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ORIGINAL ARTICLE

Economical control chart with supplementary rules to monitor the average number of defects

André Luiz Sena da Rocha · Pledson Guedes de Medeiros · Linda Lee Ho

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Abstract This study proposes a procedure for an on-line process control system to monitor the average number of defects using a Shewhart-like chart with two sets of limits (viz., control and warning limits). After the production of m units, the mth item is inspected. If the number of defects exceeds the upper control limit or if, in a sequence of the last h inspections, all inspected items exhibit a number of defects between the warning and control limits, then the process is stopped for adjustment; otherwise, production continues. The properties of an ergodic Markov chain are used to obtain an expression for the average cost per item produced. The inspection interval (m), warning and control limits (W and C, respectively), and the sequence size (h) are determined by minimizing the average cost per produced item. A numerical example illustrates the proposed procedure.

Keywords Markov chain · On-line process control · Average number of defects · Number of defects per inspected item · Warning limit · Poisson distribution

1 Introduction

According to Taguchi [15], an on-line control system may be used when the desired target values for quality characteristics can be economically controlled. Beginning with the pioneering contributions of Taguchi [14, 15], on-line process control has been used to monitor two parameters of production processes: the conforming fraction (on-line process control by attributes) and the process mean (on-line process control by variables). In the case of on-line process control by attributes, after the production of m items, the mth item is inspected. If the item is conforming, production continues; otherwise, the process is stopped for adjustment. The problem consists of determining the optimum inspection interval m such that the average cost of the control system is minimized. Nayebpour and Woodall [10] provided the main critical reference investigating Taguchi's proposal. Concerning on-line process control by variables, Ho and Quinino [7] proposed a procedure using two sets of limits (control limits and warning limits). The intervention system is also subject to a supplementary rule: if, in the last h inspections, the value of a monitored characteristic is between the control and warning limits or if, in the last inspection, the monitored characteristic exceeds the control limit, the process is stopped for adjustment; otherwise, production continues. The authors obtained an analytical expression for the average cost per produced item, which is minimized by four parameters: the interval between inspections (m), control limit ($\mu\pm C$), warning limit ($\mu\pm W$), and length of sequence (h). Other contributors have described the use of warning limits in control charts. For example, Page [11, 12] developed a control chart for variables with upper and lower warning limits and additional supplementary rules to consider a process as out of control. Gordon and Weindling [6] presented an alternative model for an $\overline{X}$ control chart with warning limits using a set of five parameters. Papers providing an economical perspective include contributions from Chiu and Cheung [2] and Chung [4]. Liu et al. [9] studied the effects of correlations in control charts with warning limits. Chou et al. [3] evaluated the effect of non-normality in a control chart for variables with warning limits. Lam et al. [8] proposed algorithms for an optimized design of an integrated control chart system to monitor a multi-stage and multi-stream manufacturing system. Recently, Wu and Jiao [19] suggested the attribute control chart MON with warning limits to monitor a process mean.

A. L. Sena da Rocha
UFERSA, Mossoró, Brazil
e-mail: andre.rocha@ufersa.edu.br

P. Guedes de Medeiros
UFRN, Natal, Brazil
e-mail: pledson@ccet.ufrn.br

L. Lee Ho (✉)
USP, São Paulo, Brazil
e-mail: lindalee@usp.br

In the aforementioned papers regarding on-line monitoring, the response variable follows either a normal distribution (on-line process control by variables) or a Bernoulli distribution (on-line process control by attributes). The literature includes few works regarding on-line control in which the number of defects in the inspected item is the response variable to be monitored and follows a Poisson distribution.

Glushkovsky [5] developed a control chart according to an economical approach (named Chart G). In his proposal, the number of non-conformities follows a Poisson distribution. Vasconcelos et al. [17] developed an on-line process control system to monitor the number of non-conformities in an inspected item. The parameters reflecting the control limit (C) and the interval inspection (m) are determined by optimization.

The present article extends the work of Vasconcelos et al. [17] with two sets of limits (viz., warning and control limits) and an additional stopping process rule based on the policies proposed by Western Electric [18]. Properties involving finite Markov chains with discrete state space to determine an optimum strategy of control in a process were used. This strategy consists of minimizing the average cost per produced item based on the inspection interval (m), warning and control limits (W and C, respectively), and length of sequence (h).

This paper is organized as follows. Section 2 presents the proposed probabilistic model. Section 3 develops the model for the average cost per unit of the control system. In Section 4, a numerical example with a sensitivity analysis involving the parameters of interest is presented to illustrate the proposed procedure. Conclusions and suggestions are given in Section 5.

2 Probabilistic model of the inspection system

Consider a situation in which items are produced one by one. The process begins under statistical control and, after a special event, is considered out of control. Monitoring consists of inspecting the mth item for each m produced item. Destructive tests are conducted on the inspected item, and it is discarded after the inspection. If the process is judged to be out of control (according to a criterion to be established), it is stopped immediately; otherwise, production continues. It is assumed that no item is produced between detection and interruption to allow for adjustment. The process starts (or restarts) production in state I (in control, with the average number of non-conformities $\lambda_0 \geq 0$), which, after the change, increases to λ_1, thereby operating the process in state II, which will to return to state I only after an adjustment. The number of time units that the process remains in state I follows a geometric distribution with parameter π, $0 \leq \pi \leq 1$. Each production cycle corresponds to the time required for production from the first item to the mth item. If the number of defects meets the intervention criterion, the process is stopped, and a new production cycle is initiated.

The inspection process can be described by a finite number of Markov chain states. The pair of integers (s,k) describes each state. The first index s represents the current state of the process. For $s=0$, all items (including the inspected one) are produced in state I (in control). For $s=1$, a change from state I to state II has necessarily occurred in the cycle under consideration, and at minimum, the inspected item is produced in state II (out of control). For $s=2$, all items (including the inspected item) are produced in state II (out of control).

Index k indicates the result of the inspection. For $k=-1$, the item inspected exhibits a higher number of defects than the control limit (C) that leads to an adjustment; that is, $[X_i > C]$. For $k=0$, the number of defects in the inspected item is lower than the warning limit (W); that is, $[X_i \leq W]$, and there will be no adjustment. For $k=1$, $[W < X_i \leq C]$ but $[X_{i-1} > C]$ or $[X_{i-1} \leq W]$, and no adjustment is carried out. For $k=2$, $[W < X_i \leq C]$, as with the previous inspected item, and no adjustment is carried out. For $k=(h-1)$, $[W < X_i \leq C]$, as with the $(h-2)$ previously inspected items, and no adjustment is carried out, and for $k=h$, $[W < X_i \leq C]$, as with the $(h-1)$ previously inspected items, and adjustment is initiated.

Let X_i (in which $i=1, 2, 3, \ldots, \infty$) be a random variable with which the number of non-conformities detected in the ith inspection is represented. X_i is assumed to follow a Poisson distribution for parameter λ. The following notations will be used to construct the model:

$$P(X_i > C/s = 0) = R_0 \quad (1)$$

$$P(X_i > C/s = 1) = R_1 \quad (2)$$

Expressions (1) and (2) represent the probabilities that the item inspected in the ith inspection exhibits a higher number of non-conformities than the control limit (C), given that the process was operating in states I and II, respectively. In both cases, the process is stopped to search for special causes.

$$P(W < X_i \leq C/s = 0) = Y_0 \quad (3)$$

$$P(W < X_i \leq C/s = 1) = Y_1 \quad (4)$$

Similarly, expressions (3) and (4) denote the probabilities that the item inspected in the ith inspection exhibits a number of non-conformities between the warning (W) and control (C) limits because the process was operating in states I and II, respectively. Expressions (5) and (6) are the probabilities that the item inspected in the ith inspection exhibits a lower number of non-conformities than the warning (W) limit given that the process is operating in states I and II, respectively.

$$P(X_i \leq W/s = 0) = G_0 \quad (5)$$

$$P(X_i \leq W/s = 1) = G_1 \quad (6)$$

In addition to the criterion of process intervention according to the control limit, a supplementary criterion based on the result of the last sequence of h inspections will be added. Explicitly, if a sequence of h inspections shows a number of non-conformities between the warning and control limits, the process will be stopped for adjustment. The use of two limits is justified because it more rapidly detects a special cause that has a moderate effect on the process.

Figure 1 represents the inspection procedure and process interruption criteria for adjustment.

The inspection system can be described by a stationary Markov chain with a set of three (h+2) discrete states represented by

$$\Omega = \{(0,-1);(0,0);(0,1)\cdots(0,h);(1,-1);(1,0);(1,1)\cdots(1,h);(2,-1);(2,0);(2,1)\cdots(2,h)\} \quad (7)$$

Consider the transition matrix **P**. The elements are probabilities of transition from state (s, k) to (s*, k*), with (s, k) and (s*, k*) $\in \Omega$ denoted by $P_{(s,k),(s^*,k^*)}$. For example, $P_{(2,h)(0,0)}$ denotes the probability that the ith inspection occurs in state (2,h) and that the (i+1)th inspection occurs in state (0,0), or simply, $P(2,h)(0,0)=P(E_{i+1}=(0;0)|E_i=(2,h))$.

In this example,

(a) All m items of the cycle are produced in state II, and the process is stopped because there is a sequence of h inspections with the number of defects between the warning and control limits.

(b) After adjustment, the process restarts in control. In the first inspection after the intervention initiated to search for special causes, the inspected item shows a number of defects fewer than or equal to the warning limit—that is, $X_i \leq W$; therefore, the process is judged to be in control, and no adjustment is made.

Elements of the transition matrix **P** are described below:

(a) The transition probabilities of $P_{(0,k)(0,k^*)}$ (i.e., the non-null probabilities) are as follows:

$$\begin{aligned} p_{(0,k)(0,-1)} &= qR_0 \\ p_{(0,k)(0,k+1)} &= qY_0 \quad \text{where } q = (1-\pi)^m \\ p_{(0,k)(0,0)} &= qG_0 \qquad k = 0,\cdots,h-1 \end{aligned} \quad (8)$$

(b) Probabilities $P_{(0,k)(1,k^*)}$ describe a change from state I to state II in the current cycle, and at minimum, the inspected item is produced in state II. In other words, there is a shift in the average number of defects from λ_0 to $\lambda_1 > \lambda_0$. The non-null probabilities are as follows:

$$\begin{aligned} p_{(0,k)(1,-1)} &= (1-q)R_1 \\ p_{(0,k)(1,k+1)} &= (1-q)Y_1 \\ p_{(0,k)(1,0)} &= (1-q)G_1 \qquad k = 0,1,\cdots,h-1 \end{aligned} \quad (9)$$

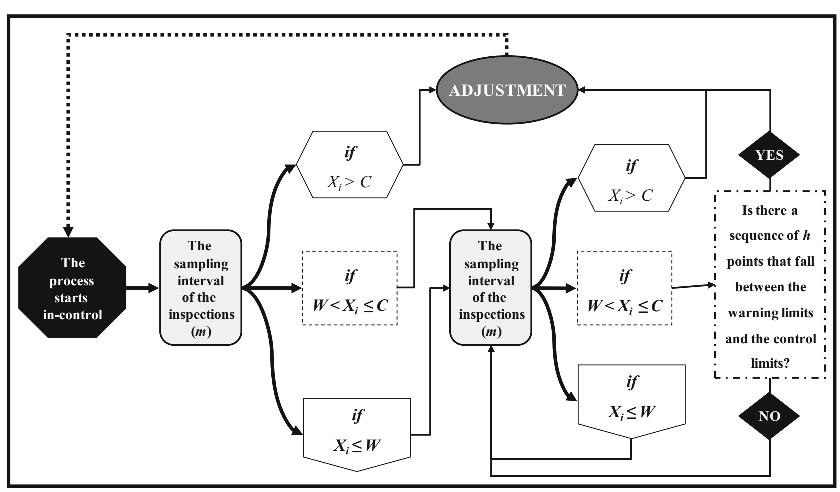
(c) If a shift occurred in previous cycles to state II, all items are produced with the average number of defects λ_1. Thus,

$$\begin{aligned} p_{(1,k)(2,-1)} &= p_{(s,k)(2,-1)} = R_1 \\ p_{(1,k)(2,k+1)} &= p_{(s,k)(2,-1)} = Y_1 \qquad s = 1,2; \\ p_{(1,k)(2,0)} &= p_{(s,k)(2,-1)} = G_1 \qquad k = 0,1,\cdots,h-1 \end{aligned} \quad (10)$$

(d) For the states s=0,1 and 2 and k=−1, h, the process always restarts in state I in the next inspection; thus, the following equalities hold:

$$\begin{aligned} p_{(s,k)(0,-1)} &= qR_0 \\ p_{(s,k)(0,1)} &= qY_0 \qquad s = 0,1,2; \\ p_{(s,k)(0,0)} &= qG_0 \qquad k = -1,h \end{aligned} \quad (11)$$

Fig. 1 Flowchart of the inspection procedure



(e) Similar to expression (10), in the next inspection, there may be a transition from state I to state II. Thus,

$$\begin{aligned} p_{(s,k)(1,-1)} &= (1-q)R_1 \\ p_{(s,k)(1,1)} &= (1-q)Y_1 \quad s = 0,1,2; \\ p_{(s,k)(1,0)} &= (1-q)G_1 \quad k = -1, h \end{aligned} \tag{12}$$

For example, consider these interruption rules for intervention in a production process: if the inspected item exhibits a greater number of defects than the control limit or if h=3 consecutive inspections in which the inspected items show a number of defects between the warning and control limits, the set of discrete states of the Markov chain is as follows:

$$\Omega = \{(0,-1);(0,0);(0,1);(0,2);(0,3);(1,-1);(1,0);(1,1);(1,2);(1,3);(2,-1);(2,0);(2,1);(2,2);(2,3)\}$$

Moreover, the process is stopped for adjustment in states {(0,−1), (0,3), (1,−1),(1,3), (2,−1), (2,3)}.

Transition matrix **P**, for this example of h=3, is given by the following:

$\boldsymbol{P} =$

	(0,−1)	(0,0)	(0,1)	(0,2)	(0,3)	(1,−1)	(1,0)	(1,1)	(1,2)	(1,3)	(2,−1)	(2,0)	(2,1)	(2,2)	(2,3)
(0,−1)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0
(0,0)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0
(0,1)	qR_0	qG_0	0	qY_0	0	$(1-q)R_1$	$(1-q)G_1$	0	$(1-q)Y_1$	0	0	0	0	0	0
(0,2)	qR_0	qG_0	0	0	qY_0	$(1-q)R_1$	$(1-q)G_1$	0	0	$(1-q)Y_1$	0	0	0	0	0
(0,3)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0
(1,−1)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0
(1,0)	0	0	0	0	0	0	0	0	0	0	R_1	G_1	Y_1	0	0
(1,1)	0	0	0	0	0	0	0	0	0	0	R_1	G_1	0	Y_1	0
(1,2)	0	0	0	0	0	0	0	0	0	0	R_1	G_1	0	0	Y_1
(1,3)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0
(2,−1)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0
(2,0)	0	0	0	0	0	0	0	0	0	0	R_1	G_1	Y_1	0	0
(2,1)	0	0	0	0	0	0	0	0	0	0	R_1	G_1	0	Y_1	0
(2,2)	0	0	0	0	0	0	0	0	0	0	R_1	G_1	0	0	Y_1
(2,3)	qR_0	qG_0	qY_0	0	0	$(1-q)R_1$	$(1-q)G_1$	$(1-q)Y_1$	0	0	0	0	0	0	0

$\mathbf{P}$ is a matrix of an ergodic Markov chain (details in [13]), and $\lim_{n\to\infty}\mathbf{P}^n = \mathbf{Z}$, in which all lines in matrix $\mathbf{Z}$ are equal to the line vector

$$\mathbf{Z} = \left[Z_{(0,-1)}, Z_{(0,0)}, \ldots, Z_{(0,h)}, Z_{(1,-1)}, Z_{(1,0)}, \ldots, Z_{(1,h)}, Z_{(2,-1)}, Z_{(2,0)}, \ldots, Z_{(2,h)}\right] \quad (13)$$

The z vector is a probability vector in the stationary state, in which $\sum_{s=0}^{2}\sum_{k=-1}^{h}\mathbf{z}_{(s,k)} = 1$ with $z_{(s,k)}\geq 0$. Each element of $\mathbf{z}$ is interpreted as the fraction of the number of inspections that is performed in each of the states after a sufficiently large number of inspections. Given that $\mathbf{P}^{(k+1)} = \mathbf{P}^{(k)}\mathbf{P}$ and $\lim_{n\to\infty}\mathbf{P}^{(k+1)} = \lim_{n\to\infty}\mathbf{P}^{(k)} = \mathbf{Z}$, then $\mathbf{Z} = \mathbf{ZP}$. Because all lines of $\mathbf{Z}$ are equal to vector $\mathbf{z}$, the equation $\mathbf{z}=\mathbf{zP}$ is also valid, which can be rewritten as follows:

$$\mathbf{z} = \mathbf{zP} \therefore \mathbf{z}(\mathbf{P}-\mathbf{I}) = \mathbf{0} \quad (14)$$

where $\mathbf{I}$ is the matrix identity and $\mathbf{0}$ is a null vector. Thus, a single vector z can be obtained from resolving the linear system (14) with the restriction $\sum_{s=0}^{2}\sum_{k=-1}^{h} z_{sk} = 1$.

3 Average cost per produced item

More assumptions are needed to determine the cost function. As the destructive inspection is performed, every inspected item is discarded. Moreover, it is assumed that the interruption is instantaneous as soon as this decision is taken. After adjustment, the process restarts in state I ($\lambda=\lambda_0$), and an item is classified as non-conforming if the number of defects is greater than the upper limit of specification (LE), which is specified by the engineering team.

Costs in the current model follow a similar structure to those reported in earlier studies by Taguchi et al. [16]; Nayebpour and Woodall [10]; Vasconcelos et al. [17]. The following costs are defined: inspection cost per item (C_I), adjustment cost (C_a), production cost of a non-conforming item (C_{nc}), and the cost of discarding an inspected item (C_d).

The cost for the state (s,k) for s=0,1,2 and k=−1,0,…,h is described as

$$\psi(s,k) = C_I + \phi(s,k) + \xi(s,d) + C_d \quad (15)$$

where $\xi(s,k)$ represents the cost of sending non-conforming items to the consumer or to subsequent steps in the process and $\varphi(s,k)$ is the cost of adjusting the process. The costs of inspection and of discarding the inspected item are fixed and included in the states. Below, other costs are detailed:

(a) Cost of sending non-conforming items to the consumer or to subsequent steps in the process—$\xi(s,k)$

– $\xi(0,k)$—All of the items are produced with $\lambda=\lambda_0$ (state I); thus,

$$\xi(0,k) = C_{nc}(m-1)(1-p_1) \quad (16)$$

where $k=-1,0,1,2,\cdots,h$ and $p_1=P(X_i<LE/\lambda=\lambda_0)$, where LE is the specification limit;

– $\xi(2,k)$—All items are produced with $\lambda=\lambda_1$ (state II), such that

$$\xi(2,k) = C_{nc}(m-1)(1-p_2) \quad (17)$$

where $k=-1,0,1,2,\cdots,h$ and $p_2=P(X_i<LE/\lambda=\lambda_1)$;

– $\xi(1,k)$—In this case, some of the m items are produced in state I ($\lambda=\lambda_0$). It is known that, at minimum, the last item of the cycle, the one inspected, was produced in state II ($\lambda=\lambda_1$). However, for the $(m-1)$ items not inspected, the quantities produced in states I and II are unknown. This change can be manifested in the first item produced (in this case, all $m-1$ are produced in state II) or may even occur only in the last item of the cycle (in this case, all $m-1$ are produced in state I). Thus, considering all possibilities,

$$\xi(1,k) = C_{nc}\left[\frac{\sum_{i='2}^{m-1}\pi(1-\pi)^{(i-1)}[(i-1)(1-p_1)+(m-1)(1-p_2)]}{1-(1-\pi)^m}\right] \quad (18)$$

where $k=-1,0,1,2,\cdots,h$ and $(i<m); i$ indicates the first item produced in state II.

(b) Cost of adjusting the process (C_a)

For the states (s,k), $s=0,1,2$ and $k=-1$, h, the process is interrupted for adjustment, such that

$$\phi(s,k) = C_a \quad (19)$$

for $k=-1$ or h; otherwise, $\varphi(s,k)=0$.

In each cycle, $(m-1)$ items are sent to the market or to the next production stages. For a sufficiently large number of inspections, **z** corresponds to the occurrence probability vector of each state in the chain. Therefore, the average cost per item inspected and not discarded in each inspection cycle is as follows:

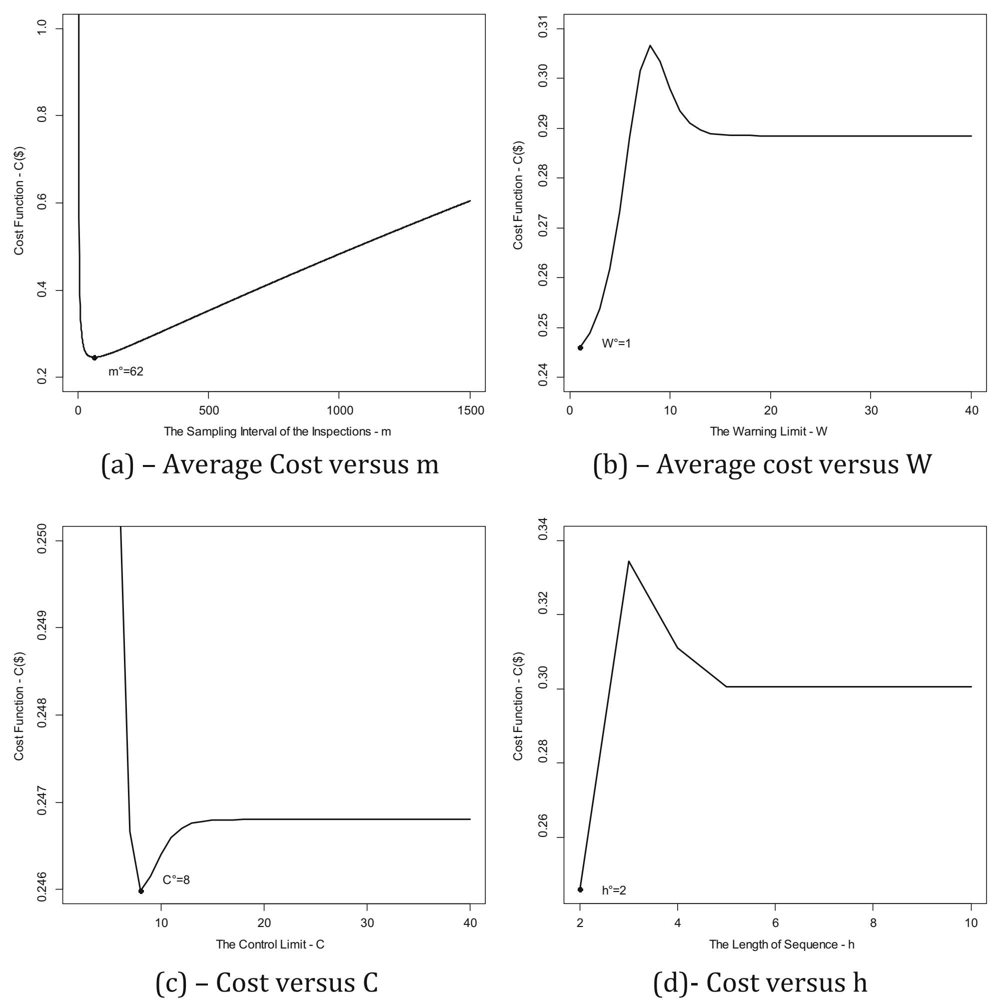
$$AV(m,W,C,h) = \frac{\sum_{s=0}^{2}\sum_{k=-1}^{h} z_{(s,k)}\left[C_I+\phi_{(s,k)}+\xi_{(s,k)}+C_d\right]}{m-1} \quad (20)$$

The problem consists of determining the values of m, W, C, and h by an optimum control policy that minimizes (21).

$$\left(m^0, W^0, C^0, h^0\right) = \arg\min_{m,W,C,h}[AV(m,W,C,h)] \quad (21)$$

The input parameters are as follows: the probability of changing from state I to state II (π), the average number of defects in the items produced in state I (λ_0) and state II (λ_1),

Fig. 2 Plots of average cost versus control limit (C), warning limit (W), length of sequence (h), and sampling interval (m)



and the limit of specification (LE). The cost parameters are as follows: the inspection cost per item (C_i), the adjustment cost (C_a), the production cost of a non-conforming item (C_{nc}), and the cost of discarding an inspected item (C_d).

Because it is not possible to obtain an analytical expression for (21), an exhaustive direct search is used to determine the optimum parameters (m^0, W^0, C^0, h^0).

4 Numerical example

To illustrate the proposed model with a numerical example, consider a process in which flash drives are produced with the following costs:

C_I \$0.025 (inspection cost)
C_a \$30 (adjustment cost)
C_{nc} \$5 (production cost of a non-conforming item);and
C_d \$1 (cost of discarding an inspected item).

The upper specification limit is considered equal to five (LE=5), and the lower specification limit is zero. The probability of a change from state I to state II is π=0.0001. The average number of defects follows a Poisson distribution of parameter λ_0=2.5 (state I) or λ_1=6.5 (state II).

A program (using R software) was developed to obtain the parameters of the optimum design (for access to the program, write to the first author) as follows: sampling interval m^0=62, warning limit W^0=1, control limit C^0=8, and length of the sequence between the warning and control limit h^0=2. For this

Table 1 Cost variations obtained from m^0, W^0, C^0, and h^0 varying one of the parameters or one of the process costs

π	m	W	C	h	C(\$)	λ_1	M	W	C	h	C(\$)
0.000001	605	1	8	2	0.2136	3	171	1	23	2	0.2223
0.00001	192	1	8	2	0.2213	5	73	1	10	2	0.2391
0.0001	62	1	8	2	0.2463	6.5	62	1	8	2	0.2463
0.001	19	1	34	2	0.3284	13	65	1	8	2	0.2466
0.005	9	1	35	2	0.4809	26	67	1	11	2	0.2447
0.009	7	1	36	2	0.5783	70	67	1	21	2	0.2446
0.01	7	1	36	2	0.5991	90	67	22	27	2	0.2446
0.05	4	1	36	2	1.1277	100	67	22	33	2	0.2446
0.09	3	1	36	2	1.4759	150	67	16	23	2	0.2446
0.1	3	1	36	2	1.5454	300	67	16	23	2	0.2446

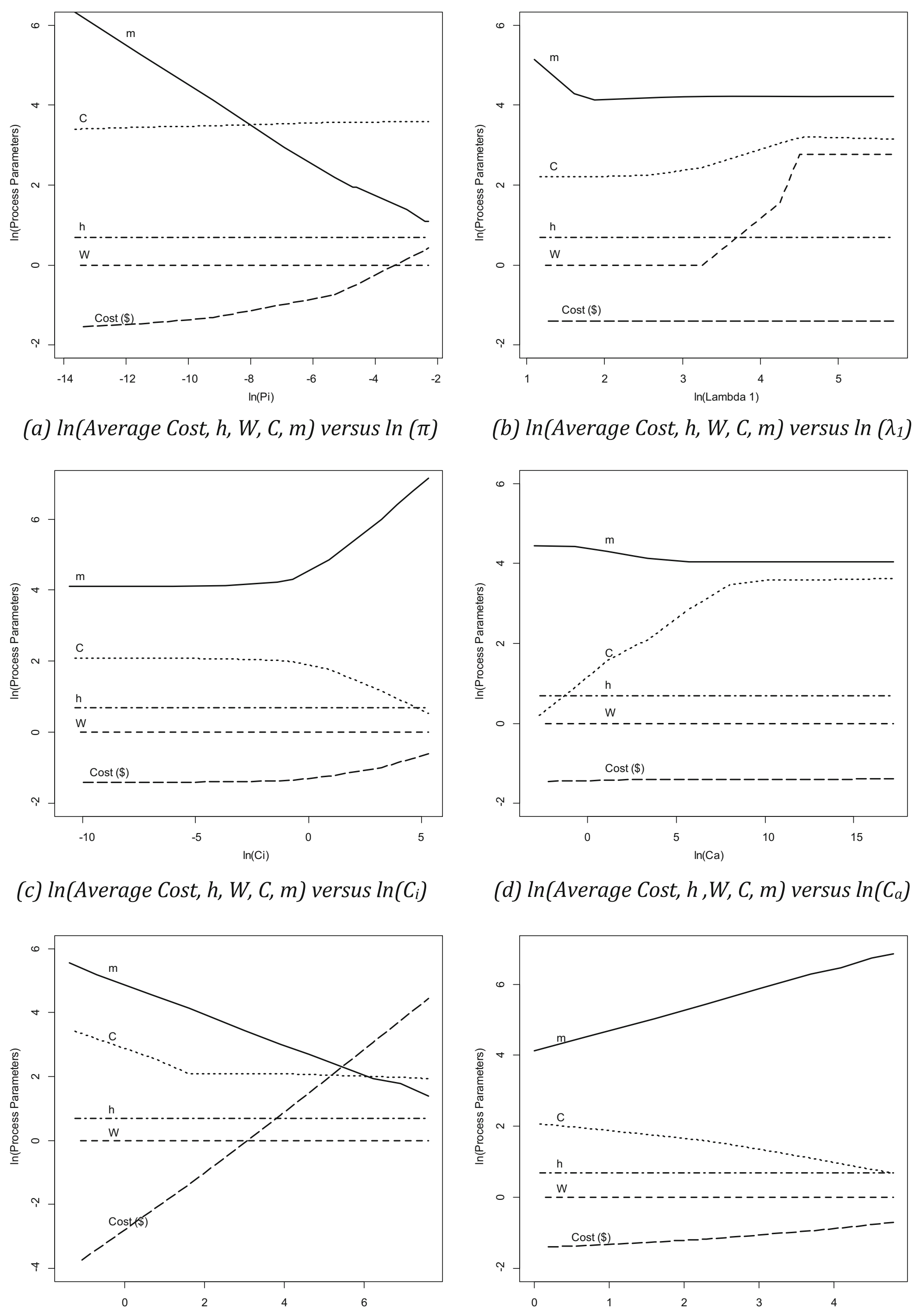
C_i	m	W	C	h	C(\$)	C_a	m	W	C	h	C(\$)
0.000025	61	1	8	2	0.2459	0.05	85	1	1	2	0.2354
0.0025	61	1	8	2	0.2459	0.5	83	1	3	2	0.2382
0.025	62	1	8	2	0.2463	3	74	1	5	2	0.2420
0.25	68	1	8	2	0.2498	15	65	1	7	2	0.2452
0.5	74	1	8	2	0.2534	30	62	1	8	2	0.2463
2.5	129	1	6	2	0.2737	300	57	1	34	2	0.2470
25	400	1	4	2	0.3607	3,000	57	1	37	2	0.2470
50	598	1	3	2	0.4097	30,000	57	1	34	2	0.2470
100	881	1	2	2	0.4768	300,000	57	1	37	2	0.2470
200	1,271	1	1	2	0.5692	30,000,000	57	1	37	2	0.2494

C_{nc}	m	W	C	h	C(\$)	C_d	m	W	C	h	C(\$)
0.25	256	1	33	2	0.0186	0	10	1	33	2	0.2161
0.5	180	1	35	2	0.0325	1	62	1	8	2	0.2463
2	94	1	9	2	0.1071	2	91	1	7	2	0.2597
5	62	1	8	2	0.2463	5	152	1	6	2	0.2846
20	31	1	8	2	0.9125	10	232	1	5	2	0.3106
50	20	1	8	2	2.2157	20	356	1	4	2	0.3448
100	15	1	8	2	4.3657	40	538	1	3	2	0.3904
500	7	1	8	2	21.3913	60	644	1	3	2	0.4243
1,000	6	1	7	2	42.5759	90	836	1	2	2	0.4640
2,000	4	1	7	2	84.8584	120	954	1	2	2	0.4976

scenario, the optimal cost per item produced is \$0.2463. For comparison purposes, an optimum policy of m^0=57 and C^0= 6, obtained by Vasconcelos et al. [17], with an average cost per item of \$0.2658 is the situation obtained for the case in which the warning limit is equal to the control limit (W=C); consequently, h=0. This cost is approximately 8 % higher than the proposed model. It is noteworthy that a cost is 8 % less expensive per item produced if the production volume is high. For example, ordering 100,000 pieces will result in a reduction of \$800,000 if the cost per item is \$100. Plots of the average cost versus m, W, C, and h can be found in Fig. 2a–d, respectively.

It is worth noting that the engineers of the project team define the specification limit (LE) and that the control limit

Fig. 3 Sensitivity analysis of process parameters in natural logarithm (ln)



(a) ln(Average Cost, h, W, C, m) versus ln (π)

(b) ln(Average Cost, h, W, C, m) versus ln (λ_1)

(c) ln(Average Cost, h, W, C, m) versus ln(C_i)

(d) ln(Average Cost, h ,W, C, m) versus ln(C_a)

(e) ln(Average Cost, h, W, C, m) versus ln(C_{nc})

(f) ln(Average Cost, h, W, C, m) versus ln(C_d)

Table 2 Levels of the factor used in the supplementary sensitivity analysis

Factor	Level (−)	Level (+)	Estimate	*p* value
π	0.0001	0.001	*−0.079*	0.015
λ_1	6.5	19.5	0.033	n.s
C_i	0.025	0.25	0.035	n.s.
C_a	30	300	0.016	n.s
C_{nc}	5	20	*−0.356*	<0.000
C_d	1	5	−0.026	n.s

(C) is determined according to statistical and/or economical criteria via optimization to define the rejection region of the null hypothesis that the process is in control. These two quantities are not directly related because an item can be classified as non-conforming according to engineering's criteria (e.g., an item with six or seven defects) despite not requiring rejection of the null hypothesis when the process remains in control.

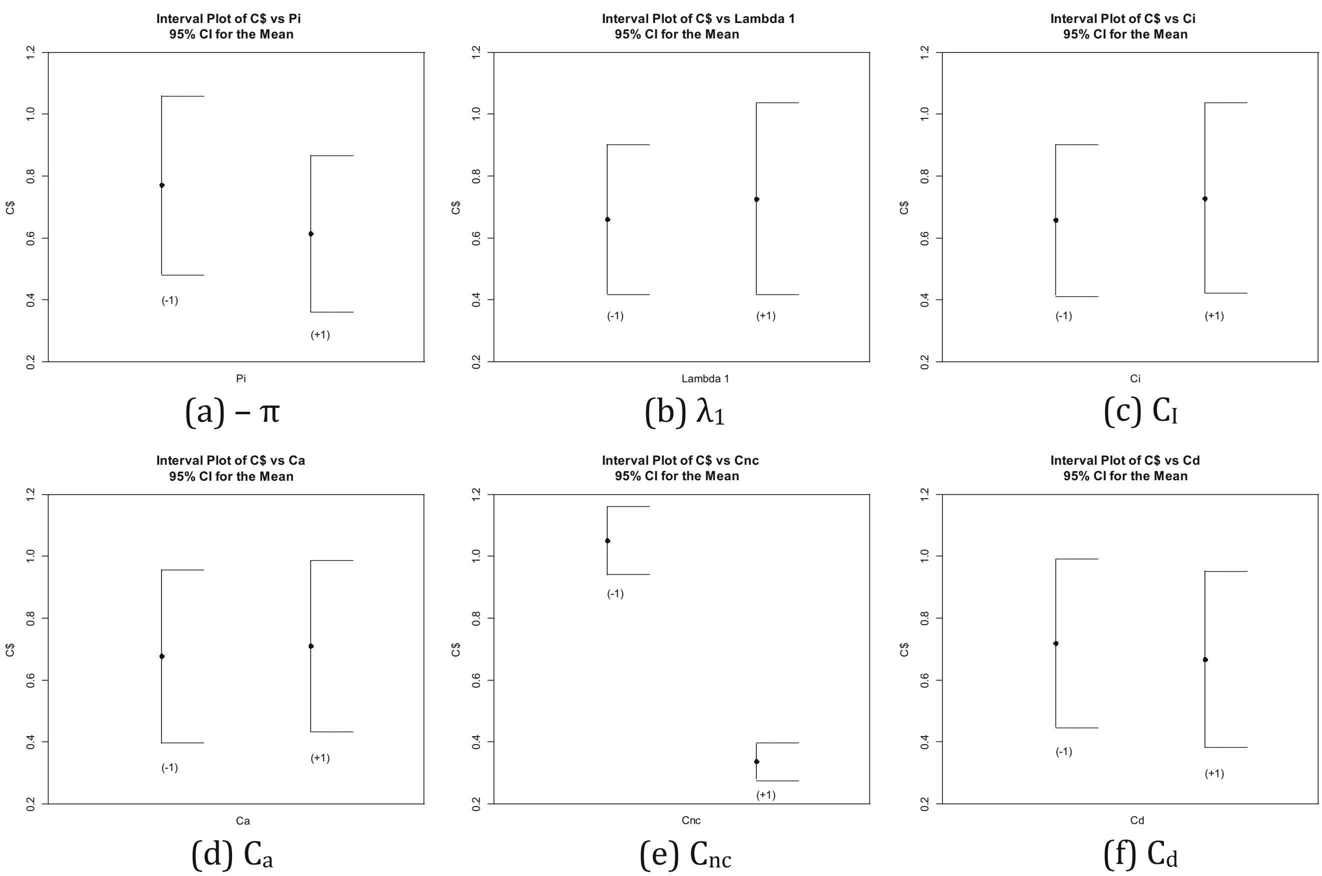
A complementary sensibility analysis (varying one parameter at a time) is conducted to analyze the behavior of the average cost AV and the parameters of design, including the inspection interval (m), the control limit (C), the warning limit (W), and the length of h in a function of the input process parameters π, λ_1, C_i, C_a, C_{nc}, and C_d in a range of previously specified values obeying the following restrictions:

- the average number of defects in state I is lower than that of state II ($\lambda_0<\lambda_1$),
- the adjustment cost is much higher than the inspection cost ($C_a>>>C_I$), and
- the production cost of a non-conforming item is higher than the cost of discarding it ($C_{nc}>C_d$).

The results of this analysis are summarized in Table 1. To accommodate the different scales of the input parameters, the logarithmic scales for both axes are employed in Fig. 3 to draw the multiple overlaid graphs.

Several interesting observations can be drawn from Fig. 3.

- The length of the sequence (h^0) is insensitive to all variations in costs and parameters insofar as it always exhibits the same optimum value.
- The warning limit (W^0) is also invariable, except when the average number of defects in the items produced in state II (λ_1) is very large ($\lambda_1 \geq 90$).
- As the probability of changing from state I to state II (π) increases, the average cost per item produced (AV) also rises, whereas the inspection interval (m^0) declines



(a) – π (b) λ_1 (c) C_I

(d) C_a (e) C_{nc} (f) C_d

Fig. 4 Interval plots of average cost versus the various factors (inspection frequency increases) (Fig. 3a). If the average number of defects in items produced in state II (λ_1) increases but is lower than the specification limit ($\lambda_1 \leq LE = 5$), the inspection interval (m) and control limit (C) decrease. When $\lambda_1 > LE$, m remains stable (Fig. 3b).

- As the inspection cost per item (C_i) increases, the average cost per item produced (AV) and the inspection interval (m^0) also increase. However, the control limit (C^0) starts to decline (Fig. 3c).
- As the adjustment cost (C_a) increases, the inspection interval (m^0) decreases (after remaining stable for $C_a > 30$). The control limit (C^0) rises when the adjustment cost also increases (Fig. 3d). Figure 3e illustrates that as the production cost of a non-conforming item (C_{nc}) rises, so does the average cost per item produced, whereas the inspection interval (m^0) decreases. As shown in Fig. 3f, the greater the cost of discarding an inspected item (C_d), the higher the inspection interval will be (m^0).

These findings are very interesting; however, this sensitivity analysis was conducted while varying only one parameter at a time, and it is important to identify which factor produces a substantial impact mainly on the average cost. Thus, a supplementary sensitivity analysis was developed to reflect a fractional factorial experimental design $2^{(6-2)}$, where six parameters are assigned as factors that can be varied simultaneously. The decision to conduct 16 runs was due to the interest in verifying the existence of main effects among the factors on the average cost per item produced. The estimates of effects with the respective p values (as well as the levels of the factors) are shown in Table 2. Note that the values specified in the beginning of this section are set as the level (−). According to Table 2 and Fig. 4, the parameters that exhibited the greatest significance in their variation are the probability of changing from state I to state II (π) and the cost to produce a non-conforming item (C_{nc}).

5 Final considerations and suggestions for future studies

In cases of the use of on-line processes to monitor the stability of the average number of non-conformities in an inspected item, the model developed in this study considers two sets of limits (control and warning) and a supplementary decision rule for process interruption. This policy provides a control strategy that is approximately 8 % more economical (per unit) than the model that solely applies the conventional decision rule proposed by Vasconcelos et al. [17]. Our proposal aimed at developing a model that obtains the following optimum parameters: m^0 (the inspection interval), C^0 (the control limit), W^0 (the warning limit), and h^0 (the size of the inspection sequence). This approach minimizes the mean cost per item produced (here, an infinite production horizon is considered). We presumed a production system that produces items exhibiting an average number of defects λ_0 when the process is operating in state I and a number of non-conformities $\lambda_1 > \lambda_0$ when it is operating in state II. The production change from state I to state II occurs with probability π, and the time (in number of units produced) that the process remains in control follows a geometric distribution for parameter π, ($0 \leq \pi \leq 1$). In a sensitivity analysis, the probability of changing from state I to state II and the production cost of a non-conforming item exhibited more significant variation.

Future studies based on Bessegato et al. [1] should conduct on-line monitoring for the number of non-conformities in the inspected item using a Shewhart graph with multiple inspection intervals and the addition of other supplementary rules for process intervention.

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