Linear Programming Using Symmetric Triangular Intuitionistic Fuzzy Numbers

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Abstract. This paper introduces Symmetric Triangular Intuitionistic Fuzzy Numbers (STriFNs) and also proposes a new type of intuitionistic fuzzy arithmetic operations on STriIFNs. A special ranking function for ordering STriIFNs has been introduced. A solution methodology for Intuitionistic Fuzzy Linear Programming Problems (IFLPPs) with STriIFNs as parameters has been proposed by using Intuitionistic Fuzzy Simplex Method and the arithmetic operations on STriIFNs. Finally, an illustrative numerical example is presented to demonstrate the proposed approach.

Keywords: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Linear Programming Problems, Intuitionistic Fuzzy Simplex Method, Symmetric Triangular Intuitionistic Fuzzy Numbers, Ranking Function.

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INTRODUCTION

Decision making problems exhibit some level of imprecision and vagueness in estimation of model parameters. Applications of fuzzy set theory in decision making and in particular to optimization problems have been extensively studied ever since the introduction of fuzzy sets by Zadeh (1965). The theory of the intuitionistic fuzzy set (IFS) [1, 2, 3] has been found to be more useful to deal with vagueness and uncertainty in decision making situations than that of the fuzzy set [4, 5].

In this paper, Symmetric Triangular Intuitionistic Fuzzy Numbers (STriIFNs) which are the extensions of triangular intuitionistic fuzzy numbers have been introduced and their arithmetic operations are also proposed. Also, a new concept to the optimization problem in an IF environment and a special ranking function to rank STriFN have been introduced. Finally, a solution methodology for an Intuitionistic Fuzzy Linear Programming Problem (IFLPP) in which the right hand side constants are STriIFNs is explained by using intuitionistic fuzzy simplex method.

SYMMETRIC TRIANGUALR INTUITIONISTIC FUZZY NUMBERS

An IFS $\tilde{A}^I$ in R is said to be a Symmetric Triangular Intuitionistic Fuzzy Number (STriIFN) if there exist real numbers *a, h* and h' where $h \le h'$ and $h, h' \ge 0$ such that the membership and non-memberhsip functions are as follows:

$$\mu_{A^I}(x) = \begin{cases} \dfrac{x-(a-h)}{h} & \text{for } x \in [a-h,\ a] \\ \dfrac{a+h-x}{h} & \text{for } x \in [a,\ a+h] \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{A^I}(x) = \begin{cases} \dfrac{a-x}{h'} & \text{for } x \in [a-h',\ a] \\ \dfrac{x-a}{h'} & \text{for } x \in [a,\ a+h'] \\ 0 & \text{otherwise} \end{cases}$$

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Symbolically, a STriIFN can be represented as $\tilde{A}^{I}_{STriIFN} = [a;\ h,\ h;\ h',\ h']$

Arithmetic Operations on STriIFNs

The purpose of defining arithmetic operations on STriIFNs is to make use of them in linear programming in an IF environment. In this section, the operations like addition, additive image, subtraction and scalar multiplication are defined on STriIFNs.

Let $\tilde{A}^{I} = [a;\ h,\ h;\ h',\ h']$ and $\tilde{B}^{I} = [b;\ k,\ k;\ k',\ k']$ be two STriIFNs. Then the basic arithmetic operations on $\tilde{A}^{I}$ and $\tilde{B}^{I}$ are given as under.

Addition

The addition of $\tilde{A}^{I}$ and $\tilde{B}^{I}$, denoted by $\tilde{A}^{I} + \tilde{B}^{I}$, is defined as

$$\tilde{A}^{I} + \tilde{B}^{I} = [a+b;\ h+k,\ h+k;\ h'+k',\ h'+k'].$$

Additive Image

The additive image of $\tilde{A}^{I}$ and additive image of $\tilde{B}^{I}$ are defined as

$$\text{Additive image of } \tilde{A}^{I} = [-a;\ h,\ h;\ h',\ h']$$

$$\text{Additive image of } \tilde{B}^{I} = [-b,\ k,\ k;\ k',\ k']$$

Subtraction

The subtraction of $\tilde{A}^{I}$ and $\tilde{B}^{I}$, denoted by $\tilde{A}^{I} - \tilde{B}^{I}$, is defined as

$$\tilde{A}^{I} - \tilde{B}^{I} = [a-b,\ h+k,\ h+k;\ h'+k',\ h'+k']$$

Scalar Multiplication

Let $k \in R$ be a scalar. The scalar multiplication of $\tilde{A}^{I}$ by the scalar k denoted by $k\tilde{A}^{I}$, is defined as

$$k\tilde{A}^{I} = \begin{cases} [ka,\ kh,\ kh;\ kh',\ kh'] & \text{if } k > 0 \\ [ka,\ \text{-}kh,\ \text{-}kh;\ \text{-}kh',\ \text{-}kh'] & \text{if } k < 0 \\ [0,\ 0,\ 0;\ 0,\ 0] & \text{if } k = 0 \end{cases}$$

RANKING FUNCTION

In fact, an efficient approach for ordering the elements of F(R), the set of STriIFNs on R, is to define a ranking function $\Re : F(R) \to R$ which maps each STriIFN into the real number.

Orders on F(R) are defined as follows:

$\tilde{A}^{I} \succeq \tilde{B}^{I}$ if and only if $\Re(\tilde{A}^{I}) \geq \Re(\tilde{B}^{I})$

$\tilde{A}^{I} \succ \tilde{B}^{I}$ if and only if $\Re(\tilde{A}^{I}) > \Re(\tilde{B}^{I})$

$\tilde{A}^{I} \approx \tilde{B}^{I}$ if and only if $\Re(\tilde{A}^{I}) = \Re(\tilde{B}^{I})$ where $\tilde{A}^{I}$, $\tilde{B}^{I}$ are in F(R).

Note:

For any two STriIFNs $\tilde{A}^{I} = [a;\ h,\ h;\ h',\ h']$ and $\tilde{B}^{I} = [b;\ k,\ k;\ k',\ k']$, the relation $\preceq$ is a partial order relation and is defined as $\tilde{A}^{I} \preceq \tilde{B}^{I}$ if and only if $2a + \frac{1}{2}(h'-h) \leq 2b + \frac{1}{2}(k'-k)$.

INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEMS

Definition: An Intuitionistic Fuzzy Linear Programming Problem (IFLPP) is defined as

$$\begin{aligned} &\text{Max } \tilde{z}^I \approx c\tilde{x}^I \\ &\text{subject to } A\tilde{x}^I \preceq \tilde{b}^I \\ &\tilde{x}^I \succeq 0 \end{aligned} \tag{1}$$

where $\tilde{b}^I \in (F(R))^m$, $\tilde{x}^I \in (F(R))^n$, $A \in R^{m\times n}$, $c^T \in R^n$ and $\Re$ is a linear ranking function and F(R) is the set of STriIFNs.

INTUITIONISTIC FUZZY SIMPLEX METHOD

For the solution of any IFLPP by IF simplex algorithm, the existence of an initial IF basic feasible solution is always assumed.The steps for the computation of an optimum solution are as follows:

Step1

Check whether the objective function of the given IFLPP is to be maximized or minimized. It is to be minimized, then it is converted into a problem of maximizing it by using the result

Minimum z = -Maximum($-z$).

Step 2

Convert all the inequations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the costs of these variables equal to zero.

Step 3

Obtain an initial intuitionistic fuzzy basic feasible solution to the problem in the form $\tilde{x}_B{}^I \approx B^{-1}\tilde{b}^I \approx \tilde{y}_0{}^I$ and $\tilde{x}_N{}^I \approx 0$. (Here, the matrix 'B' is the same as that of the technological coefficient matrix 'A' in the definition of IFLPP and so, B^{-1} is the same as the inverse of the matrix 'A'.)

The intuitionistic fuzzy objective is $\tilde{z}^I \approx c_B B^{-1}\tilde{b}^I \approx c_B\tilde{y}_0{}^I$.

Step 4

Calculate $w = c_B B^{-1}$ and $y_0 = \Re(\tilde{y}_0{}^I)$. For each non basic variable calculate $\gamma_j = z_j - c_j = c_B B^{-1}a_j - c_j = wa_j - c_j$. Let $\gamma_l = \min\{\gamma_j\}$ where $1 \le j \le n$. If $\gamma_l \ge 0$, then stop the procedure and the current solution is optimal. Otherwise go to step 5.

Step 5

Calculate $y_l = B^{-1}a_l$. If $y_l \le 0$, then stop; the optimal solution is unbounded. Otherwise determine the index of the variable $\tilde{x}_{B_r}{}^I$ leaving the basis as follows: $\dfrac{y_{r0}}{y_{rl}} = \min\left\{\dfrac{y_{i0}}{y_{il}} / y_{il} > 0 \text{ and } 1 \le i \le m\right\}$ Update $\tilde{y}_{i0}{}^I$ by replacing $\tilde{y}_{i0}{}^I - \dfrac{\tilde{y}_{r0}{}^I}{y_{rl}} y_{il}$ for $i \ne r$ and $\tilde{y}_{r0}{}^I$ by replacing $\dfrac{\tilde{y}_{r0}{}^I}{y_{rl}}$. Also, update $\tilde{z}^I$ by replacing $\tilde{z}^I - \dfrac{\tilde{y}_{r0}{}^I}{y_{rl}}(z_l - c_l)$.

Step 6

Update B by replacing a_{B_r} with a_l and go to step 4 and repeat the procedure until the optimality is reached.

Adequacy of the Suggested Method for High Dimension Problems

The proposed approach can also be used for big sizes of problems by making use of duality of the IFLPP. Though the approach can not be reversible, the IFLPP can be rewritten in its dual form as in the case of crisp LPP and then the same algorithm can be applied to the dual problem which will lead to the solution of the given IFLPP.

A Numerical Example

The following example will depict the working of the proposed IF simplex algorithm technique for solving IFLPP with STriIFNs as parameters: Max $\tilde{z}^I = 2\tilde{x}_1^{\,I} + 4\tilde{x}_2^{\,I}$

subject to $3\tilde{x}_1^{\,I} + \tilde{x}_2^{\,I} \preceq [6; 2, 2; 3, 3]$: $2\tilde{x}_1^{\,I} - 3\tilde{x}_2^{\,I} \preceq [4; 1, 1; 2, 2]$: $\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I} \succeq 0$.

By using IF simplex method, the optimal solution is given by $\tilde{x}_1^{\,I} \approx [0; 0, 0; 0, 0]$ and $\Re\,(\tilde{x}_1^{\,I}) = 0$; $\tilde{x}_2^{\,I} \approx [6; 2, 2; 3, 3]$ and $\Re\,[\tilde{x}_2^{\,I}] = 12.5$ with maximum of $\tilde{z}^I \approx [24; 8, 8; 12, 12]$ with $\Re\,[\tilde{z}^I] = 50$.

The intitial basic feasible solution is presented in Table 1.

TABLE 1. First iteration table.

Basis ($\tilde{x}_B^{\,I}$)	$\tilde{x}_1^{\,I}$	$\tilde{x}_2^{\,I}$	$\tilde{x}_3^{\,I}$	$\tilde{x}_4^{\,I}$	R.H.S	$\Re$
$\tilde{x}_3^{\,I}$	3	1^*	1	0	[6; 2, 2; 3, 3]	12.5
$\tilde{x}_4^{\,I}$	2	-3	0	1	[4; 1, 1 ; 2, 2]	8.5
$z_j - c_j \Rightarrow$	-2	-4	0	0	$\tilde{0}^I$	0

From Table 1, it is obtained that $\tilde{x}_2^{\,I}$ is an entering variable and $\tilde{x}_3^{\,I}$ is a leaving variable. By proceeding in the same way, the optimal solution is given by $\tilde{x}_1^{\,I} \approx [0; 0, 0; 0, 0]$ and $\Re\,(\tilde{x}_1^{\,I}) = 0$; $\tilde{x}_2^{\,I} \approx [6; 2, 2; 3, 3]$ and $\Re\,[\tilde{x}_2^{\,I}] = 12.5$ with maximum of $\tilde{z}^I \approx [24; 8, 8; 12, 12]$ with $\Re\,[\tilde{z}^I] = 50$.

CONCLUSION

In this study, a more general definition of STriIFN has been proposed and the arithmetic opeartions on STriIFNs have been introduced which can be used to solve a class of intuitionistic fuzzy optimization problems in which the data parameters are STriIFNs. A special ranking fucntion for ordering STriIFNs has been proposed in project environment. Finally an attempt has been made to solve the given IFLPP without converting them to crisp linear programming problems by using IF simplex method. This approach gives a complete chioce of alternatives to the decision maker in an IF environment. The authors further proposed to use the arithmetic opeartions on STriIFNs to deal regression analysis in IF environment.

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REFERENCES

1. K.T. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica – Verlag, Heidelberg, New York, 1999.
2. G.S.Mahapatra and T.K.Roy, "Reliability Evaluation Using Triangular Intuitionistic Fuzzy Numbers Operations", *International Journal of Mathematical and Statistical Sciences*, **1(1)**, 31-38 (2009).
3. H.M.Nehi, "A New Ranking Method for Intuitionistic Fuzzy Numbers", *International Journal of Fuzzy Systems*, **12 (1)**, 80 – 86 (2010).
4. A.Kumar, J.Kaur and P.Singh, "A New Method for Solving Fully Fuzzy Linear Programming Problems", *Applied Mathematical Modeling*, **35**, 817 – 823 (2011).
5. S.H.Nasseri, E.Ardil, A.Yazdani and R.Zaefarian, "Simplex Method for Solving Linear Programming Problems with Fuzzy Numbers" in *Proceedings of World Academy of Science, Engineering and Technology*, 10, 2005, pp. 284 – 288.