Influence of Outside Air Relative Humidity on Interior Humidity including Economical Aspects

Petra Bednářová[a], Roman Šubrt[b] and Petr Chládek[a]

[a]Institute of Technology and Business in České Budějovice, Department of Civil Engineering, Okružní 517/10, 37001 České Budějovice, Czech Republic
[b]Energy consulting, Alešova 21, 370 01 České Budějovice, Czech Republic

Abstract. Microclimate depends not only on ventilation intensity and inside contamination sources, but also on concentration of particular substances in the exterior. It has consequences on microclimate and, in addition, on the economical requirements of buildings management. The article informs on these dependences and on dependence calculation model.

Keywords: Air relative humidity, CO_2 concentration, mathematical and economical models.
PACS: 89

INTRODUCTION

When we monitor concentration of various gases, in common buildings these are particularly carbon dioxide and water vapour, it is logical that the size of the interior, source of these gases in the interior, airing intensity and often forgotten concentration of these gases in the exterior have influence on their interior concentration. This is given by the fact that we are usually interested in gases that either do not exist in the exterior or in carbon dioxide, the outside concentration of which is (with small deviations) basically constant. Research measurement performed in 2010 by Energy Consulting at 20 places of South Bohemia found CO2 concentration range between 374 and 486 ppm, while the median was 430 ppm and the average was 423 ppm [1]. However when we assess interior concentration of water vapour we get substantially different values in dependence on temperature and air relative humidity.

We can thus conclude that the possibility to exhaust water vapour from an interior depends substantially on the exterior and ventilation intensity or quality cannot be assessed only upon measurement of relative humidity in the interior as it is often done. Nevertheless, the ventilation has an effect not only on microclimate but also on running cost of buildings.

MATHEMATICAL MODELS

The question, which is discussed in this paper, is related to the theory of mathematical growth models. These models are used in natural sciences, especially in biology, chemistry and medicine. Such a model is always based on observation of the change in the velocity for given quantity *y*. The formulation of assumptions is following. Let *y(t)* be the size of quantity *y* at the time *t*. Denote by *b(t,y)* the increasing measure of *y* with respect to a time unit and to a quantity unit. Roughly speaking, the function *b(t,y)* is a growth velocity for the quantity *y*. Similarly, denote by *d(t,y)* the decreasing measure of *y* with respect to a time unit and to a quantity unit. For the construction of mathematical model we suppose that *y(t), b(t,y), d(t,y)* are differentiable functions. We deal with the change in the velocity of the quantity *y* during the time *h*, i.e. in the interval $\langle t, t+h \rangle$.
Then

$$y(t+h) - y(t) = b(t,y).h.y(t) - d(t,y).h.y(t) \quad \Rightarrow$$
$$\frac{y(t+h) - y(t)}{h} = y(t).[b(t,y) - d(t,y)] \quad \Rightarrow$$
$$lim_{h\to 0}\ \frac{y(t+h) - y(t)}{h} = lim_{h\to 0}\ y(t).[b(t,y) - d(t,y)] \quad \Rightarrow$$
$$\frac{dy}{dt} = y(t).\mu(t,y),$$

AIP Conf. Proc. 1648, 720003-1–720003-4; doi: 10.1063/1.4912940

where the function $\mu(t,y) = b(t,y) - d(t,y)$ is called the specific measure of growth. By solving the differential equation we obtain the information about behaviour of the quantity y.

CALCULATION OF GAS CONCENTRATION IN THE AIR

We can simplify the concentration problem modelling as the so called lake self cleaning task [2]. The situation may be modelled as shown in Fig. 1, mathematic description is then in formula (1), which is reformulating of the previous differential equation for $b(t,y) = 0$ and $d(t,y) = \frac{r}{V}$, where r is flow velocity [m^3/hr], V is the volume of the examined object [m^3] and y_1 is the concentration of the examined substance in the object [l/m^3].

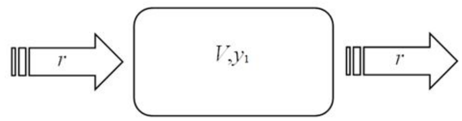

FIGURE 1. Graphic interpretation of the self cleaning lake model

Then

$$\frac{dy_1}{dt} = -\frac{r \cdot y_1}{V} \qquad (1)$$

The equation has the following solution:

$$y_1 = C \cdot e^{(-\frac{r \cdot t}{V})}, \qquad (2)$$

where t is time, C is an integral constant. The equation and its solution show that concentration y_1 will be factually zero for very long times.

This solution however only deals with the situation where certain contamination level exists in the object, clean content is driven to the object at speed r and the equation describes the progress of gradual decrease of the contamination in the object. If we want to illustrate a real situation i.e. to accept the fact that the air driven to the interior from the outside already includes some natural contamination, the previous self cleaning lake model has to be modified as illustrated in Fig. 2 and mathematically described by equation (3), which is reformulating of the previous differential equation for $b(t,y) = \frac{r}{V}$ and $d(t,y) = \frac{r}{V}$, but the contamination inside is different from the contamination outside the object.

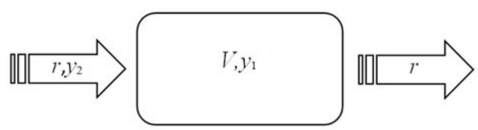

FIGURE 2. Graphic interpretation of the self cleaning lake model with contaminated inflow

where

y_2 is constant contamination brought to the interior from the exterior [$1/m^3$]. The equation is

$$\frac{dy_1}{dt} = -\frac{r \cdot y_1}{V} + \frac{r \cdot y_2}{V} \qquad (3)$$

and the solution of such an equation is

$$y_1 = y_2 + C \cdot e^{(-\frac{r \cdot t}{V})}. \qquad (4)$$

Nevertheless a situation where there is a source of contamination inside the monitored object is most likely, the graphic interpretation is then in Fig. 3 and mathematical description in equation (5).

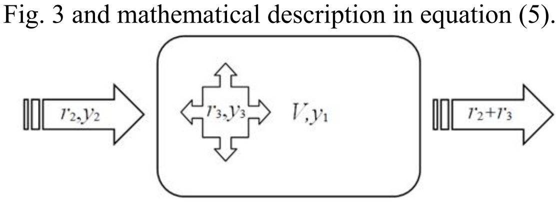

FIGURE 3. Graphic interpretation of the self cleaning lake model with contaminated inflow a source of contamination inside the monitored object

where

y_2 is still constant contamination brought by air exchange [1/m^3]

y_3 is constant contamination caused by inner source [1/m^3]

r_2 velocity of the inlet air [m^3/hr]

r_3 speed of contamination production of the inner source [m^3/hr]

$$\frac{dy_1}{dt} = -\frac{(r_2 + r_3)\cdot y_1}{V} + \frac{r_2 \cdot y_2}{V} + \frac{r_3 \cdot y_3}{V}. \tag{5}$$

This equation can be simplified for just one contamination source, i.e. the second and the third members of the equation may be included in one. This is expressed by a modified equation (6):

$$\frac{dy_1}{dt} = -\frac{r \cdot y_1}{V} + \frac{r \cdot (y_2 + y_3)}{V} \tag{6}$$

where the solution is $y_1 = y_2 + y_3 + C \cdot e^{\left(-\frac{r \cdot t}{V}\right)}$.

We want to know what the concentration of gases in the interior air will be after certain time *t* elapses. This time may however be random as well as the initial air contamination in the object. If we want to eliminate these two influences we have to monitor the object for as long as possible. By solving the equation for time approaching infinity we find that the concentration in the monitored object equals to the sum of contamination produced by the inner source and the contamination brought in.

CONTEXT WITH ENERGY SAVINGS

Of the foregoing

1) Ventilation is an indispensable function of buildings (very airtight windows are not avable ensure IAQ, as previous ones).
2) Ventilation has to be fluent or cyclic; cycle of switching has to be frequent. But it is necessary remember ventilation increases energy demands.

MODEL EXAMPLE OF VENTILATION

For an example of quantification impact of ventilation to energy consumption was elected an single family house with a enclosed volume of 600 m3 of air. The house is inhabited by family about 4 members. Every member of family spent at home approximately 12 hours.

In the case of forced ventilation is supposed use heat recovery system with efficiency about 80 % and power consumption 300 W. For the example were used followings prices of energy: 3,5 CZK/kWh for electricity and 1,2 CZK/kWh for heat energy. Also were used average climate dates for Czech Republic.

There are options of ventilation at model family house:

a) Use nature ventilation system by windows (air exchange multiplicity n = 1). Regulated by subjective feeling of inhabitants.
b) Use heat recovery ventilation system (air exchange multiplicity n = 0,4).
c) Continual ventilation, intensity of air exchanging about 25 m3/person/ hour.
d) Ventilation only during stay of persons, intensity of air exchanging about 25 m3/person/ hour.

TABLE 1. Summary of energy consumption and energy costs with different ventilation systems

	A	B	C	D
Heat consumption per year [kWh]	20812	1665	694	347
Electricity consumption per year [kWh]	-	2628	2628	1314
Price of heat energy per year [CZK]	24974	1998	832	416
Price of electricity per year [CZK]	-	9198	9198	4599
Price of energy per year [CZK]	24974	11196	10030	5015

REFERENCES

1. R. Šubrt, *Studie Microclimate in Public Buildings as the Reason for Installation of Heat Recovery*, sponsored from the State Programme for Energy Saving and use of Renewable Resources for 2011 – Part A – EFEKT Programme
2. J. Kalas and Z. Pospíšil, *Continuous Models in Biology*. MU Brno 2001, ISBN 80-210-2626-X
3. *Decree 20/2012 Col.*, amending Decree 268/2009 Col. on technical requirements for constructions
4. M. Halahyja et al., *Thermal Construction Technology*. Alfa Publishing with SNTL 1985, ISBN 6356085
5. ČSN 73 0540, pars 1 to 4 – *Thermal Protection of Buildings*
6. M. Jokl and D. Šebesta, "Factors defining ventilation system function". in *Stavební Obzor 10/2007***,** s. 310, ISSN 1805-2576
7. O. Hnilica and J. Plášek, "Influence of Attendance to the interior climate of Znojmo Rotund" in *Stavební Obzor 5/2012*, p. 142, ISSN 1805-2576