}(x) = 1 - \exp(-x/\Omega_{h3})$. Using these, the CDF of $\Gamma^{\rm gs}_{s,k^\dagger}$ for given $|\mathcal{J}_d|$ can be written as (see (18)) where $\mathbb{A}_2(k) = \binom{K}{k}(-1)^{k-1}$, $\mathbb{C}_1(j, k, x) = jkx\gamma^{\rm th}_p/\psi$, $\psi = \Omega_{g1}\Omega_{h2}/\Omega_{h3}\Omega_{g2}$, $\xi_{g1/h3} = \Omega_{g1}/\Omega_{h3}$ and (see equation (19) at the bottom of the next page) where $\mathbb{C}_2(j, k, x) = -1 + jkx\gamma^{\rm th}_{p2}/\psi$. The term $E_1((1/\gamma_r\Omega_{h3}) + (kx/\gamma\Omega_{g1}))$ arises out of the interference channel, and is the dominating factor influencing the outage degradation. Proof of (18) is presented in Appendix 1.

$$F_{\Gamma^{\rm gs}_{s,k^\dagger}||\mathcal{J}_d|}(x) = 1 - \sum_{k=1}^{K}\mathbb{A}_2(k)\left[\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{C}_1(j,k,x)\mathrm{e}^{-(j/\overline{\gamma}_r\Omega_{h2})}\Lambda_1(j,k,x) + \frac{\overline{F}_{Z||\mathcal{J}_d|}(\gamma)\mathrm{e}^{-(kx/\gamma\Omega_{g1})}}{1 + (kx\gamma_r/\gamma\xi_{g1/h3})}\right] \quad (18)$$

The exact closed form expression of $p_{o,s}^{\text{gs}}$ for PNO case can be obtained by averaging $F_{\Gamma^{\text{gs}}_{s,k^\dagger}|_{|\mathcal{J}_d|}}(\gamma_s^{\text{th}})$ using (1) as

$$p_{o,s}^{\text{gs}} = \sum_{\ell=0}^{J} Pr(|\mathcal{J}_d| = \ell) F_{\Gamma^{\text{gs}}_{s,k^\dagger}|_{|\mathcal{J}_d|=\ell}}(\gamma_s^{\text{th}}) \tag{20}$$

2. *SS scheme:* Here we derive the outage probability of the secondary with SS selection scheme for the PNO case where SINR $\Gamma_{s,k^\dagger}^{\text{ss}} = \max_{k\in\mathcal{K}}\left(\gamma_s|g_{1k}|^2/(\gamma_r|h_{3k}|^2+1)\right)$ as in (5). Since the SINRs of SUs are dependent on random variable $\gamma_s$, we first obtain the CDF of SINR $\Gamma_{s,k}^{\text{ss}} = \gamma_s|g_{1k}|^2/\,(\gamma_r|h_{3k}|^2+1)$ of $k$th SU for given $\gamma_s$. Using $|h_{3k}|^2 \sim \exp(-y/\Omega_{h3})/\Omega_{h3}$ we have (see (21)). It follows from the fact that the SNRs $\Gamma^{\text{ss}}_{s,k|_{\gamma_s}}$ are i.i.d that $F_{\Gamma^{\text{ss}}_{s,k^\dagger}|_{\gamma_s=w}}(x) = [F_{\Gamma^{\text{ss}}_{s,k}|_{\gamma_s=w}}(x)]^K$. We obtain the exact closed form expression for given set $\mathcal{J}_d$ by using PDF of $\gamma_s$ from (14) as (see (22) and (23)).

The term $\Lambda_2(j,k,x)$ accounts for interference from primary relays to SU, and is responsible for outage degradation. Proof of (22) is presented in Appendix 1. Once again, the exact closed form expression of $p_{o,s}^{\text{ss}}$ can be obtained by using (1) as

$$p_{o,s}^{\text{ss}} = \sum_{\ell=0}^{J} Pr(|\mathcal{J}_d| = \ell) F_{\Gamma^{\text{ss}}_{s,k^\dagger}|_{|\mathcal{J}_d|=\ell}}(\gamma_s^{\text{th}}) \tag{24}$$

3. *IC scheme:* In this case, interference is cancelled when possible. From (7), it is clear that the CDF of SNR $\Gamma_{s,k^\dagger}^{\text{ic}}$ for given decoding set $\mathcal{K}_d$ and $\gamma_s$ is given by

$$F_{\Gamma^{\text{ic}}_{s,k^\dagger}|_{|\mathcal{K}_d|,\gamma_s=w}}(x) = \left[1-\exp\left(-\frac{x}{w\Omega_{g1}}\right)\right]^{|\mathcal{K}_d|} \tag{25}$$

It should be noted that $\gamma_s$ is a function of $\mathcal{J}_d$ since the interference threshold depends on the number of nodes used for relay selection in the primary link. Using (14) for averaging, we obtain the conditional CDF as (see (26) at the bottom of the next page) where $\mathbb{A}_3(k) =$

$$\Lambda_1(j,k,x) = \frac{e^{\frac{1}{\gamma_r\Omega_{h3}}}E_1\left(\frac{1}{\gamma_r\Omega_{h3}}+\frac{kx}{\gamma\Omega_{g1}}\right)}{(\mathbb{C}_2(j,k,x))^2} - \frac{e^{\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}}}{\mathbb{C}_2(j,k,x)}\frac{E_2\left(\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}+\frac{kx}{\gamma\Omega_{g1}}\right)}{\left(\mathbb{C}_1(j,k,x)+\frac{kx\gamma_r}{\gamma\xi_{g1/h3}}\right)} - \frac{e^{\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}}}{(\mathbb{C}_2(j,k,x))^2}E_1\left(\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}+\frac{kx}{\gamma\Omega_{g1}}\right) \tag{19}$$

$$\begin{aligned}F_{\Gamma^{\text{ss}}_{s,k}|_{\gamma_s=w}}(x) &= \int_{y=0}^{\infty} Pr\left(\frac{w|g_{1k}|^2}{\gamma_r y+1}<x\big||h_{3k}|^2=y\right)f_{|h_{3k}|^2}(y)\,\mathrm{d}y = \int_{y=0}^{\infty}\left[1-\exp\left(-\frac{x(\gamma_r y+1)}{w\Omega_{g1}}\right)\right]\frac{\exp(-(y/\Omega_{h3}))}{\Omega_{h3}}\,\mathrm{d}y\\ &= 1-\frac{\exp(-(x/w\Omega_{g1}))}{1+(x\gamma_r\Omega_{h3}/w\Omega_{g1})}\end{aligned} \tag{21}$$

$$F_{\Gamma^{\text{ss}}_{s,k^\dagger}|_{|\mathcal{J}_d|}}(x) = 1-\sum_{k=1}^{K}\mathbb{A}_2(k)\left[\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{C}_1(j,1,x)e^{-\frac{j}{\overline{\gamma}_r\,\Omega_{h2}}}\Lambda_2(j,k,x)+\frac{\overline{F}_{Z|_{|\mathcal{J}_d|}}(\gamma)e^{-\frac{kx}{\gamma\Omega_{g1}}}}{\left(1+\frac{x\gamma_r}{\gamma\xi_{g1/h3}}\right)^k}\right] \tag{22}$$

$$\begin{aligned}\Lambda_2(j,k,x) &= \sum_{\mu=0}^{k-1}\frac{(2)_\mu(-1)^\mu e^{\frac{k}{\gamma_r\Omega_{h3}}}}{\mu!(\mathbb{C}_2(j,1,x))^{2+\mu}}\frac{\mathrm{E}_{k-\mu}\left(\frac{k}{\gamma_r\Omega_{h3}}+\frac{kx}{\gamma\Omega_{g1}}\right)}{\left(1+\frac{x\gamma_r}{\gamma\xi_{g1/h3}}\right)^{k-\mu-1}}\\ &\quad+\sum_{\nu=0}^{1}\frac{(k)_\nu(-1)^\nu e^{\frac{k}{\gamma_r\Omega_{h3}}\mathbb{C}_1(j,1,x)}}{\nu!(-\mathbb{C}_2(j,1,x))^{k+\nu}}\frac{\mathrm{E}_{2-\nu}\left(\frac{k}{\gamma_r\Omega_{h3}}\left(\frac{x\gamma_r}{\gamma\xi_{g1/h3}}+\mathbb{C}_1(j,1,x)\right)\right)}{\left(\frac{x\gamma_r}{\gamma\xi_{g1/h3}}+\mathbb{C}_1(j,1,x)\right)^{1-\nu}}\end{aligned} \tag{23}$$

$$F_{\Gamma^{\text{ic}}_{s,k^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x) = 1-\sum_{k=1}^{|\mathcal{K}_d|}\mathbb{A}_3(k)\left[\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\frac{je^{-\frac{j}{\overline{\gamma}_r\,\Omega_{h2}}}}{\overline{\gamma}_r\,\xi_{h2/g2}}\Lambda_3(j,k,x)+\overline{F}_{Z|_{|\mathcal{J}|}}(\gamma)e^{-\frac{kx}{\gamma\Omega_{g1}}}\right] \tag{26}$$



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$\binom{|\mathcal{K}_d|}{k}(-1)^{k-1}$ and

$$\Lambda_3(j,k,x) = e^{\frac{kxj}{\overline{\gamma}_r\,\Omega_{g1}\xi_{h2/g2}}} \frac{E_2\left(\frac{kx}{\gamma\Omega_{g1}} + \frac{jkx}{\overline{\gamma}_r\,\Omega_{g1}\xi_{h2/g2}}\right)}{\frac{1}{\gamma} + \frac{j}{\overline{\gamma}_r\,\xi_{h2/g2}}} \quad (27)$$

The outage probability expression $p^{\rm ic}_{o,s}$ is obtained by averaging (26) using (1) and (8) as

$$p^{\rm ic}_{o,s} = \sum_{\ell=0}^{J}\sum_{\zeta=0}^{K} \Pr(|\mathcal{J}_d| = \ell)\Pr(|\mathcal{K}_d| = \zeta) F_{\Gamma^{\rm ic}_{s,k^\dagger}|_{|\mathcal{J}_d|=\ell,|\mathcal{K}_d|=\zeta}}(\gamma_s^{\rm th}) \quad (28)$$

4. *Hybrid scheme:* The HS selects user $k^\dagger$ by considering users from both $\mathcal{K}_d$ and $\chi$. The SINR/SNR of $k^\dagger$ is given in (9). The SNR $\Gamma^{\rm ic}_{s,k_1^\dagger|\mathcal{K}_d}$ and SINR $\Gamma^{\rm ss}_{s,k_2^\dagger|\chi}$ are correlated because both are dependent on the random variable $\gamma_s$ and cardinality of $\mathcal{K}_d$. We can obtain the conditional CDF of $\Gamma^{\rm hs}_{s,k^\dagger}$ as (see (29)) where $\mathbb{A}_4(k_2) = \binom{|\chi|}{k_2}(-1)^{k_2-1}$. Using (14), we average (29) over $\gamma_s$ for given $\mathcal{J}_d$ and obtain the conditional CDF $F_{\Gamma^{\rm hs}_{s,k^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x)$ as

$$F_{\Gamma^{\rm hs}_{s,k^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x) = \int_{w=0}^{\infty} F_{\Gamma^{\rm hs}_{s,k^\dagger}|_{\gamma_s=w,|\mathcal{K}_d|}}(x) f_{\gamma_s|_{|\mathcal{J}_d|}}(w)\,{\rm d}w \quad (30)$$

The closed form expression can be obtained directly from (22) and (26) as (see (31)) where $F_{\Gamma_{s,k^\dagger}|_{|\mathcal{J}_d|,|\chi|}}(x)$ is derived similar to (22) and $\Lambda_3(j,\ k_1,\ k_2,\ x)$ is readily obtained similar to $\Lambda_2(j, k, x)$ as (see (32)). Hence we can obtain the closed form expression as

$$p^{\rm hs}_{o,s} = \sum_{\ell=0}^{J}\sum_{\zeta=0}^{K} \Pr(|\mathcal{J}_d| = \ell)\Pr(|\mathcal{K}_d| = \zeta) F_{\Gamma^{\rm hs}_{s,k^\dagger}|_{|\mathcal{J}_d|=\ell,|\mathcal{K}_d|=\zeta}}(\gamma_s^{\rm th}) \quad (33)$$

The HS outperforms the other schemes in terms of outage probability since the SNR/SINR with the HS scheme is the best of SNR of IC and SINR of SS schemes.

### 3.1.3 Asymptotic outage probability expressions:

The BS transmit power $P_s$ for transmission in the second phase is determined by (4). When $P$ is large, or when BS is located far away from PU, it is clear that $P_s = \mathcal{I}_{\rm pu}\sigma^2/|g_2|^2$ is a reasonable approximation. This asymptotic outage,

$$\begin{aligned} F_{\Gamma^{\rm hs}_{s,k^\dagger}|_{\gamma_s=w,|\mathcal{K}_d|}}(x) &= F_{\Gamma^{\rm ic}_{k_1^\dagger}|_{\gamma_s=w,|\mathcal{K}_d|}}(x) F_{\Gamma^{\rm ss}_{s,k_2^\dagger}|_{\gamma_s=w,|\chi|}}(x) = \left[1 - e^{-\frac{x}{w\Omega_{g1}}}\right]^{|\mathcal{K}_d|} \left[1 - \frac{e^{-\frac{x}{w\Omega_{g1}}}}{1 + \frac{x\gamma_r\Omega_{h3}}{w\Omega_{g1}}}\right]^{|\chi|} \\ &= 1 - \sum_{k_1=1}^{|\mathcal{K}_d|}\mathbb{A}_3(k_1) e^{-\frac{k_1 x}{w\Omega_{g1}}} - \sum_{k_2=1}^{|\chi|}\mathbb{A}_4(k_2)\frac{e^{-\frac{k_2 x}{w\Omega_{g1}}}}{\left(1 + \frac{x\gamma_r\Omega_{h3}}{w\Omega_{g1}}\right)^{k_2}} + \sum_{k_1=1}^{|\mathcal{K}_d|}\sum_{k_2=1}^{|\chi|}\mathbb{A}_3(k_1)\mathbb{A}_4(k_2)\frac{e^{-\frac{(k_1+k_2)x}{w\Omega_{g1}}}}{\left(1 + \frac{x\gamma_r\Omega_{h3}}{w\Omega_{g1}}\right)^{k_2}} \end{aligned} \quad (29)$$

$$\begin{aligned} F_{\Gamma^{\rm hs}_{s,k^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x) &= F_{\Gamma^{\rm ic}_{s,k_1^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x)) + F_{\Gamma^{\rm ss}_{s,k_2^\dagger}|_{|\mathcal{J}_d|,|\chi|}}(x)) - 1 \\ &+ \sum_{k_1=1}^{|\mathcal{K}_d|}\sum_{k_2=1}^{|\chi|}\mathbb{A}_3(k_1)\mathbb{A}_4(k_2)\left[\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{C}_1(j,1,x)\exp\left(-j\frac{1}{\overline{\gamma}_r\,\Omega_{h2}}\right)\Lambda_3(j,k_1,k_2,x) + \overline{F}_{Z_{|\mathcal{J}_d|}}(\gamma)\frac{e^{-\frac{(k_1+k_2)x}{\gamma\Omega_{g1}}}}{\left(1 + \frac{x\gamma_r}{\gamma\xi_{g1/h3}}\right)^{k_2}}\right] \end{aligned} \quad (31)$$

$$\begin{aligned} \Lambda_4(j,k_1,k_2,x) &= \sum_{\mu=0}^{k_2-1}\frac{(2)_\mu(-1)^\mu e^{\frac{k_1+k_2}{\gamma_r\Omega_{h3}}}}{\mu!(\mathbb{C}_2(j,1,x))^{2+\mu}}\frac{E_{k_2-\mu}\left(\frac{(k_1+k_2)}{\gamma_r\Omega_{h3}} + \frac{(k_1+k_2)x}{\gamma\Omega_{g1}}\right)}{\left(1 + \frac{x\gamma_r}{\gamma\xi_{g1/h3}}\right)^{k_2-\mu-1}} \\ &+ \sum_{\nu=0}^{1}\frac{(k_2)_\nu(-1)^\nu}{\nu!}\frac{e^{\frac{(k_1+k_2)}{\gamma_r\Omega_{h3}}\mathbb{C}_1(j,1,x)}}{(-\mathbb{C}_2(j,1,x))^{k_2+\nu}}\frac{E_{2-\nu}\left(\frac{(k_1+k_2)}{\gamma_r\Omega_{h3}}\left(\frac{x\gamma_r}{\gamma\xi_{g1/h3}} + \mathbb{C}_1(j,1,x)\right)\right)}{\left(\frac{x\gamma_r}{\gamma\xi_{g1/h3}} + \mathbb{C}_1(j,1,x)\right)^{1-\nu}} \end{aligned} \quad (32)$$

denoted by $p_{o,s}^{\text{asy}}$, can be obtained from by using $P \to \infty$

$$p_{o,s}^{\text{asy}} = \lim_{\gamma\to\infty} p_{o,s} \quad (34)$$

For each scheme, the CDF of SINR is easily deduced from (18), (22), (26) and (29) by taking $\gamma \to \infty$, respectively, as (see (35)–(38)) where $\Lambda_i^{\text{asy}}(j,k,x) = \lim_{\gamma\to\infty} \Lambda_i(j,k,x)$ for $i = 1$, 2 and 3 and $\Lambda_4^{\text{asy}}(j,k_1,k_2,x) = \lim_{\gamma\to\infty} \Lambda_4(j,k_1,k_2,x)$ that are directly obtained from (19), (23), (27) and (32) by substituting $1/\gamma = 0$ as (see 39)

$$\Lambda_3^{\text{asy}}(j,k,x) = \mathrm{e}^{\frac{kxj}{\overline{\gamma}_r\,\Omega_{g1}\xi_{h2/g2}}}\mathrm{E}_2\left(\frac{jkx}{\overline{\gamma}_r\,\Omega_{g1}\xi_{h2/g2}}\right), \quad \text{and}$$

(see 40)

It is observed that the outage probability exhibits a floor, and the asymptotic value can be used in system design. We can easily observe that $\Lambda_2^{\text{asy}}(j,k,x) > \Lambda_1^{\text{asy}}(j,k,x)$ for $K > 1$ because the interference term $1/\gamma_r\Omega_{h3}$ is scaled by $k$ in SS scheme. It can be verified that $\Lambda_2^{\text{asy}}(j,k,x) = \Lambda_1^{\text{asy}}(j,k,x)$ for $K = 1$. Intuitively, the IC scheme is superior to GS and SS schemes. However, the HS scheme delivers the best performance as this selects the best SNR/SINR. The asymptotic outage probabilities for respective schemes are obtained by substituting (35)–(38) into (20), (24), (28) and (33). It is noted that in all these expressions, $\overline{F}_{Z|_{|\mathcal{J}_d|}}(\gamma) \to 0$ as $\gamma \to \infty$.

It is not possible to derive exact expressions for the optimum choice of $\alpha$ from the outage expressions. The optimum value needs to be determined numerically as discussed in Section 4 so as to guarantee some primary outage performance.

### 3.2 Achievable ergodic rate

In this section, we analyse the achievable ergodic rate of the secondary network for each selection scheme. Here, we use SINR approximations for providing insightful results. $R_s^{\text{po}}$ and $R_s^{\text{pno}}$ denote the achievable rates for PO and PNO scenarios. Hence the overall achievable ergodic rate of DSA network can be defined as

$$\overline{R}_s = p_{o,p}\overline{R}_s^{\text{po}} + (1 - p_{o,p})\overline{R}_s^{\text{pno}} \quad (41)$$

where $\overline{R}_s^{\text{po}} = \mathbb{E}[R_s^{\text{po}}]$ and $\overline{R}_s^{\text{pno}} = \mathbb{E}[R_s^{\text{pno}}]$ are the ergodic rates

$$F^{\text{asy}}_{\Gamma^{\text{gs}}_{s,k^\dagger}|_{|\mathcal{J}_d|}}(x) = 1 - \sum_{k=1}^{K}\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{A}_2(k)\mathbb{C}_1(j,k,x)\mathrm{e}^{-\frac{j}{\overline{\gamma}_r\,\Omega_{h2}}}\Lambda_1^{\text{asy}}(j,k,x) \quad (35)$$

$$F^{\text{asy}}_{\Gamma^{\text{ss}}_{s,k^\dagger}|_{|\mathcal{J}_d|}}(x) = 1 - \sum_{k=1}^{K}\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{A}_2(k)\mathbb{C}_1(j,1,x)\mathrm{e}^{-\frac{j}{\overline{\gamma}_r\,\Omega_{h2}}}\Lambda_2^{\text{asy}}(j,k,x) \quad (36)$$

$$F^{\text{asy}}_{\Gamma^{\text{ic}}_{s,k^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x) = 1 - \sum_{k=1}^{|\mathcal{K}_d|}\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{A}_3(k)\mathrm{e}^{-\frac{j}{\overline{\gamma}_r\,\Omega_{h2}}}\Lambda_3^{\text{asy}}(j,k,x), \quad \text{and} \quad (37)$$

$$F^{\text{asy}}_{\gamma^{\text{hs}}_{k^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x) = F^{\text{asy}}_{\Gamma^{\text{ic}}_{s,k_1^\dagger}|_{|\mathcal{J}_d|,|\mathcal{K}_d|}}(x) + F^{\text{asy}}_{\Gamma^{\text{ss}}_{s,k_2^\dagger}|_{|\mathcal{J}_d|,|\chi|}}(x) - 1 + \sum_{k_1=1}^{|\mathcal{K}_d|}\sum_{k_2=1}^{|\chi|}\mathbb{A}_3(k_1)\mathbb{A}_4(k_2)\left[\sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathbb{C}_1(j,1,x)\mathrm{e}^{-\frac{j}{\overline{\gamma}_r\,\Omega_{h2}}}\Lambda_4^{\text{asy}}(j,k_1,k_2,x)\right] \quad (38)$$

$$\Lambda_1^{\text{asy}}(j,k,x) = \frac{\mathrm{e}^{\frac{1}{\gamma_r\Omega_{h3}}}\mathrm{E}_1\left(\frac{1}{\gamma_r\Omega_{h3}}\right)}{(-1+\mathbb{C}_1(j,k,x))^2} + \frac{\mathrm{e}^{\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}}}{(1-\mathbb{C}_1(j,k,x))}\frac{\mathrm{E}_2\left(\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}\right)}{\mathbb{C}_1(j,k,x)} - \frac{\mathrm{e}^{\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}}}{(1-\mathbb{C}_1(j,k,x))^2}\mathrm{E}_1\left(\frac{\mathbb{C}_1(j,k,x)}{\gamma_r\Omega_{h3}}\right)$$

$$\Lambda_2^{\text{asy}}(j,k,x) = \sum_{\mu=0}^{k-1}\frac{(2)_\mu(-1)^\mu\mathrm{e}^{\frac{k}{\gamma_r\Omega_{h3}}}\mathrm{E}_{k-\mu}\left(\frac{k}{\gamma_r\Omega_{h3}}\right)}{\mu!(-1+\mathbb{C}_1(j,1,x))^{2+\mu}} + \sum_{\nu=0}^{1}\frac{(k)_\nu(-1)^\nu\mathrm{e}^{\frac{k}{\gamma_r\Omega_{h3}}\mathbb{C}_1(j,1,x)}}{\nu!(1-\mathbb{C}(j,1,x))^{k+\nu}}\frac{\mathrm{E}_{2-\nu}\left(\frac{k}{\gamma_r\Omega_{h3}}\mathbb{C}_1(j,1,x)\right)}{\left(\mathbb{C}_1(j,1,x)\right)^{1-\nu}} \quad (39)$$

$$\Lambda_4^{\text{asy}}(j,k_1,k_2,x) = \sum_{\mu=0}^{k2-1}\frac{(2)_\mu(-1)^\mu\mathrm{e}^{\frac{(k_1+k_2)}{\gamma_r\Omega_{h3}}}}{\mu!(-1+\mathbb{C}_1(j,1,x))^{2+\mu}}\mathrm{E}_{k_2-\mu}\left(\frac{k_1+k_2}{\gamma_r\Omega_{h3}}\right) + \sum_{\nu=0}^{1}\frac{(k_2)_\nu(-1)^\nu\mathrm{e}^{\frac{(k_1+k_2)}{\gamma_r\Omega_{h3}}\mathbb{C}_1(j,1,x)}}{\nu!(1-\mathbb{C}_1(j,1,x))^{k_2+\nu}} \times \frac{\mathrm{E}_{2-\nu}\left(\frac{(k_1+k_2)}{\gamma_r\Omega_{h3}}\mathbb{C}_1(j,1,x)\right)}{(\mathbb{C}_1(j,1,x))^{1-\nu}} \quad (40)$$



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for PO and PNO scenarios. In the following section, we explicitly derive the ergodic rate for each selection scheme.

*3.2.1 Ergodic rate for PO case:* The achievable rate $R_s^{\text{po}} = (1-\alpha)\log_2(1+\Gamma_{s,k^\dagger}^{\text{po}})$, where $\Gamma_{s,k^\dagger}^{\text{po}} = \gamma|g_{1k^\dagger}|^2$ from (10). The ergodic rate can be derived by $\overline{R}_s^{\text{po}} = (1-\alpha)\,\mathbb{E}[\log_2(1+\gamma|g_{1k^\dagger}|^2)]$ where $1-\alpha$ is the time duration for second phase. Clearly small time duration of first phase results higher achievable rate in second phase. We approximate $1+\gamma|g_{1k^\dagger}|^2$ as $\gamma|g_{1k^\dagger}|^2$ for further analysis. Hence the approximate ergodic rate is obtained as

$$\overline{R}_s^{\text{po}} \simeq (1-\alpha)\log_2(\gamma) + (1-\alpha)\mathbb{E}[\log_2(|g_{1k^\dagger}|^2)]$$

Since the PDF of $|g_{1k^\dagger}|^2$ is $f_{|g_{1k^\dagger}|^2}(x) = K[F_{|g_{1k}|^2}(x)]^{K-1} f_{|g_{1k}|^2}(x)$, the closed form expression of ergodic rate is given by

$$\overline{R}_s^{\text{po}} \simeq \log_2(\gamma) - \frac{\gamma_e}{ln2} - \sum_{k=1}^{K}\mathbb{A}_2(k)\left(\log_2\left(\frac{k}{\Omega_{g1}}\right)\right) \quad (42)$$

where Euler–Mascheroni constant $\gamma_e = 0.57722$. The proof is straightforward except the integration $\int_0^\infty \log_e(x)\, \mathrm{e}^{-(k+1)x/\Omega_{g1}}\,\mathrm{d}x$, which is solved by using formula given in [17, eq. (4.331)]. From (42), it is apparent that the ergodic rate $\overline{R}_s^{\text{po}}$ monotonically increases with $\gamma$ and $K$. In the next section, we derive the ergodic rate in PNO scenario for various user selection schemes.

*3.2.2 Ergodic rate for PNO case:*

1. *GS scheme:* The SINR $\Gamma_{s,k^\dagger}^{\text{gs}}$ using GS scheme is given by (5). The ergodic rate is seen to be $\overline{R}_s^{\text{gs}} = \mathbb{E}[(1-\alpha)\log_2(1+\Gamma_{s,k^\dagger}^{\text{gs}})]$. In high SINR regime we approximate the expression as

$$\overline{R}_s^{\text{gs}} \simeq (1-\alpha)\mathbb{E}\left[\log_2\left(\frac{\max_{k\in\mathcal{K}}\gamma_s|g_{1k}|^2}{\gamma_r|h_{3k^\dagger}|^2+1}\right)\right]$$

by assuming $1+\Gamma_{s,k^\dagger}^{\text{gs}} \simeq \Gamma_{s,k^\dagger}^{\text{gs}}$. Clearly random variable $\gamma_s$ is dependent on the cardinality of decoding set $|\mathcal{J}_d|$, therefore the ergodic rate for given $|\mathcal{J}_d|$ is obtained as (see (43)).

The simplified expression of each term can be obtained as (see (44))
where $\rho = \gamma_{p2}^{\text{th}}\gamma\Omega_{g2}/\gamma_r\Omega_{h2}$. In high SNR regime (high $\gamma$), $\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|}]$ becomes independent of $\gamma$ as $\gamma_s = \min(Z,\gamma)$. Proof is presented in Appendix 2. It is also shown that

$$\mathbb{E}\left[\log_2(\max_{k\in\mathcal{K}}|g_{1k}|^2)\right] = -\sum_{k=1}^{K}\mathbb{A}_2(k)\left(\log_2\left(\frac{k}{\Omega_{g1}}\right)+\frac{\gamma_e}{ln2}\right) \quad (45)$$

and

$$\mathbb{E}\left[\log_2\left(\gamma_r|h_{3k^\dagger}|^2+1\right)\right] = \exp\left(\frac{1}{\gamma_r\Omega_{h3}}\right)\frac{\mathrm{E}_1(1/(\gamma_r\Omega_{h3}))}{\ln 2} \simeq \log_2(\gamma_r\Omega_{h3}) - \frac{\gamma_e}{ln2} \quad (46)$$

The closed form of approximate expression after averaging for $|\mathcal{J}_d|$ can be obtained as

$$\overline{R}_s^{\text{gs}} \simeq \sum_{\ell=0}^{J}\Pr(|\mathcal{J}_d|=\ell)\,\overline{R}_{s|_{|\mathcal{J}_d|=\ell}}^{\text{gs}} \quad (47)$$

We note that $\gamma_s$, which is the only function of $\gamma$ in the SNR becomes independent of it for high $\gamma$. Saturation of the ergodic rate is therefore expected at high SNR. From (46), we observe that the closeness of sets $\mathcal{J}$ and $\mathcal{K}$ leads to impairment in the secondary rate obtained from (47).

2. *SS scheme:* Similarly, using second equation of SINR from (5) for SS scheme, we obtain the conditional approximated expression of ergodic rate as (see (48)). The closed form of $\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|}]$ is obtained from (44) and the expectation of logarithm of maximum of $K$ SINRs is given by (see (49)) where $\psi^{(0)}(k)$ is digamma function. Derivation is presented in Appendix 2. For this, we use the approximation for each SINR as $|g_{1k}|^2/(\gamma_r|h_{3k}|^2+1) \simeq |g_{1k}|^2/\gamma_r|h_{3k}|^2$. The closed form expression of approximated ergodic rate after averaging as follows

$$\overline{R}_s^{\text{ss}} \simeq \sum_{\ell=0}^{J}\Pr(|\mathcal{J}_d|=\ell)\,\overline{R}_{s|_{|\mathcal{J}_d|=\ell}}^{\text{ss}} \quad (50)$$

The superiority of SS scheme over GS can be observed by comparing $\mathbb{E}\left[\log_2\left(\max_{k\in\mathcal{K}}\frac{|g_{1k}|^2}{\gamma_r|h_{3k}|^2+1}\right)\right]$ and $\mathbb{E}\left[\log_2\left(\max_{k\in\mathcal{K}}|g_{1k}|^2\right)\right] - \mathbb{E}[\log_2(\gamma_r|h_{3k^\dagger}|^2+1)]$.

$$\overline{R}_{s|_{|\mathcal{J}_d|}}^{\text{gs}} \simeq (1-\alpha)\left[\mathbb{E}[\log_2(\gamma_s)|_{|J_d|}] + \mathbb{E}[\log_2(\max_{k\in\mathcal{K}}|g_{1k}|^2)] - \mathbb{E}\left[\log_2\left(\gamma_r\left|h_{3k^\dagger}\right|^2+1\right)\right]\right] \quad (43)$$

$$\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|}] = \sum_{j=1}^{|\mathcal{J}_d|}\mathbb{A}_1(j)\mathrm{e}^{-\frac{j}{\gamma_r\Omega_{h2}}}[\log_2(\gamma) - \log_2(1+j\rho)] \quad (44)$$

$$\overline{R}_{s|_{|\mathcal{J}_d|}}^{\text{ss}} \simeq (1-\alpha)\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|}] + (1-\alpha)\mathbb{E}\left[\log_2\left(\max_{k\in\mathcal{K}}\frac{|g_{1k}|^2}{\gamma_r|h_{3k}|^2+1}\right)\right] \quad (48)$$

$$\mathbb{E}\left[\log_2\left(\max_{k\in\mathcal{K}}\frac{|g_{1k}|^2}{\gamma_r|h_{3k}|^2+1}\right)\right] \simeq -\log_2\left(\frac{\gamma_r}{\xi_{g1/h3}}\right) - \frac{\gamma_e}{\ln 2} - \sum_{k=1}^{K}\mathbb{A}_2(k)\frac{\psi^{(0)}(k)}{\ln 2} \quad (49)$$

 

3. *IC scheme:* Similarly, the approximated conditional ergodic rate for IC scheme where SNR is obtained from (7), is given by (see (51)). From (44), the closed form expression of $\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|=\ell}]$ is obtained. We can derive the closed form of second term using (45) as (see (52))

Hence the closed form expression of ergodic rate after averaging (51) is obtained by

$$\overline{R}_s^{\text{ic}} \simeq \sum_{\ell=0}^{J}\sum_{\zeta=0}^{K} \Pr(|\mathcal{J}_d| = \ell)\Pr(|\mathcal{K}_d| = \zeta)\,\overline{R}^{\text{ic}}_{s|_{|\mathcal{J}_d|=\ell, |\mathcal{K}_d|=\zeta}} \quad (53)$$

From the analysis, we can easily determine that the ergodic rate of the DSA system saturates at high $\gamma$. However parameters such as number of nodes, primary transmit power and relay power shift the saturation level. The ergodic rate for HS scheme is straightforward and can be easily obtained from the SS and IC schemes by using SINR given in (9).

## 4 Simulation results and discussion

In this section the analytical plots of outage are compared with those obtained by computer simulations. We assume $R_p = R_s = 1$ bps/Hz. The variance of AWGN noise is assumed to be unity. We use path-loss Rayleigh fading channel models so that channel variances are given by $\Omega = 1/d^{\epsilon}$ for nodes in $\{h_1, h_2, g_1, g_2, h_3\}$ where $d$ is the distance between the nodes, and $\epsilon$ is path loss exponent ($\epsilon = 3.2$ assumed). We assume $d_{g_1} = 1$, $d_{h_3} = 1.5$ and $d_{g_2} = 2$. For obtaining the DSA network outage probability $p_{o,s}$, we substitute (12), (14) and outage probability for each selection scheme $p_{o,s}^{\text{pno}}$ when primary is not in outage into (11). We assume a primary outage probability target of $10^{-3}$. For obtaining the optimal value of $\alpha$ for each scenario, we use the following steps:

1. A range of $\alpha$ that is $(\alpha_1^\star, \alpha_2^\star)$ is obtained that satisfies the target outage probability of primary.
2. The optimal value of $\alpha$ is obtained as

$$\alpha^\star = \min_{\alpha \in (\alpha_1^\star, \alpha_2^\star)} p_{o,s}(\alpha) \quad (54)$$

In Fig. 2, we plot the primary outage probability against $\alpha$. It can be seen that the primary outage is within a target of $10^{-3}$ when $\alpha$ takes values between $\alpha_1^\star = 0.28$ and $\alpha_2^\star = 0.71$. In the same figure, secondary outage probabilities for various selection schemes are also indicated. We need to choose a value of $\alpha$ in this range for which the secondary outage is minimum. It can be seen that the $\alpha$ for optimum outage of IC and HS schemes are close to $\alpha$ for optimum primary outage. It can be seen that for the parameters chosen, the best value of $\alpha$ for HS and IC

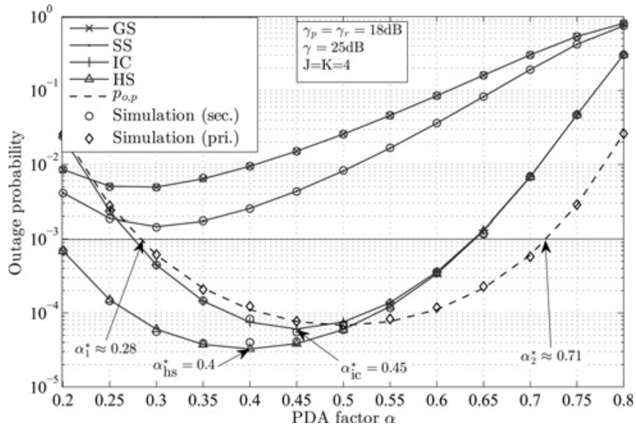


**Fig. 2** *Illustration of outage probabilities against PDA factor* $\alpha$

schemes are $\alpha_{\text{hs}}^\star = 0.4$ and $\alpha_{\text{ic}}^\star = 0.45$. In subsequent experiments, the optimum values of $\alpha$ is calculated for all selection schemes (for various parameters) and used in all simulations.

Fig. 3 demonstrates the performance of the DSA system for best PDA factor for each SNR $\gamma$ where nodes $\mathcal{J}$ are comparatively close to BS so that $d_{h_1} = 0.8$ and $d_{h_2} = 1.2$. For this location of relays, we obtain the range of $\alpha$ satisfying primary outage threshold criteria as $\alpha_1^\star = 0.25$ and $\alpha_2^\star = 0.65$. The close agreement between exact outage and asymptotic outage at high $\gamma$ is demonstrated. It can be seen that outage performance of the secondary always exhibits flooring because of interference from $\mathcal{J}$ to SUs and constrained power $P_s$ to limit the interference at PU. The flooring in IC plot is because of constrained $P_s$.

Fig. 4 illustrates the outage performances for various combination secondary nodes $J$ and $K$ for best possible values of $\alpha$. For fixed size of $\mathcal{J}$, the plot clearly shows that as expected intuitively, performance improves with

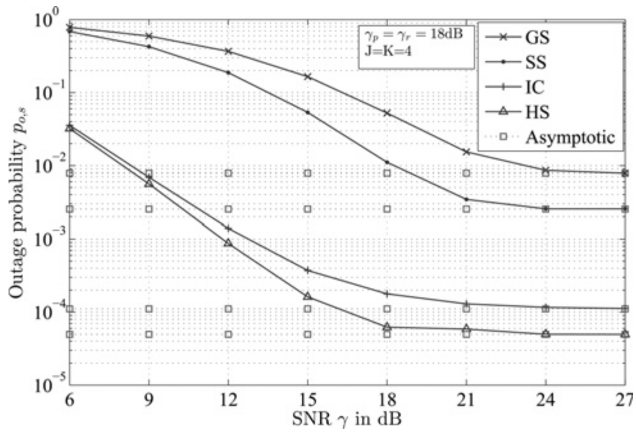


**Fig. 3** *Secondary outage probability* $p_{o,s}$ *against SNR* $\gamma$ *in dB where the best possible PDA factor* $\alpha$ *for each scheme is taken*

$$\overline{R}^{\text{ic}}_{s|_{|\mathcal{J}_d|=\ell, |\mathcal{K}_d|=\zeta}} \simeq (1-\alpha)\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|=\ell}] + (1-\alpha)\mathbb{E}\left[\log_2\left(\max_{k\in\mathcal{K}_d}|g_{1k}|^2\right)|_{|\mathcal{K}_d|=\zeta}\right] \quad (51)$$

$$\mathbb{E}\left[\log_2\left(\max_{k\in\mathcal{K}_d}|g_{1k}|^2\right)|_{|\mathcal{K}_d|=\zeta}\right] = \sum_{k=1}^{\varsigma}\binom{\zeta}{k}(-1)^k\left(\log_2\left(\frac{k}{\Omega_{g1}}\right) + \frac{\gamma_e}{\ln 2}\right) \quad (52)$$



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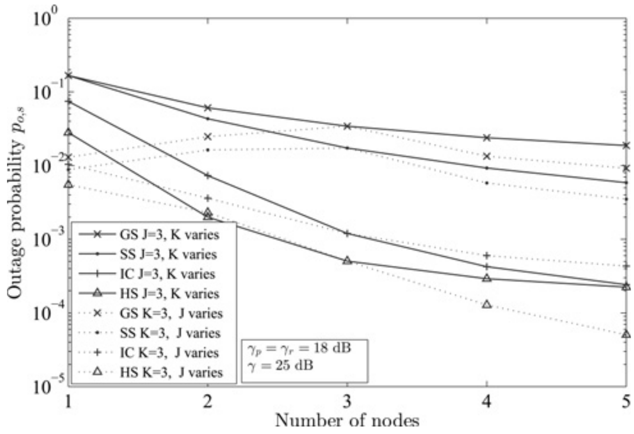


**Fig. 4** *Secondary outage probability $p_{o,s}$ against number of secondary nodes*

increasing $K$. However the constant $J$ limits $P_s$, so that saturation in outage is observed. For fixed size of $\mathcal{K}$, the outage probability of GS and SS schemes increase first then decrease. When $J$ is small, the primary is in outage more often, and the secondary uses full power. For small increase in $J$, the advantage accrued by increased interference threshold does not compensate for the fact that the primary is less often in outage. However, for larger $J$, the increase in transmit power because of larger $J$ makes up for this. Note that since IC and HS use IC, they do not show this trend with increase in $J$. We note that this observation is a direct result of the fact that $\alpha$ is chosen to keep primary outage below a threshold and minimise secondary outage, and not fixed to a value.

In Fig. 5, we demonstrate the outage performance of the secondary system for the best possible $\alpha$. For fixed secondary number of nodes $M = 8$, secondary outage is minimised when both $J$ and $K$ are large. Intuitively, this is because having a larger $J$ and $K$ ensures a higher SNR $\gamma_s$ and large gain because of selection. Moreover, the impact of distances on outage performance of secondary network is presented in Fig. 6. In this figure we have evaluated the impact of distances for the HS scheme scenario that has been presented in the paper. Firstly, the distance between cluster $\mathcal{J}$ and cluster $\mathcal{K}$ is not of much consequence when the users in set $\mathcal{K}$ employ IC. This is also true of the HS

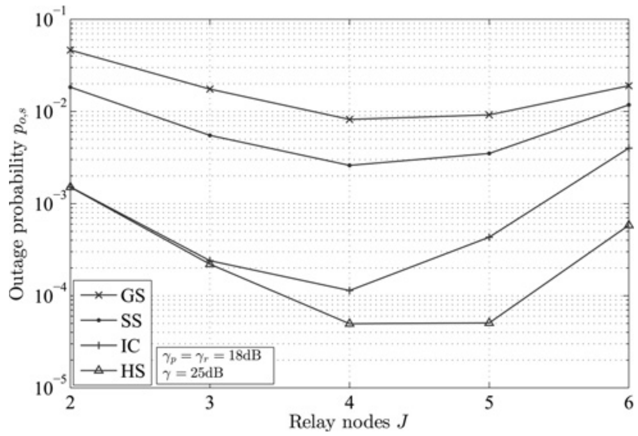


**Fig. 5** *Secondary outage probability $p_{o,s}$ against number of nodes $J$ with consideration of the best possible PDA factor $\alpha$ for each scheme*

Here total $J + K = 8$ secondary nodes are considered

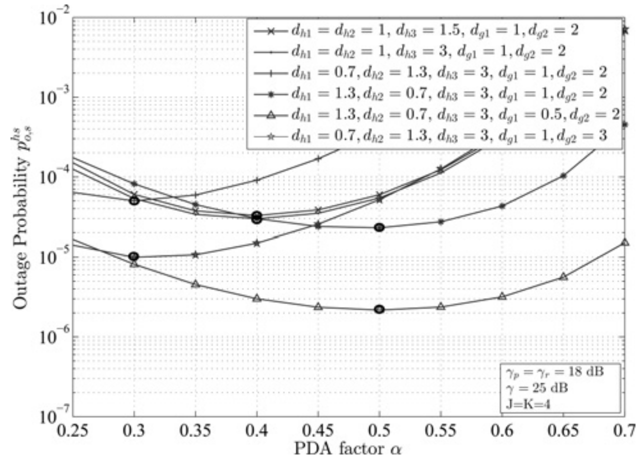


**Fig. 6** *Illustration of secondary outage probability $p_{o,s}^{\text{hs}}$ against PDA factor $\alpha$ for demonstrating the impact of distances*

For this, the optimal $\alpha_{\text{hs}}^{\star}$ for each case is also presented

case that is plotted in the figure (see first two plots of the figure as per the legend). Second, performance will always improve as cluster $\mathcal{K}$ moves closer to the BS (see the fourth and fifth plot as per the legend). Third, because of interference from the BS to PU in the second hop of the primary, and because the power used by the BS is a random variable, secondary performance is not optimised by location of relay cluster exactly in between the BS and the PU (see second and fourth plot as per the legend). Fourth, secondary performance improves when PU moves away from the BS (decreased interference implies larger power availability at the BS in the second phase). This is brought out clearly in the third and the sixth plot of the figure (as per the legend).

In Fig. 7 approximate ergodic rate are plotted against PDA factor $\alpha$ and compared with simulations where $d_{h_1} = 0.8$ and $d_{h_2} = 1.2$. It can be seen that simulations agree with the derived expressions for GS and IC schemes. However the SS scheme shows comparatively loose agreement with simulation as the multiple approximations are used for providing the insightful rate expression. All schemes show intuitive results and favor large second phase duration time. IC scheme suggests higher $\alpha$ than GS and SS schemes since a larger $\alpha$ facilitates IC. Note that performance of all schemes decreases with respect to $\alpha$ that falls within the range [0.25, 0.65] that guarantees primary outage

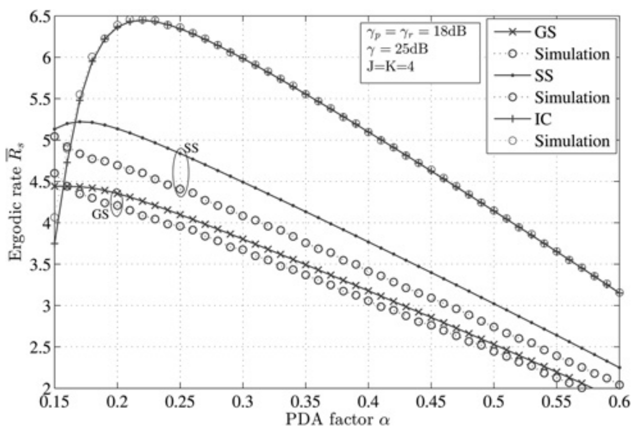


**Fig. 7** *Illustration of ergodic rate $\overline{R}_s$ against PDA factor $\alpha$ with $d_{h_1} = 0.8$ and $d_{h_2} = 1.2$*

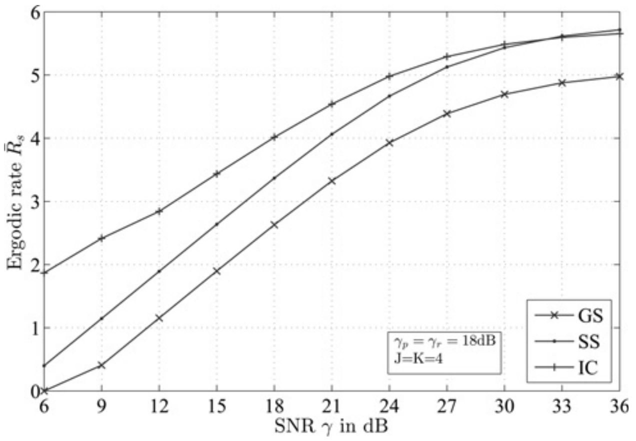


**Fig. 8** *Ergodic rate $\overline{R}_s$ against SNR $\gamma$ in dB for the best possible PDA factor $\alpha$ for each SNR with $d_{h_1} = 0.8$ and $d_{h_2} = 1.2$*

performance. In Fig. 8, we demonstrate the ergodic rate $\overline{R}_s$ against SNR $\gamma$ in dB for the best possible $\alpha$. The rate performance of SS scheme approaches the performance of IC scheme in high SNR $\gamma$ regime. Such type of performance gain is achieved only by considering the apportioned $\alpha$ according to the best secondary outage in range of primary outage constraints.

## 5 Conclusion

We proposed a new CM scheme for DL transmission where the BS uses DSA principles to multiplex symbols to a selected user located close to it, while guaranteeing performance of the distant user served by a selected relay. Expressions were derived for outage and ergodic rate performance with various user selection schemes. It was shown that duration of the two primary hops should be carefully chosen for the best secondary outage performance.

## 6 Acknowledgment

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## 8 Appendix

### 8.1 Appendix 1: Derivation of CDF and PDF of Z given in (15) and (16)

We first note that $|h_{2j}^{\star}|^2$ is the maximum of $|\mathcal{J}_d|$ independent and identical exponentially distributed random variables, each with CDF $\Pr(|h_{2j}| < x) = 1 - \mathrm{e}^{-x/\Omega_{h2}}$. Clearly, $F_{|h_{2j^\star}|^2|_{|\mathcal{J}_d|=\ell}}(x) = [\Pr(|h_{2j}| < x)]^{\ell}$ using property of order statistics [18, eq. (2.1.1)], which on using the binomial expansion results in

$$F_{|h_{2j^\star}|^2|_{|\mathcal{J}_d|=\ell}}(x) = 1 - \sum_{j=1}^{\ell} \binom{\ell}{j} (-1)^{j-1} \mathrm{e}^{-jx/\Omega_{h2}}$$

Denote by $V = \mathcal{I}_{\text{pu}} = \overline{\gamma}_r\ |h_{2j^\star}|^2 - 1$ the right hand side of (3). We obtain CDF of $V$ for $V = \mathcal{I}_{\text{pu}} > 0$ as

$$\begin{aligned} F_{V|_{|\mathcal{J}_d|}}(v) &= \Pr\left(V < v \middle| |h_{2j^\star}|^2 > \frac{1}{\overline{\gamma}_r}\right) \\ &= \frac{\Pr(V < v) - \Pr\left(|h_{2j^\star}|^2 < \frac{1}{\overline{\gamma}_r}\right)}{\Pr\left(|h_{2j^\star}|^2 > \frac{1}{\overline{\gamma}_r}\right)} \\ &= \frac{F_{|h_{2j^\star}|^2}\left(\frac{v+1}{\overline{\gamma}_r}\right) - F_{|h_{2j^\star}|^2}\left(\frac{1}{\overline{\gamma}_r}\right)}{1 - F_{|h_{2j^\star}|^2}\left(\frac{1}{\overline{\gamma}_r}\right)} \end{aligned} \quad (55)$$

After substituting values of $F_{|h_{2j^\star}|^2|_{|\mathcal{J}_d|}}(x)$ into (55), we can readily obtain the CDF of $Z = V/|g_2|^2$ for given $|g_2|^2$ as $F_{Z|_{|\mathcal{J}_d|=\ell,|g_2|^2=x}}(z) = \Pr(V < zx|_{|\mathcal{J}_d|=\ell,|g_2|^2=x})$. Since the channel gain $|g_2|^2 \sim \mathrm{e}^{-x/\Omega_{g2}}/\Omega_{g2}$, $F_{Z|_{|\mathcal{J}_d|=\ell}}(z) = \int_0^{\infty} F_{V|_{|\mathcal{J}_d|=\ell,|g_2|^2=x}}(zx) f_{|g_2|^2}(x)\,\mathrm{d}x$ can be written as (see (56))

We can solve $\mathbb{I}_1$ by using [17, eq. (3.310)] and obtain

$$\mathbb{I}_1 = \left[\frac{jz}{\overline{\gamma}_r\,\Omega_{h2}} + \frac{1}{\Omega_{g2}}\right]^{-1}$$

which can be substituted back into (56) to obtain (15). We derive (16) using $f_{Z||\mathcal{J}_d|=\ell}(z) = \mathrm{d}F_{Z||\mathcal{J}_d|=\ell}/\mathrm{d}z$.

### 8.1.1 Appendix 1: Proof of (18)

We first derive the CDF of the SINR $\Gamma^{\text{gs}}_{s,k^\dagger}$ presented in (5). To this end, we first write the CDF for given $\gamma_s$ and $|h_{3k^\dagger}|^2$ as $F_{\Gamma^{\text{gs}}_{s,k^\dagger}|_{\gamma_s=w,|h_{3k^\dagger}|^2=y}}(x) = \left[\Pr\left(|g_{1k}|^2 < \frac{x(\gamma_r y+1)}{w}|\gamma_s = w, |h_{3k^\dagger}|^2 = y\right)\right]^K$ considering the conditional gains at nodes in $\mathcal{K}$ are independent and identically distributed. Using $\Pr(|g_{1k}|^2 < x) = 1 - \mathrm{e}^{-x/\Omega_{g1}}$ we have

$$F_{\Gamma^{\text{gs}}_{s,k^\dagger}|_{\gamma_s=w,|h_{3k^\dagger}|^2=y}}(x) = \left[1 - \mathrm{e}^{-\frac{x(\gamma_r y+1)}{w\Omega_{g1}}}\right]^K$$

which can be averaged over the CDFs of variables $\gamma_s$ and $|h_{3k^\dagger}|^2$ to give (see (57))

After using the binomial expansion of $\left[1 - \mathrm{e}^{-\frac{x(\gamma_r y+1)}{w\Omega_{g1}}}\right]^K$ we obtain (see (58))

We first integrate over $|h_{3k^\dagger}|^2$ using $f_{|h_{3k^\dagger}|^2}(y) = \mathrm{e}^{-y/\Omega_{h3}}/\Omega_{h3}$ and the relation [17, eq. (3.310)] to obtain (see (59))

which is further averaged by using $f_{\gamma_s||\mathcal{J}_d|}(w)$ from (14) as (see (60))

$\text{II}_2$ is solved using $\int_\gamma^\infty f(w)\delta(w-\gamma)\,\mathrm{d}w = f(\gamma)$. For $\text{II}_1$, we substitute $f_{Z||\mathcal{J}_d|}(w)$ from (16) and use the following partial fraction expansion (see (61))
For integrating each term we use [19, eq. (5.1.4)], and obtain (18) after rearranging the variables.

### 8.1.2 Appendix 1: Proof of (22)

To find the closed form expression in (22), we first write the conditional CDF of $\Gamma^{\text{ss}}_{s,k^\dagger}$ for given $\gamma_s$ as

$$F_{\Gamma^{\text{ss}}_{s,k^\dagger}|_{\gamma_s=w}}(x) = [F_{\Gamma_{s,k}|_{\gamma_s=w}}(x)]^K$$

After substituting $F_{\Gamma_{s,k}|_{\gamma_s=w}}(x)$ from (21), we use the binomial

$$F_{Z||\mathcal{J}_d|=\ell}(z) = 1 - \sum_{j=1}^{|\mathcal{J}_d|}\binom{|\mathcal{J}_d|}{j}\frac{(-1)^{j-1}\mathrm{e}^{-\frac{j}{\overline{\gamma}_r\Omega_{h2}}}}{\Pr\left(|h_{2j\star}|^2 > \frac{1}{\overline{\gamma}_r}\right)\Omega_{g2}}\overbrace{\int_{x=0}^{\infty}\mathrm{e}^{-\frac{jzx}{\overline{\gamma}_r\Omega_{h2}} - \frac{x}{\Omega_{g2}}}\,\mathrm{d}x}^{\mathbb{I}_1} \tag{56}$$

$$F_{\Gamma^{\text{gs}}_{s,k^\dagger}||\mathcal{J}_d|}(x) = \int_{y=0}^{\infty}\int_{w=0}^{\infty}\left[1 - \mathrm{e}^{-\frac{x(\gamma_r y+1)}{w\Omega_{g1}}}\right]^K f_{\gamma_s||\mathcal{J}_d|}(w) f_{|h_{3k^\dagger}|^2}(y)\,\mathrm{d}w\,\mathrm{d}y \tag{57}$$

$$F_{\Gamma^{\text{gs}}_{s,k^\dagger}||\mathcal{J}_d|}(x) = 1 - \sum_{k=1}^{k}\binom{K}{k}(-1)^{k-1}\int_{y=0}^{\infty}\int_{w=0}^{\infty}\mathrm{e}^{-\frac{kx(\gamma_r y+1)}{w\Omega_{g1}}} f_{\gamma_s||\mathcal{J}_d|}(w) f_{|h_{3k^\dagger}|^2}(y)\,\mathrm{d}w\,\mathrm{d}y \tag{58}$$

$$F_{\Gamma^{\text{gs}}_{s,k^\dagger}||\mathcal{J}_d|}(x) = 1 - \sum_{k=1}^{K}\binom{K}{k}(-1)^{k-1}\int_{w=0}^{\infty}\frac{\mathrm{e}^{-\frac{kx}{w\Omega_{g1}}}}{\left(1 + \frac{kx\gamma_r}{w\xi_{g1/h3}}\right)} f_{\gamma_s||\mathcal{J}_d|}(w)\,\mathrm{d}w \tag{59}$$

$$F_{\Gamma^{\text{gs}}_{s,k^\dagger}||\mathcal{J}_d|}(x) = 1 - \sum_{k=1}^{k}\binom{K}{k}(-1)^{k-1}\left[\overbrace{\int_{w=0}^{\gamma}\frac{\mathrm{e}^{-\frac{kx}{w\Omega_{g1}}}}{\left(1 + \frac{kx\gamma_r}{w\xi_{g1/h3}}\right)} f_{Z||\mathcal{J}_d|}(w)\,\mathrm{d}w}^{\text{I}_1} + [1 - F_{Z||\mathcal{J}_d|}(\gamma)]\overbrace{\int_{w=\gamma}^{\infty}\frac{\mathrm{e}^{-\frac{kx}{w\Omega_{g1}}}}{\left(1 + \frac{kx\gamma_r}{w\xi_{g1/h3}}\right)}\delta(w-\gamma)\,\mathrm{d}w}^{\text{I}_2}\right] \tag{60}$$

$$\frac{1}{t^a(t+b)^c} = \sum_{m=0}^{a-1}(-1)^m\frac{(c)_m}{m!}\frac{1}{b^{c+m}}\frac{1}{t^{a-m}} + \sum_{n=0}^{c-1}(-1)^n\frac{(a)_n}{n!}\frac{1}{(-b)^{a+n}}\frac{1}{(t+b)^{c-n}} \tag{61}$$

expansion and obtain

$$F_{\Gamma^{ss}_{s,k^\dagger}|_{\gamma_s=w}}(x) = 1 - \sum_{k=1}^{K} \binom{K}{k} (-1)^{k-1} \frac{e^{-\frac{kx}{w\Omega_{g1}}}}{\left(1 + \frac{x\gamma_r \Omega_{h3}}{w\Omega_{g1}}\right)^k}$$

which can be averaged using PDF of $\gamma_s$ from (14) using a procedure similar to that used to derive (18).

## 8.2 Appendix 2

Achievable rate for each scheme is given by $R_s^{pno} = (1-\alpha)\log_2(1+\Gamma^{pno}_{s,k^\dagger})$. Assuming $\Gamma^{pno}_{s,k^\dagger} >> 1$ we obtain $R_s^{pno} \simeq (1-\alpha)\log_2(\Gamma^{pno}_{s,k^\dagger})$. With this, the ergodic rate becomes $\mathbb{E}[R_s^{pno}] \simeq (1-\alpha)\mathbb{E}[\log_2(\Gamma^{pno}_{s,k^\dagger})]$. Substituting the SINR for each case yields (43) and (48). We present an outline of the derivation of each term. Firstly (see (62))

Using CDF and PDF of $Z$ from (15) and (16), we simplify (62). We also use the definite integral

$$\int_{w=0}^{\gamma} \frac{\log_2(w)}{\left(1 + j\frac{w}{\overline{\gamma}_r\,\xi_{h2/g2}}\right)^2}\, dw = \frac{\gamma \log_2(\gamma)}{1+j\rho} - \frac{\log_2(1+j\rho)}{j\rho/\gamma} \tag{64}$$

$$F_{\Gamma^{ss}_{s,k}}(x) \simeq 1 - \frac{1}{1 + \frac{x\gamma_r\Omega_{h3}}{\Omega_{g1}}}, \text{ large } \gamma_s$$

and the CDF $F_{\Gamma^{ss}_{s,k^\dagger}(x)}$ is given by $F_{\Gamma^{ss}_{s,k^\dagger}}(x) \simeq \left[1 - \frac{1}{1+\frac{x\gamma_r\Omega_{h3}}{\Omega_{g1}}}\right]^K$. We obtain PDF $f_{\Gamma^{ss}_{s,k^\dagger}}(x) = \frac{dF_{\Gamma^{ss}_{s,k^\dagger}(x)}}{dx}$ as

$$f_{\Gamma^{ss}_{s,k^\dagger}}(x) = \sum_{k=1}^{K} \binom{K}{k} (-1)^{k-1} k \frac{\gamma_r\Omega_{h3}}{\Omega_{g1}} \frac{1}{\left(1 + \frac{x\gamma_r\Omega_{h3}}{\Omega_{g1}}\right)^{k+1}} \tag{64}$$

Clearly the expected value of $X = \log_2\left(\Gamma^{ss}_{s,k^\dagger}\right)$ is obtained by (see (65))

We solve the integration by using [17, eq. (4.331)]. Using these, (48) and (49) can be established.

$$\mathbb{E}[\log_2(\gamma_s)|_{|\mathcal{J}_d|}] = \int_{w=0}^{\gamma} \log_2(w) f_{Z|_{|\mathcal{J}_d|}}(w)\, dw + \int_{w=\gamma}^{\infty} (1 - F_{Z|_{|\mathcal{J}_d|}}(\gamma)) \log_2(w)\delta(w-\gamma)\, dw \tag{62}$$

$$\mathbb{E}[X] \simeq \int_{x=0}^{\infty} \log_2(x) f_{\Gamma^{ss}_{k^\dagger}}(x)\, dx = \sum_{k=1}^{K} \binom{K}{k} (-1)^{k-1} k \frac{\gamma_r\Omega_{h3}}{\Omega_{g1}} \int_{x=0}^{\infty} \frac{\log_2(x)}{\left(1+\frac{x\gamma_r\Omega_{h3}}{\Omega_{g1}}\right)^{k+1}}\, dx \tag{65}$$



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