J Geogr Syst (2013) 15:211–228
DOI 10.1007/s10109-012-0172-1

ORIGINAL ARTICLE

An adaptive compromise programming method for multi-objective path optimization

Rongrong Li · Yee Leung · Hui Lin · Bo Huang

Received: 3 August 2011 / Accepted: 26 June 2012 / Published online: 13 July 2012

Abstract Network routing problems generally involve multiple objectives which may conflict one another. An effective way to solve such problems is to generate a set of Pareto-optimal solutions that is small enough to be handled by a decision maker and large enough to give an overview of all possible trade-offs among the conflicting objectives. To accomplish this, the present paper proposes an adaptive method based on compromise programming to assist decision makers in identifying Pareto-optimal paths, particularly for non-convex problems. This method can provide an unbiased approximation of the Pareto-optimal alternatives by adaptively changing the origin and direction of search in the objective space via the dynamic updating of the largest unexplored region till an appropriately structured Pareto front is captured. To demonstrate the efficacy of the proposed methodology, a case study is carried out for the transportation of dangerous goods in the road network of

R. Li (✉)
Institute of Space and Earth Information Science,
The Chinese University of Hong Kong, Shatin, N.T., Hong Kong
e-mail: rongrongli@cuhk.edu.hk

Y. Leung
Department of Geography and Resource Management,
Institute of Environment, Energy and Sustainability,
The Chinese University of Hong Kong, Shatin, N.T., Hong Kong
e-mail: yeeleung@cuhk.edu.hk

H. Lin · B. Huang
Department of Geography and Resource Management,
Institute of Space and Earth Information Science,
The Chinese University of Hong Kong, Shatin, N.T., Hong Kong
e-mail: huilin@cuhk.edu.hk

B. Huang
e-mail: bohuang@cuhk.edu.hk

Hong Kong with the support of geographic information system. The experimental results confirm the effectiveness of the approach.

Keywords Multi-objective optimization · Compromise programming · Multi-objective shortest path · Pareto-optimality

JEL Classification C44 · C61 · L91

1 Introduction

Shortest path is a major stand-alone model or a subproblem of a complex model for practical network routing in a wide variety of contexts. Being an extension of the conventional shortest path problem, the multi-objective shortest path problem (MOSP) is concerned with finding a set of optimal paths with respect to multiple conflicting objectives. A good example is the search for efficient routes passing through densely populated regions that simultaneously minimize the travel cost and population exposure. MOSP is one of the core problems in multi-objective optimization (Ehrgott 2005) with numerous applications. In reality, a unique solution that can simultaneously optimize every single objective hardly exists in a MOSP involving conflicting objectives (Zitzler et al. 2003). Consequently, the focus should be on finding solutions that are near optimal or giving the best trade-offs among the conflicting objectives (Malczewski 1999).

The MOSP is known as NP-complete (Garey and Johnson 1979; Balas 1989). Many MOSPs are reduced to classic shortest path problems by using cost functions with a positively weighted linear combination of all objectives, where the weights stipulate the decision makers' preferences for individual objectives (White 1982; Cherkassky et al. 1996). However, only a few of such MOSPs have satisfactory performance. In practice, it can be very difficult to precisely and accurately specify these weights, even for someone familiar with the problem domain. Compounding this drawback is the fact that scaling among the objectives is needed and small perturbations in the weights can sometimes lead to quite different solutions. Besides, given a set of weights, this method generates a single solution rather than a set of solutions for the examination of trade-offs. More importantly, it fails to capture the efficient solutions that fall within the non-convex parts of the Pareto set.

Li and Leung (2011) developed a compromise programming (CP) method for multi-objective shortest path routing problem with a priori weights assignment. Built upon this effort, this paper proposes an adaptive method to determine the weights in an automatic and adaptive manner for similar routing problems. Compared to the CP-based approach, the adaptive method does not require one to determine weights in advance for each objective. It automatically calculates the weights based on the known information. Hence, a subset of efficient solutions can be generated without prior knowledge of the relative importance of/preferences for the objectives in question. Huang et al. (2008) devised an approach to generate a set of well-distributed Pareto candidates for a multi-objective spatial optimization problem. This approach approximates the Pareto front by gradually "learning" its shape and investing computational effort in the unexplored regions of the objective space. However, since the reference point is fixed at the origin, that is, (0, 0, …, 0), their method might yield an unadjusted Pareto front. In our approach, the starting point of the search direction is made dynamic. It is adaptively changed to locate the utopian point of the largest unexplored feasible region at which the new search direction is determined. This significant improvement, though making the problem particularly challenging, allows a more evenly distributed set of points to be generated to appropriately capture the Pareto front. We also employ a node-labeling algorithm, rather than the branch-and-bound method used by Huang et al. (2008), to solve the reformulated minimization problem. The labeling algorithms are specifically designed to make use of the network configuration. They process the links in an optimal order and run more efficiently than a standard linear program solver.

Being equipped with heuristics such as ant algorithms and genetic algorithms, geographic information system (GIS) has been widely employed to solve various optimization problems (Xiao et al. 2002; Li and Yeh 2005; Huang et al. 2006; Liu et al. 2006; Murray 2010; Delmelle et al. 2012). In this study, a GIS is used to facilitate multi-objective path optimization. To exploit the spatial-analytical capabilities of GIS, the proposed methodology is applied to solve the MOSP in a GIS environment. A case study involving dangerous goods (DG) transportation is carried out to search for optimal routes for multi-objective DG transportation in Hong Kong. Besides facilitating the implementation of the algorithm to derive optimal-route recommendations, GIS serves the multi-objective optimization problem by providing the road network and information about its surrounding environment.

The remainder of this paper is organized into several sections. An overview of the methods used to solve the MOSPs is given in Sect. 2. The formulation of MOSP and relevant notations are then outlined in Sect. 3, followed by a discussion of the proposed methodology. Details of the experimental analysis are provided in Sect. 4. Finally, Sect. 5 provides some concluding remarks.

2 Background of research

A wide variety of algorithms and methods have been developed to implement the MOSP (Ehrgott and Gandibleux 2000), such as dynamic programming, label setting, label correcting, interactive methods, approximation algorithms, genetic algorithms (GAs), and evolutionary algorithms (EAs).

Martins (1984) and Hartley (1985) propose several node-labeling algorithms, the exact algorithms based on dynamic programming, to generate the entire set of Pareto-optimal paths for a multi-criteria shortest path problem. Due to the NP-complete nature of the problem, these algorithms are non-polynomial and thus lead to considerable computational effort, even for moderately sized networks, because the set of non-dominated paths could be enormous. Martins and Santos (1999) outline a labeling algorithm for the multi-objective shortest path problem and present an analysis in terms of finiteness and optimality. Mooney and Winstanley (2006) argue that Martins' labeling algorithm works well in theory but is prohibitive to implement in practice due to memory costs. Skriver and Andersen (2000) employ a label-correcting method for a bi-criterion shortest path problem. Although efficient, their approach appears to be limited to relatively smaller networks. A study by Gandibleux et al. (2006) reports a concise description of a MOSP and clearly elucidates the most significant issues to its solution. They extend Martins' algorithm by introducing a procedure that can solve MOSP which have multiple linear functions and a max–min function. Coutinho-Rodrigues et al. (1999) introduce an interactive method for a bi-objective shortest path problem, which makes use of an efficient *k*-shortest path algorithm in identifying Pareto-optimal paths. A different interactive procedure for the MOSP based on a reference point labeling algorithm is suggested in Granat and Guerriero (2003). The multi-objective problem is converted to a parametric single-objective problem whereby the efficient paths are found. Hallam et al. (2001) design an approximation algorithm for MOSPs. The Pareto-optimal paths are selected on the basis of their selection-function value which contains heuristic and constraint information. Tsaggouris and Zaroliagis (2006) outline an improved fully polynomial time approximation scheme (FPTAS) for MOSPs, along with a generic approach to constructing FPTAS for the multi-objective optimization problem with quasi-polynomial nonlinear objectives. Their algorithm resembles the Bellman-Ford method, but it implements the label sets as arrays of polynomial size by relaxing the requirements for strict Pareto-optimality.

The linear or integer programming methods are generally straightforward and powerful, yet they often require extensive additional effort in recasting the problem in a feasible framework, and might fail in capturing the non-convex optimal solutions. As alternatives to the mathematical programming approaches, genetic algorithms (GAs) and evolutionary algorithms (EAs) have seen wide applications to various types of routing problems (Leung et al. 1998; Mooney and Winstanley 2006). However, few systematic attempts have been made to apply them directly to MOSP. Ahn and Ramakrishna (2002) present a GA for solving a single criterion shortest path routing problem. A population-sizing equation is developed to give the solutions with quality desired. Davies and Lingras (2003) implement a GA-based approach to routing shortest paths in dynamic and stochastic networks where the network information changes over time. Mooney and Winstanley (2006) propose an EA for multi-criteria path optimization problems. The EA is designed to generate MOSP solutions without the classic shortest path algorithms.

Despite a large variety of methods and algorithms that can either generate all Pareto-optimal solutions or directly generate the user-optimal solution, only a few of them deal specifically with the problem of generating a subset of the Pareto-optimal solutions without making assumptions about the decision makers' preferences. The main difficulty lies in generating a number of efficient solutions small enough to be handled by a human operator but also large enough to provide an overview of all possible trade-offs among conflicting objectives.

The transportation of dangerous goods (DGs) involves multiple stakeholder groups playing different roles and having different objectives that are generally conflicting. Designing the routes with the best possible trade-offs for conflict resolution among different objectives is of great importance for safe and efficient

DG transportation. Given the multi-objective nature of the DG routing problem, multi-objective optimization (MOP) provides a sound framework for analysis and decision making (Huang et al. 2004; Meng et al. 2005; Erkut et al. 2007; Li and Leung 2011). However, rigorous MOP methods have seldom been developed to seek optimal routes for DG transportation. This paper makes an attempt in this regard.

3 Mathematical formulation and search procedure for approximate Pareto-optimal solutions

3.1 Formulation of MOSP

Denote $G = (N, A, C)$ as a directed network, where $N = \{1, 2,\ldots, n\}$, $A = \{(i, j) | i, j \in N\}$, and $C = \{c_{ij}^k \mid k = 1, \ldots, m \text{ and } (i, j) \in A\}$ are the sets of nodes, arcs, and m-dimensional arc costs, respectively. It is assumed that G does not comprise any cycles with negative cost, and that the costs c_{ij}^k are additive along the arcs. Given a source node s and a sink node t, a path is a sequence of nodes and arcs from s to t. The cost vector for linear functions of path p is the sum of the cost vectors of its arcs. The multi-objective shortest path problem can be formulated as:

$$\min F(x) = \begin{cases} f_1(x) = \sum_{(i,j)\in A} c_{ij}^1 x_{ij}, \\ f_2(x) = \sum_{(i,j)\in A} c_{ij}^2 x_{ij}, \\ \ldots \\ f_m(x) = \sum_{(i,j)\in A} c_{ij}^m x_{ij}. \end{cases} \tag{1}$$

$$\text{s.t} \sum_{(i,j)\in A} x_{ij} - \sum_{(i,j)\in A} x_{ji} = \begin{cases} 1, & \text{if } i = s, \\ 0, & \text{if } i \neq s, t, \\ -1, & \text{if } i = t. \end{cases} \tag{2}$$

$$x_{ij} = \begin{cases} 1, & \text{if arc } (i,j) \text{ belongs to the shortest path,} \\ 0, & \text{otherwise,} \end{cases} \quad \forall \text{ arcs } (i,j) \in A. \tag{3}$$

The objectives in a MOSP are generally conflicting. Therefore, unless a well-defined utility function exists, there is no single optimal solution but rather a set of non-dominated or non-inferior solutions from which a best compromise solution can be selected (Malczewski 1999). Denote $R_p = (r_p^1, \ldots, r_p^m)$ and $R_q = (r_q^1, \ldots, r_q^m)$ as two feasible routing paths, where r_p^i and r_q^i, $i = 1, \ldots, m$, are the ith objective value for R_p the R_q, respectively, m is the number of objectives. R_p is Pareto-optimal or non-dominated if there is no other route R_q such that $f_k(R_q) \leq f_k(R_p)$, $k = 1, \ldots, m$ and $f_k(R_p) \neq f_k(R_q)$ for at least one k. In addition, R_p is weakly Pareto-optimal if there is no other feasible solution R_q such that $f_k(R_q) < f_k(R_p)$, $k = 1, \ldots, m$. Under the concept of Pareto-optimality, the efficient solutions for a MOSP are equivalent: a gain in one objective is at the cost of another. The globally optimal solution to a MOSP with conflicting objectives rarely, if ever, exists. Weakly Pareto-optimal solutions, on the other hand, are also of importance to MOSP. Although they do not strictly optimize any objective, they offer useful trade-offs among the objectives to decision makers, who can then keep or discard such solutions by comparing them with the genuine Pareto-optimal.

3.2 A framework to explore the Pareto front

Various optimization methods have been proposed to solve the multi-objective path optimization problems. By and large, these methods can be classified into two major categories: preference-based approaches and generating approaches. The preference-based approaches have been developed to allow decision makers to state their preferences a priori for all the objectives or interactively during the search procedure. Based on these preferences, different objectives are assigned different weights and aggregated into a single objective. The optimal solutions of the original multi-objective problem can then be obtained by solving the weighted sum single-objective optimization problem.

In many multi-objective decision-making processes, however, it is difficult for decision makers to state their preferences among objectives before they have an explicit notion of the actual trade-offs involved. As Zionts and Wallenius (1976) stated, decision makers in general are accustomed to responding to the trade-off questions in the context of a concrete situation (i.e., the trade-offs that are attainable from realizable situations) rather than in abstraction. Consequently, it is often desirable to generate the efficient solutions first and subsequently let decision makers select the most preferred or the best compromise solution from this set. This is the so-called generating method. A generating approach attempts to obtain a set of Pareto-optimal solutions for a given problem, with the ultimate goal of sampling a well-extended and uniformly diversified Pareto front. Various generating methods have been developed, from the exact methods such as multi-objective linear programming and dynamic programming to a series of heuristic approaches such as simulated annealing and tabu search. However, many of these methodologies are incapable of searching for the non-convex part of a Pareto front that may be of interest to decision makers. Some of them also suffer from excessive computational complexity or generating too many solutions for a straightforward choice (Huang et al. 2008).

An alternative to both the generating techniques and preference-based techniques is to define a parametric objective function that behaves like a utility function and can generate multiple Pareto-optimal paths for MOSP by varying the parameters. A careful choice of these parameters makes it possible to directly generate reasonably good paths, which provide an approximation of the set of optimal paths without much redundancy. As a result, decision makers are presented with a small set of solutions for the final choice, but they can also be reasonably confident that the key options have not been overlooked.

The best possible outcome of a multi-objective (minimization) problem would be the ideal point F^I, or the utopian point defined as $F^U = F^I - \varepsilon, \varepsilon \geq 0$ with small components, where all considered objectives achieve their optimal values simultaneously. However, when the objectives are conflicting, it is impossible to reach the ideal point or the utopian point in spite of its existence. Nevertheless, this point can serve as a reference point for the search of a feasible solution closest to it. This forms the basic notion of compromise programming (CP) (Zeleny 1973; Yu and Leitmann 1974). Based on this, a parametric objective function is adopted. This function is commonly used in CP to measure the distance between an efficient solution point and the reference point F^0, $F^0 \in \{F^I, F^U\}$. According to the definition, a general formulation of the CP problem can be expressed as:

$$\min\left\| F(X) - F^0 \right\|. \tag{4}$$

The metric $\left\|F(X) - F^0\right\|$ is a measure of the distance between the solution point and the known reference point. Whether a solution of (4) is Pareto-optimal depends on the properties of the distance measure and therefore on the properties of the norm $\|\cdot\|$. A norm $\|\cdot\|$ is called monotone, if $\|a\| \leq \|b\|$ holds for all $a, b \in R^m$ with $|a_i| \leq |b_i|$, $i = 1,\ldots, m$, and moreover $\|a\| < \|b\|$ if $|a_i| < |b_i|$, $i = 1,\ldots, m$. A norm $\|\cdot\|$ is called strictly monotone, if $\|a\| < \|b\|$ holds whenever $|a_i| \leq |b_i|$, $i = 1,\ldots, m$, and $\left|a_j\right| \neq \left|b_j\right|$ for some j. With this definition of monotone, it is easy to prove that for an optimal solution $\hat{x}$ of (4), the followings hold (Ehrgott 2005):

1. If $\|\cdot\|$ is monotone, then $\hat{x}$ is weakly Pareto-optimal. If $\hat{x}$ is a unique optimal solution of (4), then $\hat{x}$ is Pareto-optimal.
2. If $\|\cdot\|$ is strictly monotone, then $\hat{x}$ is Pareto-optimal.

The main idea of CP is to search for a feasible solution closest to the ideal point. As a measure of distance between points in multi-dimensional space, the L_p-metric is used to estimate the degree of closeness. Generally, the weighted metric $L_p = \|\cdot\|_p^\lambda$ with $p \geq 1$ is adopted, so that the CP problem is expressed as

$$\min_{x \in X}\left(\sum_{k=1}^{m} \lambda_k \left(f_k(X) - f_k^0\right)^p\right)^{\frac{1}{p}}, \quad \lambda_k > 0,\ 1 \leq p \leq \infty, \tag{5}$$

for general p, and

$$\min_{x \in X}\max_{k=1,\ldots,m}\left(\lambda_k\left(f_k(X) - f_k^0\right)\right), \quad \lambda_k > 0 \tag{6}$$

for $p = \infty$, where λ_k designates the kth positive weighting coefficient, representing the relative preference/importance attached to objective k; p is the parameter governing the distance between $F(X)$ and F^0, which acts as a weight attached to the deviation of a solution from the reference point reflecting the decision maker's perspective. L_p is strictly monotone for $1 \leq p < \infty$ and monotone for $p = \infty$.

Since the structure of the CP problem depends on the choice of the metric, we use the notation CP (p, λ). When $p = 1$, the CP (1, λ) is equivalent to the weighted sum (WS) formulation. Hence, the WS scalarization can be treated as a special case of the weighted compromise programming. When $2 \leq p < \infty$, the objective function of the CP (p, λ) is nonlinear and does not have explicit physical meaning. When $p = \infty$, the CP (∞, λ) becomes a min–max problem formulated as (6). The CP (∞, λ), referred to as the *weighted Tchebycheff approach*, is very useful in generating Pareto solutions. Bowman (1976) shows that for every Pareto solution, there exists a positive vector of weights so that the corresponding CP (∞, λ) is solved by this Pareto point. Therefore, the *weighted Tchebycheff approach* is applied, in this paper, to solve a multi-objective path optimization problem, which guarantees that a set of efficient solutions can be generated.

It should be noted that the solutions obtained by solving (6) are weakly Pareto-optimal (i.e., weakly non-dominated) when $F^0 = F^U$. The proof is as follows:

Proposition *A feasible solution $\hat{x} \in X$ is weakly non-dominated $\Leftrightarrow$ there exists a weight vector $\lambda > 0$ such that $\hat{x}$ is an optimal solution of the problem* (6)

Proof "$\Leftarrow$" Suppose $\hat{x}$ is an optimal solution of the problem (6) and $\hat{x}$ is not weakly non-dominated. Then, for a strictly positive weight vector $\lambda > 0$, there is some $x' \in X$, such that $0 < \lambda_k(f_k(x') - f_k^U) < \lambda_k(f_k(\hat{x}) - f_k^U)$. Divided by λ_k, we get $f_k(x') - f_k^U < f_k(\hat{x}) - f_k^U$ for all $k = 1,\ldots, m$, which contradicts the optimality of $\hat{x}$.

"$\Rightarrow$" The necessity property can be proved by defining appropriate weights. Let $\lambda_k = 1/(f_k(\hat{x}) - f_k^U)$, $k = 1,\ldots, m$. Since $(f_1^U, \ldots, f_m^U)$ is the utopian point, λ_k is strictly positive for all $k = 1,\ldots, m$. Suppose $\hat{x}$ is not optimal for (6) with these weights. Then, there is a feasible $x' \in X$ such that

$$\begin{aligned} &\max_{k=1,\ldots,m} \lambda_k\left(f_k(x') - f_k^U\right) \\ &\quad = \max_{k=1,\ldots,m} \frac{1}{f_k(\hat{x}) - f_k^U}\left(f_k(x') - f_k^U\right) \\ &\quad < \max_{k=1,\ldots,m} \frac{1}{f_k(\hat{x}) - f_k^U}\left(f_k(\hat{x}) - f_k^U\right) = 1 \end{aligned}$$

and therefore

$$\lambda_k(f_k(x') - f_k^U) < 1 \quad \text{forall} \quad k = 1, \ldots, m.$$

Divided by λ_k we get $f_k(x') - f_k^U < f_k(\hat{x}) - f_k^U$ for all $k = 1,\ldots, m$ and thus $f(x') < f(\hat{x})$, contradicting the fact that $\hat{x}$ is weakly non-dominated. □

In summary, any Pareto-optimal solution can satisfy the min–max formulation (6) for a given positive vector λ. On the other hand, by solving (6), a weakly non-dominated solution can be obtained. Furthermore, if this optimal solution is unique, it is then Pareto-optimal.

A geometrical interpretation shows that in two-dimensional space, the isolines of the function $\max(\lambda_1 f_1', \lambda_2 f_2') = C$ form a square wedge and that the inner part of the wedge corresponds to the set of solutions dominating the summit of the square angle (Fig. 1). The shape of the isolines is ideally suited for the exploration of both the "convex" and "concave" parts of the Pareto front, while ensuring the Pareto-optimality of the points encountered. Hence, an approximation of the Pareto front can be obtained by solving several instances of the min–max problem

$$\min \max_{k=1,\ldots,m} \left(\lambda_k(f_k(x) - f_k^0)\right), \quad \lambda_k > 0,$$

where $(f_1^0, \ldots, f_m^0)$ defines the search origin, and the reciprocal of the weight's vector $(1/\lambda_1,\ 1/\lambda_2,\ \ldots,\ 1/\lambda_m)$ designates the search direction. Figuratively, solving an

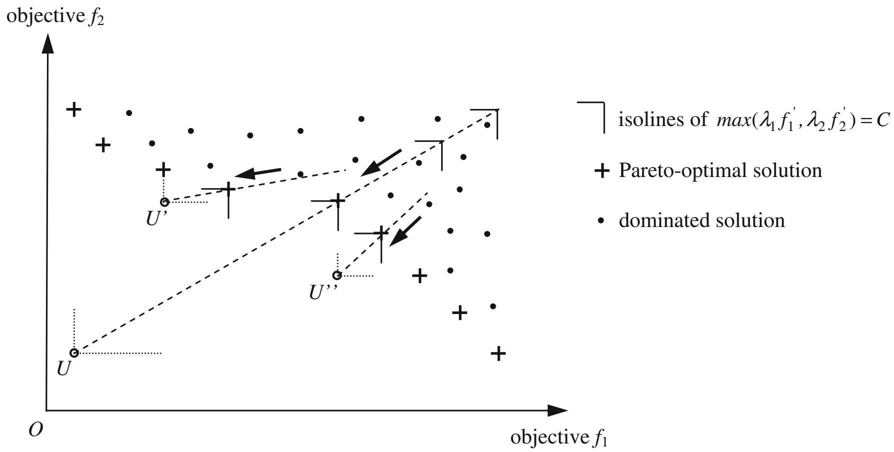

Fig. 1 Isolines of formulation (6) used to derive both the convex and concave parts of Pareto front

instance of this problem is equivalent to exploring the Pareto front along the specific line joining the reference point and the nadir point (i.e., the anti-ideal point, which is defined in such a way that it is composed of the worst values obtained for each objective) of the current exploration region. This approach can be considered as an application of the theory of achievement scalarizing function developed by Wierzbicki (1982).

3.3 Approximating the Pareto front

A significant problem in the design of our method is how to alter the weight vector λ, so that a good approximation of the Pareto front can be efficiently generated with an acceptable amount of the solutions. Studies have shown that an approximation of the Pareto front without prior knowledge of the actual one can be achieved by means of heuristic methods. In order to improve the efficiency, an adequate heuristics should seek a balance between the amount of information provided and the computational time required to obtain it. Therefore, an ideal algorithm should effectively combine the exploration of the largest unexplored regions of the objective space and the exploitation of the previously encountered solutions (Hughes 2003). In our method, once a Pareto-optimal is obtained, the search space will be partitioned into smaller pieces, and the regions that are either dominated by the known optimal solutions or without optimal solutions will be discarded. The origin and direction of search are then adjusted based on the largest unexplored space that may contain efficient solutions. The notion of volume is simply used to compare unexplored regions. A list of unexplored regions is carefully maintained in implementing the method. In this list, every unexplored region is described by its utopian point and nadir point, its estimated volume, and the known solutions lying on its boundaries. The utopian point and the nadir point of an unexplored region are defined as the lowest point and the furthest summit of the region, respectively. Each time when a new efficient solution is found, the list will be subsequently updated. The proposed procedure works as follows:

Step 1: For each objective k, search for the optimal solution f_k, and thus define the utopian point $\boldsymbol{U} = (f_1^U, f_2^U, \ldots, f_m^U)$ and the nadir point $\boldsymbol{V} = (f_1^V, f_2^V, \ldots, f_m^V)$. Based on $\boldsymbol{U}$, $\boldsymbol{V}$, and the optimal solutions obtained for each individual objective, the region to be explored can be identified. Initialize the list of the unexplored regions.
Step 2: Remove the largest unexplored region from the list and define the new search origin and the new search direction $\boldsymbol{\lambda}$ based on the attributes of $\boldsymbol{U}$ and $\boldsymbol{V}$ as $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, where $\lambda_k = 1/(f_k^V - f_k^U)$, $k = 1, \ldots, m$.
Step 3: Solve the min–max problem (6).
Step 4: If the solution found is already known, resume at Step 2; else a new solution is found. Calculate the new unexplored regions lying between this new solution and its neighbors according to their objective values, then update the list of unexplored regions and resume at Step 2.

In Step 1, the utopian point $\boldsymbol{U} = (f_1^U, f_2^U, \ldots, f_m^U)$ is computed as a result of m single-objective optimizations with each objective serving as a single-objective function. Once the utopian point is determined, the information found is then used to compute an estimate of the nadir point $\boldsymbol{V} = (f_1^V, f_2^V, \ldots, f_m^V)$. An approximation of $\boldsymbol{V}$ is defined in such a way that for each criterion k, f_k^V represents the worst value obtained during the computation of the utopian point. Except for the first iteration, the attributes of $\boldsymbol{U}$ and $\boldsymbol{V}$ in Step 2 need to be updated in each iteration according to the known Pareto-optimal lying on the largest unexplored region. To solve the min–max problem in Step 3, a tailor-made node-labeling algorithm is employed, which modifies the classic Dijkstra's algorithm (1959) by taking into account multiple attributes in the cost calculation for each link. The cost of traversing link is not the value of any single criterion but rather the largest element of the weighted "distance" between the point being explored and the reference point among all the objectives examined, that is, $c_{ij} = \max_{(i,j) \in A, k=1,\ldots,m}\left(\lambda_k\left(f_k - f_k^0\right)\right)$. The procedure of solving the min–max problem by means of the modified Dijkstra's algorithm is similar to that of the conventional Dijkstra's working on shortest path problems. The recursive steps of the algorithm can be described as follows:

> Find an arc $(i, j) \in A$, so that the cost $f(i)$ of traveling from the origin to node i increased with the cost c_{ij} of traveling along (i, j) is less than the present cost of traveling from origin to node j: $f(i) + c_{ij} < f(j)$. If such an arc exists, then node i becomes the predecessor of node j in the shortest path and the procedure resumes, otherwise the present cost of traveling from the origin to node j is the minimum cost.

The iterative search procedure terminates when a subset of Pareto-optimal solutions of the desired size has been obtained or when the proportion of the remaining unexplored subregions is sufficiently small.

4 Case study

DG route planning can be considered as a multi-objective issue as it involves various factors, such as the safety of the surrounding population and properties along the road and the safety of travelers in the road network. At the same time, the operation cost of transporting DG should not be overlooked. Hence, to design appropriate routes for DG transportation that can make a rational compromise between costs and risks is of significance to decision makers (Li and Leung 2011).

To test the applicability of the proposed method in DG routing analysis and to examine the effectiveness of the methodology, we employ the proposed algorithm to solve a real-life routing problem for transporting liquefied petroleum gas (LPG) on the road network in Hong Kong. The algorithm is used to search Pareto-optimal routes for transporting LPG from Tsing Yi LPG Terminal to the designated LPG filling stations. To benefit from the data manipulation and visualization techniques of GIS, the method is implemented in a GIS environment.

4.1 Formation of the objectives

Li and Leung (2011) proposed a set of criteria for route planning for LPG transportation in Hong Kong road network with the consideration of cost, safety, and exposure, including (1) travel time, (2) probability of an incident with release of LPG, (3) road users at risk, (4) off-road population at risk, (5) people with special needs at risk, and (6) possible negative impact on the economy. In the present study, an extra criterion with reference to the emergency response capabilities has been added. Given these criteria, up to seven different objectives aiming at the minimization of the magnitude of the aforementioned attributes were included in the routing analysis. These objectives were quantified using the framework suggested in the US DOT guidelines for DG routing (FHWA 1994). GIS is used to facilitate the quantification of objectives by creating buffer area for the identification of the impact zone in the event of a DG incident. ArcGIS 9.3 is employed as the GIS platform to support routing analysis. In ArcGIS, a buffer zone is created to simulate the potential impact area. The potential impact zone for petrochemicals is typically taken at 800 meters in all directions (FHWA 1994). Therefore, a buffer of 800 m width is generated for each road segment. Various exposures, such as the off-road population exposure, along a road segment are then calculated from the exact number of buildings (residential and commercial) within the potential impact zone. Through ArcGIS, the appropriate attributes are queried and the respective risk values are calculated.

4.2 Results and interpretations

By applying the proposed algorithm, a set of routes rendering various trade-offs between risk and cost have been generated. Of the numerous route alternatives, twelve Pareto-optimal routes from Tsing Yi LPG terminal to Tai Po station are selected for illustration purpose. These routes are presented in Fig. 2 to offer some insight into the trade-offs between different solutions. Routes 1–7 are “extreme”

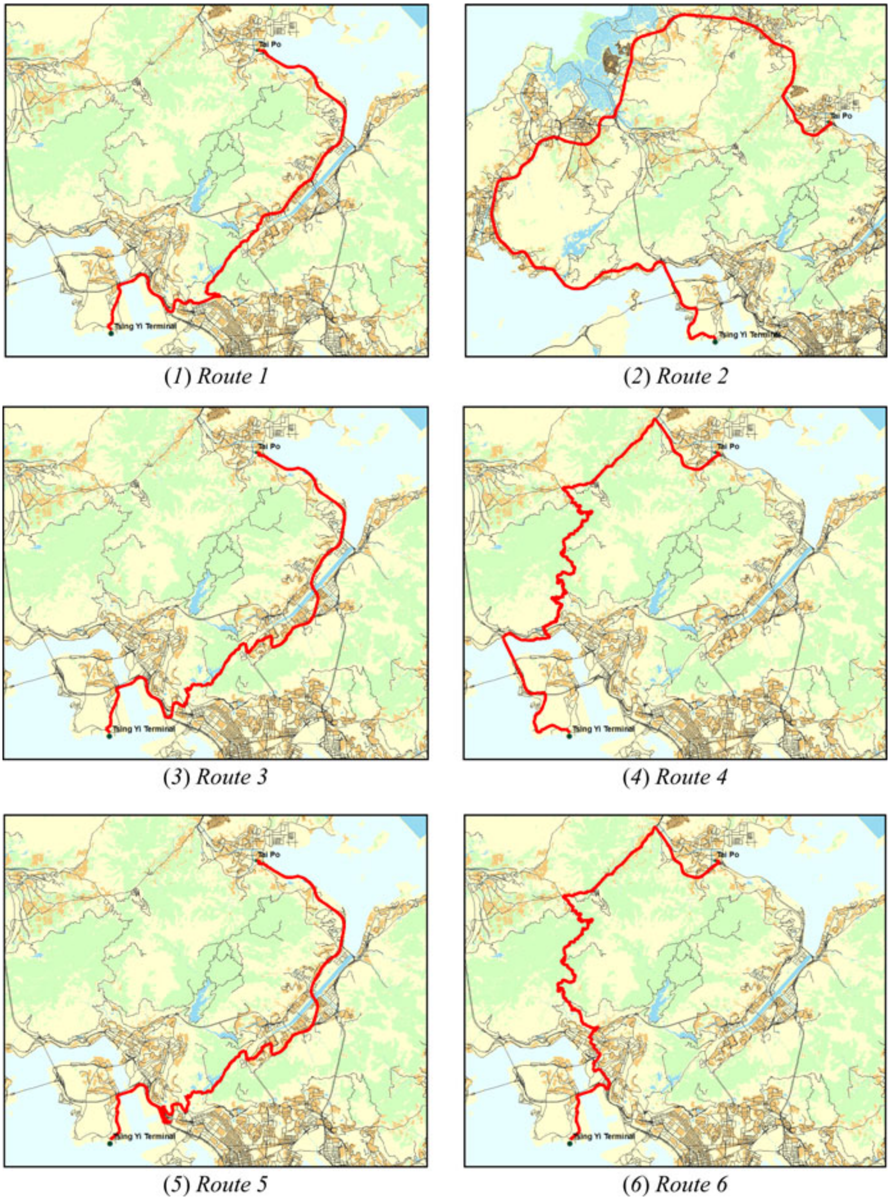

Fig. 2 Efficient routes from Tsing Yi terminal to Tai Po LPG filling station (routes 1–7 are "extreme" solutions, routes 8–12 are MOSP solutions)

ones which individually minimize each of the seven objectives, while routes 8–12 are MOSP routing solutions. It is unsurprising to find that the first six "extreme" solutions are identical to those which are reported in Li and Leung (2011), as the adaptive method employed in the present study is actually the special case of

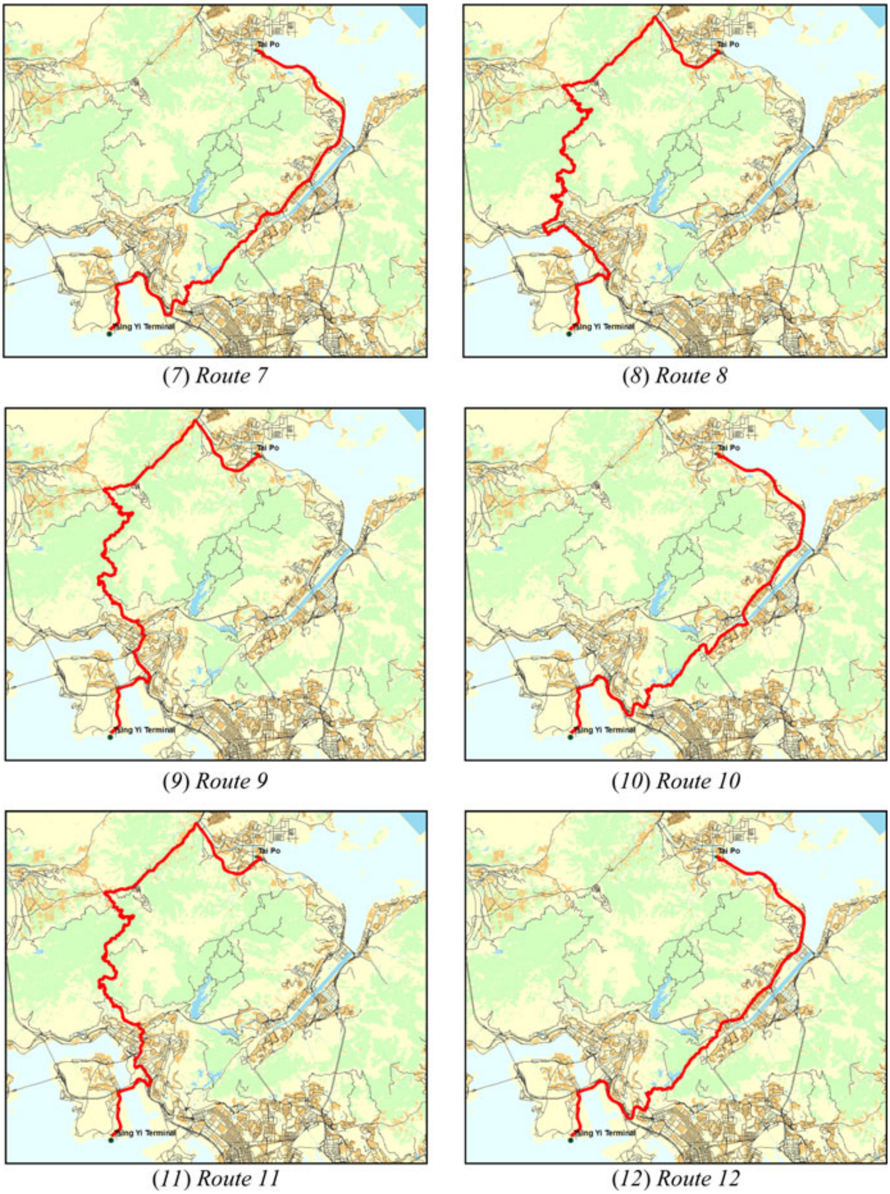

Fig. 2 continued

compromise programming with the parameter p taking the value of infinity. With respect to MOSP solutions, however, they are somewhat different from the results of Li and Leung (2011) due to the nature of multi-objective optimization. Table 1 summarizes the attributes of the twelve routing alternatives. The rows correspond to the optimal routes and the columns to the objectives. All the values in the table are unit-free due to data normalization.

Table 1 Normalized objective function values of optimal solutions (1–7 are "extreme" solutions and 8–12 are MOSP solutions)

Criterion-based solution	Travel time	Accident probability	Off-road exposure risk	Population with special needs at risk	Damage on economy	Road users at risk	Emergency response capabilities
1. Min (travel time)	3.70	2.97	0.75	1.80	1.09	6.55	2.66
2. Min (accident probability)	7.12	2.74	0.86	1.48	0.66	5.56	2.42
3. Min (off-road population exposure)	4.03	3.08	0.42	1.46	0.60	4.58	2.72
4. Min (population with special needs at risk)	6.29	5.62	0.67	0.80	0.67	3.73	3.41
5. Min (expected damage on the economy)	4.42	3.47	0.43	1.70	0.51	4.63	3.18
6. Min (road users at risk)	5.26	4.80	1.02	1.67	1.35	3.19	3.87
7. Min (risk from emergency response)	3.92	3.07	0.54	1.46	0.70	5.52	2.19
8. Focusing on societal risk	5.43	4.85	0.70	1.20	0.96	3.28	3.29
9. Focusing on both travel time and societal risk	5.07	4.56	0.89	1.47	1.04	3.36	3.52
10. Considering all criteria impartially	3.92	2.99	0.49	1.49	0.67	4.43	2.51
11. Considering all criteria impartially	5.19	4.73	0.91	1.59	1.15	3.33	3.87
12. Considering all criteria impartially	3.74	2.84	0.48	1.32	0.66	5.14	2.45

All the values in the table are unit-free due to data normalization

Route 1 has the shortest travel time. However, the risks of special population exposure and on-road exposure with this solution are quite high, resulting in the maximum exposure risks among the 12 alternatives. Although route 2 minimizes the accident probability, the large detour from Tsing Yi to Tai Po following the links with low accident rate leads to the longest travel time. Route 3 has minimum off-road exposure risk. Route 4 is the best in terms of the risk of exposure of populations with special needs. However, it bears the highest accident probability, more than 100 % greater than the minimum obtained by route 2. Route 5 minimizes the expected damage to the economy in the event of an accident. Route 6 minimizes the on-road exposure risk by following secondary roads with lighter traffic, at the cost of the highest risk of off-road exposure. Moreover, should a DG accident occur, the expected damage on the economy of this solution will be more than doubled compared to the minimum obtained by solution 5. Route 7 is the most desirable from the perspective of emergency response capabilities.

Routes 8–12 are a subset of "compromise" solutions containing comparatively milder trade-offs than the "extreme" solutions among different objectives. Route 8 considers simultaneously the road users at risk, off-road population at risk, and people with special needs at risk for the generation of optimal routes for DG shipments. While effectively addressing the government's major concerns in DG route planning, this routing solution is not equally desirable from the perspective of accident probability, which is over 70 % higher than the minimum obtained under solution 2. Route 9 incorporates operating cost with public safety, which produces a shorter travel time and a lower accident probability than route 8. On the other hand, this solution causes deterioration ranging from 2 to 21 % over route 8 on the other objectives. Routes 10–12 are obtained by taking all the criteria into consideration. Like the other two MOSP solutions, these three routes involve various trade-offs between different objectives, which, however, are not as significant as those reflected in the "extreme" ones.

4.3 Assessing the theoretical validity of the algorithm

To assess the theoretical validity of the proposed algorithm, a couple of aspects of the results have been examined. The first criterion is to estimate the goodness of the approximation, which can be measured by the proportion of the objective space that is covered by the approximate set of solutions. In practice, this notion is controlled by the size of the unexplored regions remaining for exploration. In the calculation process, it is observed that when the algorithm stops, the size of the unexplored regions accounts for less than 20 % of the whole objective space. Another criterion is to examine the efficiency of the method by estimating the dissimilarity of the generated routes, which is of importance in routing DG shipments. A dissimilarity index is calculated for every pair of routes selected. The dissimilarity of two routes R_i and R_j is defined as the symmetrical function (Akgün et al. 2000):

$$D(R_i, R_j) = 1 - \left(\frac{L(R_i \cap R_j)}{2L(R_i)} + \frac{L(R_i \cap R_j)}{2L(R_j)}\right),$$

where $R_i \cap R_j$ denotes the portion of shared arcs by the route pair R_i and R_j, and $L(\cdot)$ denotes the length of quantity, $\cdot$, in brackets. The results are displayed in Table 2. It shows that there are very few instances of high similarities between the generated routes. Most of them are quite dissimilar with each other. In particular, more than 10 % of resultant routes present extremely high dissimilarity, with the dissimilarity index higher than 90 %. Among the presented twelve routes, the minimum and maximum dissimilarities are 8.4 and 99.4 %, respectively, while the average is 65.8 % with a standard deviation of 31.5 %.

Table 2 Dissimilarity value of every pair of routes (the lower-left part of the matrix is identical to the transpose of the upper-right corner)

Route	Dissimilarity with route (%)												Mean (%)
	1	2	3	4	5	6	7	8	9	10	11	12	
1	0.0	99.1	36.2	98.9	45.9	87.4	30.6	87.7	87.3	32.8	89.5	18.9	64.95
2		0.0	98.3	73.2	99.0	88.9	99.4	89.3	88.8	98.9	88.9	98.9	92.97
3			0.0	98.0	18.4	86.9	35.6	87.3	86.8	29.0	88.9	29.8	63.20
4				0.0	98.8	43.0	99.3	32.7	42.3	98.8	42.7	98.7	75.14
5					0.0	82.2	39.5	82.6	81.9	30.6	84.0	29.2	62.92
6						0.0	87.7	21.9	11.5	85.5	8.4	83.0	62.40
7							0.0	85.8	85.3	20.3	82.8	13.7	61.83
8								0.0	20.3	85.8	22.6	83.4	63.58
9									0.0	85.3	11.4	82.8	62.16
10										0.0	80.6	14.2	60.16
11											0.0	85.1	62.26
12												0.0	57.99

To further check the effectiveness of the proposed methodology, the same routing problem was also analyzed using the compromise programming-based (CP) approach outlined in Li and Leung (2011). It is found that solutions obtained by the suggested method are, at times, very close to the routes generated by CP with a certain weights assignment. For example, given the criteria of travel time, accident probability, off-road exposure risk, people with special needs at risk, expected damage on the economy, on-road exposure risk, and emergency response capabilities, under the weight structure of (0.06, 0.11, 0.21, 0.20, 0.10, 0.22, 0.10) which places a higher importance on population exposure risks, the route produced by the CP method exhibits high similarity with Route 8 shown in Fig. 2, while for another weight assignment (0.22, 0.01, 0.25, 0.25, 0.01, 0.25, 0.01), which emphasizes both operating cost and public safety, the resulting route of CP closely resembles Route 10. CP is often used to directly generate the user-optimal solutions. The high similarity in the solutions of CP and the suggested approach indicates that the latter is capable of identifying a set of efficient solutions for decision making. This will enable us to cover all bases in multi-objective path optimization.

5 Conclusion

This paper has presented a novel perspective on multi-objective path optimization problem aiming at the search for a subset of efficient paths for the determination of an optimal solution. To avoid the pitfalls of preference-based techniques and the burden of generating a complete set of possible solutions, the proposed method adopts a weighted min–max formulation and the notion of adaptive optimization, and focuses at each time the search for optimal solutions on a particular region of the Pareto front. By adaptively altering the weights according to the largest unexplored feasible region and solving the corresponding min–max problem through tailor-made labeling algorithm, a set of relatively well-distributed Pareto-optimal solutions can be obtained. These solutions provide decision makers with an unbiased overview of the possible trade-offs among different objectives. An application in LPG route planning in Hong Kong illustrates the adaptation of the proposed framework in a GIS environment. The final results justify the effectiveness of the suggested approach.

A main challenge in implementing a multi-objective optimization problem is the accommodation of more objectives. This paper has hammered out a method for this curse-of-dimensionality issue through the calculation of all unexplored region volumes. However, further efforts need to be made to improve the computational efficiency. In addition, the node-labeling algorithm employed also needs to be refined in order to more effectively handle nonlinear objective functions and, possibly, the constraints in real-time operations.

Acknowledgments This research was supported by the earmarked grant (CUHK 449711) of the Hong Kong Research Grants Council. The authors would like to thank the anonymous reviewers for valuable comments.

References

Ahn CW, Ramakrishna R (2002) A genetic algorithm for shortest path routing problem and the sizing of populations. IEEE Trans Evol Comput 6(6):566–579

Akgün V, Erkut E, Batta R (2000) On finding dissimilar paths. Eur J Oper Res 121(2):232–246

Balas E (1989) The prize collecting traveling salesman problem. Networks 19(6):621–636

Bowman VJ (1976) On the relationship of the Tchebycheff norm and the efficient frontier of multiple-criteria objectives. Lect Notes Econ Math Syst 135:76–85

Cherkassky BV, Goldberg AV, Radzik T (1996) Shortest paths algorithms: theory and experimental evaluation. Math Program 73(2):129–174

Coutinho-Rodrigues J, Clímaco J, Current J (1999) An interactive bi-objective shortest path approach: searching for unsupported nondominated solutions. Comput Oper Res 26(8):789–798

Davies C, Lingras P (2003) Genetic algorithms for rerouting shortest paths in dynamic and stochastic networks. Eur J Oper Res 144(1):27–38

Delmelle EM, Li SP, Murray AT (2012) Identifying bus stop redundancy: a gis-based spatial optimization approach. Comput Environ Urban Syst. doi:10.1016/j.compenvurbsys.2012.01.002

Dijkstra EW (1959) A note on two problems in connection with graphs. Numer Math 1(1):269–271

Ehrgott M (2005) Multicriteria optimization, 2nd edn. Springer, Berlin, Heidelberg and New York

Ehrgott M, Gandibleux X (2000) A survey and annotated bibliography of multi-objective combinatorial optimization. OR Spectr 22(4):425–460

Erkut E, Tjandra SA, Verter V (2007) Hazardous materials transportation. In: Laporte G, Barnhart C (eds) Handbooks in operations research and management science. Elsevier Science, North-Holland, pp 539–621

FHWA (1994) Guidelines for applying criteria to designate routes for transporting hazardous materials. Report FHWA-SA-94-083, Federal Highway Administration, USA

Gandibleux X, Beugnies F, Randriamasy S (2006) Martins' algorithm revisited for multi-objective shortest path problems with a MaxMin cost function. 4OR-Q J. Oper Res 4(1):47–59

Garey M, Johnson D (1979) Computers and intractability: a guide to the theory of NP-completeness. Freeman, San Francisco

Granat J, Guerriero F (2003) The interactive analysis of the multicriteria shortest path problem by the reference point method. Eur J Oper Res 151(1):103–118

Hallam C, Harrison K, Ward J (2001) A multiobjective optimal path algorithm. Digit Signal Process 11(2):133–143

Hartley R (1985) Vector optimal routing by dynamic programming. In: Serafini P (ed) Mathematics of multiobjective optimization. Springer, Berlin, Heidelberg and New York, pp 215–224

Huang B, Cheu RL, Liew YS (2004) GIS and genetic algorithms for HAZMAT route planning with security considerations. Int J Geogr Inf Sci 18(8):769–787

Huang B, Liu N, Chandramouli M (2006) A GIS supported Ant algorithm for the linear feature covering. Decis Support Syst 42(2):1063–1075

Huang B, Fery P, Xue LL, Wang YJ (2008) Seeking the Pareto front for multiobjective spatial optimization problems. Int J Geogr Inf Sci 22(5):507–526

Hughes EJ (2003) Multi-objective binary search optimisation. In: Proceedings of 2nd international conference on evolutionary multi-criterion optimization, EMO'03, pp 102–117

Leung Y, Li G, Xu ZB (1998) A genetic algorithm for the multiple destination routing problems. IEEE Trans Evol Comput 2(4):150–161

Li RR, Leung Y (2011) Multi-objective route planning for dangerous goods using compromise programming. J Geogr Syst 13(3):249–271

Li X, Yeh AGO (2005) Integration of genetic algorithms and GIS for optimal location search. Int J Geogr Inf Sci 19(5):581–601

Liu N, Huang B, Chandramouli M (2006) Optimal siting of fire stations using GIS and ANT algorithm. J Comput Civ Eng 20(5):361–369

Malczewski J (1999) GIS and multicriteria decision making analysis. Wiley, New York

Martins EQV (1984) On a multicriteria shortest path problem. Eur J Oper Res 16(2):236–245

Martins EQV, Santos JLE (1999) The labelling algorithm for multiobjective shortest paths. Technical Report, Departmento de Matematica, Universidade de Coimbra, Portugal

Meng Q, Lee DH, Cheu RL (2005) The multiobjective vehicle routing and scheduling problem with time window constraints in hazardous material transportation. J Transp Eng ASCE 131(9):699–707

Mooney P, Winstanley A (2006) An evolutionary algorithm for multicriteria path optimization problems. Int J Geogr Inf Sci 20(4):401–423

Murray AT (2010) Advances in location modeling: GIS linkages and contributions. J Geogr Syst 12(3):335–354

Skriver A, Andersen K (2000) A label correcting approach for solving bicriterion shortest-path problems. Comput Oper Res 27(6):507–524

Tsaggouris G, Zaroliagis C (2006) Multi-objective optimization: improved FPTAS for shortest paths and non-linear objectives with applications. Lect Notes Comput Sci 4288:389–398

White DJ (1982) The set of efficient solutions for multiple objective shortest path problems. Comput Oper Res 9(2):101–107

Wierzbicki AP (1982) Mathematical basis for satisfying decision making. Math Model 3(5):391–405

Xiao N, Bennett DA, Armstrong MP (2002) Using evolutionary algorithms to generate alternatives for multiobjective site search problems. Environ Plan A 34(4):639–656

Yu PL, Leitmann G (1974) Compromise solutions, dominations structures, and Salukvadze's solution. J Optim Theory Appl 13(3):362–378

Zeleny M (1973) Compromise programming. In: Cochrane JL, Zeleny M (eds) Multiple criteria decision making. University of South Carolina Press, Columbia, pp 262–301

Zionts S, Wallenius J (1976) An interactive programming method for solving the multiple criteria problem. Manag Sci 22(6):652–663

Zitzler E, Thiele L, Laumanns M, Fonseca CM, Da Fonseca VG (2003) Performance assessment of multiobjective optimizers: an analysis and review. IEEE Trans Evol Comput 7(2):117–131