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Multi-way relaying for cooperative indoor power line communications

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Abstract: Sustaining high data rates over an indoor broadband power line communication (PLC) link is a challenging task due to the significant attenuation of high frequency signals transmitted over power cables. Repeaters are often used to alleviate the problem of signal attenuation, however, this comes at a loss of multiplexing gain. On the other hand, modern relaying concepts, as used in wireless systems, are not as effective in PLC setups due to the absence of diversity improvement. In this study, the authors address this issue and propose a new approach to improve the multiplexing gain of relay-aided PLC systems. The authors consider a multi-user indoor PLC scenario where all users want to share their data. For this setup, the authors then advocate the application of amplify-and-forward (AF) multi-way relaying (MWR). Considering the practical constraints of an indoor PLC system, the authors study the achievable data rates of AF MWR, and compare them to those achievable with direct (non-cooperative) transmission and conventional relaying schemes. The simulations demonstrate that depending on the network topology, MWR can result in significant performance improvement. The authors further discuss how their rate analysis can be exploited to decrease the energy consumption in the system.

1 Introduction

Although power line communication (PLC) [1] is a competitive technology for high-speed indoor communications, its implementation faces some practical challenges rooted in the 'horrible channel' [2]. In particular, the PLC channel is frequency and time selective causing significant path-loss-like attenuation for high-frequency signals. Thus, achievable data rates decay with increasing transmitter–receiver distance.

Inspired by cooperative communications for wireless systems, several studies [3–10, 11] have discussed the application of relaying for PLC to overcome the signal attenuation and improve the data rate. While at the first glance relay-aided PLC looks promising, the authors in [3, 4, 12] report that relaying could be less effective for PLC than for wireless systems. This comes from the fact that PLC networks typically have a tree topology, hence, the received replicas of the signal from the relayed and direct paths are interdependent. This means that the diversity gain in PLC systems is not as significant as in wireless setups. This phenomenon is referred to as *keyhole effect* in [6]. That said, opportunistic relaying [3, 13] has been adopted for PLC systems, so that relaying is used merely when it is beneficial. Another strategy is to exploit two-way relayed PLC systems based on network coding that provides a better multiplexing gain than conventional relaying [4, 7]. Two-way relaying is shown to provide significant performance improvement over non-cooperative transmission in the setups that both amplify-and-forward (AF) and decode-and-forward (DF) one-way relaying (OWR) fail to do so [4].

This work focuses on improving the performance of relay-aided PLC systems through increasing the system's multiplexing gain. More specifically, we consider an indoor multi-user PLC setup where several users want to share their data, for example file sharing, video conferencing and multi-player gaming. For such a scenario, we advocate the application of multi-way relaying (MWR) [14] to improve the multiplexing gain of the system compared with previous relay-aided PLC methods, referred to as OWR. Note that here, we focus on AF MWR and use it as a framework to illustrate the application of MWR to improve spectral efficiency in PLC systems. However, the application of MWR for PLC systems can be readily extended to the case where the relay employs DF approach.

First, we explain how MWR can be used in an indoor PLC setup and discuss its implementation considerations. Then, to quantitatively compare MWR with direct transmission and OWR, we study the achievable common and sum rates of all these transmission schemes. To this end, we incorporate the characteristics of a PLC channel by considering frequency-dependent channel gains, derived based on the network topology, as well as frequency-dependent background and impulsive noise in our rate analysis. In addition, we consider a power spectral density (PSD) mask for the transmit power of the users and the relay making sure that electromagnetic compatibility requirements are not violated. Our analysis is then followed by computer simulations for a practical indoor PLC scenario. Our simulation results show that depending on the number of communicating users, topology of the network and type of connected loads, and the relative position of the users, AF MWR is able to outperform direct transmission in scenarios for which AF OWR does not provide an improvement. This performance improvement is more pronounced in the case of impulsive noise that is specific to PLC systems and has not been studied for wireless MWR scenarios. Thus, our proposed AF MWR is able to avoid possible rate loss of AF relaying compared to direct transmission reported in [4, 13].

As a byproduct of our rate study, we also suggest an approach to decrease energy consumption in the network. That is, for data transmission with a common rate for all users, we show that users with better channel conditions can back off their transmit power without degrading the common rate performance. For direct transmission and OWR, an optimal solution for the power allocation problem is found. For MWR, finding the optimal solution is intractable, and hence, a suboptimal solution is proposed.

2 Preliminaries

Here, we describe the system setup and then introduce different transmission strategies for the considered PLC setup.

2.1 Network topology and communication setup

Typically, electricity is delivered to a building through cables coming from a step-down transformer to the building electrical panel (EP). Then, the wiring goes from the EP to N electrical subpanels (ESP) located in different parts of the building. We name the ESPs by ESP1, ESP2, etc. Each of these ESPs is in turn connected to several outlets (OL) that are connected to a load or are open-circuit. OLs connected to a specific ESP are named by OL1, OL2, etc. A schematic of the electric wiring is presented in Fig. 1. Note that while this figure depicts a star network, this does not mean that the scope of this paper is limited to such topology.

Here, several PLC users (modems) are connected to the OL exploiting the electric wiring for communications. More specifically, k_j users are connected to the jth ESP, $j = 1, 2, \ldots, N$, and the total number of PLC users in the system is $K = \sum_{j=1}^{N} k_j$. We denote the ith user, $i = 1, 2, \ldots, K$, by u_i. It is assumed that each of these K users wants to share its data with all other users. Example applications of such scenario are file sharing, video conferencing, and multi-player gaming. To assist the users, a relay node is used. To this end, users transmit their data to the relay in an uplink phase. The relay then amplifies its received signal and broadcasts it to all users in a downlink transmission. Uplink and downlink transmissions continue till the point that each user has the data of all other users. Note that the relay not only amplifies the users' signals and retransmits them over the PLC channel to compensate for the signal attenuation, but also moderates the communication between the users by sending synchronisation signals to them. The synchronisation signal is a vital component of a multi-user PLC system to make sure that each user sends only within its associated transmission time. In addition to synchronisation, the relay estimates the uplink (user to relay) channel gains and sends this channel information to the users. These gains are needed at the users for successful decoding of the data. Unlike uplink gains, each user is in charge of estimating its downlink (relay to user) channel gain. Estimation of the channel gains are achieved by exchanging pilot signals between the relay and the users. While the relay node can theoretically be placed anywhere in the network, we consider it to be inside the EP. This gives an easy access to it for installation or maintenance purposes and enables the relay to receive relatively strong signal from all users.

The PLC modems are half-duplex and either transmit or receive signals. The data communication happens over a frequency bandwidth W using multicarrier modulation, which is the dominant modulation format for broadband PLC systems [1]. The number of subcarriers is denoted by L and equal transmit power is applied for all subcarriers. That is, u_i transmits with a total power of $P_i \le P_{\max}$ satisfying a PSD mask constraint of $P_{\max}/W$. The relay transmits with a fixed power P_r using a constant PSD of $P_r/W \le P_{\max}/W$.

A rate tuple $\mathcal{R} = (R_1, R_2, \ldots, R_K)$ is achievable if any arbitrary user u_i can reliably send its information to all other users with rate R_i. Instead of the rate tuple, common rate and sum rate measures are commonly used. The common rate is

$$R_c = \min_i R_i, \quad (1)$$

representing the rate with which information can be shared equally among all users. Moreover, the sum rate is defined as

$$R_s = \sum_{i=1}^{K} R_i. \quad (2)$$

2.2 Channel gain

Here, we use a method based on the transmission line theory to find the channel frequency response. To this end, each network component (e.g. cables, loads, open-circuit OLs) is modelled by a two-port network through its ABCD matrix. The elements of the ABCD matrix relate the input current and voltage of a two-port network to its output current and voltage [15]. This allows the computation of the ABCD matrix of a set of cascaded two-port networks by multiplying their individual ABCD matrices. Having the ABCD matrix of the whole network, the channel frequency response can be obtained. See [1, Ch. 2] [15, 16] for further details.

2.3 Noise model

We consider two types of noise that are common in PLC systems.

(i) *Background noise:* For the noise PSD $S_n(f)$, we use the following model [15]

$$10 \log_{10}\left(\frac{S_n(f)}{1\,\text{mW/Hz}}\right) = a + b\left|\frac{f}{1\,\text{MHz}}\right|^c, \quad (3)$$

where a, b and c are constants derived from measurements. For simplicity, we assume that the noise PSDs at the relay and the users are equal.

(ii) *Impulsive noise:* This type of noise hits the channel at random and is usually spread over a large bandwidth. Here, we use the

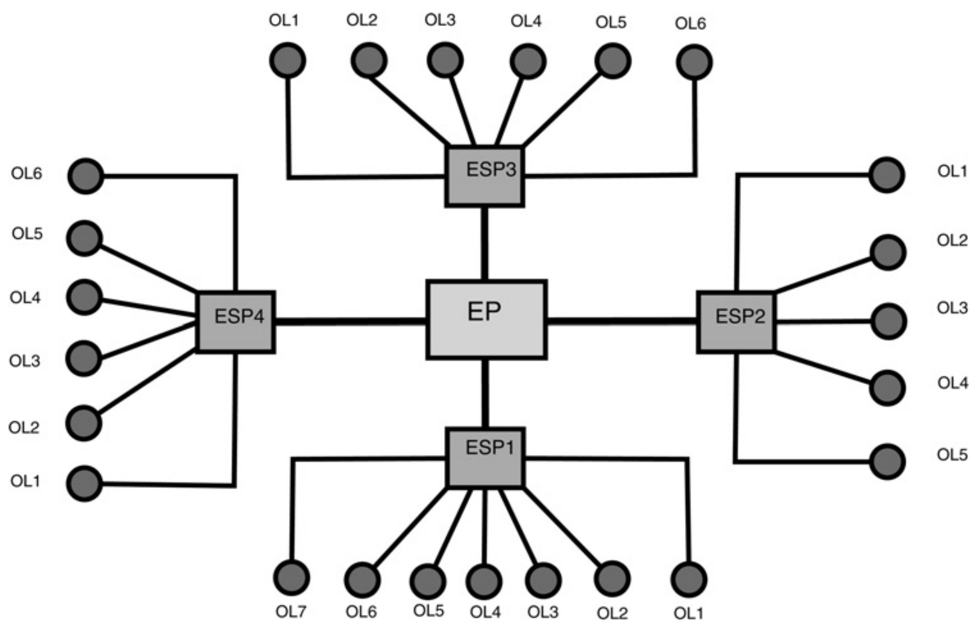

Fig. 1. *Topology of an indoor electric wiring*

model from [17] to incorporate the effect of impulsive noise. To this end, it is assumed that each PLC link has two modes. In the first mode, only background noise is present, while in the second mode, the impulsive noise hits the channel. The probabilities of being in the first and second modes are ρ and $1-\rho$ respectively. During the impulsive noise mode, the noise PSD is $S_n^{ip}(f)$ while it is $S_n(f)$ in the non-impulsive mode. Note that ρ can be found using the width distribution and inter-arrival time distribution of the impulses [15].

2.4 Transmission strategies

To send/receive data to/from other users, different cooperative or non-cooperative approaches can be employed. For cooperative approaches, we consider AF relaying in this paper [The analysis can be conducted for DF relaying, however, for brevity we do not include it in this paper. Note that DF relaying needs a more complex relay that makes its comparison with direct transmission less fair compared to AF.]. Please note that for both cooperative and non-cooperative schemes, nodes should be synchronised to make sure they transmit at the right time.

(i) *Direct transmission:* Users share their data via K transmission phases. In each of these phases, one of the users transmits its message and the rest of the users are in the receiving mode. To decode the data of a transmitting user, all other users should know about the end-to-end channel gain between the transmitting user and themselves. This is done through pilot signaling between any pair of users.
(ii) *One-way relaying:* Here, there are K equal *uplink* and K equal *downlink* phases. In each uplink phase, one user sends its data to the relay. Then, the relay amplifies the received signal such that the PSD mask is met and broadcasts it in one downlink phase. After receiving the relay's signal, each user is able to decode the data of the source user. For this purpose, the knowledge of the end-to-end channel gains including the effect of the relay should be available at the users (e.g. through pilot symbols).
(iii) *OWR with overhearing* (*OWRO*)*:* This scheme is similar to OWR except that in the uplink phase, users also listen to the channel. Then, they combine the overheard signal from the uplink phase with the received signal from the downlink phase to gain better data rates. Users need to know about the end-to-end channel gains with and without the relaying effect.
(iv) *Multi-way relaying:* In an MWR system [14], data sharing is accomplished through one uplink and one downlink phase. Instead of individual transmissions, all users simultaneously transmit to the relay in the uplink phase. As a result, the relay receives the superposition of the signals. After receiving the superimposed signal in the uplink phase, the relay amplifies and forwards it to all users. Then, having its own data, each user cancels out its own signal from the relayed signal and finds the other users' data using a joint or successive decoding technique. Note that choosing each of these decoding techniques depends on the level of computational complexity that the nodes are able to handle. While joint decoding gives a better overall performance, it has higher complexity. In the following, we assume joint decoding at the users. For data decoding at the users, each user should know its end-to-end channels (including the relay's effect) with all other users.

2.5 Approach for the rate analysis

We assume multicarrier modulation with orthogonal subcarriers and independent detection of subcarrier signals. The achievable rate at the lth subcarrier with centre frequency f_l depends on $|H(f_l)|^2/S_n(f_l)$ where $H(f_l)$ is the complex-valued channel gain observed on subcarrier l. The overall achievable rate is given by the sum of the subcarrier rates.

Furthermore, we should note that the cooperative schemes require twice the number of channel uses compared with direct transmission. More specifically, while data sharing is accomplished through K channel uses for direct transmission, cooperative schemes need $2K$ channel uses (K uplink and K downlink transmissions). This difference between the schemes should be accounted for when we study their achievable rates.

3 Achievable data rates of direct transmission

Consider the lth subcarrier at frequency f_l for a given multicarrier symbol transmitted from u_i. Denoting the transmitted symbol by X_i^l, the received signal sample Y_j^l at u_j is

$$Y_j^l = H_{i,j}(f_l)X_i^l + N_j^l, \tag{4}$$

where $H_{i,j}(f_l)$ is the gain of the communication channel from u_i to u_j and N_j^l is the noise sample. From (4), the received signal power and noise power on the lth subcarrier are $P_i|H_{i,j}(f_l)|^2/L$ and $WS_n(f_l)/L$ respectively. Thus, the received SNR at the u_j on the lth subcarrier is $(P_i|H_{i,j}(f_l)|^2)/(WS_n(f_l))$. Now, using (4) and by summation over all subcarriers, u_i can reliably transmit with rate R_i to all u_j's, $j \neq i$, if

$$R_i \le \frac{W}{LK} \min_{j, j\neq i} \sum_{l=1}^{L} \log_2(1 + P_i\omega_{i,j}(l)) \tag{5}$$

where

$$\omega_{i,j}(l) = \frac{|H_{i,j}(f_l)|^2}{\Gamma W S_n(f_l)} \tag{6}$$

and $\Gamma > 1$ represents the effect of imperfect modulation and coding on the data rate. For any u_i, we denote the value of $j \in \{1, 2, \ldots, K\}\backslash i$ that results in the minimum in (5) by m_i. Now, using (1) and (2), we have the following results on the common rate, R_c^D, and sum rate, R_s^D, of direct transmission:

$$R_c^D \le \frac{W}{LK} \min_i \sum_{l=1}^{L} \log_2(1 + P_i\omega_{i,m_i}(l)) - 0.2\,\text{cm} \tag{7}$$

and

$$R_s^D \le \frac{W}{LK} \sum_{i=1}^{K} \sum_{l=1}^{L} \log_2(1 + P_i\omega_{i,m_i}(l)). \tag{8}$$

The right-hand sides of (7) and (8) are increasing functions of P_i's. Thus, the maximum upper bounds on R_c^D and R_s^D are achieved when $P_i = P_{\max}$. The maximum common rate and sum rate for direct transmission follow from equality in (7) and (8) and we denote them by $R_{c,\max}^D$ and $R_{s,\max}^D$ respectively. Here, we denote the user index that yields the minimum in (7) by i^*, that is

$$R_{c,\max}^D = R_{i^*} = \frac{W}{LK} \sum_{l=1}^{L} \log_2\left(1 + P_{\max}\omega_{i^*,m_{i^*}}(l)\right). \tag{9}$$

3.1 Minimising transmit power

Our common rate analysis can be used to achieve an energy-efficient transmission strategy at the users. Assume that we want to maintain the maximum achievable common rate, that is, $R_{c,\max}^D$, while minimising energy consumption at the users. This can be formulated as the following optimisation problem:

$$\min\left\{\sum_{i=1}^{K} P_i | R_i = R_{c,\max}^D, \quad P_i \le P_{\max} \quad \forall i\right\}. \tag{10}$$

We can show that the optimal solution for the above problem is achieved when u_{i^*} transmits with full power $P_{\max}$ and any other user, say u_j, sets its power to $P_j \le P_{\max}$ such that

$$\prod_{l=1}^{L} (1 + P_j\omega_{j,m_j}(l)) = 2^{(LKR_{c,\max}^D/W)}. \tag{11}$$

3.2 Effect of impulsive noise

We suggest a method to account for the presence of impulsive noise in a PLC channel by considering two noise modes as discussed in Section 2: only background noise contaminates the transmitted message in the first mode while in the second mode both background and impulsive noise perturb the communication. However, there exists a correlation between impulsive noise events at different users. Impulsive noise is caused by loads connected to the power grid and thus noise events are observed by multiple users simultaneously, but with a different strength. Capturing the exact inter-dependence between the observed noise at different users would require finding precise models for the noise source that is out of the scope of this work. However, to capture the essence of the impulsive noise effect, we use the generic two-state model and consider two extreme cases for the correlation between the observed impulsive noise at different users.

(i) *Complete dependency:* Here, the received signals at all users are either contaminated only by background noise or are simultaneously affected by an impulsive noise event. The rate analysis for the situation where only background noise exists is as discussed above. To involve the effect of the impulsive noise, the analysis is similar except that $S_n(f_l)$ is replaced by $S_n^{ip}(f_l)$. Using the results from [17], the data rate achievable by u_i satisfies

$$R_i \le \frac{W}{LK}\min_{j,j\neq i}\sum_{l=1}^{L}[\rho\log_2(1+P_i\omega_{i,j}(l))+(1-\rho)\log_2(1+P_i\lambda_{i,j}(l))] \tag{12}$$

where

$$\lambda_{i,j}(l)=\frac{|H_{i,j}(f_l)|^2}{\Gamma W S_n^{ip}(f_l)}$$

Now, using (12) in (1) and (2), the common rate and sum rate are derived accordingly.

(ii) *Complete independency:* The other extreme case is independent impulsive noise events at each different user. It means that when u_i transmits its data to other users, the received signals at any k users are affected by impulsive noise with probability $(1-\rho)^k\rho^{K-k-1}$, $0\le k< K$. Now, the achievable transmit rate of u_i is bounded as

$$R_i \le \sum_{k=0}^{K-1}\sum_{\mathcal{A}\in\mathcal{S}_k}(1-\rho)^k\rho^{K-k-1}R^{D}_{i,\mathcal{A}}, \tag{13}$$

where $\mathcal{S}_k$ is the set of all subsets of $\{1,2,\ldots,K\}\backslash i$ with k members and

$$R^{D}_{i,\mathcal{A}}=\frac{W}{LK}\min_{j\in\mathcal{A},h\notin\mathcal{A}}\{\log_2(1+P_i\lambda_{i,j}(l)),\ \log_2(1+P_i\omega_{i,h}(l))\}.$$

Using (13) in (1) and (2), the common and sum rate are found.

4 Achievable data rates of OWR

We now consider the use of the relay in a conventional fashion, that is, OWR. When u_i sends its message in the ith uplink phase, the relay's received signal at the lth subcarrier is

$$Y_r^l = H_i(f_l)X_i^l + N_r^l \tag{14}$$

where $H_i(f_l)$ is the uplink gain from u_i to the relay at f_l and N_r^l is the noise sample at the relay. Now, the relay forms $X_r^l=\alpha_i(f_l)Y_r^l$ such that

$$\alpha_i(f_l)=\sqrt{\frac{P_r}{P_i|H_i(f_l)|^2+WS_n(f_l)}}, \tag{15}$$

ensuring the PSD mask constraint over this subcarrier is not violated. In the ith downlink phase, the relay broadcasts X_r^l to all users. The received signal at u_j in this phase is

$$\begin{aligned}Y_j^l &= G_j(f_l)X_r^l+N_j^l\\ &=\alpha_i(f_l)H_i(f_l)G_j(f_l)X_i^l+\alpha_i(f_l)G_j(f_l)N_r^l+N_j^l,\end{aligned} \tag{16}$$

where $G_j(f_l)$ denotes the downlink channel gain from the relay to u_j. From (16), u_i is able to transmit its data reliably to any u_j, $i\neq j$, with a rate R_i if

$$R_i \le \frac{W}{2LK}\min_{j\neq i}\sum_{l=1}^{L}\log_2(1+\Omega_j(\alpha_i(f_l),S_n(f_l))) \tag{17}$$

where

$$\Omega_j(x,y)=\frac{P_i|G_j(f_l)|^2|H_i(f_l)|^2x^2}{\Gamma Wy[1+|G_j(f_l)|^2x^2]} \tag{18}$$

Using (1) and (2), the common rate and sum rate of OWR, respectively denoted by R_c^{OWR} and R_s^{OWR}, are derived from (17). Further, it can be shown that the right-hand side of the inequality in (17) is an increasing function of P_i. Thus, the maximum achievable common rate of OWR, $R_{c,\max}^{OWR}$, is

$$R_{c,\max}^{OWR}=\frac{W}{2LK}\min_i\sum_{l=1}^{L}\log_2\times\left(1+\frac{P_rP_{\max}|G_{m_i}(f_l)|^2|H_i(f_l)|^2}{\Gamma WS_n(f_l)[P_{\max}|H_i(f_l)|^2+WS_n(f_l)+P_r|G_{m_i}(f_l)|^2]}\right)$$

where for any u_i, m_i is the value of $j\in\{1,2,\ldots K\}\backslash i$ that results in the minimum in (17).

The multiplexing gain of OWR compared to direct transmission is determined by the ratio of their pre-log factors of the achievable data rate. Thus, the multiplexing gain of OWR is 1/2 of that of direct transmission since OWR requires more channel uses than direct transmission.

4.1 Minimising transmit power

Similar to the case of direct transmission, we can improve energy efficiency by optimising the transmit powers. It can be shown that the optimal solution is achieved when the bottleneck user u_{i^*}, that is, $R_{c,\max}^{OWR}=R_{i^*}$, transmits with full power and any other u_i transmits with power P_i^* such that

$$\prod_{l=1}^{L}\log_2\left(1+\frac{P_rP_i^*|G_{m_i}(f_l)|^2|H_i(f_l)|^2}{\Gamma WS_n(f_l)[P_i^*|H_i(f_l)|^2+WS_n(f_l)+P_r|G_{m_i}(f_l)|^2]}\right)=2^{(2LKR_{c,\max}^{OWR}/W)}. \tag{19}$$

4.2 Effect of impulsive noise

(i) *Complete dependency:* To incorporate the effect of impulsive noise in this case, four end-to-end channel modes should be considered depending on the occurrence of the impulsive noise in the uplink or downlink. The first mode reflects the situation where no impulsive noise hits the channel. The second (third) mode models the channel when only the transmission in the uplink (downlink) is affected by the impulsive noise. Finally, the fourth mode represents the case where both uplink and downlink are hit by the impulsive noise. According to our assumed model for impulsive noise, the probability for being in each of these modes is ρ^2 for the first mode, $\rho(1-\rho)$ for the second and third modes, and $(1-\rho)^2$ for the fourth mode. Thus, the achievable data rate for

u_i in the presence of impulsive noise and complete dependency between the impulsive noise events is

$$R_i \le \frac{W}{2LK}\Bigg[\rho^2 \min_{j\neq i}\sum_{l=1}^{L}\log_2(1+\Omega_j(\alpha_i(f_l), S_{\rm n}(f_l))) + \rho(1-\rho)\min_{j\neq i}\sum_{l=1}^{L}\log_2(1+\Lambda_j(\alpha_{i,{\rm ip}}(f_l), S_{\rm n}(f_l), S_{\rm n}^{\rm ip}(f_l))) + \rho(1-\rho)\min_{j\neq i}\sum_{l=1}^{L}\log_2(1+\Lambda_j(\alpha_i(f_l), S_{\rm n}^{\rm ip}(f_l), S_{\rm n}(f_l))) + (1-\rho)^2\min_{j\neq i}\sum_{l=1}^{L}\log_2(1+\Omega_j(\alpha_{i,{\rm ip}}(f_l), S_{\rm n}^{\rm ip}(f_l)))\Bigg] \qquad (20)$$

where

$$\alpha_{i,{\rm ip}}(f_l) = \sqrt{\frac{P_{\rm r}}{P_i|H_i(f_l)|^2 + WS_{\rm n}^{\rm ip}(f_l)}} \qquad (21)$$

and

$$\Lambda_j(x, y, z) = \frac{P_i|G_j(f_l)|^2|H_i(f_l)|^2x^2}{\Gamma W[y + |G_j(f_l)|^2x^2z]}. \qquad (22)$$

(ii) *Complete independency:* Here, the independence of the impulsive noise events at the users only refers to the downlink transmission from the relay to the users, as there is only one destination (the relay) in the uplink direction. Thus, the channel modes can be partitioned into two sets: the first set includes the cases where the uplink is free from impulsive noise and the second one represents the occurrence of impulsive noise in the uplink. Now, the achievable data rate of u_i satisfies

$$R_i \le \sum_{k=0}^{K-1}\sum_{\mathcal{A}\in\mathcal{S}_k}(1-\rho)^k\rho^{K-k-1}(\rho R_{i,\mathcal{A}}^{\rm OWR,1} + (1-\rho)R_{i,\mathcal{A}}^{\rm OWR,2})$$

where

$$R_{i,\mathcal{A}}^{\rm OWR,1} = \frac{W}{2LK}\min_{j\in\mathcal{A}, m\notin\mathcal{A}}\Big\{\log_2(1+\Lambda_j(\alpha_i(f_l), S_{\rm n}^{\rm ip}(f_l), S_{\rm n}(f_l))), \log_2(1+\Omega_m(\alpha_i(f_l), S_{\rm n}(f_l)))\Big\},$$

$$R_{i,\mathcal{A}}^{\rm OWR,2} = \frac{W}{2LK}\min_{j\in\mathcal{A}, m\notin\mathcal{A}}\Big\{\log_2(1+\Omega_j(\alpha_{i,{\rm ip}}(f_l), S_{\rm n}^{\rm ip}(f_l))), \log_2(1+\Lambda_m(\alpha_{i,{\rm ip}}(f_l), S_{\rm n}(f_l), S_{\rm n}^{\rm ip}(f_l)))\Big\},$$

represent the bounds on u_i's rate when u_j's, $j\in A$, are affected by impulsive noise in the downlink and there is, respectively, background and impulsive noise in the uplink.

5 Achievable data rates of OWRO

The data transmission from u_i to u_j for the overheard and relayed paths can be modeled similar to (4) and (14) respectively. However, the values of $H_{i,\ j}(f_l)$ and $H_i(f_l)$ for OWR with overhearing are generally different from those for direct transmission and simple OWR. More specifically, the effect of relay impedance on $H_{i,\ j}(f_l)$ should be taken into account for OWRO while there is no relay impedance when direct transmission is used. Moreover, when the users overhear the transmit signal of u_i in the uplink, their receiving impedance affects $H_i(f_l)$.

After receiving the relay's signal in the downlink, each user applies maximum-ratio combining to combine the overheard and relayed signals. Thus, u_i is able to reliably transmit its data with rate R_i to them if

$$R_i \le \frac{W}{2LK}\min_{j\neq i}\sum_{l=1}^{L}\log_2(1+\Omega_j(\alpha_i(f_l), S_{\rm n}(f_l)) + P_i\omega_{i,j}(l)). \qquad (23)$$

The common rate and sum rate of this strategy are obtained by substituting R_i from (23) into (1) and (2). Since the right-hand side of (23) is an increasing function of P_i, the maximum common rate and sum rate are achieved when $P_i = P_{\max}$ and (23) holds with equality.

Similar to OWR, the multiplexing gain of OWR with overhearing is 1/2. Further, non-bottleneck users can back off their power to minimise the power consumption in the system. Due to space constraint, we skip mathematical representation for the optimal power values as well as the effect of impulsive noise in this case. However, related simulation results are presented in Section 7.

6 Achievable data rates of MWR

In an uplink phase, the received signal at the relay is

$$Y_{\rm r}^l = \sum_{i=1}^{K}H_i(f_l)X_i^l + N_{\rm r}^l. \qquad (24)$$

The relay amplifies its received signal and forms its message $X_{\rm r}^l = \alpha(f_l)Y_{\rm r}^l$. To enforce the PSD constraint, we have

$$\alpha(f_l) = \sqrt{\frac{P_{\rm r}}{\sum_{i=1}^{K}P_i|H_i(f_l)|^2 + WS_{\rm n}(f_l)}}. \qquad (25)$$

In the downlink, the relay broadcasts $X_{\rm r}^l$ to all users over the power line. After cancelling out its own signal, the received signal at user u_j is

$$Y_j^l = \alpha(f_l)G_j(f_l)\sum_{i\neq j}H_i(f_l)X_i^l + \alpha(f_l)G_j(f_l)N_{\rm r}^l + N_j^l. \qquad (26)$$

The signal model in (26) resembles that of a multiple-access (MAC) channel where u_j wants to decode the message of all other users and $\alpha(f_l)G_j(f_l)N_{\rm r}^l + N_j^l$ is the effective noise. Thus, the data of all other users can be reliably decoded at u_j if their transmit data rates satisfy the following constraint [18]

$$\sum_{k\in\mathcal{S}_j}R_k \le \frac{W}{2L}\sum_{l=1}^{L}\log_2\left(1+\frac{\alpha^2(f_l)|G_j(f_l)|^2\sum_{k\in\mathcal{S}_j}P_k|H_k(f_l)|^2}{\Gamma WS_{\rm n}(f_l)[1+\alpha^2(f_l)|G_j(f_l)|^2]}\right), \quad \forall\mathcal{S}_j\subseteq\{1, 2, \ldots, K\}\backslash j. \qquad (27)$$

Since the received signal at all other users has a similar MAC form, for a rate tuple $\mathcal{R}$ to be achievable, its elements should satisfy K polyhedron constraints similar to (27), where each of them is associated with data receiving at one of the users.

Now, to find the constraint on the user's common rate called $R_{\rm c}^{\rm MWR}$, we set all R_i's equal to $R_{\rm c}^{\rm MWR}$ resulting in $\sum_{k\in\mathcal{S}_j}R_k = |\mathcal{S}_j|R_{\rm c}^{\rm MWR}$, where $|\mathcal{S}_j|$ is the cardinality of $\mathcal{S}_j$. All users reliably transmit and receive with this rate when $R_{\rm c}^{\rm MWR}$ satisfies a constraint similar to (27) at all users. Hence,

$$R_{\rm c}^{\rm MWR} \le \min_j\frac{W}{2L|\mathcal{S}_j|}\times\sum_{l=1}^{L}\log_2\left(1+\frac{\alpha^2(f_l)|G_j(f_l)|^2\sum_{k\in\mathcal{S}_j}P_k|H_k(f_l)|^2}{\Gamma WS_{\rm n}(f_l)[1+\alpha^2(f_l)|G_j(f_l)|^2]}\right). \qquad (28)$$

Note that $\max_j|\mathcal{S}_j| = K-1$, hence, the smallest pre-log factor for MWR is $1/2(K-1)$. Thus, the MWR multiplexing gain is not

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worse than $K/2(K-1)$, which is still smaller than the multiplexing gain of direct transmission, but larger than the gain of OWR.

Unlike direct transmission and OWR, finding the maximum achievable common rate of MWR is not straightforward. This is due to the dependency of the achievable rate of one user on the transmit power of other users. In fact, it can be shown that transmitting with maximum power at the users, that is, $P_i = P_{\max}$ for all i, does not guarantee achieving the maximum common rate in MWR. As a result, some of the users can back off their transmit power to improve the common rate of the system. However, we are not able to find a tractable solution for transmit power at the users to achieve the maximum common rate.

Finding the users' sum rate is more challenging than finding the common rate. Using the properties of the polyhedron rate regions at all users, we are however able to find the following upper bound on the achievable sum rate

$$R_s^{\mathrm{MWR}} \le \min_a \frac{1}{a\binom{K-1}{a-1}} \times \frac{W}{2L} \times \sum_{j=1}^{K}\sum_{l=1}^{L} \log_2\left(1+\frac{\alpha^2(f_l)|G_j(f_l)|^2\sum_{k\in\mathcal{S}_{j,a}}P_k|H_k(f_l)|^2}{\Gamma W S_n(f_l)[1+\alpha^2(f_l)|G_j(f_l)|^2]}\right), \quad (29)$$

where for any arbitrary j, $\mathcal{S}_{j,a}$ refers to all $\mathcal{S}_j$'s such that $|\mathcal{S}_j| = a$. See Appendix for the proof of (29). In general, the above sum rate upper bound is not achievable. However, for $K=2$ users, an achievable bound for sum rate can be presented in a simple form as

$$R_s^{\mathrm{MWR}} \le \frac{W}{2L}\sum_{l=1}^{L}\log_2\left(1+\frac{\alpha^2(f_l)P_1|G_2(f_l)|^2|H_1(f_l)|^2}{\Gamma W S_n(f_l)[1+\alpha^2(f_l)|G_2(f_l)|^2]}\right) + \frac{W}{2L}\sum_{l=1}^{L}\log_2\left(1+\frac{\alpha^2(f_l)P_2|G_1(f_l)|^2|H_2(f_l)|^2}{\Gamma W S_n(f_l)[1+\alpha^2(f_l)|G_1(f_l)|^2]}\right).$$

6.1 Minimising transmit power

Similar to previous transmission strategies, we can formulate an optimisation problem to minimise required transmit power while still maintaining the maximum common rate. However, unlike other transmission strategies, an optimal solution for a general MWR is not analytically tractable due to the dependency between the achievable data rate of each user and the transmit power of the other users. For the special case of $K=2$, however, we derive a suboptimal solution for the power optimisation problem. The analytical form of this suboptimal solution is omitted here for brevity, however, simulation results are provided in Section 7.

6.2 Effect of impulsive noise

Accommodating the impact of impulsive noise on the achievable rates of MWR is similar to OWR. For the case of complete dependency between the impulsive noise events, a four-state channel similar to (20) should be considered. Using this four-state channel model, the right-hand side of (27) is modified accordingly. Further, the rate analysis for the scenario where the impulsive noise events are completely independent is done through modifying (27) using a similar approach as the one used for OWR. Due to space constraint, we skip the formal presentation of the modified rate bounds.

7 Numerical results

Here, we have used the PLC simulator available at [19] with some modifications to accommodate the relay placement at the EP and the presence of multiple transmitters in the system.

Table 1 Description of the network topology

ESP	1	2	3	4
ESP cable cross section, mm^2	2.5	2.5	2.5	2.5
ESP to EP, m	U(5, 10)	U(5, 10)	U(5, 10)	U(5, 10)
Number of OLs	7	5	6	6
OL cable cross section, mm^2	1.5	1.5	1.5	1.5
OL to ESP, m	U(2, 4)	U(2, 4)	U(2, 4)	U(2, 4)

7.1 Simulation parameters

We consider an indoor PLC network similar to Fig. 1 where four ESPs are connected to the EP and each ESP services several OLs. For a more representative comparison between different transmission strategies, we perform our simulation for 100 different network realisations where the length of the cables connecting ESPs to the EP and OLs to the ESPs are sampled randomly according to a uniform distribution. Specifications of the cables, support of the uniform distribution as well as the number of OLs connected to each ESP are summarised in Table 1. Note that in this table, $U(a, b)$ refers to a uniform distribution over $[a, b]$. More details on the cable specifications used in our results are found in [20, Table 1]. Moreover, $\Gamma = 10$ is considered to include the effect of non-ideal modulation and channel coding.

In our simulations, data communication takes place over a frequency band from 2 to 28 MHz and the subcarrier spacing is set to 24.414 kHz, commonly used for indoor broadband PLC applications [1, Chapter 7]. Moreover, $P_{\max} = 26$ mW, which translates into a constant PSD of −60 dBm/Hz over the considered frequency band [21]. For the background noise model, it is assumed that the parameters in (3) are set as $(a, b, c) = (-145, 53.23, -0.337)$ [15]. Moreover, it is assumed that the impulsive noise causes a 30 dB increase in the noise level, that is, $(a, b, c) = (-115, 53.23, -0.337)$ when impulsive noise hits the channel.

Furthermore, we use different impedances as loads connected to OLs in the network. Frequency non-selective resistive loads of 100 Ω and 1000 Ω are referred to as R_L and R_H respectively. RLC_i, for $i = 1,2,3$, are frequency-selective impedances representing the effect of parallel RLC resonant circuits. The impedances are given by [20]

$$Z(f) = \frac{R}{1 + jQ((f/f_0) - (f_0/f))}, \quad (30)$$

where R is the resistance at resonance, Q denotes the quality factor determining the frequency selectivity of the load and f_0 represents the resonance frequency. Each load can be represented by a tuple as (R, Q, f_0) presented in Table 2. In addition, the effect of PLC modem circuitry on the channel gains is reflected through a resistance $R_m = 50$ Ω.

In the following, simulation results are presented for several network setups whose specifications are listed in Table 3 (refer to Fig. 1 for the labels of ESPs and OLs). Moreover, note that for MWR, we set the power of all users to $P_{\max}$. As we discussed in Section 6, transmitting with maximum power at the users does not guarantee achieving the maximum common rate in a MWR PLC system. Thus, the ultimate performance of MWR, given a rate-maximising power allocation at the users, is in fact better than the simulation results presented in the following.

Table 2 Parameters (R, Q, f_0) of the frequency-selective loads

RLC_1	RLC_2	RLC_3
(500 Ω, 10, 2.5 MHz)	[U(200, 400) Ω, U(20, 40), U(18, 28) MHz]	[U(100, 200) Ω, U(5, 10), U(10, 20) MHz]

Table 3 Different network setups. See Fig. 1 for location of users and loads

Setup name	Users	Loads
Setup 1	u_1: OL3 on ESP1, u_2: OL5 on ESP1	No loads
Setup 2	u_1: OL3 on ESP1, u_2: OL2 on ESP4	No loads
Setup 3	u_1: OL3 on ESP1, u_2: OL2 on ESP4	R_L: OL1 on ESP1, RLC_1: OL2 on ESP1, RLC_2: OL4 on ESP1, RLC_3: OL5 on ESP1, R_H: OL7 on ESP1, RLC_1: OL2 on ESP2, R_H: OL3 on ESP2, RLC_3: OL4 on ESP2, RLC_1: OL2 on ESP3, RLC_2: OL3 on ESP3, RLC_3: OL4 on ESP3, R_H: OL5 on ESP3, R_L: OL1 on ESP4, RLC_2: OL3 on ESP4, RLC_3: OL4 on ESP4, RLC_2: OL5 on ESP4
Setup 4	u_1: OL3 on ESP1, u_2: OL2 on ESP4, u_3: OL1 on ESP2, u_4: OL1 on ESP3	Same as Setup 3

Table 4 Achievable data rates in Mbps of different schemes for different setups defined in Table 3

Setup name	Direct	OWR	OWRO	MWR
Setup 1: common rate	95.94	42.50	52.75	77.67
Setup 2: common rate	53.75	35.94	42.94	65.30
Setup 3: common rate	25.82	16.09	19.43	30
Setup 3: sum rate	52.83	33.1	39.67	62.34
Setup 4: common rate	9.15	6.68	7.86	12.13
Setup 4: common rate with dependent impulsive noise	5.41	2.76	4.22	9.25
Setup 4: common rate with independent impulsive noise	2.21	1.17	3.33	6.63

7.2 Results and discussion

We first consider the case of $K = 2$ users. Fig. 2 shows the achievable common rate as a function of subcarrier frequency for Setup 1. The overall common rates for this setup are presented in the first row of Table 4. We observe that since users are connected to the same ESP, it is difficult for a relay located at the EP to improve performance and direct transmission achieves the highest common rate.

Fig. 3 presents the rate results for Setup 2 where the inter-user distance is longer compared to Setup 1 and the direct link experiences a larger attenuation. The common rate results are given in the second row of Table 4. We observe that while OWR and OWRO are still outperformed by direct transmission, MWR indeed provides an advantage over direct transmission. This is due to MWR's better multiplexing gain compared to OWR and OWRO and higher SNR gain compared to direct transmission.

Fig. 4 shows the per subcarrier data rate for Setup 3. Again, the overall common rate results for this setup are provided in Table 4. While adding the loads to the network drops the achievable common data rate in the system due to their effect on the channel gains especially at lower frequencies, MWR remains the best transmission strategy. In addition, MWR achieves higher sum rates than the other schemes, as seen in Table 4. Note that from Figs. 2 and 3, we observe that direct transmission sends most of its data through lower frequencies. As a result, when loads degrade the channel conditions for these lower frequencies (as seen in Fig. 4), direct transmission is affected the most.

Furthermore, it is insightful to consider the power savings due to the optimisation of transmit powers while keeping the common rate constant (at its maximum). On average (over the 100 network realisations), the non-bottleneck user can reduce its power from 26 mW to a smaller value as reported in the first row of Table 5 for different strategies. As seen, the most significant power saving is observed for MWR with approximately 18%. Note that the user with worse channel condition and the relay (for cooperative schemes) still use a transmit power of 26 mW. In addition, the associated energies per bit for different transmission schemes are also reported.

In terms of the energy consumption and energy efficiency, MWR is clearly outperforming the OWR schemes. Interestingly, direct transmission is the 'greenest' strategy, since no energy is used for relaying, at the cost of lower common rate.

Next, we consider Setup 4 with $K = 4$ users. The overall common rate for different transmission strategies are shown in Table 4. For this setup, we also study the effect of impulsive noise on the users' common data rate when $\rho = 0.9$ meaning that the probability of observing an impulsive noise at the nodes (users and the relay) is $1 - \rho = 0.1$. We consider both cases of complete independence and complete dependence of the observed impulsive noise at the users. As can be seen from Table 4, MWR outperforms other transmission schemes, with most pronounced gains for the case of impulsive noise. This observation can be explained considering that for MWR, the effect of impulsive noise is distributed over different users' signals. While in direct transmission and OWR, when impulsive noise hits the channel, it significantly degrades the signal of an individual user resulting in lower common rates.

To highlight the effect of the network loads and cable length, Fig. 5 shows the empirical cumulative distribution functions (CDF) of common rate of the different transmission strategies for Setup 4 and no impulsive noise. We observe a significant gap between the CDFs of direct transmission and OWR and OWRO

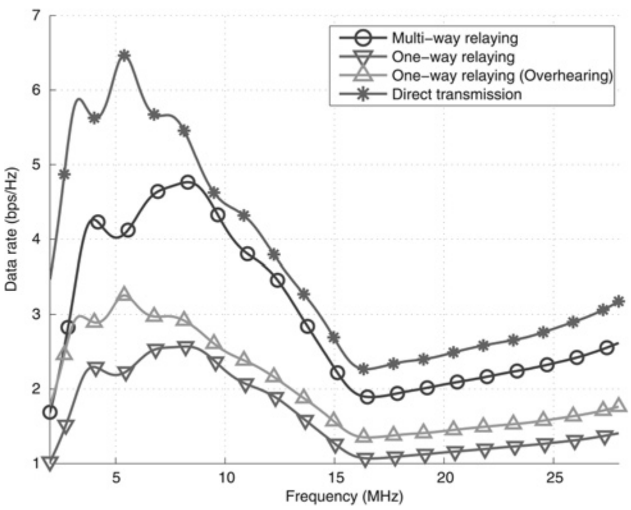

Fig. 2. *Data rate over frequency for Setup 1*

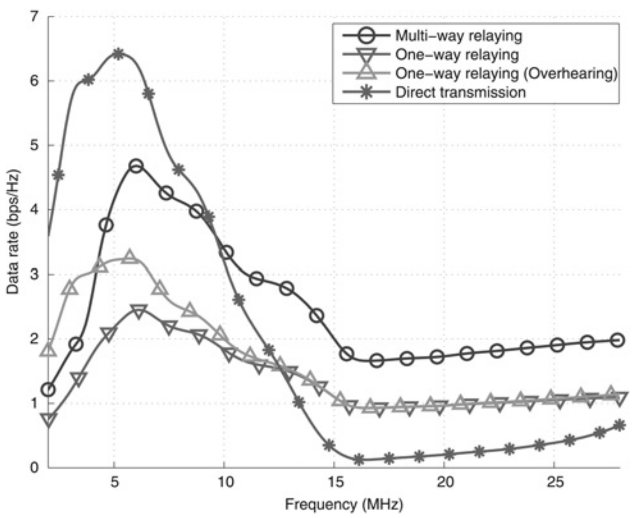

Fig. 3. *Data rate over frequency for Setup 2*

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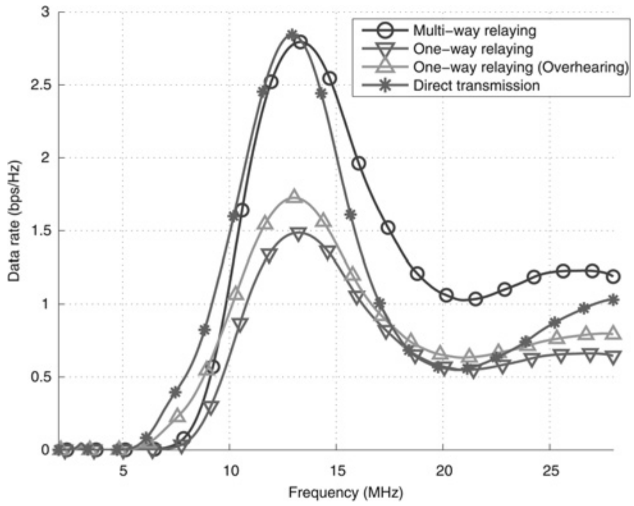

Fig. 4. *Data rate over frequency for Setup*

Table 5 Energy and power for data transmission with and without the power saving

Transmission	Direct	OWR	OWRO	MWR
Transmit power of the user with better channel using power saving (mW)	23.4	22.5	23.1	21.3
Transmit energy per bit with power saving (nJ)	0.479	1.562	1.301	0.611
Transmit energy per bit without power saving (nJ)	0.504	1.616	1.338	0.650

that is consistent with [4, 13], where it is noted that AF OWR can result in lower data rates compared to direct transmission. Notably, MWR shows a significantly better performance than direct transmission.

7.3 An Opportunistic MWR scheme

As seen in our simulation results, depending on the network topology, either MWR or direct transmission achieves higher data rates. One approach to get the best from both methods is to employ an opportunistic transmission strategy. In such scheme, a master node, for example, the relay node at the EP, calculates the achievable common rate or sum rate of MWR and direct transmission. The scheme that gives the maximum rate is then chosen and users are informed to set their transmission strategy accordingly. Since channel conditions can be considered slowly varying, computations and updates are occasional. Channel variations periodic with the mains cycle can be dealt with through a table of strategies applied over the mains cycle.

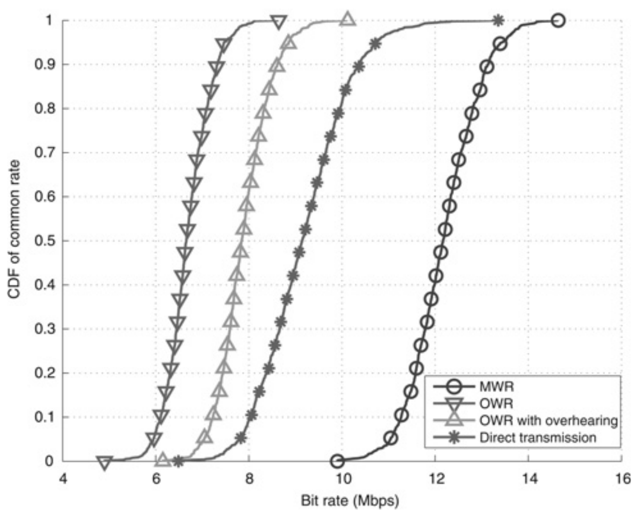

Fig. 5. *CDF of the common rate for Setup 4*

8 Conclusion

In this paper, we proposed the application of AF MWR to enhance the multiplexing gain of relay-aided PLC systems. Considering characteristics of PLC channels, we derived expressions for the achievable common rate and sum rate of MWR, OWR, OWRO and direct transmission. We applied these expressions to compare the achievable rates of these transmission strategies under different PLC network setups. Our results demonstrated that the best strategy depends on the number of users, their location in the network, and the loads connected to the network, but that for most of the test cases MWR outperformed other strategies. This can be used to devise an opportunistic transmission strategy for indoor PLC systems such that depending on the topology and condition of the network, either MWR or direct transmission is used to achieve the highest possible data rates.

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11 Appendix

11.1 Proof of (29)

From the polyhedron constraint (27) at u_j, we have

$$\sum_{k\in\mathcal{S}_{j,a}} R_k \le \frac{W}{2L}\sum_{l=1}^{L}\log_2\left(1+\frac{\alpha^2(f_l)|G_j(f_l)|^2\sum_{k\in\mathcal{S}_{j,a}}P_k|H_k(f_l)|^2}{\Gamma W S_n(f_l)[1+\alpha^2(f_l)|G_j(f_l)|^2]}\right),\quad \forall\mathcal{S}_{j,a} \tag{31}$$

where $\mathcal{S}_{j,a}$, for $1\le a\le K-1$, is the set of all $\mathcal{S}_j\subseteq\{1,2,\ldots,K\}\backslash j$ such that $|\mathcal{S}_j|=a$. Since the number of $\mathcal{S}_{j,a}$'s at user j is $\binom{K-1}{a}$, (31) actually expands to $\binom{K-1}{a}$ inequalities each of them representing one of the $\mathcal{S}_{j,a}$'s. Now, for a specific user u_i, where $i\ne j$, R_i appears in exactly $\binom{K-2}{a-1}$ of these inequalities. That said, if we sum over all $j=1, 2, \ldots, K$, we have

$$\begin{aligned}\sum_{j=1}^{K}\sum_{k\in\mathcal{S}_{j,a}} R_k &= (K-1)\binom{K-2}{a-1}\sum_{k=1}^{K}R_k = (K-1)\binom{K-2}{a-1}R_s^{\text{MWR}}\\ &= a\binom{K-1}{a-1}R_s^{\text{MWR}} \le \frac{W}{2L}\sum_{j=1}^{K}\sum_{l=1}^{L}\\ &\quad\times\log_2\left(1+\frac{\alpha^2(f_l)|G_j(f_l)|^2\sum_{k\in\mathcal{S}_{j,a}}P_k|H_k(f_l)|^2}{\Gamma W S_n(f_l)[1+\alpha^2(f_l)|G_j(f_l)|^2]}\right)\end{aligned} \tag{32}$$

This proves (29). Note that in (32), R_k does not appear in the set of equations at u_k. Thus, when we sum over all equations at all users in (32), R_k is counted at exactly $K-1$ users (all users except u_k) with $\binom{K-2}{a-1}$ times at each of them.

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