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ORIGINAL ARTICLE

Economical operation of the C_{pm} control chart for monitoring process capability index

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Abstract Process capability indices are widely used to provide the evaluation measure of a process. Especially, the process capability index C_{pm}, which is defined by the range of the process standard specification limits and the deviation from a target value, is called the Taguchi index. Boyles has investigated the statistical characteristics of the estimator $\hat{C}_{pm}$, and also proposed a technique for the C_{pm} control chart. Since the process capability index C_{pm} is based on the concept of the Taguchi's quality loss, the process capability index C_{pm} already includes an economical concept. In this article, we evaluate an operating cost consisting of the sampling cost, the sample cost, and the quality loss of failing to detect an out-of-control state when the C_{pm} control chart is used. Then, we derive an optimal operating plan by sample size and sampling interval in order to minimize the ceiling value of the operating cost based on the min–max criterion.

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1 Introduction

A manufacturing process is characterized by numerical measurements of items produced from the process. These process characteristics are assumed to be normally distributed so that the process is characterized by a process mean μ and a standard deviation σ. Generally, a process specification consists of lower and upper specification limits (LSL, USL) and a target value T somewhere between these limits. A process capability index is a unitless function of the process parameters (μ, σ) and the process specification (LSL, T, USL) for the purpose of quantifying the performance of a process [18]. The traditionally used process capability indices are C_p and C_{pk} [7, 18, 24], where

$$C_p = \frac{UCL - LCL}{6\sigma}, \tag{1}$$

$$C_{pk} = \min\left\{\frac{UCL - \mu}{3\sigma}, \frac{\mu - LCL}{3\sigma}\right\}. \tag{2}$$

C_p evaluates the related scale of the specification's tolerance with the process's tolerance, while C_{pk} simultaneously evaluates the centering degree and the dispersion degree. The estimators of C_p and C_{pk} have already been developed on evaluating the process performance in practice. On one hand, evaluating the process capability index is sometimes required when the process characteristics are assumed to be nonnormally distributed. The evaluation of process capability indices for nonnormally distributed processes has been proposed in the literature of Hosseinifard et al. [13] and Abbasi [1]. Note that the process characteristics are assumed to be normally distributed in this article.

Control charts play an important role in statistical process control tools [24]. The process performance is estimated by the quality characteristics of items produced from a process. The process mean μ and deviation σ are traditionally monitored by $\overline{x}$ and s control charts, respectively. A process capability index is also used as one of the evaluation measures for process performance.

Boyles [7] has investigated a control chart based on an estimator of the process capability index C_{pm} and insisted that the C_{pm} control chart is appropriate for monitoring the process capability, where

$$C_{pm} = \frac{UCL - LCL}{6\tau}, \tag{3}$$

$$\tau^2 = E\left[(x - T)^2\right] = \sigma^2 + (\mu - T)^2, \tag{4}$$

where T is usually set at the midpoint of the specification limits. The process capability index C_{pm} is effective in analyzing manufacturing systems because the index is composed of the specification limits of items and the deviation with respect to a specified target value T of items [20, 21].

As mentioned the above, the process capability index quantifies the performance of a process using the process parameters (μ, σ) and the process specification (LSL, T, USL). C_{pm} show the frequency with which the observed characteristics of items are within a given range of the specification. The C_{pm} control chart is different from the classical control chart, for example, the $\overline{x}$ chart, in that the stochastic characteristics are not directly estimated but the degree of fulfilling the specification for items is estimated. Then, Pearn and Shu [28] have applied the C_{pm} control chart to the practical production environment of precision electronic devices. As a result, they have verified the effectiveness of the C_{pm} control chart and concluded that the approach is useful for quality improvement decisions.

Lorenzen and Vance [22] have discussed a number of important considerations in the economic operation of control charts. Usually, some kinds of costs, such as sample cost, sampling inspection cost, and additional loss due to failure to detect an out-of-control state, have been considered in the design of economical operations. Duncan [9, 10] has studied an economical design of the $\overline{x}$ control chart. The economical design of the $\overline{x}$ control chart has also been considered by Banerjee and Rahim [5] and Parkhideh and Case [26]. When discussing the economical design of control charts, it is necessary to evaluate the loss for process quality. Conventionally, the loss for process quality has been estimated using the proportion of nonconforming items.

Taguchi [30, 31] has proposed the concept of "quality loss" based on the deviation from a target value of items instead of the quality evaluation by the proportion of nonconforming items. The quality loss is evaluated even if a produced item is judged to be conforming as an item. Therefore, the achievement of high quality can be expected when the quality loss is applied to the quality evaluation. The concept of the Taguchi's quality loss has been applied for various quality improvement decisions [2, 6, 11, 23, 33]. Arizono et al. [3] and Morita et al. [25] have applied the concept of the quality loss to the design of acceptance sampling plan. A variety of economical operations of control charts using the Taguchi's quality loss have been developed [15, 19, 32, 34].

The process capability index C_{pm} has been called the Taguchi index because the definition of C_{pm} has been essentially identical with that of the Taguchi's quality loss. Hence, it must be appropriate to consider the economical operation of the C_{pm} control chart based on the economical criterion of the Taguchi's quality loss. However, the economical operation of the C_{pm} control chart has not yet been discussed. From Eqs. 1 and 2, C_p and C_{pk} do not consider the specified target value T because C_p and C_{pk} are approaches to quality improvement for the reduction of variability. While C_{pm} is an approach to quality improvement for the reduction of variability from the target value. It is not appropriate to apply the quality loss of nominal-the-best to C_p and C_{pk}. Therefore, the economical operation of C_p and C_{pk} using the Taguchi's quality loss should not be considered. Note that the process capability index based on the quality loss has been proposed except C_{pm}. Hesieh and Tong [14] have proposed the capability index based on the quality loss. Their index is such that the quality loss is defined by the smaller-the-better quality characteristics such as the proportion of nonconforming items.

In the design of the economical operation of control charts, an additional loss due to failure to detect an out-of-control state is considered. Therefore, a particular out-of-control state to evaluate the loss is sometimes specified. For example, Takemoto et al. [32] have specified an out-of-control state to be detected and designed the economical operation of the cumulative sum $(\overline{x}, s)$ control chart for the specified out-of-control state. Also, Wu et al. [34] have assumed that the information about some out-of-control states to be shifted from the in-control state is provided. Usually, the out-of-control state to be shifted from the in-control state is unknown. On one hand, Kobayashi et al. [19] have proposed a design technique of the economical operation in the worst situation among all possible out-of-control states, where sample size is the only decision variable in the technique proposed by Kobayashi et al. [19].

In this article, we discuss the design of the economical operation of the C_{pm} control chart based on Taguchi's quality loss using the min–max criterion. The min–max criterion realizes the optimization in the worst situation among all possible out-of-control states. Then, we define the cost function including the sample size and sampling interval as decision variables and then propose the design procedure of the economical operation of the C_{pm} control chart by Taguchi's quality loss.

2 Outline of C_{pm} control chart

When the process quality characteristics obey a normal distribution $N(\mu, \sigma^2)$, the Taguchi's quality loss is defined as $k\tau^2$, where τ^2 is in Eq. 4 and k represents a proportional coefficient based on a functional limit of quality characteristics. k is a positive constant. Therefore, the process capability index C_{pm} is called the Taguchi's index or the process capability index based on the loss criterion [7, 28].

The proportional coefficient k is specified by the capability limits Δ_U, Δ_L as follows:

$$k = \frac{A}{\Delta_U^2} \quad \text{or} \quad k = \frac{A}{\Delta_L^2},$$

where A denotes the complete loss in the case that the item cannot operate normally by unsatisfying the specification. Then, $(T + \Delta_U, T, T - \Delta_L)$ corresponds with the process specification (USL, T, LSL). When $\Delta_U \neq \Delta_L$, the largest value k is adopted [30].

Let $x_i, i = 1, 2, ..., n,$ be random samples from a normal distribution $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Taguchi has proposed the estimator $\hat{\tau}^2$ of the expected loss τ^2 defined by

$$\hat{\tau}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - T)^2 = (\overline{x} - T)^2 + s^2, \tag{5}$$

where $\overline{x}$ and s^2 denote the maximum likelihood estimators of μ and σ^2 as follows:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2.$$

Since $\overline{x}$ and s^2 are the maximum likelihood estimators, the estimator $\hat{\tau}^2$ is also the maximum likelihood estimator and then the uniformly minimum variance unbiased estimator [28]. It is known that the statistic $n\hat{\tau}^2/\sigma^2$ obeys a noncentral chi-square distribution with n degrees of freedom and noncentrality parameter $n\xi^2$ [29], where ξ^2 is defined as

$$\xi^2 = \frac{(\mu - T)^2}{\sigma^2}.$$

The distribution of the estimator $\hat{C}_{pm}$ is defined as

$$\hat{C}_{pm} \equiv \frac{d}{3\hat{\tau}} \sim \frac{d}{3}\sqrt{\frac{n}{\sigma^2\chi^2_{n,n\xi^2}}}, \tag{6}$$

where $\chi^2_{n,n\xi^2}$ denotes a noncentral chi-square distribution with n degrees of freedom, noncentrality parameter $n\xi^2$, and

$$d = \frac{USL - LSL}{2}.$$

Then, we define a judgment rule for the process state. Let L be a control limit. The following judgment rule is constructed:

$$\begin{cases} \text{if} \quad \hat{C}_{pm} > L, & \text{then} \quad \text{in-control.} \\ \text{otherwise,} & \text{out-of-control.} \end{cases}$$

Now, we define the in-control state as $N(\mu_0, \sigma_0^2)$, where $T = \mu_0$ and σ_0^2 means the process variance in the in-control state. The quality loss $k\tau_0^2$ in the in-control state is given as $k\tau_0^2 = k\sigma_0^2$ from Eq. 4. When the process is in-control, we denote the distribution of the estimator $\hat{\tau}^2$ as

$$\hat{\tau}^2 \sim \frac{\tau_0^2}{n}\chi_n^2 = \frac{\sigma_0^2}{n}\chi_n^2, \tag{7}$$

where χ_n^2 means a central chi-square distribution with n degrees of freedom. Therefore, the distribution of $\hat{C}_{pm}$ in the in-control state is described as

$$\hat{C}_{pm} \sim \frac{d}{3}\sqrt{\frac{n}{\sigma_0^2\chi_n^2}}.$$

Let α be a specified type I error probability. Then, the control limit L is given as

$$L = \frac{d}{3}\sqrt{\frac{n}{\sigma_0^2 \chi_n^2(\alpha)}}, \tag{8}$$

where $\chi_n^2(\alpha)$ means the upper 100α percentile of the chi-square distribution with n degrees of freedom.

Next, let $N(\mu_1, \sigma_1^2)$ denote an out-of-control state. In this case, we address the distribution of the estimator $\hat{\tau}^2$. We denote the statistic ρ as

$$\rho = \frac{1+\xi_1^2}{1+2\xi_1^2}\frac{n\hat{\tau}^2}{\sigma_1^2}, \tag{9}$$

where

$$\xi_1^2 = \frac{(\mu_1-\mu_0)^2}{\sigma_1^2}. \tag{10}$$

Then, by letting the mean and variance of the statistic ρ correspond to those of chi-square distribution with ϕ degree of freedom, we obtain

$$\phi = \frac{n\left(1+\xi_1^2\right)^2}{1+2\xi_1^2}. \tag{11}$$

Accordingly, the chi-square distribution with ϕ degrees of freedom in Eq. 11 can be employed as the approximation distribution of ρ. Therefore, the distribution of the estimator $\hat{\tau}^2$ in the out-of-control state is represented as

$$\hat{\tau}^2 \sim \frac{\tau_1^2}{\phi}\chi_\phi^2. \tag{12}$$

This technique is based on the Patnaik transformation, in which the noncentral chi-square distribution is transformed into the central chi-square distribution [27]. Also, this technique has been employed when the distribution of $\hat{C}_{pm}$ was discussed in the previous literature [7].

Consequently, the power of the C_{pm} control charts with L in Eq. 8 is obtained as the following equation:

$$\frac{\chi_\phi^2(1-\beta)\tau_1^2}{\phi} = \frac{\chi_n^2(\alpha)\tau_0^2}{n}. \tag{13}$$

Note that the power is varied according to the out-of-control state $N(\mu_1, \sigma_1^2)$, while there are innumerable combinations of (μ_1, σ_1^2) with the same quality loss τ_1^2.

3 Definition of evaluation function

As mentioned above, we specify the control limit of the C_{pm} control chart for given α and n. In this article, we consider the economical sample size and the economical sampling interval when the C_{pm} control chart is operated for a given α. Then, the operating cost function should be constructed in order to evaluate the economical operation of the C_{pm} control chart. In this section, we define an evaluation function for operating the C_{pm} control chart.

At first, we present the expected loss $k\tau_0^2$ per unit item for the in-control state $N(\mu_0, \sigma_0^2)$ as

$$k\tau_0^2 = k\sigma_0^2. \tag{14}$$

Then, the quality loss τ_0^2 in Eq. 14 can be interpreted as the unavoidable loss due to the chance cause for the in-control state. The quality loss τ_1^2 for an out-of-control state $N(\mu_1, \sigma_1^2)$ is derived as

$$k\tau_1^2 = k\left\{\sigma_1^2 + (\mu_1-\mu_0)^2\right\}, \tag{15}$$

where τ_1^2 means the additional loss due to the assignable cause and we suppose $k\tau_1^2 > k\tau_0^2$ in general. As mentioned above, the difference $k(\tau_1^2 - \tau_0^2)$ can be interpreted as the avoidable surplus loss by means of detecting the out-of-control state.

Then, we assume that the process repeats the in-control state and the out-of-control state alternately via restoration. Therefore, we define a unit cycle by a pair of the in-control condition and the out-of-control condition. In this case, define unit time as production time per unit item. Then, an exponential distribution is often adopted as the distribution of state transition time from the in-control state to an out-of-control state [4, 8, 12]. Accordingly, since the probability density function of transition time by occurrence of the assignable cause is described as

$$f(t) = \lambda \exp\{-\lambda t\}, \tag{16}$$

we obtain the probability q that the in-control state is maintained in respective sampling intervals as

$$q = \exp\{-\lambda M\}, \tag{17}$$

where M means the batch size in the interval between successive samplings. Then, we evaluate the expected period to be in-control by mean of a geometric distribution.

Furthermore, denote the power for an out-of-control state $N(\mu_1, \sigma_1^2)$ by $1-\beta$. We evaluate the expected period from the change in the process state to the detection of the out-of-control condition by chart signals.

Now, we define the evaluation function on operating the C_{pm} control chart. Concretely, we propose the total operation cost based on the sampling cost, the sample cost on operating the C_{pm} control chart, and the additional loss due to failure to detect the out-of-control state.

Define the following expected loss C_L per unit item based on the avoidable loss when operating the C_{pm} control chart:

$$C_L = \sum_{j=1}^{\infty} k\left(\tau_1^2 - \tau_0^2\right)(j-1)\beta^{j-1}(1-\beta) = \frac{k\left(\tau_1^2 - \tau_0^2\right)\beta}{1-\beta}. \tag{18}$$

We obtain the cost C_S per unit item consisting of both the sampling cost and the sample cost as follows:

$$C_S = \left\{\sum_{i=1}^{\infty}(cn+D)iq^i(1-q) + \sum_{j=1}^{\infty}(cn+D)\,j\beta^{j-1}(1-\beta)\right\}\Big/M = \frac{cn+D}{M}\left(\frac{q}{1-q} + \frac{1}{1-\beta}\right), \tag{19}$$

where let n be the sample size, M the batch size, c the sample cost per unit item, D the sampling cost per sampling, and β the probability of failure to detect the out-of-control state. Consequently, the expected total operation cost per cycle is given as $C_S + C_L$. Then, since the cycle time CT is present as

$$CT = \frac{q}{1-q} + \frac{1}{1-\beta} = \frac{1-q\beta}{(1-q)(1-\beta)}, \tag{20}$$

we evaluate the expected total operation cost C per unit time and item as follows:

$$C = \frac{C_S + C_L}{CT} = \frac{cn+D}{M} + \frac{k\left(\tau_1^2 - \tau_0^2\right)(1-q)\beta}{1-q\beta}. \tag{21}$$

4 Derivation of economical operation

When the cost and the power on operating control charts are evaluated, it is traditionally assumed that once the process state changes into an out-of-control state, the process remains at the out-of-control state until the assignable cause has been identified and removed. Takemoto et al. [32] have specified an out-of-control state to be detected in the design of economical operation. Wu et al. [34] have assumed that the information about some out-of-control states to be shifted from the in-control state is provided in the design of economical operation.

The in-control state is known and unique. The quality loss τ_0^2 is such as to be unique to the in-control state $N(\mu_0, \sigma_0^2)$. An out-of-control state to be shifted from the in-control state is unknown and not unique. The C_{pm} control chart has the different power $1-\beta$ for respective out-of-control states $N(\mu_1, \sigma_1^2)$. Especially, there are innumerable combinations of (μ_1, σ_1^2) for a given τ_1^2 from Eq. 4, where $\tau_1^2 > \tau_0^2$. Among innumerable out-of-control states (μ_1, σ_1^2) with same loss τ_1, the worst situation in the cost should be considered. Also, the process does not always remain at an identical out-of-control state until the assignable cause has been identified and removed after the process state changes into the out-of-control state. Then, the development of economical operations for various changes of a process state is needed.

At first, we consider the combination of (μ_1, σ_1^2) in order that the expected total cost for a given τ_1^2 is maximized. The behavior of the cost function C in β for a given τ_1^2 is investigated. From Eq. 21, we have

$$\frac{dC}{d\beta} = k\left(\tau_1^2 - \tau_0^2\right)\left\{\frac{1}{1-q\beta} + \frac{q\beta}{(1-q\beta)^2}\right\} = \frac{k\left(\tau_1^2 - \tau_0^2\right)(1-q)}{(1-q\beta)^2} > 0. \tag{22}$$

Therefore, we find that the cost function C is monotonically increased in β. This fact shows the rational feature that we can decrease the expected operation cost C by decreasing the type II error probability β.

On one hand, we can evaluate the power $1-\beta$ as Eq. 13. For a given τ_1^2, the ceiling value of C is given when β is maximized. Therefore, we attempt to find the combination of (μ_1, σ_1^2) in order that β is maximized among innumerable combinations of (μ_1, σ_1^2) with the same τ_1^2. Such a combination of (μ_1, σ_1^2) is obtained as the following relation:

$$\frac{\chi_n^2(\alpha)}{n}\tau_0^2 = \min_{(\mu_1,\sigma_1^2)\in\tau_1^2} \frac{\chi_\phi^2(1-\beta)}{\phi}\tau_1^2. \tag{23}$$

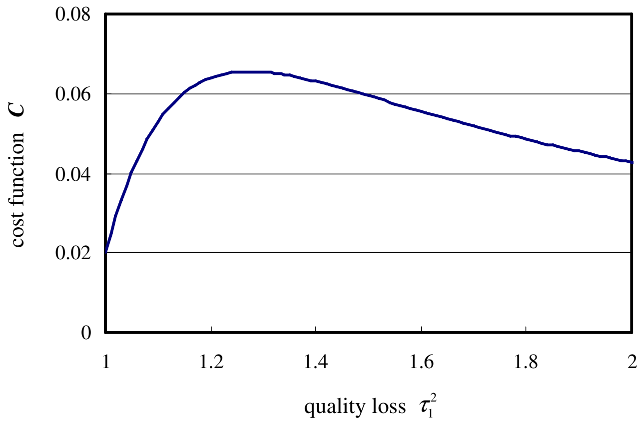

Fig. 1 The behavior of the ceiling value of C in τ_1^2 for a given $(n, M) = (10, 1000)$

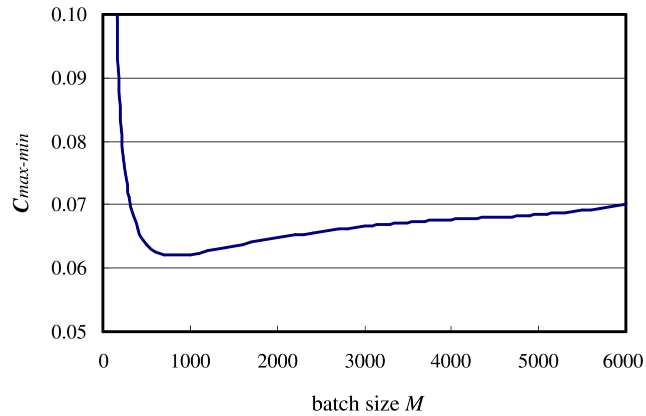

Fig. 3 The behavior of $C_{\text{max–min}}$ in batch size M

By applying the Wilson–Hilferty approximation [17], we consider the behavior of $\chi_\phi^2(1-\beta)/\phi$ in ϕ, that is, a combination of (μ_1, σ_1^2) to minimize the right side of Eq. 23. Consequently, in our previous research [25], we obtain the combination of (μ_1, σ_1^2) to minimize the right side of Eq. 23 as follows:

$$\left(\mu_1, \sigma_1^2\right) = \left(\mu_0, \tau_1^2\right) \quad \text{or} \quad \left(\mu_0 \pm \sqrt{\tau_1^2 - \sigma_0^2}, \sigma_0^2\right). \tag{24}$$

In the detail, see Morita et al. [25].

We find the maximum value $C_{\max}$ of C by comparing the respective expected costs for the quality loss τ_1^2 provided that a sample size n and a batch size M are given. Then, $C_{\max}$ is minimized by the sample size n for a given M, and then the minimum value among any $C_{\max}$ is found, where the minimum value is called $C_{\text{max–min}}$. By comparing $C_{\text{max–min}}$ for every batch size M, we find the minimum value $C^*_{\text{max–min}}$ of $C_{\text{max–min}}$. Finally, we decide the economical operation plan (n, M) of the C_{pm} control chart based on the min–max criterion. Then, the control limit L can be calculated by using Eq. 8.

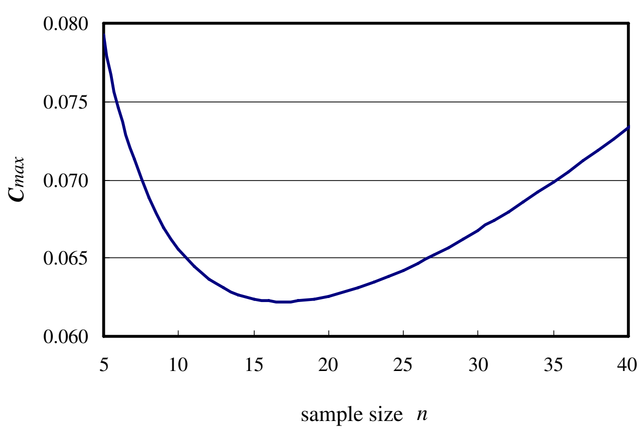

Fig. 2 The behavior of $C_{\max}$ in sample size n for a given $M = 1000$

5 Numerical examples

We consider some numerical examples in order to illustrate the derivation process of the economical operation plan (n, M) in the C_{pm} control chart. Let $d = 3\sigma_0$ and $T = \mu_0 = 0.0$, $\sigma_0^2 = 1.0$ without loss of generality, that is $\tau_0^2 = 1.0$. Then, we assume $k = 1$. As an example, the sample cost coefficient $c = 1$, the sampling cost coefficient $D = 10$, the specified type I error probability $\alpha = 0.01$, the state transition rate $\lambda = 0.00001$ in Eq. 16, and the lot size $M = 1000$. We derive the economical operation plan (n, M) in the above conditions.

At first, Fig. 1 shows the behavior of C in the quality loss τ_1^2. Note that the value of C is evaluated for the combination of (μ_1, σ_1^2) in Eq. 24. In other words, the ceiling value of C for respective τ_1^2 is shown in Fig. 1. From the result, we obtain $C_{\max}$ as the maximum value of C in Fig. 1. Next, Fig. 2 illustrates the behavior

Table 1 Te behavior of (n, M) and $C^*_{\text{max–min}}$ in c

c	n	M	$C_{\text{max–min}}$
0.50	31	1100	0.052
1.00	13	800	0.062
1.50	8	700	0.069

Table 2 The behavior of (n, M) and $C^*_{\max\text{–}\min}$ in D

D	n	M	$C_{\max\text{–}\min}$
5.0	5	400	0.054
10.0	13	800	0.062
15.0	25	1400	0.067

Table 4 The behavior of (n, M) and $C^*_{\max\text{–}\min}$ in α

α	n	M	$C_{\max\text{–}\min}$
0.010	13	800	0.062
0.025	13	1100	0.044
0.050	13	1400	0.034

of $C_{\max}$ in the sample size n. $C_{\max}$ is minimized in the sample size $n = 17$ under $M = 1000$. The operating cost is equal to or less than the minimum value of $C_{\max}$ even if the process shifts to any out-of-control condition when the operation plan $(n, M) = (17, 1000)$ is adopted. Note that the minimum value of $C_{\max}$ is equivalent to $C_{\max\text{–}\min}$ in the case of $M = 1000$. Further, we compare $C_{\max\text{–}\min}$ for respective batch sizes M in Fig. 3. Then, we derive the economical operation plan $(n, M) = (13, 800)$ of the C_{pm} control chart based on the mini-max criterion. When the economical operation plan $(n, M) = (13, 800)$ is adopted, $C^*_{\max\text{–}\min} = 0.062$ is obtained. As a result, the operation cost is always equal to or less than $C^*_{\max\text{–}\min}$ even if the process state changed to any out-of-control state as long as the C_{pm} control chart is operated in the economical plan (n, M).

Then, we have the sensitivity analysis in the economical operation of the C_{pm} control chart. Table 1 shows the behavior of economical operation plan (n, M) in the sample cost c. The parameters except c are the same as in Fig. 3. From Table 1, cheaper c leads to larger n. Then, the strict analysis by large n can lead to large M, that is, the longer sampling interval. Table 2 shows the behavior of economical operation plan (n, M) in the sampling cost D. The parameters except D is the same as in Fig. 3. From Table 2, more expensive D leads to larger n. Simultaneously, more expensive D leads to larger M, that is, the longer sampling interval. Table 3 show the behavior of economical operation plan (n, M) in the state transition rate λ. The parameters except λ are the same as in Fig. 3. From Table 3, larger λ leads to larger n and M in order to realize the quick detection of the out-of-control state. Table 4 shows the behavior of economical operation plan (n, M) in the type I error probability α. The parameters except α are the same as in Fig. 3. α implies the degree of strictness for monitoring the process capability. From Table 4, larger α leads to larger M. These results are quite reasonable.

Table 3 The behavior of (n, M) and $C^*_{\max\text{–}\min}$ in λ

λ	n	M	$C_{\max\text{–}\min}$
0.000005	11	1000	0.045
0.000010	13	800	0.062
0.000015	16	800	0.075

6 Concluding remarks

In this article, we have considered the economical operation of C_{pm} control chart when quality characteristics obey a normal distribution. We define the expected cost function based on the sampling cost, the sample cost, and the additional loss due to failure to detect the out-of-control state. Then, we have proposed the concept of the economical operation plan using the min–max criterion. Further, the decision procedure with respect to the economical operation plan (n, M) of the C_{pm} control chart has been shown throughout numerical examples.

The Taguchi's quality loss is defined based on the deviation from the target value of quality characteristics. The quality loss is evaluated even if the quality characteristics are within both specification limits. At this point, the Taguchi's quality loss is different from the traditional quality loss such as the proportion of nonconforming items. That is, the quality loss is present in the operating cost even if the item is conforming to the specification. Consequently, the Taguchi's quality loss can lead to more strict quality management in comparison with the traditional quality management. The quality management technique based on the Taguchi's quality loss is understood as the quality management technique to aim at the higher quality. Therefore, we are convinced that the proposed economical operation of the C_{pm} control chart contributes to the real industrial environment as the excellent quality management technique.

On one hand, in-control process parameters such as μ_0 and σ_0^2 are not always known and control charts are constructed using estimates in place of the parameters [16]. In this article, the target value T is a desired object and is given. While, σ_0^2 may include the error of estimation when the in-control process dispersion is estimated. The impact of error due to estimation and then the development of economical operation including the impact of error due to estimation are interesting objects in our future research.

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