

# Lithium-ion battery modelling for the energy management problem of microgrids

 ISSN 1751-8687  
 Received on 1st April 2015  
 Revised on 13th October 2015  
 Accepted on 25th October 2015  
 doi: 10.1049/iet-gtd.2015.0423  
 www.ietdl.org

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**Abstract:** This study presents a mathematical model of lithium-ion (Li-ion) batteries in the energy management (EM) problem of a microgrid (MG). In this study, the authors develop a detailed model of Li-ion batteries that considers the degradation cost associated with operation, controllable and uncontrollable charging ramps, other limits, and the operating characteristics provided by the manufacturers. The Li-ion battery degradation cost is analysed using different approaches and is compared with modelling without this cost, using a quadratic degradation cost, and using a piecewise degradation cost. Furthermore, this cost is analysed using a linear cost that takes the life expectancy based on the number of cycles of the battery into account. To analyse the proposed method and other modelling approaches, the authors examine the battery model in an EM problem in an MG. This MG, which can be connected to the main grid, also uses wind and photovoltaic as generation resources, in addition to a backup generator. The EM problem is modelled as a deterministic mixed-integer linear (or quadratic) problem; the results of eleven different cases are used in the analysis of the proposed Li-ion battery model for a 24 h planning horizon.

## Nomenclature

The notations used throughout the paper are presented below. Variables are in lowercase letters. Parameters are in capital or Greek letters.

### Indices and sets

$e$  index related to batteries ( $e \in E$ );  
 $i$  discretisation step index associated with the battery reserve;  
 $n$  index of the piecewise linearisation of the quadratic-equation cost for the battery ( $n \in N$ );  
 $t$  index of the time step ( $t \in ND$ ).

### Variables

$cb1_{et}$  cost of the piecewise linearisation of battery  $e$  at step  $t$  ( $\text{€ h}^{-1}$ );  
 $d_{pbd_{et}}$  absolute discharge power output difference between time steps  $t-1$  and  $t$  of battery  $e$  (kW);  
 $eb_{et}$  energy of battery  $e$  at step  $t$  (kWh);  
 $pb_{et}$  power charge of battery  $e$  at step  $t$  (kW);  
 $pbd_{et}$  power discharge of battery  $e$  at step  $t$  (kW);  
 $pde_t$  system deficit at step  $t$  (kW);  
 $pex_t$  excess generation at step  $t$  (kW);  
 $pgb_t$  power purchased from the main grid at step  $t$  (kW);  
 $pgs_t$  power sold to the main grid at step  $t$  (kW);  
 $rb_{et}$  reserve of battery  $e$  at step  $t$  (kW);  
 $ub_{et}^{\text{aux1}}$  binary variable that indicates whether battery  $e$  is in constant-current charge condition ( $ub_{et}^{\text{aux1}} = 1$ ) at step  $t$ ;  
 $ub_{et}^{\text{aux2}}$  binary variable that indicates whether battery  $e$  is in constant-voltage charge condition ( $ub_{et}^{\text{aux2}} = 1$ ) at step  $t$ ;  
 $ub_{et}$  binary variable that indicates whether battery  $e$  is charging ( $ub_{et} = 1$ ) at step  $t$ ;  
 $ug_t$  binary variable that indicates whether the MG is receiving energy from the main grid ( $ug_t = 1$ ) at step  $t$ ;

## Parameters

$AB_e$  battery  $e$  state-of-charge (SOC) target (%);  
 $BB_e$  battery  $e$  cost parameter [ $\text{€ Cbat h}^{-1} (\text{kWh})^{-1}$ ];  
 $BP_t$  energy purchase price in step  $t$  [ $\text{€} (\text{kWh})^{-1}$ ];  
 $CB_e$  battery  $e$  maximum charge (kW);  
 $CB2_e$  charge constant of the linear equation for battery  $e$  (kW);  
 $CD$  deficit incremental cost [ $\text{€} (\text{kWh})^{-1}$ ];  
 $CE$  incremental cost of system excess energy [ $\text{€} (\text{kWh})^{-1}$ ];  
 $CUB_{et}$  battery  $e$  degradation cost at step  $t$  ( $\text{€ h}^{-1}$ );  
 $DB_e$  battery  $e$  maximum discharge (kW);  
 $D_t$  forecast demand at step  $t$  (kW);  
 $EBF_e$  final energy of battery  $e$  (kWh);  
 $EBI_e$  initial energy of battery  $e$  (kWh);  
 $EB_e^{\text{max}}$  battery  $e$  maximum energy (kWh);  
 $EB_e^{\text{min}}$  battery  $e$  minimum energy (kWh);  
 $ED$  forecasted error associated with the demand (%);  
 $EPV$  forecasted error associated with photovoltaic generation (%);  
 $EPW$  forecasted error associated with wind generation (%);  
 $FB_e$  discharge incremental cost parameter of battery  $e$  [ $\text{€} (\text{kWh})^{-1}$ ];  
 $GB_e$  linear charge incremental cost parameter of battery  $e$  [ $\text{€} (\text{kWh})^{-1}$ ];  
 $H$  number of hours in the planning horizon ( $h$ );  
 $HB_e$  quadratic charge cost parameter of the battery  $e$  [ $\text{€ kWh} (\text{kW})^{-2} \text{h}^{-1} \text{Cbat}^{-1}$ ];  
 $IB_{en}$  linear SOC energy deviation cost parameters of battery  $e$  and linearisation  $n$  ( $\text{€ h}^{-1}$ );  
 $JB_{en}$  linear SOC energy deviation cost parameters of battery  $e$  and linearisation  $n$  ( $\text{€ h}^{-1}$ );  
 $ND$  number of time steps in the planning horizon;  
 $PBL_e$  power loss during one time step for battery  $e$  (kW);  
 $PGB_t^{\text{max}}$  grid maximum importing at step  $t$  (kW);  
 $PGB_t^{\text{min}}$  grid minimum importing at step  $t$  (kW);  
 $PGS_t^{\text{max}}$  grid maximum exporting at step  $t$  (kW);  
 $PGS_t^{\text{min}}$  grid minimum exporting at step  $t$  (kW);  
 $PV_t$  forecast of photovoltaic power at step  $t$  (kW);  
 $PW_t$  forecast of wind power at step  $t$  (kW);

$RB$	number of time steps related to the system reserve;
$SP_t$	energy selling price at step $t$ (€ (kWh) <sup>-1</sup> );
$SPC_e$	SOC set point in which charge ramp of battery $e$ changes from constant-current to constant-voltage mode (%);
$\alpha_e$	charge gradient of the linear equation of battery $e$ [kW (kWh) <sup>-1</sup> ];
$\delta_e$	incremental cost associated with pulse operation of battery $e$ [€ (kWh) <sup>-1</sup> ];
$\eta_e^{bc}$	charge efficiency of battery $e$ (%);
$\eta_e^{bd}$	discharge efficiency of battery $e$ (%).

## 1 Introduction

The integration of controllable load demand, energy storage systems (ESSs), small renewable generators, and electrical vehicles is a trend within modern power systems. The inclusion of these distributed energy resources (DERs) might reduce fossil fuel consumption, allow load peak shaving, and postpone investment in new transmission and distribution lines [1, 2]. In this new paradigm, it is important to highlight the microgrid (MG) concept, which can be roughly defined as a group of DERs that operate either connected to or disconnected from the main grid [3].

One methodological challenge that influences the operation issues of an MG is the energy management (EM) problem [4, 5]. In general, to solve this problem using centralised control, it is necessary to minimise an objective function over a planning horizon, subject to economic and technical constraints. One important result associated with EM results from scheduled operation, that is, the on/off status and the respective output active power of each controllable DER. The schedule is used as a reference for voltage and frequency control of the MG in real-time operation. Because it is necessary to minimise the objective function subject to constraints, centralised EM could be operated based on the solution to an optimisation problem.

ESSs are important within the MG, in particular, because of the intermittent characteristics of renewable generation sources, demand, reserve requirements, and islanded operation of the MG [6]. Consequently, the presence of ESSs in an MG might significantly increase the quality and reliability of the energy supply. The most common ESSs are batteries, super capacitors, flywheels, compressed air, and superconducting magnetic energy storage. Even for the same type of ESS, several characteristics in the model can be different, such as the size and technology employed, for example, as presented in [7] for chemical batteries. Regarding batteries, the most promising technology is currently lithium-ion (Li-ion) batteries, as used in electric cars, mobile phones, and notebooks [8–10].

In the literature, there are different modelling approaches to Li-ion batteries that are present in EM problems. For example, Li-ion batteries can be modelled as a generic storage system, as in [11, 12], where the incremental cost for discharge and/or charge is based on the expectancy of the number of cycles for charge and discharge. Other studies model Li-ion batteries based on the annual maintenance and acquisition cost from planning studies [13] or as in [14, 15], where the authors present a specific function for the degradation cost. In this paper, we use the equation from [14] to test the specific functions of the Li-ion degradation cost, as described in Section 3.1. In [14], in addition to the function cost, other management algorithm approaches are used to compare battery degradation over the course of several years. The operational constraints in references [11–15] are associated with the limits of charge, discharge, initial energy, final energy, and energy balance based on the efficiency and previous energy states. These (and new proposed equations) are presented in Section 3.2. The modelling considers certain aspects of the battery because the charges and discharges (with a high efficiency) will influence the state of health (SOH) and the state of charge (SOC) of the batteries. Additional physical characteristics that are usually not considered for this battery technology are moderate discharge, which can be superior to pulse discharge, and the constant current

and voltages stage characteristics for the charge [8–10, 16]. Typical characteristics that are applicable to generic batteries will also be considered, for example, times when the charge is inaccessible (because it uses self-charging management) and the battery reserve of the MG EM.

The most important contributions of this paper to Li-ion battery modelling in the EM of an MG are: (i) comparison of different approaches in terms of the associated costs; (ii) inclusion of new constraints to prevent pulse discharging and examine charging characteristics; (iii) new constraints for batteries when it is not possible to control charging; (iv) new equations for battery reserves; and (v) comparison of different numbers of time steps within a 24 h planning horizon. The EM problem is obtained by solving a deterministic mixed-integer linear (or quadratic) problem [17] in which the 24 h planning horizon is discretised into time steps of 1, 10, and 30 min. The MG, which can be operated connected to or disconnected from the main grid, includes batteries, wind and photovoltaic generation resources, and a backup generator.

This paper is organised as follows: In the next section, the modelling of a Li-ion battery is presented. Then, in Section 4, a generic optimisation model for the EM problem is described. In Section 5, we present the MG and computational experiments. Finally, Section 6 presents our conclusions.

## 2 Li-ion battery modelling

Li-ion batteries can be classified into several different types [18]. Each of these Li-ion batteries has properties suited to a specific use; however, this paper proposes a generic model.

### 2.1 Cost function

The costs used in the comparative tests of the Li-ion battery are divided in three groups: without a degradation cost; incremental costs for charge/discharge of the battery and the number of cycles; and, real degradation data, obtained in [14], as a positive semi-definite quadratic function.

Some Li-ion battery cost function approaches found in the literature are based on the number of charge/discharge cycles. For example, if a battery could perform 1500 cycles with maximum energy and the acquisition price is considered constant and is approximated by an incremental cost that depends on the discharge and/or charge, sometimes taking into account the depth of discharge. In this paper, this approach, with an incremental cost associated with discharge, will be used.

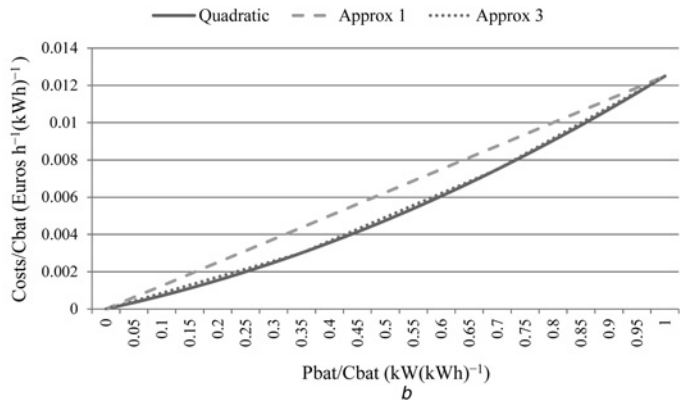
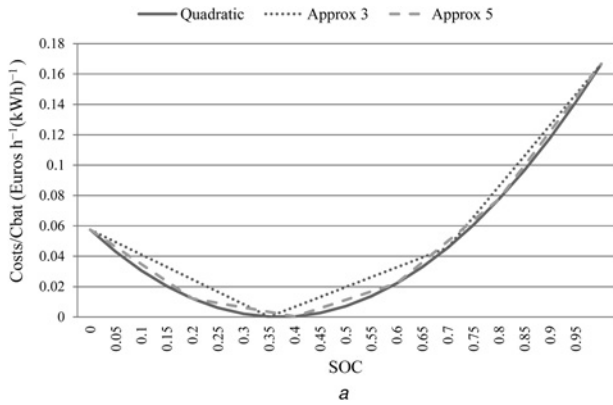
The Li-ion battery use cost due to degradation of the SOH is given by an approximate positive semi-definite quadratic function [14]. The deviation between the SOC and a set point is also considered. The degradation costs are determined for a Li-ion battery acquisition of 400 € (kWh)<sup>-1</sup>, although the parameters are proportional if the acquisition price is different. The equation is presented in (1) [14].

$$CUB_{et} = BB_e \cdot (eb_{et}/EB_e^{\max} - AB_e)^2 + FB_e \cdot pbd_{et} + GB_e \cdot pbc_{et} + HB_e/EB_e^{\max} \cdot pbc_{et}^2 \quad (1)$$

The first quadratic term of the degradation cost, associated with  $eb_{et}/EB_e^{\max}$  (i.e. SOC), is shown in Fig. 1a. The original function and two piecewise approximations with three and five linear segments are shown. The same approach is used for the remaining terms in (1), with the charge ( $pbc_{et}$ ) degradation cost, as presented in Fig. 1b. The incremental cost of discharge ( $pbd_{et}$ ) degradation ( $FB_e$ ) is considered to be zero, as long as it meets the upper discharge limit.

Figs. 1a and b are normalised and have the same behaviour as presented in [14].

Mathematically, it is suitable to use a piecewise linearisation without binary variables in (1) because the function is convex. The



**Fig. 1** Li-ion battery degradation cost functions [14]

a Degradation cost due to the deviation of the SOC  
b Degradation cost due to the deviation of the charge

linearisation process of the deviation from the specific SOC with  $N$  linear equations, dependent on the Li-ion battery energy ( $eb_{et}$ ) in (1), is given by

$$\min f = \sum_{e=1}^E \sum_{t=1}^{ND} cb1_{et} \quad (2)$$

$$\begin{aligned} \text{s.t.: } & cb1_{et} - \frac{IB_{en}}{EB_e^{\max}} \cdot eb_{et} \geq JB_{en}, \quad cb1_{et} \in \mathbb{R}^+, \\ & eb_{et} \in \mathbb{R}^+, \quad \forall t \in ND \forall e \in E \forall n \in N, \end{aligned} \quad (3)$$

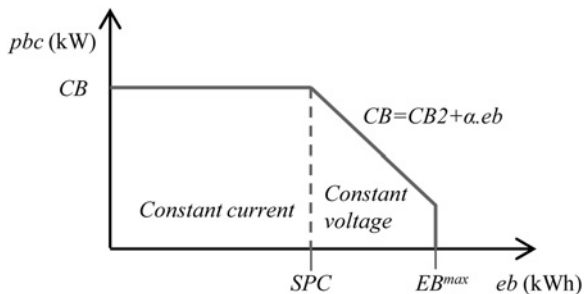
Note that in (3) the parameters  $IB_{en}$  and  $JB_{en}$  can be estimated in several ways. We choose the simplest method, that is, using two points of SOC and two costs from (1). The same approach is used for the charging cost using the variable  $pb_{et}$ .

## 2.2 Constraints

In this section we present the constraints found in the literature [5, 11–15] and new constraints (proposed in this paper) for Li-ion batteries.

Fig. 2 presents an approximation of a Li-ion battery's charge characteristics for constant current and constant voltage after a specific SOC, segregated by SPC, as a function of the SOC [19].

The behaviour presented in Fig. 2 is not perfectly linear in a real Li-ion battery; however, it is possible to obtain data from the manufacturer or tests to adapt the equations. The important factor is the change in the maximum charge power after a certain SOC. For this battery technology, the efficiency is very high for charging and discharging, and the self-discharge is very low. The influence of temperature is negligible because we assume a constant temperature.



**Fig. 2** Schematic of Li-ion charge characteristics

The energy balance, charge, discharge, and other characteristics are modelling based on the following constraints

$$\begin{aligned} & eb_{e,t+1} - eb_{et} + \left( \frac{pbd_{et}}{\eta_e^{bd}} - \eta_e^{bc} \cdot pb_{et} \right) \cdot \frac{H}{ND} \\ & = -PBL_e \cdot \frac{H}{ND}, \quad t \in \{2, \dots, ND-1\} \forall e \in E, \end{aligned} \quad (4)$$

$$\begin{aligned} & eb_{e2} + \left( \frac{pbd_{e1}}{\eta_e^{bd}} - \eta_e^{bc} \cdot pb_{e1} \right) \cdot \frac{H}{ND} \\ & = -PBL_e \cdot \frac{H}{ND} + EBI_e, \quad t \in 1 \forall e \in E, \end{aligned} \quad (5)$$

$$\begin{aligned} & eb_{eND} - \left( \frac{pbd_{eND}}{\eta_e^{bd}} - \eta_e^{bc} \cdot pb_{eND} \right) \cdot \frac{H}{ND} \\ & = PBL_e \cdot \frac{H}{ND} + EBF_e, \quad t \in ND \forall e \in E, \end{aligned} \quad (6)$$

$$\begin{aligned} & -eb_{et} + \sum_{i=1}^{RB} \left( \frac{rb_{e,t-1+i}}{\eta_e^{bd}} \right) \cdot \frac{H}{ND} \leq -EB_e^{\min}, \\ & eb_{et} \leq EB_e^{\max}, \quad \forall t \in ND \forall e \in E, \end{aligned} \quad (7)$$

$$\begin{aligned} & 0 \leq pb_{et} \leq CB_e, \quad 0 \leq pbc_{et} \leq CB2_e + eb_{et} \cdot \alpha_e, \\ & \forall t \in ND \forall e \in E, \end{aligned} \quad (8)$$

$$\begin{aligned} & 0 \leq pbd_{et} \leq DB_e, \quad 0 \leq rb_{et} \leq 2 \cdot DB_e, \\ & \forall t \in ND \forall e \in E, \end{aligned} \quad (9)$$

$$pb_{et} - ub_{et} \cdot CB_e \leq 0, \quad \forall t \in ND \forall e \in E, \quad (10)$$

$$ub_{et} \cdot DB_e + pbd_{et} \leq DB_e, \quad \forall t \in ND \forall e \in E, \quad (11)$$

$$\begin{aligned} & ub_{et} \in \{0, 1\}, \quad eb_{et} \in \mathbb{R}^+, \quad pb_{et} \in \mathbb{R}^+, \\ & pbd_{et} \in \mathbb{R}^+, \quad rb_{et} \in \mathbb{R}^+ \end{aligned} \quad (12)$$

The constraint in (4) indicates the energy balance of the battery throughout each time step, where the energy for the next step ( $eb_{e,t+1}$ ) depends on the energy in the current time step ( $eb_{et}$ ), the charge ( $pb_{et}$ ) or discharge ( $pbd_{et}$ ) power, the efficiency ( $\eta_e^{bc}$  or  $\eta_e^{bd}$ ) and the loss ( $PBL_e$ ) of energy at each time step. Note that the power is converting into energy includes the time of  $H/ND$ . The initial energy ( $EBI_e$ ) and final energy ( $EBF_e$ ) of Li-ion battery  $e$  are defined by the constraints in (5) and (6), respectively. The constraints of minimum battery energy ( $EB_e^{\min}$ ) and maximum battery energy ( $EB_e^{\max}$ ) are given in (7). In (7), the reserve constraint requires a minimum energy at each time step based on the variable  $rb_{et}$ , considering the following  $RB$  time steps. The ramp constraints for the battery charge are given in (8) for the

constant-current and constant-voltage equations. The constraints in (9) are related to discharge and reserve requirements. It is considered that the reserve can discharge twice the power of a normal discharge. Therefore, it is set to a higher value than the nominal discharge because the reserve is not frequently used. When the energy reserve is above  $DB_e$ , the state is held for only several minutes, and therefore, there is no significant damage to the battery. The constraints in (10) and (11) are used to prevent battery charge and discharge in the same time step  $t$  (a situation which could occur when there is excess of intermittent generation), where the binary variable  $ub_{et}$  is 1 (one) when charging or 0 (zero) when discharging.

The new equations for the constraints proposed above are given in (7) for minimum energy, in (8) for charging the constant voltage limits, and in (9) for reserve limits. In addition, if it is not possible to control the charge power (e.g. when the control is not accessible and it is performed by a switch), this paper proposes the following constraints

$$-pb_{et} + ub_{et}^{aux2} \cdot CB_e + ub_{et} \cdot 10,000 \leq 10,000, \quad \forall t \in ND \forall e \in E, \quad (13)$$

$$-pb_{et} + \alpha_e \cdot eb_{et} + ub_{et} \cdot 10,000 + ub_{et}^{aux1} \cdot CB2_e \leq 10,000, \quad \forall t \in ND \forall e \in E, \quad (14)$$

$$-ub_{et}^{aux1} \cdot SPC_e \cdot EB_e^{max} + eb_{et} \leq SPC_e \cdot EB_e^{max}, \quad \forall t \in ND \forall e \in E, \quad (15)$$

$$-ub_{et}^{aux2} \cdot SPC_e \cdot EB_e^{max} - eb_{et} \leq -SPC_e \cdot EB_e^{max}, \quad \forall t \in ND \forall e \in E, \quad (16)$$

$$ub_{et}^{aux1} + ub_{et}^{aux2} = 1, \quad \forall t \in ND \forall e \in E, \quad (17)$$

$$ub_{et}^{aux1} \in \{0, 1\}, \quad ub_{et}^{aux2} \in \{0, 1\}$$

Equations (13) and (14) determine the minimum and maximum charge at the same level, whereas (15)–(17) are the auxiliary binary variables used to verify that the SOC is in the current- ( $ub_{et}^{aux2}=1$ ) or voltage- ( $ub_{et}^{aux1}=1$ ) constant charge condition (because the battery will be in the constant-voltage charge condition at, for example,  $SPC_e=70\%$  of the SOC). The number 10,000 is used to guarantee a large number, although other values larger than the maximum charge could also be used. Possibly, a better mathematical approach considering the performance could be developed, however, in this paper, we focus in the technical behaviour study.

As described above, moderate discharge is better for a battery than pulse or aggregated loads. Therefore, in this paper, we propose to use the cost in (18) in the objective function and several constraints to minimise the pulse and aggregated effects, as follows

$$\sum_{e=1}^E \sum_{t=1}^{ND} \delta_e \cdot dpbd_{et}, \quad (18)$$

$$pbd_{et} - pbd_{e,t-1} - dpbd_{et} \leq 0, \quad \forall t \in ND \forall e \in E, \quad (19)$$

$$-pbd_{et} + pbd_{e,t-1} - dpbd_{et} \leq 0, \quad \forall t \in ND \forall e \in E, \quad dpbd_{et} \in \mathbb{R}^+ \quad (20)$$

Constraints (19) and (20) compute the increase and decrease in power ( $|pbd_{et} - pbd_{e,t-1}| = dpbd_{et}$ ), respectively, during two consecutive time steps, in the form of the variable  $dpbd_{et}$ . Computational tests show that, even with a small value of  $\delta_e$  (e.g.  $1 \times 10^{-4}$ ), pulse discharging is reduced.

### 3 Optimisation model

The EM optimisation problem of an MG is similar to the classical unit commitment problem [20], although it is strictly related to the DERs physical characteristics and the regulatory framework in which an MG is inserted. In this paper, the MG can buy (sell) energy from (to) the main grid with a one-day-ahead horizon. In addition, the model considers an MG with load demand, Li-ion batteries, and photovoltaic and wind generators. The backup generator is not included for connected operation and is not presented in the optimisation model. The modelling of wind and photovoltaic generators does not consider the cost of acquisition or operational costs. Thus, wind and photovoltaic generators are only included in the energy balance constraints as negative load demands; their values are supplied by a forecast model. Furthermore, it is assumed that there is a (good) forecast for intermittent generation and load demand. Accordingly, the optimisation problem used in this paper is given by

$$\min f = \sum_{t=1}^{ND} \frac{H}{ND} \left\{ \sum_{e=1}^E [CUB_{et} + \delta_e \cdot dpbd_{et}] + [BP_t \cdot pgb_t - SP_t \cdot pgs_t + CD \cdot pde_t + CE \cdot pex_t] \right\} \quad (21)$$

$$\text{s.t.: } \sum_{e=1}^E (pbd_{et} - pb_{et}) + pgb_t - pgs_t + pde_t - pex_t = D_t - PV_t - PW_t, \quad \forall t \in ND, \quad (22)$$

$$-\sum_{e=1}^E rb_{et} - pde_t \leq -D_t + PV_t + PW_t - ED \cdot D_t - EPV \cdot PV_t - EPW \cdot PW_t, \quad \forall t \in ND, \quad (23)$$

$$PGB_t^{\min} \leq pgb_t \leq PGB_t^{\max}, \quad PGS_t^{\min} \leq pgs_t \leq PGS_t^{\max}, \quad \forall t \in ND, \quad (24)$$

$$pgb_t - ug_t \cdot PGB_t^{\max} \leq 0, \quad ug_t \cdot PGS_t^{\max} + pgs_t \leq PGS_t^{\max}, \quad \forall t \in ND, \quad ug_t \in \{0, 1\}, \quad (25)$$

$$pgs_t \in \mathbb{R}^+, \quad pgb_t \in \mathbb{R}^+, \quad pde_t \in \mathbb{R}^+, \quad pex_t \in \mathbb{R}^+,$$

Constraints (4)–(12), (18)–(20).

The objective function (21) contains the batteries costs ( $CUB_{et} + \delta_e \cdot dpbd_{et}$ ), grid costs ( $BP_t \cdot pgb_t$ ), and profits ( $-SP_t \cdot pgs_t$ ) associated with grid energy transactions, and the artificial variables for deficit ( $CD \cdot pde_t$ ) and excessive ( $CE \cdot pex_t$ ) power generation. The customer electricity cost is given by the difference between the revenue associated with energy exported and the cost of energy

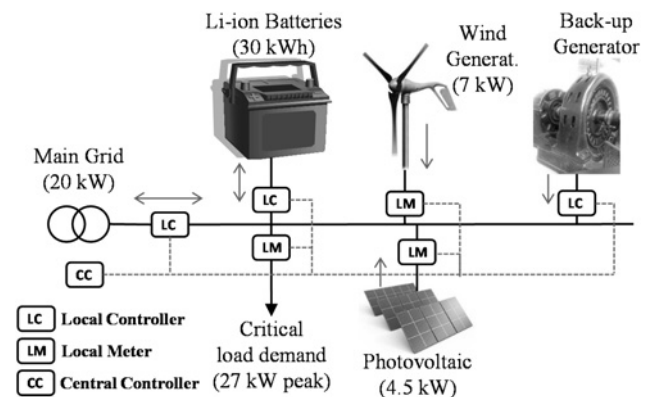


Fig. 3 MG schematic configuration

**Table 1** Li-ion battery parameters

AB	BB	CB	CB2	DB	EB <sup>max</sup>	EB <sup>min</sup>	EBI	EBF	PBL	FB	GB	HB	SPC	$\alpha$	$\delta$	$\eta_{bc}$	$\eta_{bd}$
0.37	0.42	18	60	30	30	3	11.1	11.1	0	0	$6 \times 10^{-3}$	$6.5 \times 10^{-3}$	0.7	-2	$1 \times 10^{-4}$	0.93	0.96

imported by the MG. Depending on the regulatory framework, it is necessary to replace prices by tariffs in (21). Deficit and excess generation at each step  $t$  are modelled as in [5].

Equation (22) contains the power balance constraints. Equation (23) contains the MG reserve requirements. Note that in (23) the batteries reserves must supply the load demand plus the forecasted errors, because it is the only controllable DER online for  $ND$  time steps before the backup generator starts to operate. If the battery reserve is insufficient to supply the reserve requirements, the deficit will be different from zero. In this case, it is possible to determine the cost impact and, depending on the probability of losing the main grid at that time, subtract this cost from the final result. A Li-ion battery has the ability to instantly supply more power than the normal operation discharge capacity, which is stipulated in this modelling as twice the normal operation discharge capacity. However, when this happens, the battery life will decrease more than in normal operation [18]. A rule is applied to (7): if the time step is bigger than the time of the reserve, the reserve  $rb_e$  will be multiplied by the time relation. The main difference in (21)–(25) relative to the actual literature is (23).

Equation (24) represents the limits on energy transactions with the grid; a limit on the minimum and maximum power at each time step is applied. The constraint in (25) restricts the import and export of energy with the grid at the same step  $t$ .

The formulation considers operation with a connection to the main grid. If the values of  $PGB(S)_t^{\min(max)}$  are set to zero and the costs and constraints associated with back-up generation are included, the optimisation problem will represent the islanded operation of the MG.

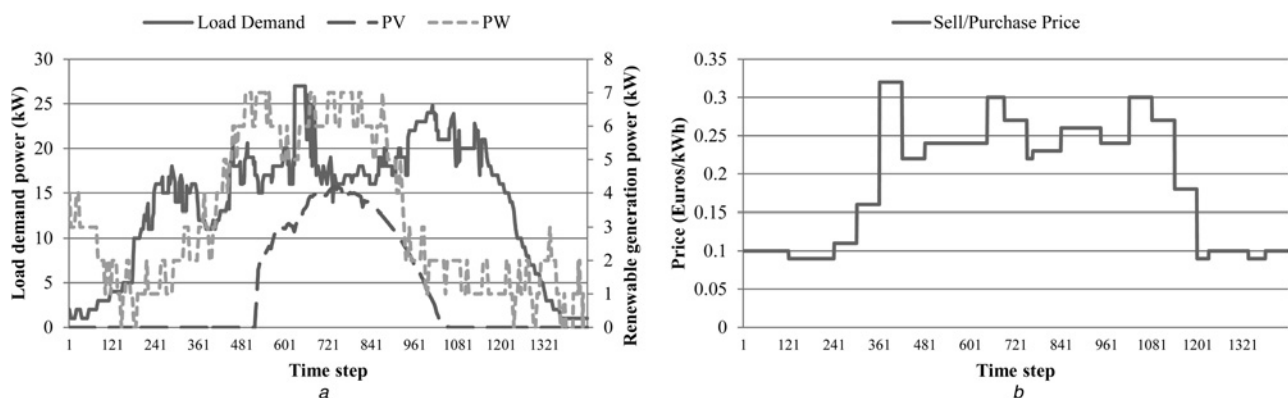
## 4 Computational experiments

To present the numerical experiments, we use an MG utilising batteries, wind and photovoltaic generators, and a backup generator. The MG is presented in Fig. 3.

The MG central controller (energy manager) is responsible for EM, sending control signals to the DERs through local controllers, and receiving information via both the local controller and the local meters. The telecommunication infrastructure is considered reliable; it is represented by the dotted line in Fig. 3.

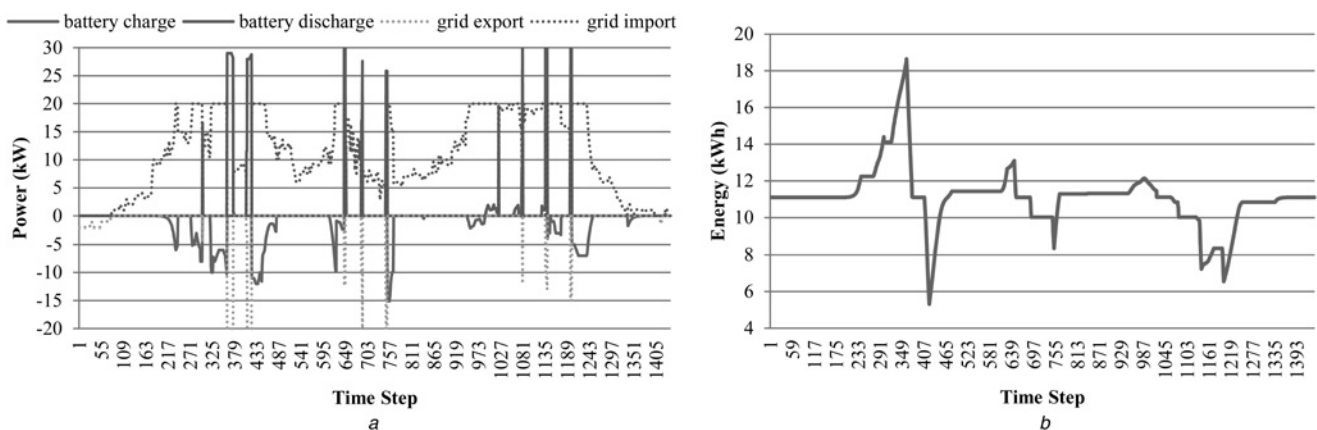
### 4.1 Input data

The planning horizon  $H$  is 24 h, discretised into 1-minute time steps (therefore,  $ND = 1440$ ). Because the generation is intermittent, it might be necessary to discretise the planning horizon in steps of



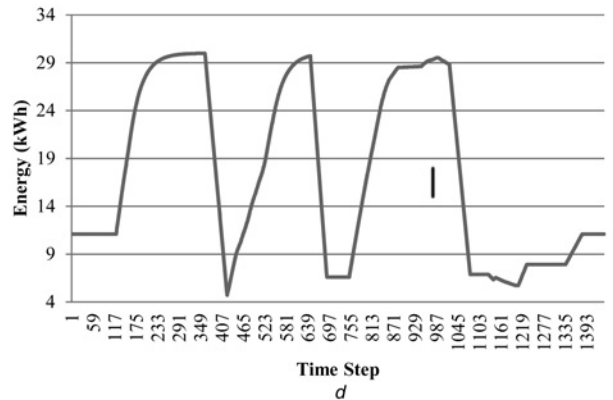
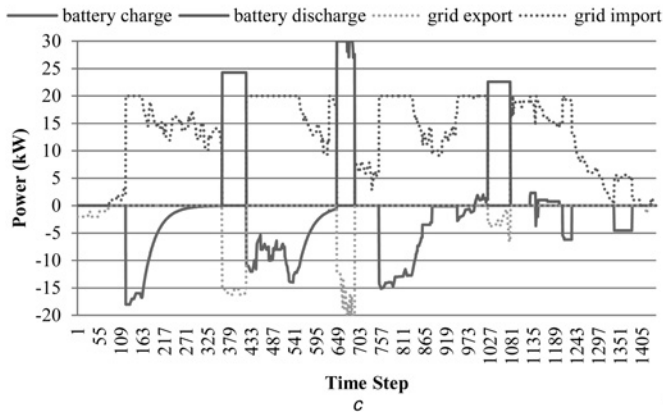
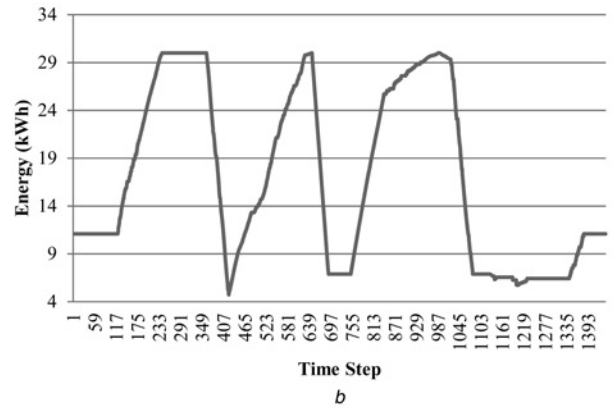
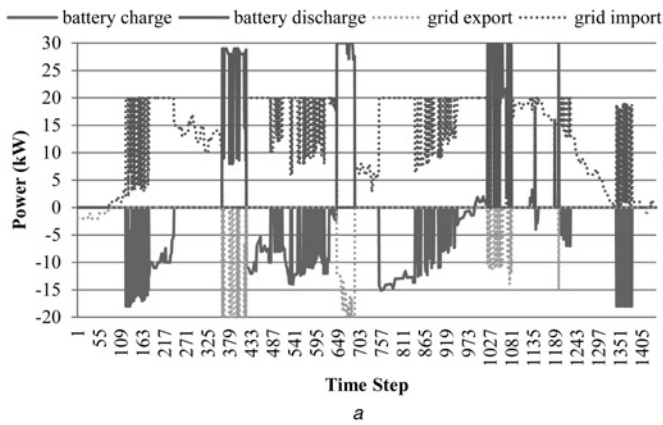
**Fig. 4** Input data

a Load demand and renewable generation power  
b Energy price input data



**Fig. 5** Case (i) MG EM and Li-ion battery energy

a MG EM  
b Li-ion battery energy



**Fig. 6** Case (ii) and (iii) MG EM and Li-ion battery energy

- a MG EM Case (ii)
- b Case (ii) Li-ion battery energy
- c MG EM Case (iii) Li-ion battery energy
- d Case (iii) Li-ion battery energy

several minutes [21]. All cases consider one-minute time steps, except when indicated. The acquisition price of the Li-ion battery is 400 € (kWh)<sup>-1</sup>. Other important data related to the Li-ion battery are presented in Table 1.

Fig. 4a shows the forecast values of load demand and renewable generation. The forecast energy prices (sell/purchase) are detailed in Fig. 4b.

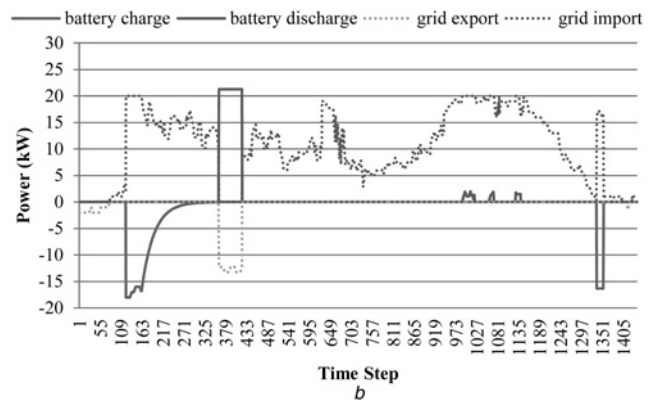
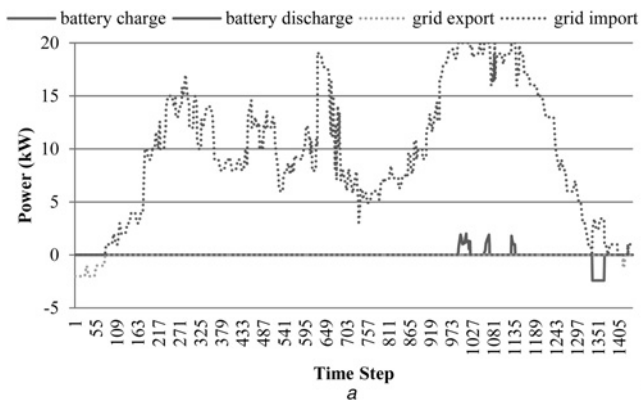
The maximum power grid interchange is 20 kW at all-time steps; the minimum is 0 kW. The deficit incremental cost (CD) is 10 € (kWh)<sup>-1</sup>, and the excess generation incremental cost (CE) is 0.01 € (kWh)<sup>-1</sup>. The forecast errors for intermittent generation are 10%

for wind and solar and 5% for demand. The time from reserve to generator start-up (RB) is considered 10 min.

#### 4.2 Computational results and analysis

The computational model is implemented in MATLAB 2011b. The tests were executed on an Intel quadcore i7 2.80-GHz CPU. The solver, Gurobi 6.0.0, was used to solve the optimisation modelling with the standard input parameters.

The results are categorised into eleven cases:



**Fig. 7** Case (iv) and (v) MG EM

- a MG EM Case (iv)
- b MG EM Case (v)

**Table 2** Cases and results

Case	Battery costs	New constraints	Time steps	Variables <sup>a</sup> (c) cont. (b) binary	Constraints <sup>a</sup>	Objective function (€)	Degradation cost (€)	Execution time (s)
(i)	quadratic	yes	1440	12,858 (c) 2880 (b)	18,466	48.77	1.07 <sup>b</sup>	6.43
(ii)	without	no	1440	9932 (c) 2880 (b)	11,277	43.35	39.45	0.46
(iii)	without	yes	1440	12,819 (c) 2880 (b)	18,427	43.40	41.91	1.01
(iv)	1500 cycles	yes	1440	12,819 (c) 2880 (b)	18,427	50.27	0.10	0.63
(v)	3000 cycles	yes	1440	12,819 (c) 2880 (b)	18,427	48.31	19.56	0.73
(vi)	piecewise <sup>c</sup>	yes	1440	15,729 (c) 2880 (b)	28,393	49.51	1.28 <sup>b</sup>	1.28
(vii)	piecewise <sup>d</sup>	yes	1440	15,733 (c) 2880 (b)	55,145	48.89	1.12 <sup>b</sup>	3.14
(viii)	piecewise <sup>e</sup>	yes	1440	15,735 (c) 2880 (b)	108,805	48.78	1.06 <sup>b</sup>	5.47
(ix)	quadratic	yes	144	1282 (c) 288 (b)	1,840	48.84	1.07 <sup>b</sup>	0.08
(x)	quadratic	yes	48	426 (c) 96 (b)	613	49.19	0.99 <sup>b</sup>	0.02
(xi)	quadratic	yes <sup>f</sup>	144	1133 (c) 576 (b)	2,396	52.43	2.91 <sup>b</sup>	1.61

<sup>a</sup>Constraints and variables after GUROBI pre-solve

<sup>b</sup>Costs are considered in the objective function

<sup>c</sup>Five energy and two charge linear piecewise equations

<sup>d</sup>20 energy and eight charge linear piecewise equations

<sup>e</sup>50 energy and 20 charge linear piecewise equations

<sup>f</sup>With the charge control just with a switch, without controlling the power of the charge

(i) Base case, which is related to the data presented previously; it considers the function costs (1) and (18), and the constraints presented in Section 3.2, excluding (13)–(17).

(ii) Base case without battery costs, without considering the constant voltage limit in (8) and (18)–(20).

(iii) Base case without the battery costs.

(iv) Base case with incremental cost of 1500 cycles at discharge.

(v) Base case with incremental cost of 3000 cycles at discharge.

(vi) Base case with piecewise linearisation cost (1) using five linear equations for the energy deviation costs and two for the charge costs.

(vii) Base case with piecewise linearisation cost (1) using 20 linear equations for the energy deviation costs and eight for the charge costs;

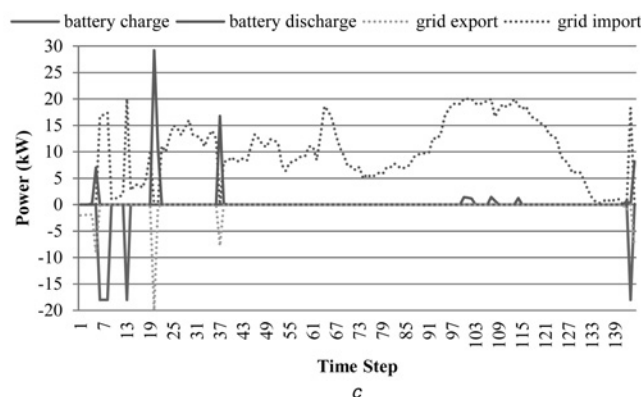
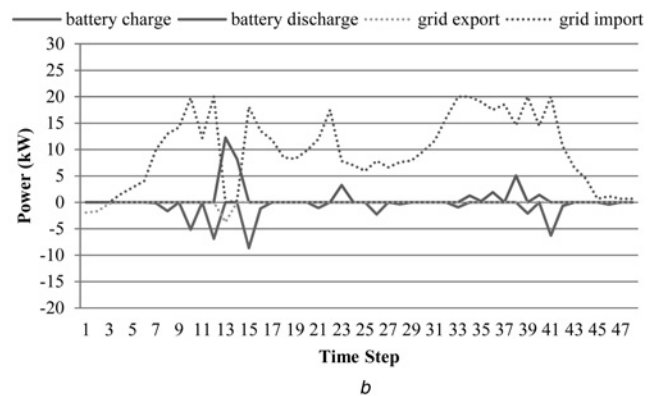
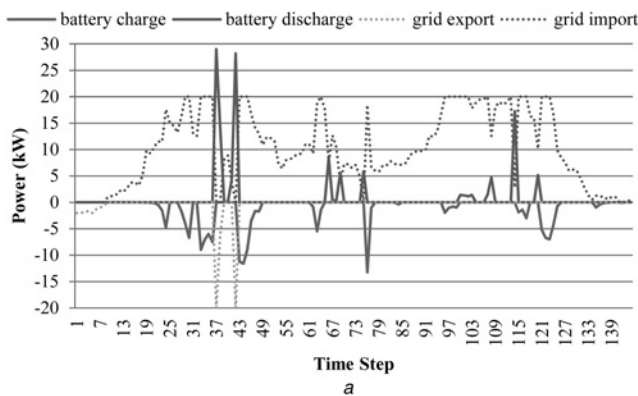
(viii) Base case with piecewise linearisation costs (1) using 50 linear equations for the energy deviation costs and 20 for the charge costs.

(ix) Base case with 144-time-step discretisation.

(x) Base case with 48-time-step discretisation.

(xi) Case (ix) including (13)–(17) in the modelling.

After several tests, these eleven cases were carefully selected to obtain the results for this paper. Case (i) considers the real degradation costs and the new proposed constraints, which are due to the physical characteristics of the Li-ion batteries. Case (ii) is the general modelling of the batteries, without costs or the physical characteristics. Case (iii) does not consider the costs, although it is more realistic than Case (ii) because it considers the physical peculiarities of the Li-ion battery as constraints. Cases



**Fig. 8** Case (ix), (x), and (xi) MG EM

a MG EM Case (ix)

b MG EM Case (x)

c MG EM Case (xi)

(iv) and (v) consider an incremental cost (instead of the real degradation cost) that is proportional to the number of cycles given by the manufactures. Cases (vi), (vii), and (viii) aim to show the effect of a piecewise model with a different number of approximations. Cases (ix) and (x) investigate the effects of different numbers of time steps discretised within a 24 h planning horizon. Finally, Case (xi) presents the results when it is not possible to control the power that charges the battery.

Continuous lines for the MG EM in Fig. 5–8 presents the battery charge and discharge, with positives values for discharge power and negative values for charge, where the dotted line indicates grid power export and import (negatives values indicate export and positive values indicate import).

Results for Case (i) are presented in Fig. 5.

Fig. 5a presents the MG EM of Case (i) with the power of the Li-ion battery and interchange with the grid. Discharge typically occurs when the price to import energy is high, as shown in Fig. 4b, or when the maximum power imported from the grid is insufficient to supply the liquid load demand (demand minus renewable generation), which happens three times between time steps 990 and 1140. As presented in Fig. 5, the charge occurs near the discharge (i.e. when the price is low), preventing the energy values from significantly deviating from the set point.

The EM cost without batteries is 60.99 €, considering the deficit incremental cost when the maximum possible import power from the grid is insufficient to supply the demand. The objective function resulting from Case (i) is 48.77 €, as presented in Table 2, which is approximately 20% less than the MG without the Li-ion battery. The degradation cost of the Li-ion battery is 1.07 €, as shown in Table 2, which is included in the final EM cost.

The results for Cases (ii) and (iii) are presented in Fig. 6.

In Case (ii), because there are no charge ramp constraints, the pulsing discharge, and any degradation costs have been established, the EM solution is very different from that of Case (i), as shown in Fig. 6a. The results show intense use of the battery, with three full cycles occurring, as presented in Fig. 6b. The EM related to Case (iii) differs from that of Case (ii) because of the inclusion of constraints into the model, as shown in Fig. 6c. The discharge and charge characteristics are different for the same number of cycles, as show in Fig. 6d. The objective function for Cases (ii) and (iii) are 43.35€ and 43.40€, respectively. These values are not accurate, because the degradation costs (39.45 € and 41.91 €) presented in Table 2 are not considered. To increase realism, these values should be added to the objective function.

The results for EM for Cases (iv) and (v) are presented in Fig. 7.

Because of the high incremental cost of Li-ion battery use, the EM in Case (iv) does not often use the battery: it is only used when the power import from the grid to supply the load is insufficient, as shown in Fig. 7a. Case (v) has half of the incremental cost; therefore, the economic use also charges the battery when the grid price is low (time steps 121 to 360) and discharges the battery when the price is high (time steps 361 to 420). This use reduces the EM from 50.27 € to 48.31 €; moreover, this price difference does not consider the real degradation cost. If the degradation cost were considered, Case (iv) would be better for the MG costs than Case (v).

Cases (vi), (vii), and (viii) use a piecewise model of the degradation costs and are similar to Case (i), with a slight difference in the dispatch and objective function values, as shown in Table 2. These differences could be greater, depending on the battery usage (e.g. a maximum of 6 € difference for the energy degradation cost, as presented in Fig. 1 for  $H=24$  with five linear equations). The difference in the computational time must also be taken into account. The accuracy of the piecewise approximation depends on the number of break points. The greater the number of break points, the higher the accuracy. Therefore, adding too many break points results in a significant increase in the computational burden [22].

Fig. 8 presents several results of Cases (ix) and (x) considering the number of time steps  $ND=144$  and  $ND=48$ . Comparing Cases (ix) and (x) with Case (i), we see the same behaviour of the solutions, although with less precision in (ix) and (x). In Case (ix), the number of time steps is the same as the amount of time necessary

to take into account the reserve requirements. As a consequence of the reserve requirement, Case (ix) results in better management than Case (x), which lacks precision in terms of peak power information. The dispatch peaks and the intermittent characteristics also are not at the same level with greater discretisation, which could result from power trade limit problems with the main grid or from exceeding physical trading limits.

Case (xi) is used to present operation for which the charge power is not possible to control, as shown in Fig. 8c. It is not possible to solve the EM with 1440 time steps in 60 s because of the increase in the binary variables and the new constraints; therefore, we use a discretisation of 144 time steps. Charging occurred five times, in the time steps 6–8, 13, and 143 with the stipulated power  $CB$ .

Table 2 summarises the cases, size of the problem, and results.

Regarding computational performance, Case (i), with quadratic cost and new constraints, has the longest execution time, as shown in Table 2. Case (viii), although it uses a detailed approximation of the objective function and the largest number of constraints, has better performance than Case (i). Case (xi) has not reached convergence by the minimum tolerance for 1440 time steps; therefore, it was performed with 144 time steps, mostly based on the modelling approach's ability to equalise the lower and upper limit of power charge. Therefore, it has a lower performance than Case (ix), which is the same, although with the ability to control the charge power.

## 5 Conclusion

The proposed equations and approaches for modelling the peculiarities of the Li-ion batteries and the battery reserve were successfully implemented and tested, as presented in the form of eleven cases. The main conclusions based on the different Li-ion battery cost modelling approaches and cases can be summarised as follows: it is necessary to include real degradation cost (or a good approximation thereof by means of a piecewise model). The use of Li-ion batteries without considering any costs or using an incremental cost based on the number of cycles gives a poor EM solution (with high degradation in some cases). In addition, the main conclusions regarding charge and discharge characteristics are: controlling the power and the period of the charge is better than only controlling when charging occurs; it is necessary to model the behaviour of the charge if the battery can be charged above a specific SOC ( $SPC_c$ ); and the inclusion of the constraints to decrease pulse discharging provides a better solution. Finally, the solution quality is a function of the number of time steps used in the planning horizon. For this reason, the length of time steps should, at least, take into account the important dynamics of the DERs, such as the reserve of 10 min, as considered in this paper. The presented modelling can also be used for planning and sizing problems regarding MGs. This contribution provides preliminary insights and denote other issues in Li-ion battery modelling such as efficiency, charge and discharge non-linearities, and the influence of temperature in cases in which it is not negligible (e.g. electric vehicles).

## 6 Acknowledgments

The authors gratefully acknowledge the financial support of the International Research Staff Exchange Scheme (IRSES) (Marie Curie) in the EU project Electricity Consumption Analysis to Promote Energy Efficiency Considering Demand Response and Non-technical Losses (ELECON) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). In particular, the authors thank Mr Hoang-Anh Dang for valuable discussions regarding Li-ion batteries.

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