International Journal of Information Technology & Decision Making
Vol. 4, No. 3 (2005) 385–394

CONTEXT-DEPENDENT DEA WITH AN APPLICATION TO TOKYO PUBLIC LIBRARIES

YAO CHEN

College of Management, University of Massachusetts at Lowell
Lowell, MA 01845, USA
Yao_Chen@uml.edu

HIROSHI MORITA

Graduate School of Information Science and Technology
Osaka University, Suita, Osaka, 565-0871, Japan
morita@ap.eng.osaka-u.ac.jp

JOE ZHU

Department of Management, Worcester Polytechnic Institute
Worcester, MA 01609, USA
jzhu@wpi.edu

Data envelopment analysis (DEA) identifies an empirical efficient frontier of a set of peer decision making units (DMUs) with multiple inputs and outputs. The efficient frontier is characterized by the DMUs with an unity efficiency score. The performance of inefficient DMUs is characterized with respect to the identified efficient frontier. If the performance of inefficient DMUs deteriorates or improves (up to the frontier), the efficient DMUs still have an unity efficiency score. However, the performance of DMUs may be influenced by the context — e.g. a product may appear attractive against a background of less attractive alternatives and unattractive when compared to more attractive alternatives. With an application to Tokyo public libraries, the current paper presents and demonstrates a context-dependent DEA which measures the relative attractiveness of libraries on a specific performance level against libraries exhibiting poorer performance. The set of libraries are grouped into different levels of efficient frontiers. Each efficient frontier (on a specific performance level) is then used as evaluation context for the relative attractiveness. The performance of the efficient libraries changes as the inefficient libraries change their performance. The context-dependent DEA can also be used to differentiate the performance of efficient DMUs. The context-dependent DEA provides finer DEA results with respect to the performance of all DMUs.

Keywords: Data envelopment analysis (DEA); attractiveness; efficient; evaluation; context.

1. Introduction

Charnes, Cooper and Rhodes[1] develop the data envelopment analysis (DEA) which is a mathematical programming method for evaluating the relative efficiency of decision making units (DMUs) with multiple outputs and multiple inputs. DEA identifies empirical efficient frontier of a set of DMUs. It is well known that adding or deleting an inefficient DMU or a set of inefficient DMUs does not alter the efficiencies of the existing DMUs and the efficient frontier. The inefficiency scores change only if the efficient frontier is altered, i.e. the performance of DMUs depends only on the identified efficient frontier. The efficient frontier is characterized by the DMUs with an unity efficiency score. If the performance of inefficient DMUs deteriorates or improves, the efficient DMUs still have an unity efficiency score. (If the inefficient DMUs improve their performance and outperform the efficient DMUs, then a new efficient frontier is established. This can be studied using sensitivity analysis described in Seiford and Zhu[4] and Zhu.[9]

The above DEA situation indicates that although the performance of inefficient DMUs depends on the efficient DMUs, efficient DMUs are only characterized by an efficiency score of one. The DEA performance (of efficient DMUs) is not influenced by the presence of inefficient DMUs. In contrast, researchers of the consumer choice theory point out that consumer choice is often influenced by the context. e.g. a product may appear attractive against a background of less attractive alternatives and unattractive when compared to more attractive alternatives.[7]

This paper presents a context-dependent DEA that measures the relative attractiveness of a particular DMU when compared to others. The relative attractiveness of DMU_x compared to DMU_y depends on the presence or absence of a third option, say DMU_z (or a group of DMUs). The context-dependent DEA is different from the super-efficiency concept where a DMU under evaluation is excluded from the reference set. In super-efficiency models, the evaluation context or third option (the reference set) changes in each evaluation.[5]

In order to obtain the relative attractiveness within the context-dependent DEA, the original DEA methodology is modified to a situation where the relative performance is defined with respect to a particular efficient context (evaluation context). The evaluation contexts are obtained by partitioning a set of DMUs into several levels of efficient frontiers. Each efficient frontier provides an evaluation context for measuring the relative attractiveness, e.g. the second-level efficient frontier serves as the evaluation context for measuring the relative attractiveness of the DMUs located on the first-level (original) efficient frontier. It can be seen that the presence or absence (or the shape) of the second-level efficient frontier affects the relative attractiveness of DMUs on the first-level efficient frontier. When DMUs in a specific level are viewed as having equal performance, the attractiveness measure allows us to differentiate the "equal performance" based upon the same specific evaluation context (or third option). In fact, as demonstrated in Zhu[10] and Seiford and Zhu,[6] a progress measure can also be obtained when different levels of efficient frontiers are obtained. A combined use of attractiveness and progress measures can further characterize the performance of DMUs.

The rest of the paper is organized as follows. The next section presents the context-dependent DEA that measures the relative attractiveness. The context-dependent DEA is then applied to a set of public libraries in Tokyo described in Cooper, Seiford and Tone.[2] The context-dependent DEA enables us to discriminate the performance of 23 Tokyo public libraries in three categories of business district of central Tokyo, shopping area around business district and residential area on the outskirts. Conclusions are given in the last section.

2. Context-Dependent DEA

Assume DMU_j $(j = 1, 2, \ldots, n)$ produces s outputs $(y_{1j}, \ldots, y_{sj})$ by using m inputs $(x_{1j}, \ldots, x_{mj})$. We define $\mathbf{J}^1 = \{DMU_j, j = 1, \ldots, n\}$ (the set of all n DMUs) and interactively define $\mathbf{J}^{l+1} = \mathbf{J}^l - \mathbf{E}^l$ where $\mathbf{E}^l = \{DMU_k \in \mathbf{J}^l | \phi^*(l, k) = 1\}$, and $\phi^*(l, k)$ is the optimal value to the following linear programming problem:

$$\begin{aligned} \phi^*(l,k) = & \max_{\lambda_j, \phi(l,k)} \phi(l,k) \\ \text{s.t.} \quad & \sum_{j \in \mathrm{F}(\mathbf{J}^l)} \lambda_j y_{rj} \geq \phi(l,k) y_{rk} \qquad r = 1, \ldots, s \\ & \sum_{j \in \mathrm{F}(\mathbf{J}^l)} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \ldots, m \\ & \lambda_j \geq 0 \quad j \in \mathrm{F}(\mathbf{J}^l) \end{aligned} \tag{2.1}$$

where x_{ik} and y_{rk} are the ith input and rth output of DMU_k, respectively, and $j \in \mathrm{F}(\mathbf{J}^l)$ means $DMU_j \in \mathbf{J}^l$, i.e. $\mathrm{F}(\cdot)$ represents the correspondence from a DMU set to the corresponding subscript index set.

When $l = 1$, model (2.1) becomes the original output-oriented CRS (constant returns to scale) model[1] and $\mathbf{E}^1$ consists of all the (radially) efficient DMUs.[a] These DMUs in set $\mathbf{E}^1$ define the first-level efficient frontier. When $l = 2$, model (2.1) gives the second-level efficient frontier after the exclusion of the first-level efficient DMUs. And so on. In this manner, we identify several levels of efficient frontiers. We call $\mathbf{E}^l$ the lth-level efficient frontier. The following algorithm accomplishes the identification of these efficient frontiers by model (2.1).

- **Step 1:** Set $l = 1$. Evaluate the entire set of DMUs, $\mathbf{J}^1$, by model (2.1) to obtain the first-level efficient DMUs, set $\mathbf{E}^1$ (the first-level efficient frontier).
- **Step 2:** Exclude the efficient DMUs from future DEA runs. $\mathbf{J}^{l+1} = \mathbf{J}^l - \mathbf{E}^l$. (If $\mathbf{J}^{l+1} = \emptyset$ then stop.)
- **Step 3:** Evaluate the new subset of "inefficient" DMUs, $\mathbf{J}^{l+1}$, by model (2.1) to obtain a new set of efficient DMUs $\mathbf{E}^{l+1}$ (the new efficient frontier).
- **Step 4:** Let $l = l + 1$. Go to Step 2.
- **Stopping rule:** $\mathbf{J}^{l+1} = \emptyset$, the algorithm stops.

Model (2.1) yields a stratification of the whole set of DMUs. This process can be easily accomplished by the DEA Excel Solver provided in Zhu (2003). From the

[a] A radially efficient DMU may have non-zero input/output slack values. In this paper, the term "efficient" refers to "radially efficient".

algorithm, we know that l goes from 1 to L, where L is determined by the stopping rule. It is easy to show that these sets of DMUs have the following properties:

(i) $\mathbf{J}^1 = \bigcup_{l=1}^{L} \mathbf{E}^l$ and $\mathbf{E}^l \cap \mathbf{E}^{l'} = \emptyset$ for $l \neq l'$.
(ii) The DMUs in $\mathbf{E}^{l'}$ are dominated by the DMUs in $\mathbf{E}^l$ if $l' > l$.
(iii) Each DMU in set $\mathbf{E}^l$ is efficient with respect to the DMUs in set $\mathbf{E}^{l+l'}$ for all $0 < l' \leq l - L$.

In the current paper, we use the output-oriented CRS model to generate the efficient frontiers in different levels. The input-oriented CRS model yields the same stratification of the whole set of DMUs.

The DEA stratification model (2.1) partitions the set of DMUs into different subgroups (efficient levels) characterized by $\mathbf{E}^l$ $(l = 1, \ldots, L)$. Based upon these evaluation context $\mathbf{E}^l$, we present our context-dependent DEA that measures the relative attractiveness of DMUs.

Consider a specific DMU_q from a specific level $\mathbf{E}^{l_o}$, $l_o \in \{1, \ldots, L-1\}$. The following model is used to characterize the attractiveness of DMU_q with respect to levels exhibiting poorer performance in $\mathbf{E}^{l_o+d}$ $(d = 1, \ldots, L - l_o)$

$$
\begin{aligned}
\Omega_q^*(d) = \max_{\lambda_j, \Omega_q(d)} & \ \Omega_q(d) && d = 1, \ldots, L - l_o \\
\text{s.t.} \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} & \lambda_j y_{rj} \geq \Omega_q(d) y_{rq} && r = 1, \ldots, s \\
\sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} & \lambda_j x_{ij} \leq x_{iq} && i = 1, \ldots, m \\
& \lambda_j \geq 0 \quad j \in \mathrm{F}(\mathbf{E}^{l_o+d})
\end{aligned} \tag{2.2}
$$

where x_{iq} and y_{rq} are the ith input and rth output of DMU_q, respectively.

It is easy to show that $\Omega_q^*(d) < 1$ for each $d = 1, \ldots, L - l_o$. Note that $\mathbf{J}^{l_o+d} = \bigcup_{l=\alpha}^{L-l_o} \mathbf{E}^{l_o+l}$. Therefore, the context-dependent model (2.2) is equivalent to the following linear programming model:

$$
\begin{aligned}
\Omega_q^*(d) = & \max_{\mu_j, \lambda_j, \Omega_q(d)} \Omega_q(d) \\
\text{s.t.} & \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} \mu_j y_{rj} + \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d+1})} \lambda_j y_{rj} \geq \Omega_q(d) y_{rq} && r = 1, \ldots, s \\
& \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} \mu_j x_{ij} + \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d+1})} \lambda_j x_{ij} \leq x_{iq} && i = 1, \ldots, m \\
& \mu_j, \ \lambda_j \geq 0.
\end{aligned} \tag{2.3}
$$

$\Omega_q^*(d+1)$ is obtained by solving the following problem:

$$\begin{aligned}
\Omega_q^*(d+1) &= \max_{\lambda_j, \Omega_q(d+1)} \Omega_q(d+1) \\
\text{s.t.} \quad & \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d+1})} \lambda_j y_{rj} \geq \Omega_q(d+1) y_{rq} \qquad r = 1, \ldots, s \\
& \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d+1})} \lambda_j x_{ij} \leq x_{iq} \qquad i = 1, \ldots, m \\
& \lambda_j \geq 0 \quad j \in \mathrm{F}(\mathbf{E}^{l_o+d+1}).
\end{aligned} \tag{2.4}$$

It can be seen that any optimal solution to (2.4) is a feasible solution to (2.3). Thus, $\Omega_q^*(d+1) \leq \Omega_q^*(d)$. However, if $\Omega_q^*(d+1) = \Omega_q^*(d)$, then $\mathbf{E}^{l_o+d} \cap \mathbf{E}^{l_o+d+1} \neq \emptyset$. Therefore, $\Omega_q^*(d+1) < \Omega_q^*(d)$.

Definition 1. $\mathrm{A}_q^*(d) \equiv \frac{1}{\Omega_q^*(d)}$ is called the (output-oriented) *d-degree* attractiveness of DMU_q from a specific level $\mathbf{E}^{l_o}$.

In model (2.2), each efficient frontier of $\mathbf{E}^{l_o+d}$ represents an evaluation context for evaluating the relative attractiveness of DMUs in $\mathbf{E}^{l_o}$. Note that $\mathrm{A}_q^*(d)$ is the reciprocal of the optimal value to (2.2), therefore $\mathrm{A}_q^*(d) > 1$. The larger the value of $\mathrm{A}_q^*(d)$, the more attractive the DMU_q is, because this DMU_q makes itself more distinctive from the evaluation context $\mathbf{E}^{l_o+d}$. We are able to rank the DMUs in $\mathbf{E}^{l_o}$ based upon their attractiveness scores and identify the best one.

Similarly, we can have the input-oriented version of the context-dependent DEA. Consider the following linear programming problem for $DMU_q = (x_q, y_q)$ in a specific level $\mathbf{E}^{l_o}$, $l_o \in \{1, \ldots, L-1\}$:

$$\begin{aligned}
H_q^*(d) &= \min H_q(d) \qquad d = 1, \ldots, L - l_o \\
\text{s.t.} \quad & \sum_{j \in F(\mathbf{E}^{l_o+d})} \lambda_j x_{ij} \leq H_q(d) x_{iq} \qquad i = 1, \ldots, m \\
& \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} \lambda_j y_{rj} \geq y_{rq} \qquad r = 1, \ldots, s \\
& \lambda_j \geq 0 \qquad j \in \mathrm{F}(\mathbf{E}^{l_o+d}).
\end{aligned} \tag{2.5}$$

Note that dividing each side of the constraint of (2.5) by $H_q(d)$ yields:

$$\begin{aligned}
& \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} \tilde{\lambda}_j x_{ij} \leq x_{iq} \\
& \sum_{j \in \mathrm{F}(\mathbf{E}^{l_o+d})} \tilde{\lambda}_j y_{rj} \geq \frac{1}{H_q(d)} y_{rq} \\
& \tilde{\lambda}_j = \frac{\lambda_j}{H_q(d)} \geq 0 \qquad j \in \mathrm{F}(\mathbf{E}^{l_o+d}).
\end{aligned}$$

Therefore model (2.5) is equivalent to model (2.2), and we have $H_q^*(d) = \frac{1}{\Omega_q^*(d)}$ for $DMU_q \in \mathbf{E}^{l_o}$, $l_o \in \{1, \ldots, L-1\}$. Further, we have for a specific $DMU_q \in \mathbf{E}^{l_o}$, $l_o \in \{1, \ldots, L-1\}$, (i) $H_q^*(d) > 1$ for each $d = 1, \ldots, L - l_o$ and (ii) $H_q^*(d+1) > H_q^*(d)$.

Definition 2. $H_q^*(d)$ is called (input-oriented) *d-degree* attractiveness of DMU_q from a specific level $\mathbf{E}^{l_o}$.

The bigger the $H_q^*(d)$, the more attractive the DMU_q. Model (2.5) determines the relative attractiveness score for DMU_q when outputs are fixed at their current levels.

3. An Application

In this section, we apply the context-dependent DEA to measure the relative attractiveness of 23 public libraries in Tokyo.[2] Table 1 presents the data for the public libraries in the 23 Wards of the Tokyo Metropolitan Area. The inputs are (i) floor area (unit = 1,000 m^2), (ii) the number of books (unit = 1,000), (iii) staffs (unit = 1,000) and (iv) population (unit = 1,000). The outputs are (i) the number of registered residents (unit = 1,000) and (ii) the number of borrowed books (unit = 1,000). The last column represents the CRS efficiency score. The scores are obtained using the input-oriented CRS model. It can be seen that six libraries are CRS efficient.

We next illustrate the stratification model (2.1) by the 23 DMUs (libraries) in Table 1. By calculating (1) for $l = 1$, we obtain $\phi^*(l, k) = 1$, for $k = 5$, 6, 9, 17, 19 and 23, and the first-level efficient frontier is $\mathbf{E}^1 = \{DMU_j | j = 5, 6, 9, 17, 19 \text{ and } 23\}$ (the original DEA frontier). Next, we exclude the DMUs in set $\mathbf{E}^1$ from $\mathbf{J}^1$ and obtain $\mathbf{J}^2 = \{DMU_j | j = 1\text{–}4, 7, 8, 10\text{–}16, 18, 20\text{–}22\}$. We have $\phi^*(2, k) = 1$ for $k = 2$, 4, 12, 13, 15, 18, 20–22. Therefore, the efficient frontier of $\mathbf{J}^2$ is $\mathbf{E}^2 = \{DMU_j | j = 2$, 4, 12, 13, 15, 18, 20–22$\}$ (second-level efficient frontier). By repeating this process, we finally obtain $\mathbf{E}^5 = \{DMU_j | j = 1\}$ (the fifth-level efficient frontier) and $L = 5$.

Table 2 reports the five levels of efficient frontiers. Library 1 ($L1$) has the least inefficiency indicated by the CRS efficiency score and form the last level of efficient frontier. Although $L14$ has a larger efficiency score than does $L4$, $L14$ is on the level 3 and $L4$ is on the level 2. This indicates that the levels obtained using (2.1) do not necessarily follow the order of the original CRS efficiency scores.

As pointed out in Cooper, Seiford and Tone,[2] the public libraries in Tokyo can be classified into three categories (see column 9 in Table 1). The first category consists of libraries in the business district of central Tokyo. The second category consists of libraries in the shopping area around business district. The third category consists of libraries in the residential area on the outskirts. Based upon Table 2, we have (i) all the category 2 libraries are in the first two levels, (ii) 4 (two thirds) and 2 (one thirds) of the category 1 libraries are in the first two and last two levels,

Table 1. Public library data.

No.	Ward	Area	Books	Staff	Population	Regist.	Borrow.	Category	CRS Efficiency
$L1$	Chiyoda	2249	163523	26	49196	5561	105321	business district	0.350
$L2$	Chuo	4617	338671	30	78599	18106	314682	business district	0.792
$L3$	Taito	3873	281655	51	176381	16498	542349	business district	0.573
$L4$	Arakawa	5541	400993	78	189397	30810	847872	business district	0.719
$L5$	Minato	11381	363116	69	192235	57279	758704	business district	1
$L6$	Bunkyo	10086	541658	114	194091	66137	1438746	business district	1
$L7$	Sumida	5434	508141	61	228535	35295	839597	shopping area	0.697
$L8$	Shibuya	7524	338804	74	238691	33188	540821	shopping area	0.580
$L9$	Meguro	5077	511467	84	267385	65391	1562274	shopping area	1
$L10$	Toshima	7029	393815	68	277402	41197	978117	shopping area	0.705
$L11$	Shinjuku	11121	509682	96	330609	47032	930437	shopping area	0.569
$L12$	Nakano	7072	527457	92	332609	56064	1345185	shopping area	0.758
$L13$	Shinagawa	9348	601594	127	356504	69536	1164801	shopping area	0.747
$L14$	Kita	7781	528799	96	365844	37467	1348588	shopping area	0.722
$L15$	Koto	6235	394158	77	389894	57727	1100779	shopping area	0.844
$L16$	Katsushika	10593	515624	101	417513	46160	1070488	shopping area	0.582
$L17$	Itabashi	10866	566708	118	503914	102967	1707645	residential area	1
$L18$	Edogawa	6500	467617	74	517318	47236	1223026	residential area	0.787
$L19$	Suginami	11469	768484	103	537746	84510	2299694	residential area	1
$L20$	Nerima	10868	669996	107	590601	69576	1901465	residential area	0.849
$L21$	Adachi	10717	844949	120	622550	89401	1909698	residential area	0.787
$L22$	Ota	19716	1258981	242	660164	97941	3055193	residential area	0.785
$L23$	Setagaya	10888	1148863	202	808369	191166	4096300	residential area	1

Table 2. Levels.

	Level 1	Level 2	Level 3	Level 4	Level 5
Frontier libraries	5, 6, 9, 17, 19, 23	2, 4, 12, 13, 15, 18, 20, 21, 22	7, 10, 14	3, 8, 11, 16	1
CRS efficiency range	1	0.719–0.849	0.697–0.722	0.569–0.582	0.35

Table 3. Attractiveness score for the libraries in the first level.

DMU	Evaluation Context (efficient frontier)	
	Second-Level $\mathbf{E}^2$	Third-Level $\mathbf{E}^3$
	1st-Degree ($A_q^*(1)$)	2nd-Degree ($A_q^*(2)$)
5	1.507	1.961
6	1.786	2.206
9	1.620	1.989
17	1.310	1.737
19	1.292	1.552
23	2.049	2.703

respectively, and (iii) only one category 2 library is on the first level frontier and all the remaining ones are equally distributed over the frontiers on levels 2–4. Thus, based upon the performance levels, the category 3 libraries in the residential area on the outskirts have the best performance.

We now turn to the attractiveness scores for the libraries on first two levels. Table 3 reports the attractiveness scores for the libraries in the first level based upon model (2.2). When $\mathbf{E}^2$ is chosen as the evaluation context, we have that $L23$ is the best library because $L23$ has the largest attractiveness score of 2.049. We can rank these libraries in the order of $L23$, $L6$, $L9$, $L5$, $L17$ and $L19$ ($2.049 > 1.786 > 1.620 > 1.507 > 1.310 > 1.292$).

Since there is only one level difference between the libraries under evaluation in $\mathbf{E}^1$ and the evaluation context $\mathbf{E}^2$, the above attractiveness scores are called first degree. If the libraries in $\mathbf{E}^3$ are chosen as the evaluation context, we obtain the second degree attractiveness scores for the first-level libraries. The last column in Table 4 reports the scores. It can be seen that the same ranking is obtained. This indicates that $L23$ in the residential area on the outskirts is the best library, followed by $L6$ (business district) and $L9$ (shopping area).

We next take a look at the library on the second level frontier. Table 4 reports the attractiveness scores when the third and fourth levels are chosen as the evaluation background, respectively. $L15$ is ranked as the best library under both evaluation contexts. When $\mathbf{E}^3$ is chosen as the evaluation context, $L2$ is ranked second. However, when $\mathbf{E}^4$ is chosen as the evaluation context, $L2$ is ranked seventh. The ranking position is changed for $L12$, $L18$, $L20$, $L21$ and $L22$ when the evaluation

Table 4. Attractiveness score for the libraries in the second level.

DMU	Evaluation Context (efficient frontier)	
	Third-Level $\mathbf{E}^3$	Fourth-Level $\mathbf{E}^3$
	1st-Degree ($A_q^*(1)$)	2nd-Degree ($A_q^*(2)$)
2	1.492	1.619
4	1.217	1.496
12	1.290	1.823
13	1.296	1.688
15	1.548	2.109
18	1.239	1.678
20	1.238	1.674
21	1.335	1.908
22	1.257	1.516

context is changed. This demonstrates that the performance of the library can be dependent on the evaluation background.

4. Conclusions

The current paper presents a new DEA approach — context-dependent DEA. The context-dependent DEA captures situations where the performance of DMUs depends on the presence or absence of a third option. Such a DEA performance is called relative attractiveness. The method is demonstrated to measure the attractiveness of a set of public libraries in Tokyo with respect to a given evaluation context. Different strata of efficient frontiers rather than the traditional first-level efficient frontier are used as evaluation contexts. The context-dependent DEA performance depends not only on the efficient frontier, but also the inefficient DMUs. This change makes DEA more versatile and allows DEA to locally and globally identify better options. In particular, the attractiveness measure can be used to (i) identify DMUs that have outstanding performance, and (ii) differentiate the performance of DEA efficient DMUs.

Other possible extentions and applications to the context-dependent DEA method include the incorporation of preference[6] and slack-based context-dependent DEA measures.[3]

Acknowledgement

The paper was finished while Joe Zhu was visiting the Osaka University under the Invitation Research Fellowship from the Japan Society for the Promotion of Science (JSPS). The financial support from the JSPS is appreciated.

References

1. A. Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, *European J. Operational Research* **2** (1978) 429–441.
2. W. W. Cooper, L. M. Seiford and K. Tone, *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software* (Kluwer Academic Publishers, Boston, 2000).
3. H. Morita, K. Hirokawa and J. Zhu, A slack base measure of efficiency in context dependent data envelopment analysis, *OMEGA* **33**(4) (2005) 357–362.
4. L. M. Seiford and J. Zhu, Sensitivity analysis of DEA models for simultaneous changes in all the data, *J. Operational Research Society* **49** (2005) 1060–1071.
5. L. M. Seiford and J. Zhu, Infeasibility of super-efficiency DEA models, *INFOR* **37**(2) (1999) 174–187.
6. L. M. Seiford and J. Zhu, Context-dependent data envelopment analysis: Measuring attractiveness and progress, *OMEGA* **31**(5) (2003) 397–480.
7. I. Simonson and A. Tversky, Choice in context: Tradeoff contrast and extremeness aversion, *J. Marketing Research* **29** (1992) 281–295.
8. G. A. Stewart, The best of the 24-pin printers, *BYTE* (April 1988), pp. 118–128.
9. J. Zhu, Super-efficiency and DEA sensitivity analysis, *European J. Operational Research* **129**(2) (2001) 443–455.
10. J. Zhu, *Quantitative Models for Performance Evaluation and Benchmarking: Data Envelopment Analysis with Spreadsheets* (Kluwer Academic Publishers, Boston, 2003).